

Exploring the Identification of Autoregression Model by General Least Deviation Method

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The data used to support the findings of this study are available from the corresponding author upon request

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All authors contributed equally to this work. Each author participated in the conceptualization, methodology, data analysis, and manuscript preparation.

ORIGINAL STUDY

Exploring the Identification of Autoregression Model by General Least Deviation Method

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Abstract

We are considering a novel method for analyzing time series data that relies on quasi-linear recurrence relations. Unlike neural networks, this approach allows for directly formulating high-quality quasi-linear difference equations that accurately represent the studied process. Techniques for determining the parameters of a single equation have been devised and validated. This work discusses and tests a technique for identifying the parameters of a quasi-linear recurrence equation. This approach is employed to tackle the issue of regression analysis including observable variables that are mutually dependent. It enables the utilization of the Generalized Least Deviations Method (GLDM). This model was utilized in a computational experiment to determine the parameters of quasi-linear differential equations that describe the spread of Covid-19 infection. The model underwent testing on three distinct types of processes: (1) monotonous (predicting cumulative cases); (2) oscillatory (predicting daily cases). The specified parameters allow the model to generate long-term predictions.

Keywords: Forecasting, Quasi-linear recurrence relation, Optimal value, Loss function identification, Least deviations method, Time series, Covid-19

1. Introduction

In December 2019, Chinese officials publicly disclosed the initial official information regarding transmitting the human coronavirus within their country as a community-acquired illness [11,12]. COVID-19 often leads to severe symptoms, particularly in older persons and individuals with underlying medical disorders such as cardiovascular diseases (CVDs), diabetes, chronic respiratory diseases (COPDs), cerebrovascular illness, and cancer [13]. When comparing the SARS-COV-2 coronavirus to prior pandemics, it is evident that SARS-COV-2 had a significantly greater impact than the SARS coronavirus pandemic. COVID-19 exhibits a similar level of mortality as prior flu pandemics. However, in comparison to the swine flu pandemic, also known as H1N1 (2009) and the Spanish flu (1918), COVID-19

appeared to be substantially more severe due to a higher number of hospitalizations [14]. In 2014, Ebola surfaced as a virus with a median mortality rate of 50%. A substantial amount of knowledge has been acquired in measuring different factors and creating methods to identify and predict the state and resources of certain systems. Therefore, a critical priority area is enhancing the precision and swiftness in determining these projected values. This condition is especially crucial for distinctive, heavily burdened mechanical systems, as specified by [1]. The solutions to this difficulty often arise from the dynamic properties of different systems. Accurately identifying these characteristics is aided by selecting an appropriate diagnostic mathematical model that connects the object's state and diagnostic feature space. Dynamic models, represented by difference equations, phenomenological, structural, regression, and other

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forms, are considered appropriate for this objective. The selection of a particular model depends on the specified attributes and the intrinsic nature of the process being analyzed. These models are crucial in accurately diagnosing and predicting system responses under various settings.

The resolution to this difficulty often arises from the dynamic attributes of different systems. Accurately identifying these characteristics is aided by selecting an appropriate diagnostic mathematical model that connects the object's state and diagnostic feature space. Dynamic models, represented by difference equations, phenomenological, structural, regression, and other forms, are considered appropriate for this objective. The selection of a particular model depends on the specified attributes and the intrinsic nature of the process being analyzed. These models are crucial in accurately diagnosing and predicting system actions in various settings.

Parameter identification, a crucial process in numerous disciplines, has traditionally relied on statistical approaches, neural networks, or mathematical models. Currently, these methods are utilized in industrial settings and endeavors to predict the advancement of the Covid-19 pandemic. For example [5], assesses the precision of predicting the pandemic's advancement by employing various established models, creating software to apply these techniques, and doing computer tests using Covid-19 time series data. The study's findings indicate that its forecasting system applies to any time series covering diverse events. Nevertheless, the models examined in that study are restricted to predicting outcomes shortly. Thus, creating a mathematical method that enables the direct derivation of accurate quasi-linear difference equations is imperative, which may effectively characterize the given process. Analysts from Sequoia Capital labeled the coronavirus as the defining and unexpected event of 2020, comparing it to a black swan [2,3]. A black swan occurrence is defined by three primary characteristics: (1) its unpredictability; (2) its substantial impact; and (3) the tendency to retrospectively rationalize the event, making it seem less random and more predictable than it initially appeared [4]. The Cauchy distribution is well-suited for recognizing and forecasting the spread of a coronavirus pandemic.

Our study discusses the techniques used to identify the parameters of a single equation. The research is centered around developing an algorithm to identify the parameters of a quasi-linear recurrence equation. This approach is used to solve the regression analysis issue involving observable variables that are mutually dependent. It allows for

implementing the Generalized Last Deviations Method (GLDM). This model was used for computational tests, allowing long-term forecasting using the identified parameters. Unlike neural networks, this approach explicitly constructs high-quality quasi-linear difference equations that accurately represent the studied process. This study did not use private criteria because they were unavailable, and a loss function without mathematical expectation and variance was selected.

The algorithms considered by [5] are implemented to determine the coefficients $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_m \in \mathbb{R}$ of an m -th order quasi-linear autoregressive model

$$y_t = \sum_{j=1}^{c(r)} \lambda_j g_j \left(\left\{ y_{t-q} \right\}_{q=1}^r \right) + \varepsilon_t, t=1, 2, \dots, T \quad (1)$$

using up-to-date information on the values of state variables $\{y_t \in \mathbb{R}\}_{t=1-r}^T$ at time instants t . Here, $g_j : (\{y_{t-q}\}_{q=1}^r) \rightarrow \mathbb{R}$ for $j = 1, 2, \dots, c(r)$ are given $c(r)$ functions, and $\{\varepsilon_t \in \mathbb{R}\}_{t=1}^T$ are unknown errors.

The gradient projection method employed for constructing our algorithms was considered by [6] for minimax optimization problems with inequality constraints. This developed algorithm constructs a convolution of partial criteria. In [7], the relaxed gradient projection method is introduced, which incorporates modifications that significantly enhance the rate of objective improvement. This method does not necessitate precise parameter setup, making it more straightforward and stable for routine use. Additionally [8], discusses a new gradient projection algorithm that utilizes a line-search along the feasible direction and an adaptive step-length selection based on recent strategies, specifically the alternation of the well-known Barzilai–Borwein rules. The convergence of this approach is also discussed by the authors. However, most of the approaches considered do not account for the “black swan” phenomenon. Therefore, the development of a new approach to address this situation remains an urgent task.

2. Literature review

Several literature research aim to forecast the transmission of the COVID-19 virus and examine its current spread. This section discusses the existing literature publications and research contributions. An implementation of a mathematical model using sequential Monte Carlo simulation was utilized to determine the initial transmission rate of the virus. This was achieved by calculating the daily average reproduction number, R_t , while considering other characteristics such as the proportion of cases and

the probability of confirmed cases. The likelihood of an epidemic can be heightened if the transmission is uniform throughout a population [15]. A mathematical model was developed using the principles of isolation and contact tracing to effectively manage the propagation of the virus. The duration between the appearance of symptoms and isolation was determined, which enhances the likelihood of transmission. Uncertainty exists over the identification of symptoms in the early stages, and the threshold for testing is low, leading to increased delays and a higher likelihood of more people being impacted [16]. Emerging technologies such as Artificial Intelligence (AI), Machine Learning (ML), Deep Learning (DL), and Big Data can be utilized to generate diverse predictions on numerous elements in the battle against COVID-19. The technology can be employed in various significant domains, such as early disease diagnosis, contact tracking, therapeutic and vaccine research, and forecasting future instances [17,18]. The COVID-19 data was clustered using an unsupervised machine learning approach called K-Means. This algorithm utilizes several factors and concepts to make predictions. The model

the pandemic's transmission. Through extensive training on large datasets, deep learning models can automate the diagnosis, treatment, and monitoring of patients, providing valuable assistance to healthcare practitioners in multiple domains [21].

3. Methods

The GLDM method depends on four algorithms. These algorithms were developed by [9,10]. The DualWLDMSoluter algorithm maximizes forecasts by utilizing a dual-weighted least deviation methodology. The process begins with the initialization of gradients, active constraints, and weights. It then proceeds to compute the gradient projection of the loss function. The program continuously alters current constraints and weights until further breakthroughs are no longer achievable. The objective is to determine the optimal weights that satisfy constraints and maximize the weighted sum of values. The process is iterated until the optimal weights and operational constraints are determined, resulting in the most precise forecast possible given the prevailing conditions.

Algorithm1. DualWLDMSoluter

Require:

$$\nabla_L$$

$$\{p_t \in \mathbb{R}^+\}_{t=1}^T$$

Gradient projection on \mathcal{L}
Weight factors

Ensure:

$w^* = \operatorname{argmax}_{w \in \mathbb{R}^T} \sum_{i=1}^T w_i \cdot y_i$
 $R^* = \{t \in T: |w_t^*| = p_t\}$
 1: $w \leftarrow \{w_i = 0; i = 1, 2, \dots, T\}; R \leftarrow \emptyset; g = \nabla_L$
 2: **while** $(\alpha_* \neq 0)$ **do**
 3: $\{(\alpha_*, t_*) \leftarrow \operatorname{argmax}\{\alpha \mid 0: -p_t \leq w_t + \alpha g_t \leq p_t\}\}$
 4: $w \leftarrow w + \alpha_* g; g_{t_*} \leftarrow 0; R \leftarrow R \cup \{t_*\};$
 5: **end while**
 6: $w^* = w, R^* = R$
 7: **return** (w^*, R^*)

Optimal dual solution
Active restrictions

facilitates the analysis of countries that have been impacted and are at risk of being impacted in the foreseeable future [19]. A clustering algorithm was utilized to determine the rate of disease transmission in Singapore by analyzing the travel history from China. In order to mitigate the spread of the virus, the clusters have the capability to forecast the local transmission rates that are expected to be impacted. Nevertheless, a substantial amount of data sets is required to create a sufficient model that can achieve higher accuracy in predictions [20]. AI-driven predictive models and their resultant consequences are recognized in various domains where AI can enhance understanding and management of

The WLDM-estimator algorithm technique is specifically developed to enhance forecasts using a weighted least deviation approach. The necessary inputs include the matrix representing a linear subspace, the gradient projection, weight factors, and specified state variables. The main objective is to identify the most favorable primal solution and limitations. The algorithm begins by calling the DualWLDMSoluter algorithm to acquire the optimal dual solution and active constraints. By utilizing these elements, it creates the required matrix and solution vector. The last phase entails calculating the most favorable primary solution, guaranteeing a resilient and effective prediction given the specified circumstances.

Algorithm 2. WLDM-estimator**Require:**

$$S = \{S_t \in \mathbb{R}^N\}_{t \in T}$$

$$\nabla_L$$

$$\{p_t \in \mathbb{R}^+\}_{t=1}^T$$

$$\{y_t \in \mathbb{R}^+\}_{t=1}^{T=1-r}$$

Ensure:

$$A^* \in \mathbb{R}^{c(r)}$$

$$z^* \in \mathbb{R}^T$$

$$1: (w^*, R^*) \leftarrow \text{DualWLDMSoluter}(\nabla_L, \{p_t \in \mathbb{R}^+\}_{t=1}^T)$$

$$2: S^* \leftarrow \{S_t: t \in R^*\}; y^* \leftarrow \{y_t: t \notin R^*\}$$

$$3. (A^*)^\top y^* (S^*)^{-1}$$

$$4. z^* \leftarrow (A^*)^\top S - y$$

return (λ^*, z^*) The matrix representation of a linear subspace \mathcal{L} Gradient projection onto the subspace \mathcal{L}

Weighting coefficients

Specified state variable values

Optimal primary solution

Constraints

System (46) matrix

System (46) solution

Find restriction

The GLDM-estimator algorithm technique optimizes forecasts using the GLDM. The necessary inputs include the matrix representing a linear subspace, the gradient projection, weight coefficients, and specified state variables. The algorithm aims to identify the most favorable GLDM solution and its accompanying residuals. The process begins by establishing the initial weight

elements and estimating the solution using the WLDMSoluter algorithm method. The weights are adjusted iteratively according to a certain formula. The procedure iterates until the estimates reach a state of convergence, leading to the optimal solution and residuals for the GLDM. This guarantees an efficient forecast under the specified parameters.

Algorithm 3. Estimator for Generalized Least Deviations Method (GLDM)**Require:**

$$S = \{S_t \in \mathbb{R}^N\}_{t \in T}$$

$$\nabla_L$$

$$\{p_t \in \mathbb{R}^+\}_{t=1}^T$$

$$\{y_t \in \mathbb{R}^+\}_{t=1}^{T=1-r}$$

Ensure:

$$A^* \in \mathbb{R}^{c(r)}$$

$$z^* \in \mathbb{R}^T$$

$$1: p \leftarrow \{p_t = 1: t = 1, 2, \dots, T\}$$

$$2. (A^{(1)}, z^{(1)}) \leftarrow$$

$$3. \leftarrow \text{WLDMSoluter}(S, \nabla_L, \{p_t\}_{t=1}^T, \{y_t\}_{t=1-r}^T)$$

$$4. \text{For all } (t = 1, 2, \dots, T)$$

$$5. \quad p_t \leftarrow \left(\frac{1}{1 + (z_t^{(1)})^2} \right)$$

$$6. \text{End for}$$

$$7. (A^{(2)}, z^{(2)}) \leftarrow \text{WLDMSoluter}(S, \nabla_L, \{p_t\}_{t=1}^T, \{y_t\}_{t=1-r}^T)$$

$$8. q \leftarrow 2$$

$$9. \text{While } (A^{(q)} \neq A^{(q-1)}) \text{ do}$$

$$10. \quad \text{For all } (t = 1, 2, \dots, T) \text{ do}$$

$$11. \quad p_t^{(q)} \leftarrow \left(\frac{1}{1 + (z_t^{(q)})^2} \right)$$

$$12. \quad \text{End for}$$

$$13. \quad ((A, z)) \leftarrow$$

$$\text{WLDMSoluter}(S, \nabla_L, \{p_t^{(q)}\}_{t=1}^T, \{y_t\}_{t=1-r}^T)$$

$$14. \quad (A^{(q+1)}, z^{(q+1)}) \leftarrow (A, z)$$

$$15. \quad q \leftarrow q + 1$$

$$16. \text{End For}$$

$$17. z^* \leftarrow z^{(q)}, (A^*) \leftarrow A^{(q)}$$

return (A^*, z^*) The matrix associated with the linear subspace \mathcal{L} Projection of the gradient on \mathcal{L}

Weighting coefficients

Specified state variable values

Optimal solution for the (GLDM)

Error terms

Find restriction

The Predictor algorithm is designed to make predictions using the provided state variables and the WLDM solution. The system computes the forecast, average prediction error, average absolute prediction error, and minimum forecast horizon. The algorithm sets the initial index and proceeds to iteratively estimate values until the difference between the predicted and observed values surpasses a specified threshold. Subsequently, it revises the projected timeframe and computes the prediction discrepancies. The process iterates until convergence, and the method terminates by providing the average prediction error, average absolute prediction error, and minimal forecast horizon.

4. Experimental results

In order to build the solution of a quasi-linear difference equation, a computational experiment was carried out to construct the solution of a Cauchy problem. This computational experiment solved the unknown recurrence equation of the time series and carried out the identification of this equation, demonstrating the high quality of the proposed algorithm.

4.1. Monotonous process

We consider the process shown in Fig. 1. It represents the number of cumulative Covid-19

Algorithm 4. Predictor

Require:

$$\{y_t \in \mathbb{R}^+\}_{t=1}^{T=1-r}$$

$$A = \{\lambda_i\}_{i=1}^{c(r)}$$

Ensure:

```

    PY[1:T][1:T]
    E
    D
    minFH
    1 Initialize Strt ← 0
    2. While FH[Strt] < r do
    3.     Strt ← Strt + 1
    4.     PY[Strt][0] ← Y[Strt]
    5.     for t ← Strt + 2 to m do
    6.         py ← 0
    7.         for j ← 0 to c
    8.             A1 ← G[j][PY[Strt][t -
    9.             1], PY[Strt][t - 2]]
    10.            py ← py + a[j] × A1
    11.        End for
    12.        PY[Strt][t] ← py
    13.        if |PY[Strt][t] - Y[Strt + t]| > SZ
    14.            break
    15.        End if
    16.    End for
    17. End While
    18. LastStrt ← t
    19. minFH ← FH[Strt]
    20. for t ← 3 to Strt do
    21.     If FH[t] < minFH Then
    22.         minFH ← FH[t]
    23.     End if
    24. End for
    25. E ← 0, D ← 0
    26. For t ← 3 to minFH do
    27.     D ← D + |Y[t + Strt] - PY[Strt][t]|
    28.     E ← E + (Y[t + Strt] - PY[Strt][t])
    29. End For
    30. D ←  $\frac{D}{\text{minFH}}$ 
    31. E ←  $\frac{E}{\text{minFH}}$ 
    32. return (D, E, minFH)

```

The specified values of the state variables
Weighted Least Deviations Method (WLDM)
solution

Prediction for y_t
Average prediction error
Average absolute prediction error
Minimum forecast horizon

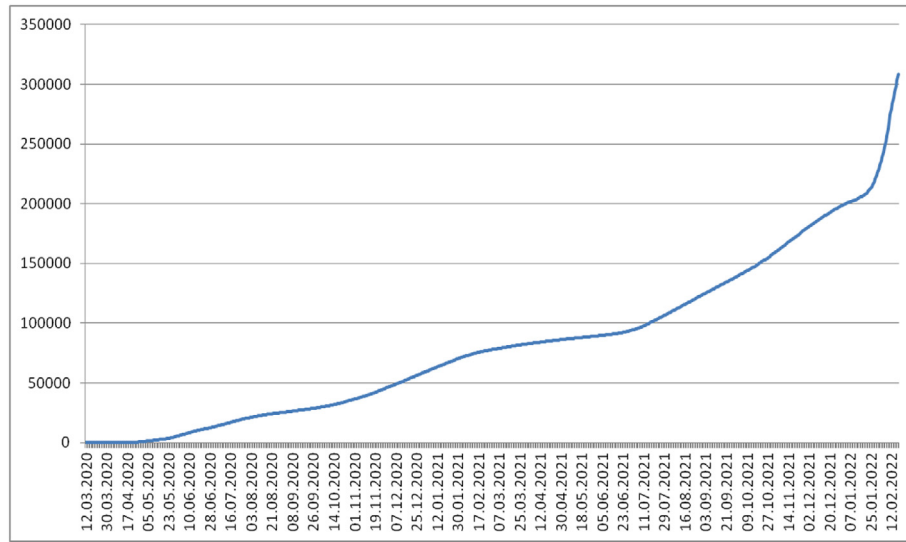


Fig. 1. The observed time series.

infection cases for the Chelyabinsk region. This dataset spans 700 days.

For this experiment, we use time series data of lengths 150, 300, 500, and 700 days to model a second-order quasi-linear difference equation:

$$y_t = (\lambda_1 y_{t-1} + \lambda_2 y_{t-2}) + (\lambda_3 y_{t-1}^2 + \lambda_4 y_{t-1} y_{t-2} + \lambda_5 y_{t-2}^2).$$

The algorithm identifies the coefficients $\lambda_1, \dots, \lambda_5$. The identification results are presented in Table 1.

According to the data, the experiment with 300 points yields the loss function with the lowest value, indicating significant coefficients. It is found that a longer observed time series facilitates a more accurate study of the time series' dependence on the loss function's value.

4.2. Oscillatory process

We consider the process shown in Fig. 2. It represents the number of currently infected Covid-19 patients across four regions of the Russian

Federation (Moscow, St. Petersburg, Chelyabinsk, and Sverdlovsk regions). This dataset spans 700 days.

The graph depicted in Fig. 2 shows the Covid-19 infection trends in four different regions: Chelyabinsk, Sverdlovsk, Moscow, and St. Petersburg. The x-axis represents the timeline from March 12, 2020, to January 31, 2022, while the y-axis shows the number of reported Covid-19 cases, ranging from 0 to 500,000.

The data for each region is represented by different colored lines: - Chelyabinsk region is represented by the blue line. - Sverdlovsk region is represented by the red line. - Moscow is represented by the green line. - St. Petersburg is represented by the purple line.

The graph highlights the fluctuating nature of Covid-19 cases over the specified period. Notably, Moscow exhibits the highest peaks compared to other regions, with significant spikes observed at multiple points throughout the timeline. St. Petersburg also shows noticeable peaks, although not as high as those in Moscow.

Table 1. The identification results.

Factors	150	300	500	700
λ_1	1.982416e00	2.056573e00	1.999967e00	1.999837
λ_2	-9.824160e-01	-1.056573e00	-9.999669e-01	-9.998369e-01
λ_3	7.459095e01	-4.031902e01	-2.774247e00	6.249941e01
λ_4	7.606037e01	-4.092965e01	-2.805568e00	6.284127e01
λ_5	-1.506450e02	8.124591e01	5.579729e00	-1.253407e02
Loss fun.	2.811987e-01	5.663879e-02	3.828931e-02	2.077376e-01
Forecasting horizon	149	299	499	699
E	-3.640266e-03	9.866220e-01	9.919839e-01	-1.472563e-01
D	9.350328e-01	9.866220e-01	9.919839e-01	9.850607e-01

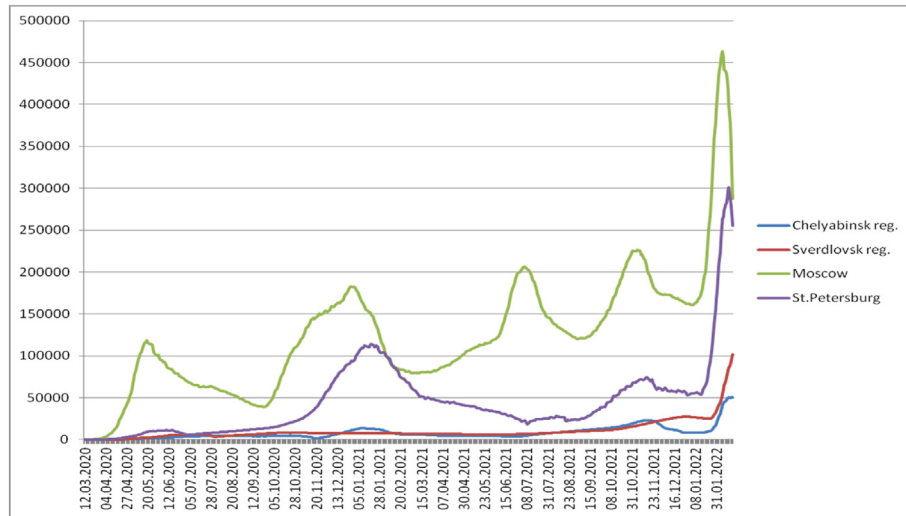


Fig. 2. The recorded time series for the oscillatory process.

Table 2. The identification results for oscillatory process.

Factors	Moscow	St. Petersburg	Chelyabinsk	Sverdlovsk
λ_1	1.951373e+000	2.024332e+000	2.035072e+000	1.956923e+000
λ_2	-9.477643e-001	-1.024285e+000	-1.035071e+000	-9.569233e-001
λ_3	-7.272139e-002	-2.690416e+003	3.586406e+001	1.841075e+002
λ_4	-1.532655e-001	-2.798081e+003	3.593035e+001	1.835499e+002
λ_5	2.155495e-001	5.487480e+003	-7.179591e+001	-3.676588e+002
Loss fun.	9.916341e-001	9.052773e+001	2.045520e+000	5.169706e+000
Forecasting horizon	699	699	699	699
E	-4.155277e-003	-1.932453e-001	-2.907818e-002	9.265717e-002
D	9.840693e-001	9.593555e-001	9.862808e-001	9.827086e-001

In contrast, the Chelyabinsk and Sverdlovsk regions have comparatively lower numbers of cases, with smaller peaks throughout the observed period.

This visual representation provides a clear comparison of the Covid-19 infection trends across the four regions, highlighting the varying impacts of the pandemic in each area. Such data is crucial for understanding the spread and control of the virus, enabling targeted interventions and resource allocation.

In this experiment, the same number of days for each time series was considered. Changing the length of the time series was deemed unnecessary as useful information might be discarded. A second-order model was also used:

$$y_t = (\lambda_1 y_{t-1} + \lambda_2 y_{t-2}) + (\lambda_3 y_{t-1}^2 + \lambda_4 y_{t-1} y_{t-2} + \lambda_5 y_{t-2}^2).$$

The algorithm identifies the coefficients $\lambda_1, \dots, \lambda_5$. The identification results are presented in Table 2.

The experiment for the oscillatory process yields error values similar to those of the monotonous process. The model coefficients are also significant.

5. Conclusion

Finding the equations' primary goal is to make it feasible to forecast potential future values of associated endogenous variables using the model values of those variables. The method under consideration makes it possible to estimate a realistic forecasting horizon, which in the cases under consideration is one day shorter than the beginning data length for both monotonous and oscillatory processes. It is clear from Tables 1 and 2 that average errors have low values even for very wide forecasting horizons and do not depend on the length of the initial vector if it is bigger than 100. Our mistakes are greater for the shortest vectors.

Regarding the model's performance, we can state that, even for situations like the spread of Covid-19, it performs comparably to neural network models or traditional statistical models. When compared to these models, it has one important advantage: it allows one to understand the model coefficients in terms of the study question. The approach discussed in the article is an additional substitute for creating digital twins of various process kinds. This method,

in contrast to neural networks, allows for the explicit extraction of excellent quasi-linear difference equations that suitably characterize the process under consideration. The following are the directions for future research: (1) forecasting the multidimensional time series using the aforementioned techniques; (2) employing parallel technology to expedite the execution of algorithms for large datasets and specialized data kinds to improve accuracy.

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