AL-Bahir
Journal for Engineering and Pure Sciences
BJEPS

Volume 4 | Issue 1 Article 8

MAGNETIC FIELD'S EFFECT ON TWO-PHASE FLOW OF JEFFREY AND NON-JEFFREY FLUID WITH PARTIAL SLIP AND HEAT TRANSFER IN AN INCLINED MEDIUM

Sunday Lot
Department of Mathematics, Tai Solarin University of Education, Ijebu-Ode, Nigeria

Waheed Oluwafemi Lawal
Department of Mathematics, Tai Solarin University of Education, Ijebu-Ode, Nigeria

Adedapo Chris Loyinmi
Department of Mathematics, Tai Solarin University of Education, Ijebu-Ode, Nigeria

Recommended Citation
Lot, Sunday; Lawal, Waheed Oluwafemi; and Loyinmi, Adedapo Chris (2024) "MAGNETIC FIELD'S EFFECT ON TWO-PHASE FLOW OF JEFFREY AND NON-JEFFREY FLUID WITH PARTIAL SLIP AND HEAT TRANSFER IN AN INCLINED MEDIUM," *Al-Bahir*: Vol. 4: Iss. 1, Article 8.
Available at: https://doi.org/10.55810/2313-0083.1054

MAGNETIC FIELD'S EFFECT ON TWO-PHASE FLOW OF JEFFREY AND NON-JEFFREY FLUID WITH PARTIAL SLIP AND HEAT TRANSFER IN AN INCLINED MEDIUM

Source of Funding

The authors declare that no funds, grants, or other support were received to prepare this manuscript

Conflict of Interest

The authors declare that they have no competing interests

Data Availability

No datasets were generated or analyzed during the current study

Author Contributions

Authors contributed equally

REVIEW

Magnetic Field's Effect on Two-phase Flow of Jeffrey and Non-Jeffrey Fluid With Partial Slip and Heat Transfer in an Inclined Medium

Sunday Lot*, Waheed O. Lawal, Adedapo C. Loyinmi

Department of Mathematics, Tai Solarin University of Education, Ijebu-Ode, Nigeria

Abstract

In this paper, the effect of magnetic field on two −phase flow of Jeffrey and non-Jeffrey fluids in an inclined medium is investigated. The flow in both medium is assumed to be set in motion by constant pressure gradient. The electrical conductivity in the non-Jeffrey fluid in phase I is considered to be zero, so that the constant magnetic field strength B_o in the transverse direction only affects the Jeffrey fluid in phase II. The equations governed the flow of the fluid were solved using perturbation method. The effect of magnetic field, Jeffrey and thermal slip parameters on the temperature and velocity profile were examined through several graphs. It is noticed that the increase in magnetic field, decreased the fluid velocity and increased the temperature profile in phase II while it has partial effect in the velocity and decreased the temperature of phase I. Also, the increase in the thermal slip parameter has no effect on the velocity of both phases but, decreased the temperature profile of the non-Jeffrey fluid in phase I and increased that of the Jeffrey fluid in phase II.

Key words: Heat transfer, Inclined medium, Jeffery, Magnetic field, Two-phase flow, Fluid

1. Introduction

One of the states of matter is phase. It can be a fluid or solid. The simultaneous flow of different phases is multiphase flow. Multiphase flow study in energy−related industries and applications is imperative. The easiest case of multiphase flow is two-phase flow. The interactive flow of different phases with common interfaces in the medium, with a phase responsible for the volume or mass of matter is a two-phase flow.

Jeffry fluids are non-Newtonian fluids. Examples are oil, gels, adhesives and paints. While, non-Jeffrey fluid are the Newtonian fluid. Examples, water, alcohol, sugar. The two immiscible fluids flow such as water and oil, which are essential in regaining processes of oil, can be an example. Transfer of heat analysis aspect of immiscible fluid and the fluids flow are very significant in petroleum transportation and extraction. For example, an oil-filled rock always contains different fluids which are in the pores. Water occupies part of the space volume and the rest can either be oil or gas, the two together. [1], Studied well-known Reynolds modes of viscosity in an asymmetric channel under the effect of Jeffrey fluid with variable viscosity. Peristaltic transport of a Jeffrey fluid under the effect of slip in an inclined asymmetric channel. The result shows that, the peristaltic flow of a Jeffrey fluid in an inclined asymmetric channel is under take when the no-slip condition at the channel wall is no longer valid [2−6]. Flow of Jeffrey fluid through narrow tubes, shows that the flow exhibits the anomalous Fahraeus-Lindquist effect and the effective viscosity decreases with Jeffrey parameter and core magnetic parameter but increases with tube haematocrit and tube radius [7−10]. Characteristics of Jeffrey fluid model for peristaltic flow of Chyme, shows that the magnetic field highly influence the peristaltic flow problem [11]. The pulsatile flows of a Jeffrey fluid in a circular tube having internal porous lining. Shows that the Jeffrey parameter enhances the fluid

Received 30 August 2023; revised 1 November 2023; accepted 9 November 2023.
Available online 13 December 2023

* Corresponding author.
E-mail address: lotsunday89@gmail.com (S. Lot).

https://doi.org/10.55810/2313-0083.1054

velocity and mass flux in the channel [12]. The free convection flow of Jeffrey fluid through a vertical deformable porous stratum, the skin friction get reduced when the porous materials is deformable on and the effect of increasing the Jeffrey parameter is to increase the skin friction in the deformable porous stratum [13]. Effect of the thickness of the porous material on the parallel plate channel flow of Jeffrey fluid when the walls provided with porous lining. The penetrable layer thickness in the channel with rate redesigns with growing potential gains of porous layer thickness [14–18]. The response surface optimization for the electro magneto hydrodynamic Cu −polyvinyl alcohol/water Jeffrey Nano fluid flow with and exponential heat source. Response surface methodology is utilized to determine the output response variables of model dependencies [19].

The importance of heat transfer and two−fluid flow in nuclear and chemical industries have been studied extensively. The following are some important for the designing of two fluid systems; determination of the void fraction, reaction quality, identification of the two fluid flow areas, two-fluid heat transfer coefficient and drop pressure. In modelling problems of its kind, several complexities are added through the presence of a second immiscible fluid phase as to the nature of transport phenomenal interaction and the conditional interface of the phases. Two phase magneto hydro-dynamic flow and heat transfer in an inclined channel. Among others, an approximate solution was obtained using perturbation method [20–24]. The suitable values of the ratio of viscosities, thermal conductivities, height and the angle of inclination increased or decreased the velocity and temperature. The influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum established that the shear-thinning reduces the wall shear stress [25,26]. Peristaltic flow of Williamson fluid in an inclined asymmetric channel with slip and heat transfer, the pressure rise decreases with an increase in slip parameter [27–29].

Magnetic field is a region around a magnetic material or moving electric charge within which the force of magnetism act. The flow of electrically conducting Jeffrey fluid through a medium with transverse magnetic field in the presence of heat transfer has been attracted to many researchers. The peristaltic transport of a Jeffrey fluid under the effect of slip in an inclined asymmetric channel. MHD mixed convective flow viscoelastic and viscous fluid in a vertical porous channel. The study is useful in understanding the influence of buoyancy and a magnetic field on enhances oil recovery and filtration system [30–33]. The effect of magnetic field and wall slip condition on peristaltic transport of Newtonian fluid in an asymmetric channel, the effect of phase difference, Knudsen number and magnetic on the pumping characteristics and velocity field are discussed [34]. A peristaltic transport of caisson fluid in contact with Newtonian fluid in a circular tube with permeable walls. Shows that the shear − thinning reduces the wall shear stress [35]. The thermal and velocity slip effects on peristaltic flow with carbon nanotubes in an asymmetric channel under the effects of magnetic field. The magnitude of pressure gradient increases with the increase in G_r and ϕ and the velocity field for SWCNT is greater than that compared to MWCNT in view of N and ϕ [36]. The MHD two-phase fluid flow and heat transfer with partial slip in an inclined channel. The fluid velocity is decreased with an increase in magnetic field in both cases of no-slip

Nomenclature

Symbols

a	Ratio coefficient expansion of phases
u_i^*	Dimensionless velocity of phases (ms^{-1})
u	Velocity of the fluid (ms^{-1})
β_i	Expansion of thermal coefficient (K^{-1})
c	Specific heat (at constant pressure) ($Jkg^{-1}K^{-1}$)
η_1	Velocity slip parameter of the phases (ms^{-1})
I_i	Thermal slip constant(K)
ε	Product of Prandtl and Eckert number
E_c	Eckert number
T	Dimensionless temperature of the phases (K)
B_o	Magnetic field (T)
μ_i	Phases dynamic viscosities (Pa.s)
g	Gravitation acceleration (ms^{-2})
ρ_i	Phases fluid densities (Kgm^{-3})
v_i	Kinematics viscosities of the phases (m^2s^{-1})
σ_2	Electrical conductivity of fluid of phases (Sm^{-1})
h	The ratio of heights of phases (m)
ϕ	Angle of the channel with horizontal (Radians)
y_i	Thermal slip parameters of phases (K)
λ	Jeffrey parameter
G_r	Grashof number
y_i^*	Dimensionless variable
P_r	Prandtl-number
h_i	Height of the phases (m)
Re	Reynolds-number
k_i	Thermal conductivity of fluid ($Wm^{-1}K^{-1}$)
M	Hartman-number
k	Phases ratio coefficient of thermal conductivity of fluid ($Wm^{-1}k^{-1}$)
n	Phases ratio of densities of fluid (Kgm^{-3})
P	Non −dimensional gradient pressure (Pam^{-1})
m	Dynamics viscosity ratio of the coefficient (Pa.s)
T_i	Temperature of the fluid of phases (K)
T_w	Surface plates temperature (K)
u_i	x-direction (velocity of phases) (ms^{-1})
$(\bar{u}_i)$	Velocity average (ms^{-1})
x, y, z	spatial coordinate

condition and slip condition [37]. The magnetic field and gravity effect on peristaltic transport on a Jeffrey fluid in an asymmetric channel and the result indicated that the peristaltic transport of fluid with figures without magnetic field and gravity field has the same behaviour in the same field [38]. Effect of a magnetic field on unsteady free convection oscillatory systems. When temperature and spaces concentration fluctuate with time around a non-zero constant, "couette flow" across a porous medium [39–42]. The unsteady magneto-hydrodynamics (MHD) heat and mass transfer for a viscous incompressible fluid through a vertical stretching surface embedded in a Darcy – Forchheimer porous medium in the presence of anon-uniform heat source/sink and field –order chemical reaction. The result is helpful in significant variations of gravitational force [43]. The entropy generation optimization of cilic regulated MHD ternary hybrid Jeffrey nanofluid with Arrhenius activation energy and induced magnetic field. The findings can be applied to enhance heat transfer efficiency in biomedical devices, optimizing cooling systems [44–47]. Exponential space and thermal-dependent heat source effects on electro-magneto-hydrodynamic Jeffrey fluid flow over a vertical stretching surface. The results may be helpful in many engineering and industrial application like manufacturing lubrication, natural gas networks and spray processes [48–52].

To the researcher's knowledge, the problem of the magnetic field effect on the two-phase flow of Jeffrey and non-Jeffrey fluid with the partial slip and heat transfer in an inclined medium has not been studied before. Therefore, the objective of the present study was to investigate the magnetic field effect on the steady two-phase flow of Jeffrey and non-Jeffrey fluid with partial slip and heat transfer. The novelty of the present study is to use the magnetic field, thermal slip and Jeffrey parameter to examine the velocity and temperature profile of the non- Jeffrey and Jeffrey fluid in an inclined medium and to also, examine the relationship and differences between effect of magnetic field and thermal slip parameter on the velocity and temperature of the two fluid. The Jeffrey fluid in the lower medium is electrically conducted while the non- Jeffrey fluid in the upper medium has electrical conductivity equal to zero. Therefore, the constant magnetic field applied perpendicular to the phases only affects the lower medium phase.

2. Problem formulation

In consideration of the steady two-phase flow of Jeffrey and non-Jeffrey fluid in an medium at an inclined angle (ϕ) to x-axis. A system of cartesian coordinates is selected in a way that the horizontal-axis considered along the medium and vertical-axis is normal to it as shown in Fig. 1. Non-Jeffrey electrically conducting fluid is filled in phase I $0 \leq y \leq h_1$ with thermal conductivity (k_i), viscosity (μ_I) and density (ρ_I). The Jeffrey electrically conducting fluid is filled in phase II $h_2 \leq y \leq 0$ with electrical conductivity (σ_2), thermal conductivity (k_2) density (ρ_2) and viscosity (μ_2). In the direction perpendicular to the medium an external magnetic field "B_0" is applied. The pressure gradient $(\frac{\partial P}{\partial x})$ of both phases flow remains constant. The governing momentum and energy equation [37] under this assumption are given as below:

Phase I

$$\mu_1 \frac{\partial^2 u_1}{\partial y^2} + \rho_1 g \beta_1 \sin\phi (T_1 - T_{w1}) = \frac{\partial P}{\partial x} \quad (1)$$

$$\frac{\partial^2 T_1}{\partial y^2} + \frac{\mu_1}{k_1}\left(\frac{\partial u_1}{\partial y}\right)^2 = 0 \quad (2)$$

Phase II

$$\frac{\mu_2}{1+\lambda}\frac{\partial^2 u_2}{\partial y^2} + \rho_2 g \beta_2 \sin\phi (T_2 - T_{w2}) - \sigma B_0^2 u_2 = \frac{\partial P}{\partial x} \quad (3)$$

$$k_2 \frac{\partial^2 T_2}{\partial y^2} + \frac{\mu_2}{1+\lambda}\left(\frac{\partial u_2}{\partial y}\right)^2 + \sigma B_0^2 u_2^2 = 0 \quad (4)$$

The acceleration due to gravity is represented by (g) while (ρ_1) and (ρ_2) the fluid densities (k_1) and (k_2) the coefficient of the thermal conductivities (u_1)

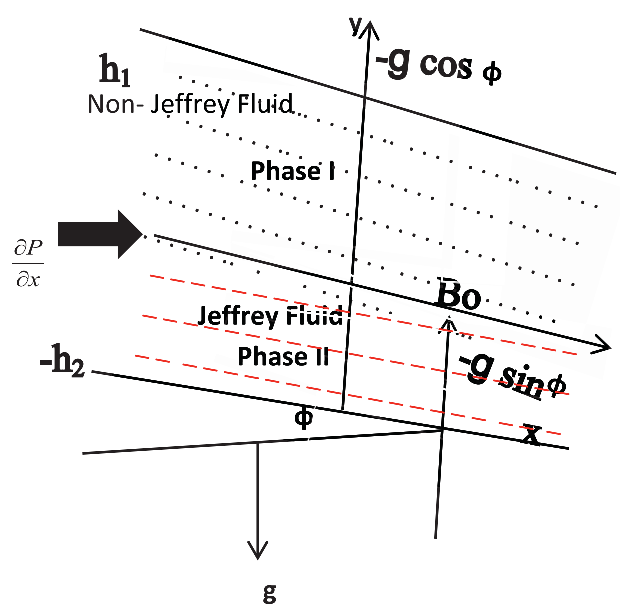

Fig. 1. Physical configuration and coordinate system.

and (u_2) are the velocities of x-component (T_1), (β_1) and (β_2) the thermal expansion coefficient for both phases, (T_1) and (T_2) the temperatures and (λ) is the Jeffrey parameter. Referring to Fig. 1, the velocity interface and boundary condition are:

$$at \quad y=h_1,\ u_1=\eta_1\frac{\partial u_1}{\partial y} \quad at \quad y=-h_2 \quad ,u_2=\frac{\eta_2}{1+\lambda}\frac{\partial u_2}{\partial y}$$

$$at \quad y=0,\ u_1=u_2,\ at \quad y=0,\ \mu_1\frac{\partial u_1}{\partial y}=\frac{\mu_2}{1+\lambda}\frac{\partial u_2}{\partial y} \tag{5}$$

The constant different temperatures maintained by the wall are T_{W1} and T_{w2} with $y=h_1$ and $y=h_2$ respectively, where η_1 and η_2 are the velocity slip lengths, the and T_2 boundary conditions are given by:

$$at \quad y=h_1,\ T_1=T_{w1} - d_1\frac{\partial T_1}{\partial y},\ at \quad y=-h_2, T_2=T_{w2}+d_2\frac{\partial T_2}{\partial y}$$

$$at \quad y=0,\ T_1=T_2,\ at \quad y=0, \quad k_1\frac{\partial T_1}{\partial y}=k_2\frac{\partial T_2}{\partial y} \tag{6}$$

where the thermal slip constant is l_1 and l_2. The dimensionless variables and parameters are introduced as follows;

$$u_1^{\cdot}=\frac{u_1}{u_1},\ u_2^{\cdot}=\frac{u_2}{u_2}, \quad y_1^{\cdot}=\frac{y_1}{h_1}, \quad y_2^{\cdot}=\frac{y_2}{h_2}, \quad \theta=\frac{(T-T_{w2})}{(T_{w1}-T_{w2})}$$

$$m=\frac{\mu_1}{\mu_2}, k=\frac{k_1}{k_2}, n=\frac{\rho_2}{\rho_1}, a=\frac{\beta_2}{\beta_1}, G_r=\frac{\rho g\beta_1 h_1^3(T_{w1}-T_{w2})}{v_1^2},$$
$$M=B_o h\sqrt{\frac{\sigma_2}{\mu_2}}, P_r=\frac{\mu_1 c_p}{k_1}$$

$$P=\left(\frac{h_1^2}{\mu_1 u_1^{\cdot}}\right)\left(\frac{\partial P}{\partial x}\right), R_e=\frac{u_1^{\cdot}h_1}{v_1}, E_c=\frac{u_1^{\cdot 2}}{c_p(T_{w1}-T_{w2})} \tag{7}$$

Here the non-dimensional gradient of the pressure (P), Eckert number (E_c), Grashof number (G_r) Prandtl number (P_r), Reynolds number (R_e), Hartmann number (M), and average velocity ($u_1^{\cdot}$).

The problem takes the following form in the new variables:

Phase I

$$\frac{\partial^2 u_1}{\partial y^2}+\frac{G_r\sin\phi}{R_e}\theta_1=P \tag{8}$$

$$\frac{\partial^2\theta_1}{\partial y^2}+P_r E_c\left(\frac{\partial u_1}{\partial y}\right)^2=0 \tag{9}$$

Phase II

$$\frac{1}{1+\lambda}\frac{\partial^2 u_2}{\partial y^2}+\frac{G_r\sin\phi}{R_e}\theta_2-M^2u_2=P \tag{10}$$

$$\frac{\partial^2\theta_2}{\partial y^2}+P_r E_c\left(\frac{k}{m}\right)\left(\frac{1}{1+\lambda}\right)\left(\frac{\partial u_2}{\partial y}\right)^2-M^2P_rE_cu_2^2=0 \tag{11}$$

with the dropped asterisks it is understood that all the quantities are now dimensionless. The boundary conditions for interface, velocities and temperature (5) and (6) in non-dimensional form are:

$$at \quad y=1,\ u_1=\gamma_1\frac{\partial u_1}{\partial y}, at\ y=-1, u_2=\left(\frac{\gamma_2}{1+\lambda}\right)\left(\frac{\partial u_2}{\partial y}\right) \tag{12}$$

$$at \quad y=0,\ u_1(0)=u_2(0) at\ y=0,\ \frac{\partial u_1}{\partial y}=\frac{1}{m}\left(\frac{1}{1+\lambda}\right)\frac{\partial u_2}{\partial y} \tag{13}$$

$$at \quad y=1,\ \theta_1=1-\lambda_1\left(\frac{\partial\theta_1}{\partial y}\right) at\ y=-1, \theta_2=\lambda_2\frac{\partial\theta_2}{\partial y} \tag{14}$$

$$at \quad y=0,\ \theta_1=\theta_2\ at \quad y=0, \quad \frac{\partial\theta_1}{\partial y}=\left(\frac{1}{k}\right)\frac{\partial\theta_2}{\partial y} \tag{15}$$

Where $\gamma_2=\frac{\eta_2}{h_2}$ are the parameters of the thermal slip and $\eta_1=d_1h_1,\ \eta_2=d_2h_2$ are parameters of the velocity slip.

3. Method of solution

The perturbation method was used to solve the equations governing the momentum (8) and (10) also with the equations of the energy (9) and (11) subject to the interface and boundary and conditions (12), (13), (14) and (15) for the temperature and velocity distributions. Due to the dissipation terms included, the equation of energy is non-linear and coupled. The approximation of the solution of (8) – (11) with the boundary condition (12), (13), (14) and (15) for small values of ε <1 (=PrEc) is valid because in most of the practical problems, the Eckert number is very small and in order 10^{-5} as [22]. The assumed solutions are of the form:

$$(u_i,\theta_i)=\sum_{j=0}^{\infty}(u_{ij},\theta_{ij})\varepsilon^j \tag{16}$$

Where (u_i) and (θ_i) are the perturbations in (u) and (θ) respectively. Neglecting the terms of O (ε^2), with equation (16) in equations (8)–(15) after comparing the similar powers of ε. The following equations are obtained:

Phase I

Order-zero equations

$$\frac{\partial^2 u_{10}}{\partial y^2}+\frac{G_r \sin\phi}{R_e}\theta_{10}=P \tag{17}$$

$$\frac{\partial^2 \theta_{10}}{\partial y^2}=0 \tag{18}$$

First−order equations

$$\frac{\partial^2 u_{11}}{\partial y^2}+\frac{G_r \sin\phi}{R_e}\theta_{11}=0 \tag{19}$$

$$\frac{\partial^2 \theta_{11}}{\partial y^2}+\left(\frac{\partial u_{10}}{\partial y}\right)^2=0 \tag{20}$$

Phase II

Order - zero equations

$$\frac{1}{1+\lambda}\frac{\partial^2 u_{20}}{\partial y^2}+\frac{G_r \sin\phi}{R_e}\theta_{20}-M^2 u_{20}=P \tag{21}$$

$$\frac{\partial^2 \theta_{20}}{\partial y^2}=0 \tag{22}$$

First−order equations

$$\frac{1}{1+\lambda}\frac{\partial^2 u_{21}}{\partial y^2}+\frac{G_r \sin\phi}{R_e}\theta_{21}-M^2 u_{21}=0 \tag{23}$$

$$\frac{\partial^2 \theta_{21}}{\partial y^2}+\left(\frac{k}{m}\right)\left(\frac{1}{1+\lambda}\right)\left(\frac{\partial u_{20}}{\partial y}\right)^2-M^2 u_{20}^2=0 \tag{24}$$

The interface and dimensionless form of the boundary conditions of (12), (13), (14) and (15) become:

$$at\ y=0,\ u_{10}=u_{20}\ and\ \frac{\partial u_{10}}{\partial y}=\frac{1}{m}\left[\frac{1}{1+\lambda}\right]\frac{\partial u_{20}}{\partial y} \tag{25}$$

$$at\ y=1,\ \theta_{10}=(1-\eta_1)\frac{\partial\theta_{10}}{y}\ and\ at\ y=-1,\ \theta_{20}=\eta_2\frac{\partial\theta_{20}}{\partial y} \tag{26}$$

$$at\ y=1,\ u_{11}=\gamma_1\frac{\partial u_{11}}{\partial y}\ at\ y=-1,\ u_{21}=\left(\frac{\gamma_2}{1+\lambda}\right)\left(\frac{\partial u_{21}}{\partial y}\right)$$

$$at\ \ y=0,\ u_{11}=u_{21}\ \ at\ \ y=0,\ \ \frac{\partial u_{11}}{\partial y}=\frac{1}{m}\left(\frac{1}{1+\lambda}\right)\frac{\partial u_{11}}{\partial y} \tag{27}$$

And

$$at\ \ y=1,\ \theta_{11}=(1-\eta_1)\frac{\partial\theta_{11}}{\partial y}\ \ at\ \ y=-1,\ \ \theta_{21}=\eta_2\frac{\partial\theta_{21}}{\partial y}$$

$$at\ \ y=0,\ \theta_{11}=\theta_{21}\ \ at\ \ y=0,\ \ \frac{\partial\theta_{11}}{\partial y}=\frac{1}{k}\frac{\partial_{21}}{\partial y} \tag{28}$$

Solutions of equations (18) and (22) (17) and (21) using boundary conditions (25) and (26) are:

$$\theta_{10}=c_1 y+c_2 \tag{29}$$

$$u_{10}=\frac{G_r\sin(\phi)\left(\frac{1}{6}y^3c_1+\frac{1}{2}c_2y^2\right)}{R_e}+\frac{1}{2}Py^2+c_3y+c_4 \tag{30}$$

$$\theta_{20}=c_5 y+c_6 \tag{31}$$

$$u_{20}=e^{\sqrt{M}\sqrt{1+\lambda_y}}c_7+e^{-\sqrt{M}\sqrt{1+\lambda_y}}c_8+\frac{G_r\sin(\phi)(c_5y+c_6)-PR_e}{R_e} \tag{32}$$

Also, the solution of equations (19) and (24) and (20) and (23) with boundary conditions (27) and (28) are:

$$\theta_{11}=-\frac{1}{30}c_9y^6-\frac{1}{20}c_{10}y^5-\frac{1}{12}c_{11}y^4-\frac{1}{6}c_{12}y^3-\frac{1}{2}c_3^2y^2+c_{13}y+c_{14} \tag{33}$$

$$u_{11}=c_{17}y^8+c_{18}y^7+c_{19}y^6+c_{20}y^5+c_{21}y^4-c_{22}y^3-c_{23}y^2+c_{15}y+c_{16} \tag{34}$$

$$\theta_{21}=\frac{1}{12}c_{30}y^4-\frac{1}{6}c_{24}y^3+c_{40}y^2+\left(c_{38}e^{\sqrt{M}\sqrt{1+\lambda_y}}+c_{34}\right)y+c_{36}e^{2\sqrt{M}\sqrt{1+\lambda_y}}+e^{\sqrt{M}\sqrt{1+\lambda_y}}c_{37}-\frac{1}{4}\frac{c_{33}e^{-\sqrt{M}\sqrt{1+\lambda_y}}}{M(1+\lambda)}+c_{35}+c_{39}e^{-\sqrt{M}\sqrt{1+\lambda_y}} \tag{35}$$

$$u_{21}=c_{57}+c_{43}e^{2\sqrt{M}\sqrt{1+\lambda_y}}+(c_{44}+c_{61})e^{\sqrt{M}\sqrt{1+\lambda_y}}+(c_{45}+c_{60})^{-\sqrt{M}\sqrt{1+\lambda_y}}+c_{46}y^4+c_{47}y^3+\left(c_{58}+c_{49}e^{\sqrt{M}\sqrt{1+\lambda_y}}\right)y^2+\left(c_{59}+c_{52}e^{\sqrt{M}\sqrt{1+\lambda_y}}+c_{53}e^{-\sqrt{M}\sqrt{1+\lambda_y}}\right)+c_{55}e^{-2\sqrt{M}\sqrt{1+\lambda_y}} \tag{36}$$

$$u_1=\left(c_{17}y^8+c_{18}y^7+c_{19}y^6+c_{20}y^5+c_{21}y^4+c_{22}y^3+c_{23}y^2+c_{15}y+c_{16}\right)\varepsilon+\left(\frac{1}{6}y^3c_1+\frac{1}{2}c_2y^2\right)+\frac{1}{2}Py^2+c_3y+c_4 \tag{37}$$

$$u_2 = \Big(c_{57} + c_{43}e^{2\sqrt{M}\sqrt{1+\lambda_y}} + (c_{44}+c_{61})e^{\sqrt{M}\sqrt{1+\lambda_y}} + (c_{45}+c_{60})e^{-\sqrt{M}\sqrt{1+\lambda_y}} + c_{64}y^4 + c_{47}y^3\Big) + \Big(c_{58} + c_{49}e^{\sqrt{M}\sqrt{1+\lambda_y}}\Big)y^2 + \Big(c_{59} + c_{52}e^{\sqrt{M}\sqrt{1+\lambda_y}} + c_{53}e^{-\sqrt{M}\sqrt{1+\lambda_y}}\Big)y + c_{55}e^{-2\sqrt{M}\sqrt{1+\lambda_y}}\Big)\varepsilon + e^{\sqrt{M}\sqrt{1+\lambda_y}}c_7 + e^{-\sqrt{M}\sqrt{1+\lambda_y}}c_8 + (c_5 y + c_6) \tag{38}$$

$$\theta_1 = \varepsilon\left(-\frac{1}{30}c_9y^6 - \frac{1}{20}c_{10}y^5 - \frac{1}{12}c_{11}y^4 - \frac{1}{6}c_{12}y^3 - \frac{1}{2}c_3^2y^2 + c_{13}y + c_{14}\right) + c_1y + c_2 \tag{39}$$

$$\theta_2 = \varepsilon\left(\frac{1}{12}c_{30}y^4 - \frac{1}{6}c_{24}y^3 + c_{40}y^2 + \left(c_{38}e^{\sqrt{M}\sqrt{1+\lambda_y}} + c_{34}\right)y + c_{36}e^{2\sqrt{M}\sqrt{1+\lambda_y}} + c_{37}e^{\sqrt{M}\sqrt{1+\lambda_y}} - \frac{1}{4}\frac{c_{33}e^{-2\sqrt{M}\sqrt{1+\lambda_y}}}{M(1+\lambda)} + c_{35} + c_{39}e^{-\sqrt{M}\sqrt{1+\lambda_y}}\right) + c_5y + c_6 \tag{40}$$

4. Result and discussion

The problem of the magnetic field effect on the two-phase flow of Jeffrey and non-Jeffrey fluid with partial slip and heat transfer in an inclined medium with the velocity/thermal slip condition is investigated analytically. Due to the small values of ε up to order one, the method of regular perturbation is used, giving the appropriate solutions analytical of compiled non-linear equations (8)–(11) and the boundary conditions (12)–(15). The thermal slip parameter that is the pertinent parameters Υ_1 and Υ_2, Jeffery parameter λ, velocity slip parameter η_1, η_2

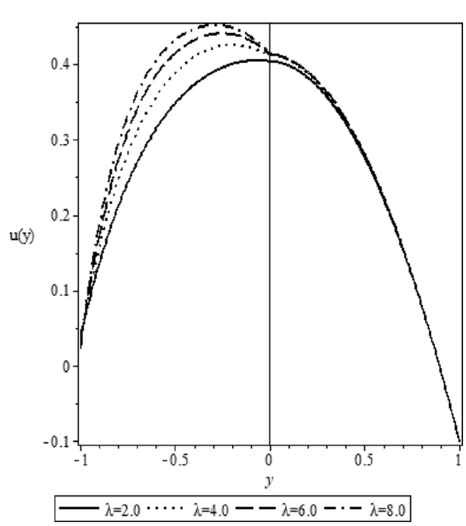

Fig. 2. Jeffrey parameter effect on velocity distribution.

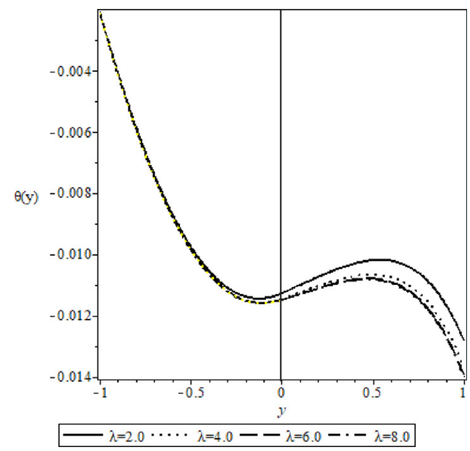

Fig. 3. Jeffrey parameter effect on temperature distribution.

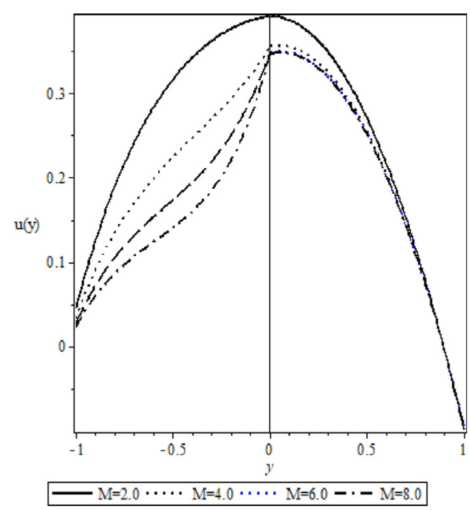

Fig. 4. Magnetic field effect on velocity distribution.

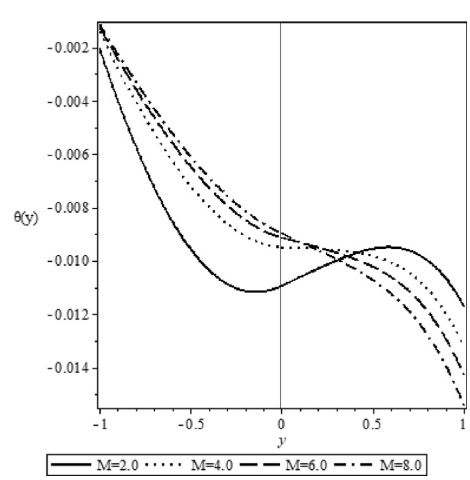

Fig. 5. Magnetic field effect on temperature distribution.

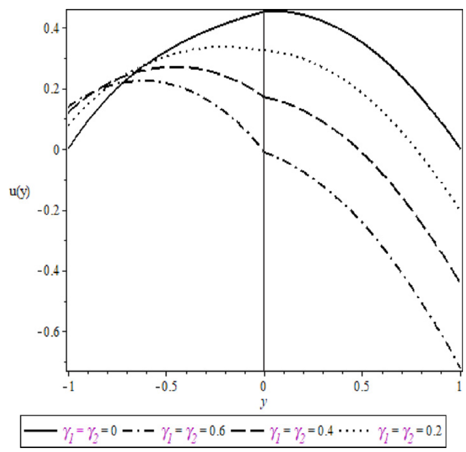

Fig. 6. Thermal slip parameter effect on velocity distribution.

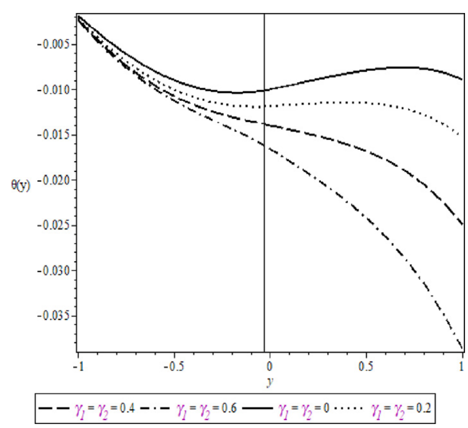

Fig. 7. Thermal slip parameter effect on temperature distribution.

and magnetic parameter M are discussed and plotted for the fluid velocity and the temperature profile. The following parameters are fixed throughout the competition in the study (Gr = 5, $\varepsilon = 0.05$, m = 0.5, $\phi = \pi/6$, K = 1, Re = 5, k = 1 and P = −1).

The Fig. 2, shows the Jeffery parameter effect on the fluid velocity u (y) in the case of velocity slip condition $\eta_1 = \eta_2 = 0.1$ and the thermal slip $\Upsilon_1 = \Upsilon_2 = 0.1$. By observation the Jeffrey fluid velocity in phase II increases as the Jeffery parameter increases and shows insignificant in the non- Jeffrey fluid in phase I.

In Fig. 3, the observation is that the temperature profile Θ (y) decreased in the non- Jeffrey fluid in phase I and showed no significant effect in the Jeffrey fluid in phase II with the increase in the Jeffery parameter. This is due to the fluid kinetic energy which varies to its temperature, at higher temperatures evaporation proceeds more quickly. The remaining fluid reduces in kinetic energy and the fluid temperature decreases as the faster-moving fluid escapes.

In Fig. 4, the magnetic field effect on the velocity of the fluid with slip condition $\eta_1 = \eta_2 = 0.1$, Jeffery parameter $\lambda = 1$ and thermal slip $\Upsilon_1 = \Upsilon_2 = 0.1$ respectively. It is found that the velocity decreases in phase II and shows a partial effect in phase I as the magnetic field value increases. It is found that the magnetic field has the power to slow down the movement of the fluid in the medium since the flow direction is normal to it and it gives an increase to the resistance force.

Fig. 5, shows that in phase II, there is a decrease in temperature and an increase in temperature in phase I as the magnetic field values increase. The flow direction is normal with the magnetic field. This makes it the probability to decrease the Jeffrey electrically conducting fluid temperature in phase II and increase the temperature of the non-Jeffery fluid in phase I.

In Fig. 6, it was found that the thermal slip parameters increase as the velocity of the fluid u (y) increases on both the Jeffrey and non- Jeffrey in the phases. It is found that the increase in the thermal slip of both fluids supports their increase in velocity.

Fig. 7, Shows that there is a decrease in temperature profile on both phases with the increase in thermal slip parameter $\Upsilon_1 = \Upsilon_2$. This is because the increase in the thermal slip of both fluids is against the temperature profile of both phases.

5. Validation of code

The validation Table 1 above is for data validation of the present study with the previous results in [37]. The Jeffrey fluid in phase II of the present study and the velocity equation (U_{20}) of both studies were used to carry out the verification. Taken $\lambda = 0$ at M = 2.0 and M = 4.0.

Table 1. Comparison of the present velocity distribution (U_{20}) with the velocity distribution in [37] for variations of magnetic field parameter at $\lambda = 0$

Y	M = 2.0		M = 4.0	
	U_{20} in [37]	U_{20} in Present Study	U_{20} in [37]	U_{20} in Present study
−1.0	0.045265236	0.045263029	0.025438225	0.02537512
−0.5	0.324573216	0.324571335	0.237312345	0.237314211
0	0.402326321	0.402325268	0.346555219	0.346537497

6. Conclusion

The problems of magnetic field effect on two phase flow of Jeffrey and non −Jeffrey fluid with partial slip and transfer of heat in inclined medium are investigated. The non-linear flow equations obtained are compiled. The regular perturbation method with the perturbation parameter (ε) is used to solve governed equation. Due to the increase and decrease in the Jeffrey and non- Jeffrey fluid velocity and temperature profile as the magnetic field, thermal slip and Jeffrey parameter increase, the following conclusions are made:

The magnetic field decreases the velocity distribution and increase the temperature profile of the Jeffrey fluid but has partial effect on the velocity distribution and decreased the temperature profile of the non- Jeffrey fluid.

The increase in the thermal slip parameter, increase the velocity and temperature of both Jeffrey and non − Jeffrey fluid of two phases.

Finally, the increase in Jeffrey parameter, increase the velocity slip and shows no significant effect in the temperature profile of Jeffrey fluid while it shows no significant effect in the velocity slip and decreased temperature profile in non − Jeffrey fluid.

Funding

The authors declare there is no external funding for this research.

Acknowledgement

We are thankful to the reviewers for their constructive suggestions which led to definite improvement in the version of the paper.

References

[1] Nadeem S, Akbar NS, Naturforsch Z. Effect of Jeffrey fluid with variable viscosity in the form of a well-known Reynolds model of viscosity in an asymmetric channel. Z Naturforsch 2009;64:713−22.

[2] Loyinmi AC, Oredein AI. The unsteady variable viscosity free convection flow on a porous plate. J Niger Assoc Math Phys 2011;19:229−32. https://www.ajol.info/index.php/jonamp/article/view/91459.

[3] Loyinmi AC, Lawal OW. The asymptotic solution for the steady variable-viscosity free convection flow on a porous plate. J Niger Assoc Math Phys 2011;19:273−6.

[4] Lawal OW, Loyinmi AC, Hassan AR. Finite difference solution for Magneto hydrodynamics thin film flow of a third grade fluid down inclined plane with ohmic heating. J Math Assoc Niger 2019;46(1):92−7.

[5] Lawal OW, Loyinmi AC. The effect of magnetic field on MHD viscoelastic flow and heat transfer over a stretching sheet. Pioneer J Adv Appl Math 2011;3:83−90.

[6] Lawal OW, Loyinmi AC. Magnetic and porosity effect on MHD flow of a dusty visco-elastic fluid through horizontal plates with heat transfer. J Niger Assoc Math Phys 2012;21:95−104.

[7] Lawal OW, Loyinmi AC. Oscillating flow on a visco-elastic fluid under exponential pressure gradient with heat transfer. Pioneer J Adv Appl Math 2011;3:33−82.

[8] Lawal OW, Loyinmi AC. Oscillating flow on a visco-elastic fluid under exponential pressure gradient with heat transfer. Pioneer J Adv Appl Math 2011;3:33−82.

[9] Multhuraj R, Srinvas S. Peristaltic transport of a Jeffery fluid under the effect of slip in an inclined a symmetric channel. Int J Appl Mech 2010;2:437−55.

[10] Nallapus S, Radhakrishnamacharya G. Flow of Jeffery fluid through narrow tubes. Int J Scient Eng Res 2013;4:468−73.

[11] Akbar NS, Nadeem S, lee C. Characteristics of Jeffery fluid model for peristaltic flow of Chyme. Int J Res Phys 2013;3:152−60.

[12] Jyothi KL, Devaki P, Screenadh S. Pulsatile flow of a Jeffery fluid in a circular tube having internal porous lining. Int J Math Achieves 2013;4:75−82.

[13] Screenadh S, kumaraswami MM, Naidu K, Parandhama A. Free convection flow of Jeffrey fluid through a vertical deformable porous septum. J Appl Fluid Mech 2016;6:69−75.

[14] Kumaraswamy KN, Sudhakara E, Sreenadh S, Arunachala PV. Effect of the thickness of the porous material on the parallel plate channel flow of Jeffrey fluid when the walls are provided with non-erodible porous linning. Int J Scient Innov Math Res 2009;7:627−36.

[15] Idowu KO, Loyinmi AC. The analytical solution of non-linear Burgers-Huxley equations using the Tanh method. Al-Bahir J Eng Pure Sci 2023;3(1):68−77. https://doi.org/10.55810/2312-5721.1038.

[16] Idowu KO, Akinwande TG, Fayemi I, Adam UM, Loyinmi AC. Laplace homotopy perturbation method (LHPM) for solving system of N-dimensional non-linear partial differential equation. Al-Bahir J Eng Pure Sci 2023;3(1):11−27. https://doi.org/10.55810/2313-0083.1031. 2023.

[17] Loyinmi AC, Akinfe TK. Exact solution to the family of Fisher's reaction-diffusion equations using Elzaki homotopy transformation perturbation method. Eng Rep 2020;2:e12084. https://doi.org/10.1002/eng2.12084.

[18] Loyinmi AC, Idowu KO. Semi −analytical approach to solving Rosenau-Hyman and Korteweg-de Vries equations using integral transform. Tanzan J Sci 2023;49:26−40. https://doi.org/10.4314/tjs.v49il.3.

[19] Anup K, Bhupendra KS, Rishu G, Nidhish KM, Bhatti MM. Response surface optimization for the electromagnetohydrodynamic Cu −polyvinyl alcohol / water Jeffrey Nano fluid flow with and exponential heat source. J Magn Magn Mater 2023;576:170751 (artich ID 896121).

[20] Malasheffy MS, Umavathi JC. Two-phase magneto hydrodynamic flow and Heat transfer in an inclined channel. Int J Eng Sci 1997;3:545−60.

[21] Loyinmi AC, Oredein AI, Prince SU. Homotopy adomian decomposition method for solving linear and nonlinear partial differential equations. Tasued J Pure Appl Sci 2018;1:254−60.

[22] Loyinmi AC, Lawal OW, Sottin DO. Reduced differential transform method for solving partial integro-differential equation. J Niger Assoc Math Phys 2017;43:37−42.

[23] Akinfe KT, Loyinmi AC. The implementation of an improved differential transform scheme on the Schrodinger equation governing wave-particle duality in quantum physics and optics. Res Phys 2021. https://doi.org/10.1016/j.rinp.2022.105806.

[24] Akinfe TK, Loyinmi AC. An algorithm for solving the Burgers-Huxley equation using the Elzaki transform. SN Appl Sci 2020;2:1−17. https://doi.org/10.1007/s42452-019-1652-3.

[25] Akinfe TK, Loyinmi AC. A solitary wave solution to the generalized Burgers-Fisher's equation using an improved differential transform method: a hybrid approach scheme

approach. Heliyon 2021;7:e07001. https://doi.org/10.1016/j.heliyon.2021.e07001.
[26] Vajravelu K, screenadh S, Lakshminarayana P. The influence of heat transfer on peristaltic transport of a Jeffery fluid in a vertical porous stratum. Int J Commun Nonlinear Sci Numer Simul 2011;16:3107–25.
[27] Akbar NS, Hayat T, Nadeem S, Obaidet S. Peristaltic flow of a Williamson fluid of a Williamson fluid in an inclined a symmetric channel with slip and transfer. Int J Heat Mass Transfer 2012;55:1855–62.
[28] Akinfe KT. A reliable analytic technique for the modified proto-typical Kelvin-Voigt viscoelastic model by means of the hyperbolic tangent function. Partial Diff Equ Appl Math 2023. https://doi.org/10.1016/j.padiff.2023.10523.
[29] Akinfe TK, Loyinmi AC. An improved differential transform scheme implementation on the generalized Allen-Cahn equation governing oil pollution dynamics in oceanography. Partial Differ Equ Appl Math 2022;6:100416. https://doi.org/10.1016/j.padiff.2022.100416.
[30] Sivaraj R, Rush KB, Prakash K. MHD mixed convective flow viscoelastic and viscous fluid in a vertical porous channel. Int J Appl Applied Math 2011;7:99–116.
[31] Loyinmi AC, Erinle-Ibrahim LM, Adeyemi AE. The new iterative method (NIM) for solving telegraphic equation. J Niger Assoc Math Phys 2017;43:31–6.
[32] Lawal OW, Loyinmi AC, Erinle-Ibrahim LM. Algorithm for solving a generalized Hirota-Satsuma coupled KDV equation using homotopy perturbation transformed method. Sci World J 2018;13:23–8.
[33] Lawal OW, Loyinmi AC. Application of new iterative method for solving linear and nonlinear initial boundary value problems with non-local conditions. Sci World J 2019; 14:100–4.
[34] Ebaid A. Effect of magnetic field and wall slip conditions on the peristaltic transport of a Newtonian fluid in an asymmetric channel. Phys Lett 2008;372:4493–9.
[35] Vajravelu K, Sreenadh S, Hemadi R, Murugesan K. Peristaltic Transport of a caisson fluid in contact with a Newtonian fluid in a circular tube with permeable wall. Int J Fluid Mech Res 2009;36(3):244–54.
[36] Akbar NS, Nadeem S, Khan ZH. Thermal and velocity ship effect on the MHD peristaltic flow with carbon nanotubes in symmetric channel. Application of radiation therapy applications. Nanosci 2014;4:849–57.
[37] Abbas Z, Hassan J, Sajid M. MHD two phase fluid and heat transfer with partial slip in an inclined channel. Therm Sci 2016;20:1435–46.
[38] Abd-All AM, Abo-Dahab SM, Albalaw MM. Magnetic field and gravity effect on Peristaltic transport on a Jeffrey fluid in an a symmetric channel. Abstract and applied analysis; 2014. https://doi.org/10.1155/2014/896121.
[39] Bhupendra KS, Pawan KS, Sudhir KC. Effect of a magnetic field on unsteady free convection oscillatory systems. Int J Appl Mech Eng 2022;1:188–202.
[40] Lawal OW, Loyinmi AC, Sowumi SO. Homotopy perturbation algorithm using Laplace transform for linear and nonlinear ordinary delayed differential equation. J Niger Assoc Math Phys 2017;41:27–34.
[41] Lawal OW, Loyinmi AC. Approximate solutions of higher dimensional linear and nonlinear initial boundary problems using new iterative method. J Niger Assoc Math Phys 2017; 41:35–40.
[42] Loyinmi AC, Akinfe TK, Ojo AA. Qualitative analysis and dynamical behavior of a Lassa haemorrhagic fever model with exposed rodents and saturated incidence rate. Sci Afr 2021;14:e01028. https://doi.org/10.1016/j.sciaf.2021.e01028.
[43] Sharma BK, Rishu G. Unsteady magneto- hydrodynamics (MHD) heat and mass transfer for a viscous incompressible fluid through a vertical stretching surface embedded in a Darcy – Forchheimer porous medium in the presence of anon-uniform heat source/ sink and field – order chemical reaction. Propuls Power Res 2022;2:276–92.
[44] Agbomola JO, Loyinmi AC. Modelling the impact of some control strategies on the transmission dynamics of Ebola virus in human-bat population: An optimal control analysis. Heliyon 2022;8:e12121. https://doi.org/10.1016/j.heliyon.2022.e12121.
[45] Idowu OK, Loyinmi AC. Qualitative analysis of the transmission dynamics and optimal control of covid-19. EDUCATUM J Sci Math Technol 2023;10(1):54–70. https://doi.org/10.37134/ejsmt.vol10.1.7.2023.
[46] Nidhish KM, Bhupendra KS, Laura MP. The entropy generation optimization of cilice regulated MHD ternary hybrid Jeffrey nanofluid with Arrhenius activation energy and induced magnetic field. Sci Rep 2023;13:14483.
[47] Akinfe KT, Loyinmi AC. Stability analysis and semi-analytical solution to a SEIR-SEI Malaria transmission model using He's variational iteration method. 2022. Preprints.
[48] Lawal OW, Loyinmi AC, Ayeni OB. Laplace homotopy perturbation method for solving coupled system of linear and nonlinear partial differential equation. J Math Assoc Niger 2019;46(1):83–91.
[49] Sharma BK, Anup K, Rishu G, Bhatt MM. Exponential space and thermal- dependent heat source effects on electro-magneto-hydrodynamic Jeffrey fluid flow over a vertical stretching surface. Int J Modern Phys B 2022;36:2250220.
[50] Erinle-Ibrahim LM, Adewole AI, Loyinmi AC, Sodeinde OK. An optimization scheme using linear programming in a production line of rites food limited, Ososa. FUDMA J Sci 2020;4:502–10.
[51] Agbomola J, Loyinmi A. A mathematical model for the dynamical behaviour of Ebola transmission in human-bat population: implication of immediate discharge of recovered individuals. 2022. Preprints.
[52] Rashidi MM, Kavyani N, Abelman S. Investigation of entropy generation in MHD and slip flow over a rotating porous disk with variable properties. Int J Heat Mass Transfer 2014;70:892–917.