Cancellation and Weak Cancellation S-Acts

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ORIGINAL STUDY

Cancellation and Weak Cancellation S-Acts

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Abstract

The purpose of this work is to introduce and study a generalization of the cancellation (weak cancellation) of modules to acts. Then some notions related of modules was extend to arbitrary acts. Several statements that are equivalent to the cancellation (weak cancellation) property have been studied. Some characterizations of cancellation (weak cancellation) acts was presented and derive many of their properties. Using the condition of multiplication acts various results of cancellation (weak cancellation) acts are investigated.

Keywords: Cancellation acts, Cyclic acts, Finitely generated act, Multiplication acts, Weak cancellation acts

1. Introduction

Let S be a semigroup. Then a nonempty set A is called left S-act if there exists a mapping: $S \times A \to A$ written by $(s, a) \mapsto sa$, satisfying $(s_1 s_2)a = s_1(s_2 a)$ for all $s_1, s_2 \in S$ and $a \in A$ [1]. In a similar manner, the definition of right S-acts. The term "S-act" will henceforth always refer to the left S-act. A semigroup with identity is called a monoid. A nonempty subset B of S-act A is said to be subact (briefly, S-subact) if for all $s \in S$ and $b \in B$, then $sb \in B$. An element $\theta \in A$ is said to be fixed of A if $s\theta = \theta$ for all $s \in S$. Through this paper an S-act A has an unique fixed element such that if a semigroup S containing zero element such that $0a = \theta$ and $s\theta = \theta$ for every $a \in A$, $s \in S$ then θ will be named the zero of A. Let A be a S-act and U be a nonempty subset of A. If for all element $a \in A$ can be represented as $a = su$ for some $u \in U, s \in S$ then U a generating set of A. In other words, U is a generating elements of A if $<U> = \cup_{u \in U} Su = A$. An S-act A is cyclic if $A = <a>$ where $a \in A$. The set U is said to be a basis of S-act A if it is generating elements of A and every element $a \in A$ has a unique representation in the form $a = us$ where $u \in U$ and $s \in S$, i.e., if $a = u_1 s_1 = u_2 s_2$ then $u_1 = u_2$ and $s_1 = s_2$. Let S be a semigroup and ρ be a equivalence relation on S. Then ρ is called a left (resp. right) congruence on S if $s\rho t$ implies $(us) \rho (ut)$ (resp. $(su) \rho (tu)$) for all $s, t, u \in S$. An equivalence relation ρ on a semigroup is called a congruence if and only if $s\rho t$ and $u\rho v$ implies $(su) \rho (tv)$ for all $s, t, u, v \in S$. In particular, let A be a left S-act and ρ be a equivalence relation on A then ρ is a congruence on A if $a\rho a'$ implies $(sa)\rho(sa')$ for $a, a' \in A$ and $s \in S$. Let B be a S-subact of S-act A and ρ is congruence on A, then ρ_B is a congruence on A such that $\rho_B = \rho \cap B \times B$. For equivalence relation ρ the class of $s \in S$ with respect to ρ is indicated by $[s]_\rho$. For a congruence ρ on S the multiplication in S/ρ is written by $[s]_\rho [t]_\rho = [st]_\rho$ for $s, t \in S$. Thus S/ρ is a semigroup that called the factor semigroup of S by ρ. Let A be a S-act and ρ be a congruence on A. Then $A/\rho = \{[a]_\rho \mid a \in A\}$ and a left multiplication by elements of S on A/ρ is defined by $s [a]_\rho = [sa]_\rho$ for every $s \in S$. Let S and S' be semigroups. The mapping $f : S \to S'$ is said to be a semigroup homomorphsm of S into S' if $f(s_1 s_2) = f(s_1) f(s_2)$ for each $s_1, s_2 \in S$. In this case $\ker f = \{(s_1, s_2) \in S \times S: f(s_1) = f(s_2)\}$ is said to be kernel congruence of homomorphism f. Let S be a semigroup and ρ be a congruence on S. The mapping $\pi_\rho : S \to S/\rho$ is defined by $s \mapsto [s]_\rho$ is a homomorphism and it is called the canonical epimorphsm [1]. For S-act A the ideal $\{s \in S \mid sa = \theta$ for all $a \in A\}$ of S is said to be the annihilator of S-act A in S (briefly, ann(A)). If ann(A) = 0; A is called faithful. Let B be a S-subact of S-act A. Then $[B : A] = \{s \in S \mid sa \in B$ for each $a \in A\}$. Clearly, $[B : A]$ is ideal of a semigroup S [2,3]. An S-act A is called a multiplication if for all S-subact B of A, there exists an ideal K of a semigroup S such that B=KA [4,5].

The main aim of this paper is to study the concept of cancellation (weak cancellation) S-acts as a generalization of cancellation (weak cancellation) modules that was introduced by Adil GN, Ali SM

[6,7]. Thus some results and concepts of modules was extend to arbitrary S-acts, analogous to the notions of cancellation (weak cancellation) modules introduced by Adil GN, Ali SM [6]. Several results and characterizations about these concepts have been studied. In particular, some results and equivalent conditions for this concept was given such as: an S-act is cancellation if and only if it's weak cancellation and faithful S-act. Also has been proven that the homomorphic image of cancellation S-act need not be a cancellation but the inverse image is hold. On other hand, it has been proven that if M is cancellation S-act, then it is finitely generated and several related notions was investigated to establish a connection between multiplication S-acts and cancellation S-acts. The relation between cancellation S-acts and some types of S-acts was discussed. Finally, in this paper S means a semigroup with zero element and the word "ideal" is used for atwo sided ideal.

2. Main results

The concept of cancellation (weak cancellation) was introduced by Adil GN, Ali SM [6,8] as follows: if M is an R-module, where R is a commutative ring with unity. Then M is said to be a cancellation R-module (resp. weak cancellation R-module) if $CM = DM$ for ideals C, D of R, then $C = D$ (resp. $C + \text{ann}(M) = D + \text{ann}(M)$). A natural generalization of this concept to S-acts was studied.

Definition 2.1:
An S-act A is said to be cancellation S-act if $CA = DA$ for ideals C, D of a semigroup S then $C = D$. An ideal K of a semigroup S is a cancellation if it is cancellation S-subact of S-act S.

Examples 2.2.

i. Let $S = A = \{a, b, c\}$. Then A is an S-act under the mapping $S \times A \to A$ that is defined by the following table:

.	a	b	c
a	a	a	a
b	a	a	a
c	a	b	c

Clearly, A is cancellation S-act.

ii. Let $S = A = \{a, b, c\}$. Define the mapping: $S \times A \to A$ by the following table:

.	a	b	c
a	a	a	a
b	a	a	b
c	a	a	a

Then A is a S-act. Now, $K = \{a\}$, $L = \{a, b\}$, $C = \{a, c\}$ and $D = S$ are ideals of S. Here, $AM = CM = \{a\}$ But $A \neq C$. Thus M is not cancellation S-act.

iii. If $\mathbb{Z}$, $\mathbb{Q}$ are the sets of all integer and rational numbers, respectively. Then $\mathbb{Q}$ is an $\mathbb{Z}$-act. It's clear that $(p\mathbb{Z})\mathbb{Q} \subseteq \mathbb{Q}$. If $q \in \mathbb{Q}$, then $q = \frac{x}{y} = \frac{px}{py} = p\frac{x}{py} \in (p\mathbb{Z})\mathbb{Q}$ such that $x, y \in \mathbb{Z}$. Thus $\mathbb{Q}$ is not cancellation $\mathbb{Z}$-act.

Remark 2.3.
An S-act A is a cancellation if and only if it is a faithful.

Proposition 2.4.
Let K be an ideal of a semigroup S and A be a cancellation S-act. Then KA is a cancellation S-subact if and only if K is cancellation ideal.

Proof: ($\Rightarrow$) Let $DK = CK$, where D and C are ideals of S. Then $D(KA) = C(KA)$ and since KA is a cancellation then $D = C$.

($\Leftarrow$) Let $D(KA) = C(KA)$, where D and C are ideals of S. Since A is cancellation then $DK = CK$ and by hypothesis $D = C$.

Let A be S-act. An element $a \in A$ is called a torsion if there exist $r, s \in S$ where $r \neq s$ such that $ra = sa$. Otherwise, a is non-torsion element.

Proposition 2.5.
Every cyclic S-act generate by non-torsion element is a cancellation S-act.

Proof: Suppose that $a \in A$ is non-torsion element and C, D are ideals of S, where $C<a> = D<a>$. Then $ca \in D<a>$ for all $c \in C$ thus there exists $d \in D$ such that $ca = da$. Since a is non-torsion $c = d$. Therefore, $C \subseteq D$. In the same way, $D \subseteq C$. Thus $C = D$.

The following example, shows that the condition of generated M by non-torsion element is necessary: Let $\mathbb{Z}_2$ be $\mathbb{Z}$-act. Then it's clear that $\mathbb{Z}_2 = (\overline{1})$. Now, $(2)\mathbb{Z}_2 = 0$ and $(0)\mathbb{Z}_2 = 0$. Thus $(2)\mathbb{Z}_2 = (0)\mathbb{Z}_2$ but $(2) \neq (0)$.

The following Proposition gives an equivalent of cancellation property:

Theorem 2.6.
Let S be a semigroup. Then the following are equivalent for an S-act A.

i. A is cancellation S-act.
ii. If C and D are ideals of S such that $CA \subseteq DA$, then $C \subseteq D$.
iii. If $s \in S$ and D is ideal of S such that $(s)A \subseteq DA$, then $s \in D$.
iv. Let C be an ideal of S. Then $[CA:A] = C$.

Proof: (i) $\Rightarrow$ (ii) Let $CA \subseteq DA$. Then $DA = CA \cup DA = (C \cup D)A$ and by hypothesis, $D = C \cup D$. Thus $C \subseteq D$.

(ii)⇒(iii) Suppose $s \in$ S and D is ideal of S where (s)A⊆DA. Then (s)⊆D, and hence $s \in$ D.

(iii) ⇒(iv) Let $x \in$ [CA: A]. Then x A⊆ CA thus x ∈C. Conversely, let $y \in$ C then y A⊆CA. So $y \in$ [CA: A]. Hence, [CA:A] = C.

(iv)⇒(i) Suppose CA = DA where C, D are ideals of S. Then CA⊆ DA and hence C⊆ [DA:A] = D. In the same way D⊆C. Thus, C = D.

Proposition 2.7.

Let A be a S-act over a semigroup S. Then for all ideals C and D of S, [C:D] = [CA:DA] if and only if A is a cancellation.

Proof: (⇒) Let $s \in$ [C:D] then s D ⊆ C. Thus s DA ⊆ CA. So, $s \in$ [CA:DA]. Conversely, let $t \in$ [CA:DA] and hence t DA⊆CA. By Theorem 2.6, t D ⊆ C. Then $t \in$ [C:D].

(⇐) Assume that DA ⊆ CA whenever C, D are ideals of a semigroup S. Then [CA: DA] = S and hence [C:D] = S, thus D ⊆ C. By Theorem 2.6, A is a cancellation S-act.

Proposition 2.8.

Every free S-act is a cancellation.

Proof: Suppose A is a free S-act with basis $\{x_1, x_2, x_3, \ldots\}$ and C,D be ideals of S where CA ⊆ DA. Let $c \in$ C. Then $cx_1 \in$ DA. Hence $cx_1 = \cup_{i=1}^{n} d_i x_i$. Thus, $d_i = 0$ for all $i \neq 1$, Says that $c \in$ D. So C D. By Theorem 2.6, M is a cancellation S-act.

If M is a cancellation S-act, then a homomorphic image of M need not be a cancellation S-act. For example, if $\mathbb{Z}$ is the set of integer numbers and $\mathbb{Z}_2$ is the integers mod 2, define a mapping $f: \mathbb{Z} \to \mathbb{Z}_2$ by $f(n) = [n]$. It is obvious, f is a homomorphism of $\mathbb{Z}$ onto $\mathbb{Z}_2$, then $\mathbb{Z}$ is a cancellation $\mathbb{Z}$-act but $\mathbb{Z}_2$ is not a cancellation S-act.

Lemma 2.9.

If $\{B_i\}_{i \in I}$ is a collection of ideals of a semigroup S and A be a S-act. Then $\underset{i \in I}{\cup} (B_i A) = (\underset{i \in I}{\cup} B_i)A$.

Proof: Clear.

If $\{f_i: A \to B;$ A, B are S-acts $\}$ is a collection of homomorphisms, then $\mathscr{F} = \cup_{i \in I} f_i$ (A), is a notation it's used throughout this work, where $\mathscr{F}$ is an S-subact of B. If B=S, then the symbol $\mathscr{F}_S$ is used instead of $\mathscr{F}$.

The following Theorem proves that the inverse image of a cancellation S-acts is a cancellation.

Theorem 2.10.

Let A, B be S-acts and $\mathscr{F}$ be a cancellation S-subact of B. Then A is a cancellation.

Proof: If C, D are ideals of a semigroup S such that CA = DA, then f_i (CA) = f_i (DA) and hence $\underset{i \in I}{\cup} f_i$(CA) = $\underset{i \in I}{\cup} f_i$(DA). But $f_i($ CA) = $Cf_i($ A) and f_i (DA) = Df_i (A). Thus, $C[\underset{i \in I}{\cup} f_i$ (A)] = $\underset{i \in I}{\cup} f_i$(CA) = $\underset{i \in I}{\cup} f_i$(DA) = D $[\underset{i \in I}{\cup} f_i$ (A)]. So $C\mathscr{F} = D\mathscr{F}$. Hence C = D.

Corollary 2.11.

If A is S-act and $\mathscr{F}_S$ is a cancellation ideal of a semigroup S. Then A is cancelation.

Corollary 2.12.

Let B, A be S-acts where B is a homomorphic image of A. If B is a cancellation. Then A is cancellation.

Proposition 2.13.

If ρ is a congruence on S-act A, then A is cancellation if and only if $A/_\rho$ is cancellation S-act.

Proof: (⇒) Let C and D be ideals of a semigroup S, such that C $\left[A/_\rho \right]$ = D[$A/_\rho$]. Let $c \in$ C. Then c $[a]_\rho \in$ C[$A/_\rho$]. So there is $d \in$ D and $[a']_\rho \in [A/_\rho]$ such that c $[a]_\rho = d$ $[a']_\rho$. Hence $[ca]_\rho = [da']_\rho$. Thus $ca = da' \in$ DA. By Theorem 2.6, $c \in$ D .Then C ⊆ D. Similarly, D⊆ C. Therefore, $A/_\rho$ is a cancellation.

(⇐) Since $A/_\rho$ cancellation S-act and it is homomorphic image of A then by Corollary 2, A is cancellation.

Now, another type of cancellation properties was introduced, it's called weak cancellation property that it's satisfy all cyclic act.

Definition 2.14.

Let A be an S-act. Then A is said to be a weak cancellation whenever if C, D are ideals of S such that CA = DA, then C ∪ ann(A) = D∪ ann(A).

Example 2.15.

Let S = A = { ∅,{x},{y},{x,y}}. Define a mapping: S ×A →A by (C, D) ↦ C∩D. Then A is S-act. Hence, A is weak cancellation.

Remark 2.16.

i. It's clear that cancellation S-act is weak cancellation. But the following example proves that the converse is not hold:

Let S = A = { a, b , c, d}. Then A is a S-act by the mapping: S × A → A which defined by the following table:

.	a	b	c	d
a	a	a	a	a
b	a	a	a	b
c	a	a	b	a
d	a	a	c	d

Here, $k_1 = \{a\}$, $K_2 = \{a,b\}$, $K_3 = \{a, c\}$ $K_4 = \{a, b, c\}$, $K_5 = \{a, c, d\}$ and $K_6 =$ S are the ideals of S. Here, A is weak cancellation S-act. On other hand, $K_2 M = K_4 M$ but $K_2 \neq K_4$. Thus, M is not cancellation S-act.

ii. An S-act. A is a cancelation if and only $i f$ it is weak cancelation and faithful.

By specific condition the following Proposition shows that every weak cancellation S-act is cancellation.

Proposition 2.17.

Let A be a weak cancellation S−act and ann(A) ⊆ K for all ideal K of a semigroup S. Then M is cancellation S−act.

Proof: Clear.

Proposition 2.18.

Every cyclic S-act generated non-torsion element is weak cancellation.

Proof: It's analogous to the proof of Proposition 2.5.

Now, in the following Theorem a characterization of weak cancellation S-acts was given.

Theorem 2.19.

Let A be a S-act and S be a semigroup. Then the following are equivalent.

i. A is a weak cancellation S-act.
ii. If C, D are ideals of S with CA⊆DA, then C⊆ D ∪ ann(A).
iii. If $d \in$S and D is ideal of S such that (*d*)A⊆DA, then $d \in$ D ∪ ann(A).
iv. If C is ideal of S and ann(A) ⊆ C then [CA:A] = C.

Proof: (i) ⇒ (ii) Let C, D be ideals of a semigroup S such that CA ⊆ DA. Then DA = CA ∪ DA = (C ∪ D)A. By hypothesis D∪ ann(A) = (C ∪ D)∪ ann(A). Thus C D ∪ ann(A).

(ii) ⇒ (iii) Let (*d*)A⊆DA. Then (*d*) ⊆ D ∪ ann(A) and hence $d \in$ D ∪ ann(A).

(iii) ⇒ (iv) Let $x \in$ [CA:A]. Then x A ⊆ CA. By hypothesis, $x \in$ C∪ ann(A). Thus $x \in$ C. Conversely, let $y \in$ C then y A ⊆ CA. Hence, $y \in$[CA:A].

(iv) ⇒ (i) Suppose CA = DA where C, D are ideals of a semigroup S. Then CA⊆DA and hence C⊆ [DA:A] = D. Hence, C∪ann(A)⊆ D∪ann(A). Similarly, D∪ ann(A)⊆C∪ann(A). Thus, C∪ann(A) = D∪ ann (A).

Proposition 2.20.

Let S be a semigroup. An S-act A is a weak cancelation if and only if [C:D] = [CA:DA] for all ideals C and D of semigroup S, such that ann(A) ⊆ C.

Proof: (⇒) Let $x \in$[C:D] then x D ⊆C and hence x DA⊆CA. So $x \in$[CA:DA]. Conversely, let $y \in$ [CA:DA] then y DA ⊆CA. By Theorem 2.19, y D ⊆ C ∪ ann(A) = C. Hence $y \in$ [C : D].

(⇐) Suppose that DA ⊆ CA. Then [CA: DA] = S and hence [C:D] = S implies D ⊆C. Thus, D ⊆ C ∪ ann(A). By Theorem 2.19, A is a weak cancellation S-act.

The example below shows that the homomorphic image of a weak cancellation S-act not necessary a weak cancellation S-act.

Example 2.21.

If A = S = ℤ and B = {1, 2}, then A is a S-act by the usual multiplication of integer numbers and B is an S-act by a mapping: $S \times B \rightarrow$ B written by (s, *b*) ↦ 2 for all s ∈ S, $b \in$ B. It's clear that A is a* weak cancellation. For *S*-act B, if C = *uS* and D = *vS* are ideals of S where $u \neq v \in S$ such that CB = DB = {2}. But C ≠ D thus B is not cancellation S-act. On other hand, define a mapping f: A → B by:

$$f(a) = \begin{cases} 1 & \text{if } a \text{ is odd} \\ 2 & \text{if } a \text{ is even} \end{cases}$$

Here, f is a homomophic image of A. Therefore, A is a weak*cancelation. But B is not a weak cancelation S-act (note that B is faithful).

A similar result to Theorem 2.10, will be shown in the next Theorem for the property of weak cancellation.

Theorem 2.22.

Let A and B be S-acts. If $\mathscr{F}$ is a weak cancelation S-subact of B and ann($\mathscr{F}$) = ann(A), then A is weak cancelation S-act.

Proof: Let CA = DA such that C and D are ideals of S. Then $\underset{i \in I}{\cup} f_i(CA) = \underset{i \in I}{\cup} f_i(DA)$. But C[$\underset{i \in I}{\cup} f_i$ (A)] = D [$\underset{i \in I}{\cup} f_i$ (A)]. So, C$\mathscr{F}$ = D$\mathscr{F}$. Since $\mathscr{F}$ is cancellation. Then C∪ann($\mathscr{F}$) = D∪ann($\mathscr{F}$) and hence, C∪ ann(A) = D∪ann(A).

Corollary 2.23.

Let A be a S-act and $\mathscr{F}_S$ be a weak cancellation ideal of a semigroup S, where ann($\mathscr{F}_S$) = ann(A). Then A is weak cancelation S-act.

Corollary 2.24.

Let A, B be a S-acts, where B is a homomorphic image of A. If B is a weak cancellation S-act and ann(B) = ann(A). Then, A is a weak cancellation S-act.

Proposition 2.25.

Let ρ be a congruence on S-act A. Then A is weak cancelation if and only if $A/_\rho$ is weak cancelation S-act.

Proof: Clear.

Theorem 2.26.

Let S be a semigroup and A be a S-act with $\overline{S} = S/\rho$ where ρ is a congruence on S. If A is a cancelation $\overline{S}$-act, then A is a weak cancelation S-act. The convers is true if A is faithful.

Proof: Suppose that KA = DA such that K and D are ideals of S. Let $k \in$K and $a', a \in$A then $ka = da'$ where $d \in$D. Then $[k]a_\rho = [d]_\rho a'$. Let π_ρ:S →$\overline{S}$ be a canonical mapping such that, π(D) = $\overline{D}$ ⇒π(D)A = $\overline{D}$ A. Thus $[d]_\rho$A = $\overline{D}$ A and hence $[k]_\rho$ A = $\overline{D}$A. By Theorem 2.6, $[k]_\rho \in \overline{D}$ then there is $d' \in \overline{D}$ such that $[k]_\rho = \overline{\text{d}} = [d']_\rho$.

Thus $k = d' \in D$. So KD. Similarly, $D \subseteq K$ and hence D = K. Therefore, K $\cup$ ann(A) = D $\cup$ ann(A).

Conversely, suppose A is a weak cancelation S-act and B* and* C are ideals of a semigroup $\overline{S}$, where BA=CA. Then B$=w/\rho w$ and C = L$/\rho_L$ such that W and L are ideals of S. Let $w \in W$ then $[w]_{\rho W} \in$ B. Thus there exists $l \in L$ and $m', m \in A$ where $[w]_{\rho W} m = [l]_{\rho_L} m' \Rightarrow [wm]_{\rho W} = [lm']_{\rho_L} \Rightarrow wm = lm'$ which implies that $wm \in$ LA. By Theorem 2.19, $w \in$ L $\cup$ ann(A). By faithfulness of A, $w \in$ L. Then W$\subseteq$L. Similarly, L$\subseteq$W. Hence W = L. Therefore, $W/\rho w = L/\rho L$ and hence B$=$C. Thus A is cancellation $\overline{S}$-act.

Now, our attention turn to the concept of cancellation S-acts in a multiplication S-acts. Before this the following notion was needed in our work:

For S-act A, define τ (A) $= \underset{a \in A}{\cup} [Sa : A]$. It's clear that τ (A) is ideal of a semigroup S. In case of A is ideal of a semigroup S, then $A \subseteq \tau$ (A).

The following Proposition was introduced to be useful in studying of cancellation (weak cancellation) S-acts.

Proposition 2.27.

If S is a commutative semigroup and A is a multiplication S-act, then $A = \tau$ (A)A. More generally for all S-subact B of A; $B = \tau$ (A)B.

Proof: Suppose $a \in$ A, Then S $a = [Sa : A]$A. Thus, $A = \underset{a \in A}{\cup} Sa = \underset{a \in A}{\cup} ([Sa : A] A) = (\underset{a \in A}{\cup} [Sa : A])A = \tau$ (A) A. Let B be a S-subact of A. Then there exists an ideal K of a semigroup S such that B=KA. Hence, B=KA= K(τ (A) A)= (τ (A)K)A = τ (A)B.

Proposition 2.28.

Let S be a monoid and A be a multiplication S-act. If A is a cancellation, then A is finitely generated.

Proof: By hypothesis, A $= \tau$ (A)A. Thus S $= \tau$ (A) and hence there exists $a_i \in$ A, $i = 1, \ldots, n$ such that $1 \in \cup_{i=1}^{n} [Sa_i : A]$. Then S$= \cup_{i=1}^{n} [Sa_i : A]$ implies that A = SA $= \cup_{i=1}^{n} [Sa_i : A]$A $= \cup_{i=1}^{n} ([Sm_i : A]A) \subseteq \cup_{i="1}^{n} Sa_i \subseteq$A. Therefore, $\cup_{i=1}^{n} Sa_i$ = A.

Proposition 2.29.

Let S be an monoid and A be a multiplication S-act. If A is a weak cancellation, then τ (A) is finitely generated.

Proof: Assume that A is a multiplication S-act, then A $= \tau$ (A)A, and hence τ (A) $\cup$ ann(A) = S $\cup$ ann(A). Since ann(A)$\subseteq \tau$(A) implies that τ (A) = S. Thus, M is finitely generated.

Proposition 2.30.

Let S be a commutative monoid and B be a weak cancellation S-subact of multiplication S-act A with ann(A) = ann(B). Then A is weak cancellation.

Proof: Assume that A is a multiplication S-act. Then B=KA for some ideal K of a semigroup S. Now, let CA = DA for ideals C, D of a semigroup S.

Then, C(KA) = D(KA). Thus CB = DB. So, C $\cup$ann(B) = D $\cup$ ann(B). Therefore, C $\cup$ann(A) = D $\cup$ ann(A). Moreover, A is finitely generated (Proposition 2.29).

Corollary 2.31.

If S is a commutative monoid, and B is a cancellation S-subact of a multiplication S-act A, then A is cancellation.

Proof: Since B is a cancellation S-subact of A. Thus, B is a weak cancellation and ann(B) = 0 (By Remark 2.16 (ii)). Then, ann(B) = ann(A) = 0 and by Proposition 2.30, A is a weak cancellation and ann(A) = 0. Hence, A is cancellation.

Proposition 2.32.

Let S be a monoid and A be a multiplication S-act. If A is weak cancellation, then it is finitely generated.

Proof: Clear.

Proposition 2.33.

Let K be ideal of a semigroup S and B be a S-subact of a multiplication S-act A. If A is a cancellation, then K[B:A] = [KB:A].

Proof: $s \in$ K[B:A], then $s = kr$ such that $k \in$ K and $r \in$ [B:A]. Thus r A $\subseteq$ B $\Rightarrow$ K(r A) $\subseteq$ KB $\Rightarrow s$ A$=$ (kr) A $\in$ KB $\Rightarrow s \in$ [KB:A]. Conversely, $t \in$ [KB: A] $\Rightarrow t$ A $\subseteq$[KB: A]A $\subseteq$ KB$=$K[B:A]A. Hence, [KB: A]A $\subseteq$ K [B: A] A. Since A is a cancellation, [KB: A]$\subseteq$ K[B: A]. Therefore, K[B:A] = [KB:A].

3. Conclusions

The main idea is to introduce the concept of a cancellation acts. Thus some results, properties and Theorems were discussed. Many of characterizations about these concept and its properties was given. Using the concept of multiplication acts various results of cancellation (weak cancellation) acts are investigated. The relation between cancellation acts and weak cancellation acts was discussed. In addition, the faithfulness concept of acts it's important for some useful results in this work.

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