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REVIEW

Volatility Modelling in GARCH Frameworks: A Comparative Analysis of Non-Gaussian Error Distributions With Skewed Parameters

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Abstract

Forecasting volatility in financial time series remains challenging due to their asymmetric nature and excess kurtosis. This study evaluates and compares the performance of four variant of GARCH models incorporating skewed non-Gaussian error innovation distribution. The performances of these GARCH family of models under the skewed error innovation distributions were evaluated for three different unique data sets to have a more robust assessment of the performance of these skewed error innovation distributions. This study leverage on daily closing prices of Bitcoin, Naira to Dollar Exchange rates and daily Nigeria All Share Index between January 1, 2015 and January 26, 2024. Model fit was assessed using Log-likelihood and Akaike Information Criterion (AIC). Forecasting accuracy was evaluated with Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE). The results confirmed the stationarity of returns and presence of ARCH effects at $p < .05$ validating the use of these volatility models. The skewed parameters in most models were significant, justifying the use of skewed innovation densities. Out-of-sample forecast showed that the skewed student-t distribution consistently outperformed other skewed innovations. Model performance varied by asset since GJR-GARCH(1,1) with Skewed Student-t was optimal for all share index based on MSE = 6.7355, RMSE = 2.5953, MAE = 1.7536. EGARCH(1,1)-sstd having MSE = 1.5576, RMSE = 1.2481, MAE = 0.8506 for USD-Naira and GARCH(1,1)-sged with MSE = 0.00009, RMSE = 0.00928, MAE = 0.00804 for Bitcoin. This study therefore signified the superiority of the skewed Student-t distribution in most of the cases considered. These findings offer valuable insights for investors on when and how to invest their assets.

Keywords: GARCH, Volatility, Assets, Skewed innovation density, Financial time series

1. Introduction

One of the main assumptions of the Ordinary Least Square (OLS) in regression analysis is homoscedasticity. However, this assumption breaks down when dealing with financial time series, such as stock prices, exchange rates, cryptocurrency, and other financial time series. To address this issue, models robust against homoscedasticity were proposed; these models are commonly referred to as heteroscedasticity models. Engle made the first attempt at this problem with the Autoregressive Conditional Heteroscedasticity (ARCH) model [1]. Other extensions of these models have been proposed as a result of the shortcomings of the ARCH models, which include, among other things, the model parsimony issue and the inability to incorporate volatility clustering. These include: Exponential Generalized Autoregressive Conditional Heteroscedasticity model (EGARCH), Asymmetric Power ARCH (APARCH), Threshold GARCH (TGARCH), Integrated GARCH (IGARCH), Threshold ARCH (TARCH), GJR-GARCH which is proposed by Ref. [2] Glosten, Jagannathan & Runkle among several other models.

Notwithstanding the evolution of several variations of these volatility models, a crucial factor remains the robustness of the error innovation

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employed in determining their parameters [3]. Consequently, attempts have been undertaken to create an error innovation distribution that may reflect the dynamics of financial time series volatility. Other forms of error innovation distribution with fatter tails, like the Student t-distribution and the generalized error distribution were proposed [4] in response to the limitations of using the normal distribution in volatility modeling, which include its inability to capture some asymmetric behavior of financial assets. The normal distribution was the first of its kind. Due to some of the stylized facts about financial time series which include excess kurtosis, a major factor responsible for its asymmetric behaviour, the skewed form of normal, Student and generalized error distribution were developed.

2. Review of empirical studies

A study by Ref. [5] carried a critical analysis of the risk and return of four mostly traded, cryptocurrencies and found a strong spillover effect among different cryptocurrencies with Bitcoin and Ether, being the top two cryptocurrencies with the highest market capitalisation. The impact of news in predicting returns volatility was investigated by Ref. [6] while [7] investigated the relationship between volatilities of cryptocurrencies and other financial assets. Similarly [8], employed GARCH models to estimate the volatility of Bitcoin, Ethereum and Ripple. Similarly [9], analysed the relationship between the implied volatility of both United States and European financial markets as measured by the VIX and VSTOXX and price volatility of a broad range of cryptocurrencies while [10] analysed the volatility in cryptocurrencies. A study was conducted by Ref. [11] about the performance of simple GARCH model with application to four Bangladeshi Companies on Dhaka Stock Exchange (DSE). The volatility in returns of Nigeria stock exchange was investigated by Ref. [12] and it was established that the GJR-GARCH (1,1) with generalized error distribution (GED) show volatility persistence, fat tail distribution, and leverage effect. Other studies have also been carried out on volatility models using GARCH family of models using either exchange rate data, cryptocurrency or stock data [5,11,13–22].

While these previous have made reasonable efforts in modelling the volatility in financial time series, only very few of them considered skewed error innovation distributions [15] and even the one that considered skewed distribution only applied it to a single data set. Also, considering the present realities in the crypto market, the floating of the Naira policy by the present administration in Nigeria, the dynamics in this market might have changed and hence a new modelling of volatility is very critical with recent data. Due to the fact that volatility modelling is data driven, it is believed that using different unique data will provide a more holistic assessment of the performance of these skewed error innovation distributions within GARCH Family of models. Therefore, this study compared the performance of GARCH family of models under different skewed error innovation distributions with different financial time series data (Bitcoin, Nigeria stock market and Naira to USD exchange rate).

3. Methodology

3.1. Source of data

The data used in this study comprised of daily closing prices of Bitcoin, Naira to USD Exchange rate and Nigeria Stock All share index between 1/01/2015 and 25/02/2024. Bitcoin data was obtained Yahoo finance website (www.yahoofinance.com), Naira to USD data was obtained from the Central Bank of Nigeria while data on All Share Index were obtained from investment.com (https://ng.investing.com).

3.2. Daily return series generation based on pricing

Daily returns series for each of the three cryptocurrencies was generated from daily closing price series using the formula below:

$$R_t = \ln\left(\frac{DCP_t}{DCP_{t-1}}\right), t = 2, \ldots, n-1 \quad (1)$$

where, DCP_t is the daily closing price at the present day while DCP_{t-1} is the previous closing price.

3.3. Normality of return series

The Jacque Bera test was employed to determine whether the daily return series of a certain cryptocurrencies was normal. Jacque Bera test is given as:

$$JcB = \frac{n}{6}\left[k^2 + \frac{(\lambda-3)^2}{4}\right] \quad (2)$$

Where, k is the skewness λ is the kurtosis and n is the number of observation. The test statistics is approximately χ^2_2 and the null hypothesis is rejected if the probability value is less than 0.05

3.4. Stationarity test for daily return series

The following hypotheses were tested for stationarity in the daily return series using the Augmented Dickey Fuller (ADF) test:

$H_0 : \xi = 1$

$H_0 : \xi < 1$

The test statistic,

$$t-\text{ratio} = \frac{\sum_{r=2}^{n} DCP_{t-1} e_t}{\hat{\sigma}^2 \sqrt{\sum_{r=2}^{n} DCP_{t-1}^2}} \quad (3)$$

where,

$$\hat{\xi} = \frac{\sum_{r=2}^{n} DCP_{t-1} p_t}{\sum_{r=2}^{n} DCP_{t-1}^2} \quad (4)$$

and,

$$\hat{\sigma}^2 = \frac{\sum_{r=2}^{n} (DCP_t - \hat{\xi} DCP_{t-1})^2}{n-1} \quad (5)$$

If the probability value is less than 0.05 (p < .05), the null hypothesis is rejected. Here, n is the sample size, which is the number of observations for returns and DCP_t is the closing price at day t. Similarly, DCP_{t-1} is the closing price at the day t-1.

3.5. Test for the presence heteroscedasticity

To test for ARCH effect, the Lagrange Multiplier test was performed. The Lagrange Multiplier test expressed the error terms as a linear combination of the prior error terms. The expression for this test is:

$$\epsilon_t^2 = \beta_0 + \beta_1 \epsilon_{t-1}^2 + \ldots + \beta_p \epsilon_{t-p}^2, t = \text{n} + 1 \ldots \text{N} \quad (6)$$

where, ϵ_t^2, ϵ_{t-1}^2, …, ϵ_{t-p}^2 are the error terms, $\beta_0, \beta_1, \ldots, \beta_p$ are the coefficients of the regression equation in equation (6) while ϵ_t is the error term in equation (6), n is a positive integer, and N is the sample size.

Here, are the null and alternative hypotheses:

$H_0 : \beta_1 = \ldots = \beta_p = 0$ versus

$H_1 : \beta_i \neq 0$ for some $i \in \{1, .., p\}$

The test statistic is given as:

$$F = \frac{(SSR_0 - SSR_1)/p}{SSR_1(T - 2p - 1)} \quad (7)$$

where,

$SSR_1 = \sum_{t=n+1}^{N} \epsilon_t^2$, where $\hat{\epsilon}_t$ is the sum of square error in equation (6).

$$SSR_0 = \sum_{t=n+1}^{N} (\epsilon_t^2 - \varpi)^2,$$

where SSR_0 is the sum of square total.

$$\varpi = \frac{1}{N} \sum_{t=n+1}^{N} \epsilon_t^2$$

The distribution of the test statistic is assumed to follow χ^2 distribution. The decision rule is to reject Ho: if $F > \chi_m^2(\alpha)$ and N is the number of observation.

3.6. Skewed error innovation used in volatility modeling

In volatility modeling, the following error innovation distributions are frequently employed:

3.6.1. Normal distribution/Gaussian distribution

$$f(w_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{w_t}{2}} \quad -\infty < z_t < \infty \quad (8)$$

3.6.2. Skewed normal distribution

$$f(w_t) = \frac{1}{\psi\pi} e^{-\frac{(w_t - \epsilon)^2}{2\psi^2}} \int_{-\infty}^{\alpha} \frac{w_t - \epsilon}{\psi} e^{-\frac{t^2}{2}} dw_t, -\infty < z_t < \infty \quad (9)$$

where, ϵ ψ and α are the location, scale and shape parameters respectively.

3.6.3. Student t-distribution

$$f(w_t) = \frac{\Gamma\left(\frac{\phi+1}{2}\right)}{\sqrt{\phi\pi\Gamma(\phi/2)}} \left(1 + \frac{w_t^2}{\phi}\right)^{-\left(\frac{\phi+1}{2}\right)}, -\infty < z_t < \infty \quad (10)$$

3.6.4. Standardized skewed student t-distribution

$$f(w_t,\mu,\sigma,\phi,\lambda) = \begin{cases} bc\left[1+\frac{1}{\phi-2}\left(\frac{b\left(\frac{\eta_t-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right]^{-\frac{\rho+1}{2}}, w_t < -\frac{a}{b} \\ bc\left[1+\frac{1}{\phi-2}\left(\frac{b\left(\frac{\eta_t-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right]^{-\frac{\rho+1}{2}}, w_t \geq -\frac{a}{b} \end{cases} \tag{11}$$

3.6.5. Generalized error distribution (GED)

$$f(w_t,\mu,\sigma,\rho) = \frac{\sigma^{-1}\rho e^{\left[-0.5\left|\left(\frac{w_t-\mu}{\sigma}\right)\lambda\right|^{\rho}\right]}}{\lambda 2^{\left[1+\left(\frac{1}{\rho}\right)\right]}\Gamma\left(\frac{1}{\rho}\right)} - \infty < \eta_t < \infty \tag{12}$$

$\rho > 0$ is tail thickness parameter and

$$\lambda = \sqrt{2^{\left(-\frac{2}{\rho}\right)\Gamma\left(\frac{1}{\rho}\right)} \Big/ \Gamma\left(\frac{3}{\rho}\right)} \tag{13}$$

3.6.6. Skewed generalized error distribution

$$f(w_t/\rho,\epsilon,\theta,\delta) = \frac{\rho}{2\theta\Gamma\left(\frac{1}{\rho}\right)}\exp\left[-\frac{|w_t-\delta|^{\rho}}{[1+sign(w_t-\delta)\epsilon]^{\rho}\theta^{\rho}}\right]$$

$$\theta > 0, -\infty < w_t < \infty, \rho > 0, -1 < \epsilon < 1 - \infty < z_t < \infty \tag{14}$$

where,

$$\theta = \Gamma\left(\frac{1}{\rho}\right)^{0.5}\Gamma\left(\frac{3}{V}\right)^{-0.5}S(\epsilon)^{-1}$$

3.7. GARCH type models considered in the study

The volatility models taken into account in this study are volatility models with constant mean which are defined as follows:

3.7.1. Generalized autoregressive conditional heteroscedasticity (GARCH) model

$$R_t = \mu + \epsilon_t, \quad {\pi_t}^2 = \omega + \sum_{j=1}^{p}\beta_j\pi_{t-j}^2 + \sum_{i=1}^{q}\alpha_i\epsilon_{t-i}^2. \quad \epsilon_t = \pi_t w_t \tag{15}$$

Where, ω, β_j, $\alpha_i \geq 0$ are the parameters of the volatility model to be estimated. Also, π_t is the volatility, R_t is the returns while ϵ_t the error term.

3.7.2. Glosten, Jagannathan and Runkle generalized autoregressive conditional heteroscedasticity (GJR-GARCH) model

$${\pi_t}^2 = \omega + \sum_{i=1}^{p}\left(\alpha_i\epsilon_{t-1}^2 + \gamma_1 I_{t-i}\epsilon_{t-1}^2\right) + \sum_{j=1}^{q}\beta_j\pi_{t-i}^2. \ \epsilon_t = \eta_t z_t \tag{16}$$

$$I_{t-i} = \begin{cases} 1, \epsilon_{t-i} < 0 \\ 0, \epsilon_{t-i} \geq 0 \end{cases}$$

ω is constant term, α_i is ARCH term while β_j is the GARCH term, γ_1 is the leverage term, $\omega \geq 0$, α_i and $\beta_j \geq 0$ and π_t measures the volatility.

3.7.3. Exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model

$$R_t = \mu + \epsilon_t, \ln\left(\pi_t^2\right) = \omega + \sum_{i=1}^{p}\alpha_i\left[\lambda\epsilon_{t-i} + \gamma\left\{\left|\epsilon_{t-i}\right| - \sqrt{\frac{2}{\pi}}\right\}\right] + \sum_{j}\beta_j\ln\left(\pi_{t-j}^2\right) \tag{17}$$

where, ω is constant term, α_i is ARCH term while β_j is the GARCH term and, γ is the leverage term and π_t is the volatility.

3.7.4. Integrated generalized autoregressive conditional heteroscedasticity (IGARCH) model

$$R_t = \mu + \epsilon_t, {\pi_t}^2 = \omega + \sum_{j=1}^{q}\beta_j\pi_{t-j}^2 + \sum_{i=1}^{p}\alpha_i\epsilon_{t-i}^2. \ \epsilon_t = \pi_t z_t \tag{18}$$

where, ω is constant term, α_i is ARCH term while β_j is the GARCH term and $\sum_{j=1}^{p}\alpha_j + \sum_{i=1}^{q}\beta_j = 1$.

3.8. Model fitness test

The logarithmic likelihood function and the Akaike Information Criteria (AIC), which are defined as follows, will be used to evaluate the fitness of the GARCH model and its extension, which were estimated using the six innovation distributions and the suggested innovation distribution.

$$AIC = -\frac{2\log(LL) + 2\beta}{n} \tag{19}$$

where, LL is the likelihood of the model and β is the number of parameters in the model and n is the number of observation.

3.9. Forecasting accuracy of the model

Forecasting accuracy metrics like Means Square Error (MSE) and Root Mean Square Error (RMSE) will be used to assess the forecasting accuracy of the GARCH model and its extension estimated using any of the distribution for error innovation explored in this work as posited by Olatayo and Taiwo [26].

$$MSE = \frac{1}{n-1}\sum_{t=2}^{r}\left(\widehat{\pi}_t - \pi_t\right)^2 \tag{20}$$

$$MSE = \sqrt{\frac{1}{n-1}\sum_{t=2}^{r}\left(\widehat{\pi}_t - \pi_t\right)^2} \tag{21}$$

Where, n is the number of observation and $\widehat{\pi}_t$ and π_t^2 are the estimated and actual volatility. When estimating the parameters of GARCH models and its extension, the distribution with the lowest RMSE will be declared the optimal distribution for error innovation among the competing distribution.

4. Results and discussion of the finding

Table 1 shows the result of the descriptive analysis of returns of the selected volatility data. Results revealed that the three selected time series and the result shows that Stock market and Bitcoin were negative skewed with the skewness of −0.112063 and −0.791352 respectively. For USD Dollar to Naira exchange rate, it was skewed to the right (Skewness = 0.118422). The daily returns of these financial time series were found not to be normally distributed ($p < 0.05$).

Result obtained from Augmented Dickey Fuller (ADF) shows that p-values obtained were all less than 0.05 ($p < 0.05$) which indicated no evidence of non-stationarity in these returns series. The ARCH test indicate the presence of heteroscedasticity since all the p-value are less than 0.05 ($p < 0.05$). Hence, the study suggests the use of volatility models in its estimation (see Table 2).

Result in Tables 3–5 present the summary of the comparison of both the fitness and forecasting performances of skewed error innovation distribution within GARCH family of models. Result in Table 3 reveals that among the competing models, the IGARCH (1,1)-snorm was found to outperformed others in terms of fitness performance (LL = −3883.490, AIC = 2.6108) while GARCH (1,1)-sstd (LL = 27540.71, AIC = -11.769) and EGARCH(1,1)-sstd (LL = 9754.865, AIC = −5.8864). The result of the forecasting performances favoured GJR-GARCH (1,1) with skewed Student t distribution for Nigeria All Share Index (MSE = 6.7355, RMSE = 2.5953, MAE = 1.7536), EGARCH(1,1)-sstd (MSE = 1.5576, RMSE = 1.2481, MAE = 0.8506) for USD-Naira exchange rate and GARCH(1,1)-sged for Bitcoin (MSE = 0.00009, RMSE = 0.00928, MAE = 0.00804). Result in Table 6 reveals that in all data sets, the skewness parameter is significant

Table 1. Summary descriptive statistics for the returns of Stock (exchange), Dollar-Naira (exchange rate) and Bitcoin.

Data sets	N	Mean	Max.	Min.	SD	Skewness	Kurtosis	Jacque Bera	p-value
Stock Exchange	3008	0.0172	4.2291	−5.7695	0.9820	−0.1121	7.6227	2684.6090	0.0000
Dollar-Naira	4709	0.0232	100.2785	−100.3325	2.2587	0.118422	1663.7680	5.41×10^8	0.0000
Bitcoin	3312	0.0006	0.0978	−0.2018	0.0162	−0.7914	14.5764	18839.4000	0.0000

n = number of observations.

Table 2. Augmented Dickey Fuller (ADF) test result summary and test of heteroscedasticity.

Data	ADF Test Statistic	Probability values	Comment	ARCH test F-stat.	p-value
Stock Exchange	−50.82751	0.0001[a]	Stationary without differencing	242.3469	0.0000[a]
Naira-Dollar	−40.85351	0.0000[a]	Stationary without differencing	1563.548	0.0000[a]
Bitcoin	−58.8118	0.0001[a]	Stationary without differencing	56.21621	0.0000[a]

[a] Significant at 1 % ($p < .01$).

Table 3. Comparative analyses of the performances of skewed error innovation distribution within GARCH family of models for Nigeria All Share Index data.

Models	SEID	Fitness performance		Forecasting performance			Model diagnostic checking
		LL	AIC	MSE	RMSE	MAE	P-Value of ARCH Test
GARCH (1,1)	Snorm	−3864.984	2.5990	8.4594	2.9085	2.5456	0.3155
	Sstd	−3413.571	2.2966	7.0698	2.6589	1.8859	0.2541
	Sged	−3548.543	2.3872	10.9136	3.3036	2.1677	0.2506
GJR- GARCH (1,1)	Snorm	−3864.967	2.5997	7.3288	2.7072	2.2019	0.3176
	Sstd	−3413.409	2.2971	**6.7355**	**2.5953**	**1.7536**	0.2482
	Sged	−3548.312	2.3877	23.4697	4.8446	4.5362	0.2552
EGARCH (1,1)	Snorm	−3865.586	2.6001	12.4321	3.5259	2.4558	0.5725
	Sstd	−3539.604	2.3819	11.9115	3.4513	2.1894	0.3242
	Sged	−3404.742	2.2913	42.9573	6.5542	6.0689	0.2528
IGARCH (1,1)	Snorm	**−3883.490**	**2.6108**	19.0411	4.3636	3.8871	0.1204
	Sstd	−3413.507	2.2958	8.0346	2.8345	2.5456	0.2491
	Sged	−3548.495	2.3865	24.1359	4.9128	4.6328	0.2493

The bolded values are the least values of MSE, RMSE and MAE.

Table 4. Comparative analyses of the performances of skewed error innovation distribution within GARCH family of models for USD to Naira exchange rate.

Models	SEID	Fitness performance		Forecasting performance			Model diagnostic checking
		LL	AIC	MSE	RMSE	MAE	P-Value of ARCH Test
GARCH (1,1)	Snorm	8231.475	−3.5163	1.6358	1.2790	0.8030	0.9999
	Sstd	**27540.71**	**−11.769**	2.1662	1.4718	1.2270	0.9999
	Sged	24140.91	−10.789	2.2167	1.4889	1.2493	0.9999
GJR- GARCH (1,1)	Snorm	7718.518	−3.2967	2.1701	1.4731	1.2293	0.9990
	Sstd	26525.27	−11.335	1.6356	1.2789	0.8029	0.20217
	Sged	265052.19	−11.722	2.1823	1.4773	1.2350	0.65312
EGARCH (1,1)	Snorm	7028.922	−3.0019	1.6245	1.2746	0.8067	0.9999
	Sstd	23242.14	−9.9317	**1.5576**	**1.2481**	**0.8506**	0.9999
	Sged	13130.58	−5.6096	1.6043	1.2666	0.9035	0.9999
IGARCH (1,1)	Snorm	8039.079	−3.4328	1.6511	1.2850	0.8634	0.9999
	Sstd	17579.28	−7.5120	2.1809	1.4768	1.2332	0.9999
	Sged	17570.22	−7.5010	2.2219	1.4906	1.2511	0.9999

The bolded values are the least values of MSE, RMSE and MAE.

Table 5. Comparative analyses of the performances of skewed error innovation distribution within GARCH family of models for Bitcoin.

Models	SEID	Fitness performance		Forecasting performance			Model diagnostic checking
		LL	AIC	MSE	RMSE	MAE	P-Value of ARCH Test
GARCH (1,1)	Snorm	9259.080	−5.5882	0.000133	0.01154	0.01077	0.5662
	Sstd	9733.984	−5.8744	0.000130	0.01140	0.01050	0.6765
	Sged	9723.934	−5.8644	**0.00009**	**0.00928**	**0.00804**	0.6988
GJR- GARCH (1,1)	Snorm	9262.117	−5.5894	0.00012	0.01085	0.00996	0.5342
	Sstd	9727.548	−5.8699	0.00009	0.00956	0.00842	0.7243
	Sged	9735.078	−5.8744	0.000101	0.010087	0.00907	0.7097
EGARCH (1,1)	Snorm	9273.594	−5.5964	0.00012	0.01115	0.00996	0.6848
	Sstd	**9754.865**	**−5.8864**	0.000101	0.01007	0.009031	0.4904
	Sged	9746.626	−5.8814	0.00011	0.010320	0.00936	0.7354
IGARCH (1,1)	Snorm	9249.650	−5.5831	0.00017	0.01293	0.01252	0.5684
	Sstd	9734.016	−5.8750	0.000102	0.01011	0.00909	0.6784
	Sged	9724.404	−5.8692	0.00011	0.01069	0.009439	0.7017

The bolded values are the least values of MSE, RMSE and MAE.

($p < 0.05$) which justified the use of skewed distributions in modelling volatility dynamics of these financial time series. The leverage effect was not significant in Nigeria All Share index ($\gamma_1 = -0.049331$, $p = .4981$, $p > 0.05$) while for Naira- USD, the leverage effect was positive and significant ($\gamma_1 = 0.000013$, $p = .000$, $p < 0.05$). This implies that for Naira- USD exchange rate, the impact of negative returns on future volatility is greater than that of positive returns. In all sets of data, the GARCH terms were positive and significant

Table 6. Summary of optimal GARCH family of models for the three set of data.

Data set	Best model	ω (p-value)	α_1 (p-value)	β_1 (p-value)	γ_1 (p-value)	Skewness	Shape
Nigeria ASI	GJR-GARCH (1,1)-sstd	0.161662** (0.0000)	0.463259** (0.0000)	0.560921** (0.0000)	−0.049331 (0.4981)	1.032820** (0.0000)	2.86442** (0.0000)
Naira-USD	EGARCH (1,1)-sstd	−4.96169** (0.0000)	−0.000012** (0.0000)	0.843414** (0.0000)	0.000013** (0.000000)	2.0079** (0.000000)	2.013240** (0.0000)
Bitcoin	GARCH (1,1)-sged	0.000003 (0.2604)	0.105401** (0.0000)	0.893599** (0.0000)	–	0.995556** (0.0000)	3.202766** (0.000000)

Significant at 1% ($p<0.01$).

indicating that high level of past volatility tends to persist into the future in all these financial time series.

This study has established that the daily returns of these financial time series are not stationary and non-normally distributed. This asymmetric distribution of these financial time series has also been established by other similar studies ([23,24]). The non-normality of the daily returns of Stock exchange is also consistent with that of the study by Ref. [18] while that of Bitcoin is also corroborated by that of the finding by Ref. [15]. The volatility of USD to naira exchange rate which favoured EGARCH model is consistent with that of [25] which favoured asymmetric GARCH. Also, out of the three financial series considered, the skewed Student t distribution outperformed other skewed innovation distribution in two cases (Stock exchange and USD to Naira exchange rate) while skewed generalized error (SGED) outperformed other innovation distribution for Bitcoin. The finding that favoured the SGED for predicting the volatility of Bitcoin is consistent with that of [16] which also favoured the SGED.

5. Discussion

In terms of fitness performance, the study discovered that the Skewed Student-t Distribution (SSTD) performed better than all other distributions in all of the models used, including GARCH (1,1), GJR-GARCH (1,1), IGARCH (1,1), and EGARCH (1,1), with the highest loglikelihood value and the lowest Akaike Information Criterion (AIC) value. The Skewed Generalized Error distribution (SGED) outperformed other models in GARCH (1,1) in the forecasting performance using the estimate of Root Mean Sum of Error (RSME), while SSTD also performed better in three models (GJR-GARCH (1,1), EGARCH (1,1), and IGARCH (1,1)). According to the analysis of the Bitcoin's fitness performance, SSTD is the best. This result is consistent with Atoi's (2014) GARCH model study on the volatility of the Nigerian stock market, which found that the student-t distribution provided better fit and forecasting ability than the normal distribution (NORM) and generalized error distribution (GED). In consideration of Dollar-Naira exchange rate, the Skewed Student-t Distribution (STTD) recorded the lowest value for all four of the GARCH models that were taken into consideration, according to the forecasting performance using the least Root Mean Square Error (RMSE) value. In terms of fitness performance, the Skewed Normal Distribution (SNORM) performed best in EGARCH (1,1) and GJR-GARCH (1,1), while the Skewed Student-t

Distribution (STTD) performed better in GARCH (1,1) and IGARCH (1,1), respectively.

Loglikelihood and Akaike Information Criterion (AIC) method was used to test the fitness performance of the models where SSTD outperformed others in the entire four models considering their highest loglikelihood value and least AIC value.

The better fitness performance on GTB Stock is shared by the skewed student-t distribution (SSTD) and the skewed normal distribution (SNORM). SSTD preferred GJR-GARCH (1,1) and EGARCH (1,1), whereas SNORM performed better with GARCH (1,1) and IGARCH (1,1). The models' fitness performance was evaluated using the Akaike Information Criterion (AIC) method and the Root Mean Square Error (RMSE) with the lowest value. TSSTD perform better in three out of the four model with only Skewed Generalized Error Distribution (SGED) which favoured EGARCH (1,1), considering their highest loglikelihood value and least AIC value.

6. Conclusion

This study has assessed the performance of three skewed error innovation distribution: skewed normal, skewed Student t distribution and skewed generalized distribution in predicting the volatility of three financial time series (Stock Exchange, Bitcoin and USD to Naira exchange rate). The study established that the skewed Student t distribution outperformed other skewed innovation distribution in two cases (Stock Exchange and USD to Naira exchange rate) while the skewed generalized error distribution was found to be best for Bitcoin. Therefore, this study underscored the superiority of the skewed Student distribution over skewed normal and skewed generalized error in predicting the volatility of these financial time series.

Dataset

The study's data set included Bitcoin's each day close rates obtained from Yahoo finance website (www.yahoofinance.com), GTB Stock obtained from investment.com (https://ng.investing.com/equities/GTB-historical-data) and Naira to Dollar Exchange rate obtained from the Central Bank of Nigeria website between 1/01/2015 and 26/01/2024.

Ethics information

The participant personal information is not confidential, consent is given for future participant in research, data should be handling as it is, peer view is allowed and intellectual integrity uphold.

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Conflict of interest

Nil.

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