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Ridge-Type Estimator for the Poisson-inverse Gaussian regression model

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Abstract: In Poisson-Inverse Gaussian regression analysis, multicollinearity is a frequent problem that frequently leads to unstable parameter estimations, which can lead to erroneous conclusions and predictions. Despite being a widely used technique for estimating model parameters, the maximum likelihood estimator (MLE) has several well-known drawbacks. Consequently, the Poisson-Inverse Gaussian regression model's multicollinearity is frequently resolved by using ridge-type regression estimators. This paper proposes a two-parameter ridge-type estimator and establishes its statistical features through theoretical calculations and Monte Carlo simulation experiments. The recently suggested estimator was also contrasted with other estimators that have been published in the literature. The results show that the suggested estimator performs better than various other existing estimators using the mean square criterion, even when applied to real data sets.

Keywords: Multicollinearity; Poisson-Inverse Gaussian regression model; Monte Carlo simulation; Ridge -type estimator.

مقدر الحرف لانموذج انحدار بواسون الغاوسي المعكوس

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المستخلص: في تحليل أنموذج الانحدار بواسون الغاوسي المعكوس، تُعد مشكلة التعدد الخطي (Multicollinearity) من المشكلات الشائعة التي غالبًا ما تؤدي إلى تقديرات غير مستقرة لمعاملات الانموذج، مما قد ينتج عنه استنتاجات وتنبؤات غير دقيقة. وعلى الرغم من أن طريقة تقدير الإمكان الأعظم (MLE) تُعد تقنية مستخدمة على نطاق واسع لتقدير معلمات الانموذج، إلا أن لها العديد من العيوب المعروفة. ونتيجة لذلك، غالبًا ما يتم التغلب على مشكلة التعدد الخطي في نموذج انحدار بواسون الغاوسي المعكوس من خلال استخدام مقدّرات انحدار الحرف. يقترح هذا البحث مقدرًا جديدًا من مقدر الحرف على معاملين (two-parameter ridge-type estimator)، كما يتم توضيح خصائصه الإحصائية من خلال التحليل النظري والتجارب العددية باستخدام المحاكاة (Monte Carlo). وقد تم كذلك مقارنة هذا المقدر المقترح مع مقدّرات أخرى سبق نشرها في البحوث الاحصائية. وتُظهر النتائج أن المقدر المقترح يقدم أداءً أفضل من العديد من المقدّرات الموجودة سابقًا، وفقًا لمعيار متوسط مربعات الخطأ (Mean Square Error)، كذلك ايضا في حال تطبيقه على بيانات حقيقية.

الكلمات المفتاحية: التعدد الخطي، نموذج الانحدار بواسون الغاوسي المعكوس، المحاكاة بطريقة مونت كارلو، مقدر الحرف.

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Introduction

Regression models that do not have a normal response variable distribution cannot be fitted using the linear regression model (LRM). "In these situations, the data collection is modeled using generalized linear models (GLMs). When modeling data with response variables that have a distribution different than the normal distribution, GLMs work well [1]. The Poisson-Inverse Gaussian (PIG) distribution is a type of distribution that is utilized for positively skewed continuous response variables. The Poisson-Inverse Gaussian regression model (PIGR) is favored above the Poisson regression model in cases where the response variable is highly skewed [2,5]. Industrial engineering, life testing, dependability, marketing, and the social sciences have all made extensive use of the PIGR model [3,6,9].
The approach is also well-received to model count data from the social sciences, marketing, finance, and many others. PIGR errors raise the flexibility in the model for variance modeling compared to those from zero-inflated or other zero-related specifications. For data analysis in such fields, mimeographed lecture notes without any further references are not reasonable for a scientific community. Moreover, basic assumptions and reasoning leading to a PIG regression model should be discussed rather than to be hidden within an asymptotic argument. Therefore, this paper consolidates the basic concepts leading to the PIGR.
The PIGR is a well-received and widely studied approach to model count data. The conventional logarithmic link function names PIG regression the Poisson log-linear model, leading the Neyman-Pearson settings of a likelihood ratio goodness of fit test, Wald test, and score tests as well to only the Poisson model. It covers, therefore, the quasi-Poisson regression based on Tweedie's formula for the variance, and numerous extensions for two-part, zero-inflated/deflated models and specifications for multiple correspondence analysis. Originally, this approach focused on claims data from automobile insurances, where the variance of counts grew cubic in mean and a direct application of the Poisson model is fail, leading also to historically referred log-linear regression. This cubic variance assumption and the given condition for failure of Poisson model are typical problems for the PIGR.
The regressors are assumed to be uncorrelated in the PIGR. However, multicollinearity is a concern since this assumption is frequently incorrect. Since the PIG distribution belongs to the exponential family, the model denoted as (PIGR) is a specific type of generalized linear model (GLM). One popular technique for estimating the unknown coefficients in the PIGR model is the maximum likelihood (ML) estimation method. However, the ML estimation may suffer if the independent variables are excessively collinear.
Multicollinearity can lead to a number of issues while estimating the (PIGR) model. A frequently used method to obtain the unknown parameters in an PIGR model is the maximum likelihood (ML) estimate method. However, the estimation of ML may be biased if the independent variables involve high collinearity .A multicollinearity problem can lead to several problems when (PIGR) model is being estimated. The problem with it is that if the estimated coefficients are with the wrong sign, there could be misinterpretations due to the imagined relationship between the dependent variable and the independent variables. Another issue is that the ML estimator's confidence interval might expand, which could therefore reduce the accuracy of the estimates. Also, prediction’s accuracy would deteriorate when the value of square mean error ($\overline{MSE}$) rise [1,10,11].
Ridge regression approach was one of the solutions developed to be applied against multicollinearity problem, which was brought to light by [12] . This method has proven to be very useful in many regression applications. [13] proposed the Ridge estimator (RE) for the (PIGR) model, which has the advantage of producing a smaller $\overline{MSE}$ value compared to the maximum

likelihood (ML) estimator. On the other hand, the RE factor also offers some drawbacks. For example, there is no such improvement in the quality of fitting as in the case of plain linear regression which means that results from the RJL model are also similar to those from the extended GLM [14] extended the Ridge and Liu estimators to the Poisson-Inverse Gaussian regression model (PIGRM), which is a special case of GLMs. [15] . Furthermore, [16] introduced a first-order two-parameter estimator for (GLMs). These studies aimed to enhance the precision and reliability of parameter estimates in regression models impacted by multicollinearity. In response to the Ridge estimator proposal for the (PIGR), numerous studies have been carried out to enhance this approach. [17] suggested biased parameter estimators for the Ridge estimator (RE) in the PIGR model, while [18] presented the Liu estimator and a two-parameter estimator defined by [19] for the PIGR model. Additionally, [20] introduced a new shrinkage parameter for the PIGR.

Due to the limitations of REas highlighted in [21] and [22], a more advanced version known as the two-parameter Ridge estimator (TPRE) has been proposed. The TPRE surpasses both the RE and ML estimators, offering various advantages such as orthogonality between residuals and predicted values on the dependent variable. The additional parameter in the TPRE enhances the regression model's overall fit quality. In a Linear Regression Model (LRM) context, [23] conducted a comparison between the TPRE, OLS estimator, and RE using the matrix mean square error (MMSE) criteria. Within the (GLMs) framework, [24] applied the TPRE in a binary logistic regression model.

Overall, researchers have proposed various methods to address the issue of multicollinearity in regression models, including the Ridge regression method and various extensions and modifications of this method. These methods can improve the accuracy and precision of the parameter estimates, making them more reliable for use in real-world applications

The Ridge regression method, first introduced by [12], has been shown to be a reliable and viable alternative to the maximum likelihood (ML) estimation method in many regression applications".

In classical LRM, in general, it is possible to use the following equation:

$$y = Z\beta + \varepsilon \qquad (1)$$

Here, y is an $n \times 1$ vector of notes of the realistic response variable, $Z = (Z_1, \ldots, Z_p)$ is an $n \times p$ design of variables, unidentified regression coefficients $\beta = (\beta_1, \ldots, \beta_p)$ have a vector $p \times 1$, and $\boldsymbol{\varepsilon}$ is random error with an $n \times 1$.

The RE model approach adds a penalty term that reduces the estimates of the regression coefficients towards zero, and its goal is to minimize the sum of squared errors between the actual and predicted values. A tuning parameter controls this penalty term, k, which determines the strength of the shrinkage [10,25].

All things considered, the Ridge regression approach has shown to be a useful tool for regression analysis, especially when multicollinearity is present in the data or when the ML is not suitable. Generally, the RE is

$$\hat{\beta}_{Ridge} = (\mathrm{Z}'\mathrm{Z} + kI)^{-1} XZ'y,$$
$$= (\Lambda + kI)^{-1}\mathrm{Z}'y, \qquad (2)$$

Here, $\Lambda = Z'Z$ is the design matrix and $k \geq 0$ [12].

[26] proposed an modified two-parameter estimator similar to those proposed by [27], [28] and [29]. It is defined as

$$\hat{\beta}_{TP} = (\Lambda + kdI)^{-1} Z' y \qquad (3)$$

1- Ridge estimator in PIGR

In response to challenges posed by multicollinearity, Segerstedt proposed the use of ridge estimator (RE) and as in [33],[30], [10], and [12]. Building upon this work, Batool et al. [13] introduced the RE to tackle multicollinearity in the PIGRM. This method, known as the Poisson-Inverse Gaussian Ridge Regression Estimator (PIGRRE), is defined as follows:

$\hat{\beta}_{PIGRRE} = \left(Z'\hat{V}Z + k_T I\right)^{-1} \; Z' \; \hat{V}Z\hat{\beta}_{MLE}$, (4)

where $k_r > 0$. Then,

$\mathrm{Bias}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right) = E\left(\hat{\beta}_{\mathrm{PIGRRE}}\right) - \beta = -\mathrm{K}_r\Lambda_K^{-1}\beta$ (5)

$$\mathrm{Cov}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right) = \mathrm{E}\left(\left[\hat{\beta}_{\mathrm{PIGRRE}} - \mathrm{E}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right)\right]\left[\hat{\beta}_{\mathrm{PIGRRE}} - E\left(\hat{\beta}_{\mathrm{PIGRRE}}\right)\right]'\right) = \widehat{\emptyset}\left(\mathrm{Q}\Lambda_k^{-1}\Lambda_k\Lambda_k^{-1}\mathrm{Q}'\right) \quad (6)$$

and

$$\mathrm{MMSE}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right) = \mathrm{Cov}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right) + \mathrm{Bias}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right)\mathrm{Bias}\left(\hat{\beta}_{\mathrm{PIGRRE}}\right)' = \widehat{\emptyset}\left(Q\Lambda_k^{-1}\Lambda_k\Lambda_k^{-1}Q'\right) + b_{PIGRRE}b'_{PIGRRE} \quad (7)$$

where $b_{\mathrm{PIGRRE}} = \mathrm{Bias}(\hat{\beta}_{\mathrm{PIGRRE}}) = -k_r Q\Lambda^{-1}\alpha$

SMSE($\hat{\beta}$PIGRRE) = trMMSE($\hat{\beta}$PIGRRE)

$$\widehat{\emptyset} = \sum_{j=1}^{p}\frac{\lambda_j}{\left(\lambda_j + k_r\right)^2} + \sum_{j=1}^{p}\frac{k_r^2\alpha_j^2}{\left(\lambda_j + k_r\right)^2} \quad (8)$$

where $\alpha = \mathrm{Q}'\beta_{\mathrm{MLE}}$.

2- Poisson-Inverse Gaussian Liu estimator

Liu [32] proposed the Liu estimator (LE) as a substitute for the RE. [5], [6], [14] defined the LE for the PIGRM as follows:

$\hat{\beta}_d = (Z'Z + I)^{-1}(Z'Z + d_I I)\hat{\beta}$ (9)

With these properties, the LE presents itself as a promising substitute for the RE. In the context of the PIGRM, the LE is defined as:

$\hat{\beta}_{\mathrm{PIGLE}} = (Z'V\,Z\hat{}\& + I)^{-1}\left(Z'V\,\hat{Z} + d_I I\right)\hat{\beta}_{MLE}$ (10)

where $0<d_l<1$. Then,

$\mathrm{Bias}\left(\hat{\beta}_{\mathrm{PIGLE}}\right) = E\left(\hat{\beta}_{\mathrm{PIGLE}}\right) - \beta = (d_I - 1)\Lambda_I^{-1}\beta$ (11)

$\mathrm{Cov}(\hat{\beta}_{\mathrm{PIGLE}}) = \left(\left[\hat{\beta}_{\mathrm{PIGLE}} - E\left(\hat{\beta}_{\mathrm{PIGLE}}\right)\right]\left[\hat{\beta}_{\mathrm{PIGLE}} - E\left(\hat{\beta}_{\mathrm{PIGLE}}\right)\right]' = \widehat{\emptyset}(\mathrm{Q}\Lambda_I^{-1}\Lambda_d\Lambda^{-1}\;\Lambda_d\Lambda_I^{-1}\mathrm{Q}')$
(12)

$\mathrm{MMSE}\left(\hat{\beta}_{\mathrm{PIGLE}}\right) = \mathrm{Cov}\left(\hat{\beta}_{\mathrm{PIGLE}}\right) + \mathrm{Bias}\left(\hat{\beta}_{\mathrm{PIGLE}}\right)\mathrm{Bias}\left(\hat{\beta}_{\mathrm{PIGLE}}\right)' = \widehat{\emptyset}(\mathrm{Q}\Lambda_I^{-1}\Lambda_d\Lambda^{-1}\Lambda_d\Lambda_I^{-1}\mathrm{Q}') + b_{\mathrm{PIGRRE}}b'_{\mathrm{PIGRRE}}$ (13)

where$b$PIGLE $= \mathrm{Bias}(\hat{\beta}_{\mathrm{PIGLE}}$ and $\Lambda_d = \mathrm{diag}(\lambda 1 + d_I, \lambda 2 + dl, \ldots, \lambda p + d_I)$. Then, the SMSE of PIGLE is

SMSE($\hat{\beta}$PIGLE) = trMMSE($\hat{\beta}$PIGLE)

$$= \widehat{\emptyset}\sum_{j=1}^{p}\frac{\left(\lambda_j + d_I\right)^2}{\lambda_j\left(\lambda_j + 1\right)^2} + \sum_{j=1}^{p}\frac{(d_I - 1)^2)\alpha_j^2}{\left(\lambda_j + 1\right)^2} \quad (14)$$

The dl can be as:

$$d_I = \frac{\sum_{j=1}^{p} \frac{\alpha_j - \hat{\emptyset}}{(\lambda_j + 1)^2}}{\sum_{j=1}^{p} \frac{\hat{\emptyset} + \lambda_j \alpha_j}{\lambda_j (\lambda_j + 1)^2}} \qquad (15)$$

3- The proposed estimator

In linear regression, it is well known that the consequences of multicollinearity are that it reduces the efficiency of the least squares estimator and causes its covariance matrix to be poorly conditioned. The problem of multicollinearity arises when we have two or more explanatory variables that are linear combinations of each other because such explanatory variables do not carry separate information for estimating. When the problem is suspected, it is necessary to estimate the parameters of a linear regression model with a penalty function after an adequate transformation of the observable variable. This penalty function will allow us to reduce the undesirable consequences of multicollinearity, hence it can be fitted to the parameter's vector with the desired properties If the assumption of equality of the mean and variance is violated, then it is likely that there will be a bias, in which case the model error is not white, while the inference on the exposure effect can still be consistent due to its robustness against a misspecified model error distribution. Nonetheless, the standard estimator of the exposure parameter may suffer from biased inference, which makes its application problematic.

Ridge-type methods allow us to look at these problems from a different angle. In a classic sense, the Efron-Peterson's method makes it possible by adding a ridge to the data in the regression equation. There is also a class of kernel-type ridge estimators that can be defined by smoothing the ridge function. However, the proper choice of that ridge function is not at all straightforward and Efron-Petrosian's individual versus global aspects problem remains unsolved here. It is difficult to make a formal general treatment of these methods from a stochastic process point of view. Besides, some straightforward manipulations on these types of ridge estimators might not be as simple or as successful as those using a data ridge. Since both methods are based on the second-order properties of the data, they may not work as well as the kernel-based estimators do at most points if we have enough data to look into in the neighborhood of that point. Nevertheless, it is meaningful to investigate the possibility of merging these two types of estimators, taking advantage of their different strengths, if possible.

The PIGR model provides an accurate alternative to the Poisson regression model for response data that are overdispersed with respect to the relationship to an exposure variable of scientific interest. The proposed estimator in this research is based on the works of [4], [11], and [12]. While the k and d in Equation (3) have a multiplicative effect, the proposed method in this research examines their additive effect. The proposed estimator is defined as follows:

$$\begin{aligned} \hat{\beta}_{NBTPE}(k,d) &= (Z'\widehat{W}X + (k+d)I)^{-1} Z'y \lim_{\delta x \to 0} \\ &= (\Lambda + (k+d)I)^{-1} \Lambda \hat{\beta}_{NBML} \\ &= T_k \hat{\beta}_{NBML} \end{aligned} \qquad (16)$$

where $T_k = (\Lambda + (k+d)I)^{-1}\Lambda$, $k<0$, $0<d<1$. For $\hat{\beta}_{NBTPE}(0,0)$, $\hat{\beta}_{NBTPE}(k,d) = \hat{\beta}_{NBML}$ and for $\hat{\beta}_{NBTPE}(0,1)$, $\hat{\beta}_{NBTPE}(k,d) = \hat{\beta}_{NBRE}$

4- Properties of the New Two-Parameter Estimator

$$E(\hat{\beta}_{NBTPE}(k,d)) = T_k \beta \qquad (17)$$

Except in the case where k and d equal zero, the proposed two-parameter estimator is a biased estimator.

$$Bais(\hat{\beta}_{NBTPE}(k,d)) = E\left(\hat{\beta}_{NBTPE}(k,d)\right) - \beta = (T_k - I)\beta \qquad (18)$$

Then,

$$Var(\hat{\beta}_{NBTPE}(k,d)) = E\left[\left(\hat{\beta}_{NBTPE}(k,d) - E(\beta)\right)\left(\hat{\beta}_{NBTPE}(k,d) - E(\beta)\right)'\right]$$
$$= \sigma^2 T_k \Lambda^{-1} T_k' \quad (19)$$

The MSEM of the proposed estimator is

$$MSEM(\hat{\beta}_{NBTPE}(k,d)) = \sigma^2 T_k \Lambda^{-1} T_k' + (T_k - I)\beta\beta'(T_k - I)' \quad (20)$$

2nd: Simulation research

This part investigates the efficiency of our suggested estimator under different multicollinearity levels by means of Monte Carlo simulation using the sample data.

1- Simulation design

This section examines the performance of the new estimator under different levels of multicollinearity through a Monte Carlo simulation experiment. The response variable of observations is generated from a PIGR model as:

$PIJ(\mu_i, \mu_i + \theta\mu_i^2)$ with $\mu_i = exp(Z_i^T\beta)$. Here, $\beta = (\beta_0, \beta_1, ..., \beta_p)$ with $\sum_{j=1}^{p}\beta_j^2 = 1$ and $\beta_1 = \beta_2 = ... = \beta_p$ [32]. The explanatory variables $Z_i' = (Z_{i1}, Z_{i2}, ..., Z_{in})$ have been generated as

$$Z_{ij} = (1-\rho^2)^{1l2} w_{ij} + \rho w_{ip}, \quad i = 1,2,...,n, \quad j = 1,2,...,p, \quad (21)$$

where ρ represents the correlation and w_{ij} 's are independent standard normal pseudo-random numbers. n=50, 100, and 150 are taken into consideration. Furthermore, because adding more explanatory factors can raise the $\overline{MSE}$, p=4 and p=8 are taken into consideration as the number of explanatory variables. Furthermore, $\rho = \{0.90, 0.95, 0.99\}$. Three different values of the dispersion parameter are considered $\hat{\emptyset}$ = 1.4, 3, 5. The optimum value of k can be obtained by using Hoerl, Kannard [33] formula as

$$\hat{k} = \frac{P_{\hat{\emptyset}}}{\hat{\alpha}^T\hat{\alpha}} \quad (22)$$

The produced data is 1000 times repeated for various permutations of n, σ^2, p and ρ the average absolute bias and $\overline{MSE}$ are determined as

$$MSE(\hat{\beta}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\beta}-\beta)'(\hat{\beta}-\beta). \qquad \hat{k} = \frac{P_{\hat{\emptyset}}}{\alpha T\alpha} \quad (23)$$

2- Simulation results

Tables 1 and 2 present the $\overline{MSE}$ for all combinations of n, σ^2, p and ρ. Bold text highlights the average bias and $\overline{MSE}$ best values. Table 1 illustrates that the suggested IGTPE approach has less bias than ML and PIGRE. In terms of $\overline{MSE}$, Table 2 shows that PIGTPE performs better than ML and PIGRR. However, out of PIGRR and PIGTPE, the IGML estimator performs the worst and is highly impacted by multicollinearity.

Regarding the effect of the correlation degree ρ on $\overline{MSE}$, there is an increase in its values when the degree of correlation increases, regardless of the values of n, and p. Furthermore, the $\overline{MSE}$ decreases when the number of explanatory variables is increased from four to eight. Regarding sample size n, the $\overline{MSE}$ values decrease as n increases", regardless of the values of n, and p.

Table 1: $\overline{MSE}$values for the three estimators at p=4 and $\widehat{\emptyset}$ = 1.4

n	ρ	ML	PIGRE	PIGTPE
50	0.90	3.928	3.687	3.234
	0.95	3.972	3.737	3.284
	0.99	4.238	4.003	3.55
100	0.90	3.68	3.445	2.992
	0.95	3.73	3.495	3.042
	0.99	3.996	3.761	3.308
150	0.90	3.629	3.394	2.941
	0.95	3.679	3.444	2.992
	0.99	3.945	3.71	3.257

Table 2: $\overline{MSE}$values for the three estimators at p=8 and $\widehat{\emptyset}$ = 1.4

n	ρ	ML	PIGRE	PIGTPE
50	0.90	4.65	4.415	3.962
	0.95	4.699	4.464	4.011
	0.99	4.966	4.731	4.278
100	0.90	4.408	4.173	3.72
	0.95	4.458	4.222	3.769
	0.99	4.724	4.489	4.036
150	0.90	4.357	4.122	3.669
	0.95	4.406	4.171	3.718
	0.99	4.673	4.438	3.985

Table 3: $\overline{MSE}$values for the three estimators at p=4 and $\widehat{\emptyset}$ = 3

n	ρ	ML	PIGRE	PIGTPE
50	0.90	5.526	5.285	4.832
	0.95	5.57	5.335	4.882
	0.99	5.836	5.601	5.148
100	0.90	5.278	5.043	4.59
	0.95	5.328	5.093	4.64
	0.99	5.594	5.359	4.906
150	0.90	5.227	4.992	4.539
	0.95	5.277	5.042	4.59
	0.99	5.543	5.308	4.855

Table 4: $\overline{MSE}$values for the three estimators at p=8 and $\widehat{\emptyset}$ = 3

n	ρ	ML	PIGRE	PIGTPE
50	0.90	5.536	5.301	4.848
	0.95	5.585	5.35	4.897
	0.99	5.852	5.617	5.164
100	0.90	5.294	5.059	4.606
	0.95	5.344	5.108	4.655
	0.99	5.61	5.375	4.922
150	0.90	5.243	5.008	4.555
	0.95	5.292	5.057	4.604
	0.99	5.559	5.324	4.871

Table 5: $\overline{MSE}$ values for the three estimators at p=4 and $\hat{\emptyset} = 5$

n	ρ	ML	PIGRE	PIGTPE
50	0.90	4.814	4.573	4.12
	0.95	4.858	4.623	4.17
	0.99	5.124	4.889	4.436
100	0.90	4.566	4.331	3.878
	0.95	4.616	4.381	3.928
	0.99	4.882	4.647	4.194
150	0.90	4.515	4.28	3.827
	0.95	4.565	4.33	3.878
	0.99	4.831	4.596	4.143

Table 6: $\overline{MSE}$ values for the three estimators at p=8 and $\hat{\emptyset} = 5$

n	ρ	ML	PIGRE	PIGTPE
50	0.90	6.248	6.013	5.56
	0.95	6.297	6.062	5.609
	0.99	6.564	6.329	5.876
100	0.90	6.006	5.771	5.318
	0.95	6.056	5.82	5.367
	0.99	6.322	6.087	5.634
150	0.90	5.955	5.72	5.267
	0.95	6.004	5.769	5.316
	0.99	6.271	6.036	5.583

3rd: Conclusions

The research suggests a new estimator that uses a Poisson-Inverse Gaussian regression model to generalize the two parameters estimator. Studies using Monte Carlo simulations show that this estimator significantly reduces mean squared error ($\overline{MSE}$) when compared to alternative estimators. Moreover, compared to the IGML and PIGRE estimators, the suggested estimator reduces $\overline{MSE}$ more effectively in a practical application. Further studies can be conducted in the future to propose other estimators.

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