المجموعات المهيمنة ومتعددات الحدود المهيمنة لبيانات المتحدة ببيان تام من الدرجة

(2) II r

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الخلاصة

إذا كان $G=(V,E)$ بيان بسيط . وكانت D مجموعة جزئية من V فأن D تسمى مجموعة مهيمنة للبيان G، إذا كان كل رأس في $V-D$ يجاور على الأقل رأس واحد في D . وليكن G بيان مكون من بيانين تامين من الدرجتين t,m بوسطة بيان تام من الدرجة r ، لذلك تكون درجة البيان G هي $n=m+t-r$ ولتكن $d(G_n,i)$ هي عدد كل المجموعات المهمنة التي عدد عناصرها i وان متعددة الحدود G_n تكون $D(G_n,x)=\sum_{i=1}^{n} d(G_n,i)x^i$. في هذا البحث وجدنا علاقة لإيجاد $d(G_n,i)$ وبعض الخواص فيها واستعمالها في ايجاد متعددة الحدود المهيمنة $D(G_n,x)=\sum_{i=1}^{n} d(G_n,i)x^i$ للبيان المتكون من اتحاد بيانين تامين وبعض الخواص فيها

Dominating Sets and Domination Polynomial of k_r-gluing of Graphs II

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Abstract

Let $G=(V,E)$ be a simple graph. A set $D\subseteq V$ is a dominating set of G, if every vertex in $V-D$ is adjacent to at least one vertex in D. Let G be k_r-gluing of G_1 and G_2 and denote by $C[G_1 \cup_r G_2]$ the family of all k_r-gluing of G_1 and G_2. Let K_t be complete graph with order t and K_m be complete graph with order m and let G be k_r-gluing of K_t and K_m with order $n=m+t-r$. Let G_n^i be the family of dominating sets of G_n with cardinality i, and let $d(G_n\,,i)=|G_n^i|$. In this paper, we construct G_n^i, and obtain a recursive formula for $d(G_n\,,i)$. Using this recursive formula, we consider the polynomial $D(G_n,x)=\sum_{i=1}^{n} d(G_n\,,i)x_i$, which we call domination polynomial of k_r-gluing of graphs and obtain some properties of this polynomial.

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1 Introduction

Let $G = (V, E)$ be a simple graph of order $|V| = n$. A set $D \subseteq V$ is a dominating set of G, if every vertex in $V - D$ is adjacent to at least one vertex in D. The domination number (G) is the minimum cardinality of a dominating set in G. For a detailed treatment of this parameter, the reader is referred to[11]. It is well known and generally accepted that the problem of determining the dominating sets of an arbitrary graph is a di_cult one (see [6]). Alikhani and Peng found the dominating set and domination polynomial of cycles and certain graph [1],[2]. Alikhani found On the domination polynomials of non P4-free [3]. Dod, Kotek, Preen and Tittmann found Bipartition Polynomials, the Ising Model, and Domination in Graphs [4]. Kahat and Khalaf found the dominating set and domination polynomial of stars, wheels, complete graph with missing and kr-gluing of Graphs see [7], [8], [9], [10]. Kotek, JPreen and Tittmann found Domination Polynomials of Graph Products [12]. graphs Let G_n be graph

with order n and let G_n^i be the family of dominating sets of a graph G_n with cardinality i and let $d(G_n\ , i) = |G_n^i|$. We call the polynomial $D(G_n, x) = \sum_{i=r(G)}^{n} d(G_n\ , i)x_i$, the domination polynomial of graph G [2]. Let G be k_r-gluing of G_n and G_n and denote by $C[G_1 \cup_r G_2]$the family of all k_r-gluing of G_1 and G_2 [5]. Let K_t be complete graph with order t and K_m be complete graph with order m and let G be k_r-gluing of K_t and K_m with order $n = m + t - r$. Let Gi n be the family of dominating sets of a complete graph G_n with cardinality i and let $d(G_n\ , i) = |G_n^i|$. We call the polynomial $D(G_n, x) = \sum_{i=1}^{n} d(G_n\ , i)x_i$, the domination polynomial of G_n graph.

In the next section we construct the families of dominating sets of G_n with cardinality i by the families of dominating sets of G_{n-1} with cardinality i and $i - 1$. We investigate the domination polynomial of G_n graph in Section 3. As usual we use $\binom{n}{i}$for the combination $n\ to\ i$.

2 Dominating sets of k_r-gluing of K_t and K_m

We shall investigate dominating sets of Let G_n be k_r-gluing of K_t and K_m. To prove our main results we need the following lemmas:

Lemma 1 [7].

The following properties hold forall graph G.

(i) $|G_n^n|$ = 1 (ii) $|G_n^{n-}|$ = n (iii) $|G_n^i|$= 0 if $i > n$ (iv) $|G_n^0|$= 0

Theorem 1 [10]
Let G_n be k_r-gluing of K_t and K_m with order $n = m + t - r$, then $d(G_n\,, i)$=$\binom{n}{i} - \binom{m-r}{i} - \binom{t-r}{i}$, $\forall n, m, r, t \in Z^+$, and $i = 1, 2, \dots, n$

Let G_n be k_3-gluing of K_6 and K_m with order $n = m + 3$. Using Theorem 1, we obtain the coeffcients of $D(G_n, x)$ for $6 \le n \le 15$ in Table 1. Let $d(G_n\,, i)$= $|G_n^i|$. There are interesting relationships between the numbers $d(G_n\,, i)$($1 \le i \le n$) in the table.

i			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n	t	m															
6	6	3	6	15	20	15	6	1									
7	6	4	3	18	34	35	21	7	1								
8	6	5	3	24	55	70	56	28	8	1							
9	6	6	3	30	82	126	126	84	36	9	1						
10	6	7	3	36	115	209	252	210	120	45	10	1					
11	6	8	3	42	154	325	461	462	330	165	55	11	1				
12	6	9	3	48	199	480	786	923	792	495	220	66	12	1			
13	6	10	3	54	250	680	1266	1709	1715	1287	715	286	78	13	1		
14	6	11	3	60	307	931	1946	2975	3424	3002	2002	1001	364	91	14	1	
15	6	12	3	66	370	1239	2877	4921	6399	6426	5004	3003	1365	455	105	15	1

Table 1. d(G_n, i) The number of dominating sets of G_n with cardinality i sech that G_n be K_3-gluing of K_6 and K_m

In the following theorem, we obtain some properties of $d(G_n\,, i)$: G_n be k_3-gluing of K_6 and K_m

Theorem 2
The following properties hold of $d(G_n\,, i)$, for every $n \in Z^+$:
(i) $d(G_n, 1) = 3\ \forall n \ge 7$.
(ii) $d(G_n, 2) = d(G_{n-1}, 2) + 6$.
(iii) $d(G_n, 3) = d(G_{n-1}, 3) + d(G_{n-1}, 2) + 3$.
(iv) $d(G_n, 4) = d(G_{n-1}, 4) + d(G_{n-1}, 3) + 1$.
(v) $d(G_n, i) = d(G_{n-1}, i) + d(G_{n-1}, i - 1)\ \forall i \ge 5$.
(vi)$\gamma(G_n) = 1\ \forall n \ge 7$
(vii)$d(G_6, i) = d(K_6, i)$
Proof.
Let G_n be k_3-gluing of K_6 and K_m with order $n = m + 3$, then
(i)By Theorem 1 d(G_n, 1)= $\binom{n}{1} - \binom{m-3}{1} - \binom{3}{1}$, since $n = m + 3$ then $m = n - 3$, therefore $d(G_n, 1)$= $\binom{n}{1} - \binom{n-6}{1} - \binom{3}{1}$= $n - n + 6 - 3 = 3$

(ii) By Theorem 1 $d(G_n,2)=\binom{n}{2}-\binom{n-6}{2}-\binom{3}{2}=\frac{n(n-1)}{2}-\frac{(n-6)(n-7)}{2}-3=6n-24$,
and $d(G_{n-1},2)+6=\binom{n-1}{2}-\binom{n-7}{2}-\binom{3}{2}+6=\frac{(n-1)(n-2)}{2}-\frac{(n-7)(n-8)}{2}-3+6=6n-24$ then $d(G_n,2)=d(G_{n-1},2)+6$.
(iii) By Theorem 1 $d(G_n,3)=\binom{n}{3}-\binom{n-6}{3}-\binom{3}{3}=\frac{n(n-1)(n-2)}{6}-\frac{(n-6)(n-7)(n-8)}{6}-1=3n^2-24n+55$, and $d(G_{n-1},3)+d(G_{n-1},2)+3=\binom{n-1}{3}-\binom{n-7}{3}-\binom{3}{3}+\binom{n-1}{2}-\binom{n-7}{2}-\binom{3}{2}+3=\frac{(n-1)(n-2)(n-3)}{6}-\frac{(n-7)(n-8)(n-9)}{6}-1+\frac{(n-1)(n-2)}{2}-\frac{(n-7)(n-8)}{2}-3+3=3n^2-24n+55$,
then $d(G_n,3)=d(G_{n-1},3)+d(G_{n-1},2)+3$
(iv)By Theorem 1 $(G_n,4)=\binom{n}{4}-\binom{n-6}{4}-\binom{3}{4}=\frac{n(n-1)(n-2)(n-3)}{24}-\frac{(n-6)(n-7)(n-8)(n-9)}{24}-0=n^3-13.5n^2+68.5n-126$,
and $d(G_{n-1},4)+d(G_{n-1},3)+1=\binom{n-1}{4}-\binom{n-7}{4}-\binom{3}{4}+\binom{n-1}{3}-\binom{n-7}{3}-\binom{3}{3}+1=n^3-13.5n^2+68.5n-126$
then $d(G_n,4)=d(G_{n-1},4)+d(G_{n-1},3)+1$.
(v)By Theorem 1we have $d(G_n,i)=\binom{n}{i}-\binom{n-6}{i}=\frac{n}{n-1}\binom{n-1}{i}-\frac{n-6}{n-i-6}\binom{n-7}{i}$
and we have $d(G_{n-1},i)+d(G_{n-1},i-1)=\binom{n-1}{i}-\binom{n-7}{i}+\binom{n-1}{i-1}-\binom{n-7}{i-1}=\binom{n-1}{i}-\binom{n-7}{i}+\frac{i}{n-i}\binom{n-1}{i}-\frac{i}{n-i-6}\binom{n-7}{i}=\left(1+\frac{i}{n-i}\right)\binom{n-1}{i}-\left(1+\frac{i}{n-i-6}\right)\binom{n-7}{i}=\frac{n}{n-1}\binom{n-1}{i}-\frac{n-6}{n-i-6}\binom{n-7}{i}=d(G_n,i)$.
(vi) since $K_3\subseteq K_m$ and $K_3\subseteq K_6$, then $\{v\}$ is dominating set of (G_n) $\forall v\in K_3$, therefor $\gamma(G_n)=1$.
(vii) $d(G_6,i)=\binom{6}{i}-\binom{0}{i}=\binom{6}{i}=d(K_6,i)$ by Lemma 1 (iii)■

Let G_n be k_4-gluing of K_6 and K_m with order $n=m+3$. Using Theorem 1, we obtain the coeffcients of $D(G_n,x)$ for $6\le n\le 15$ in Table 2. Let $d(G_n,i)=|G_n^i|$. There are interesting relationships between the numbers $d(G_n,i)(1\le i\le n)$ in the table.

i			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n	t	m															
6	6	4	6	15	20	15	6	1									
7	6	5	4	20	35	35	21	7	1								
8	6	6	4	26	56	70	56	28	8	1							
9	6	7	4	32	83	126	126	84	36	9	1						
10	6	8	4	38	116	209	252	210	120	45	10	1					
11	6	9	4	44	155	325	461	462	330	165	55	11	1				
12	6	10	4	50	200	480	786	923	792	495	220	66	12	1			
13	6	11	4	56	251	680	1266	1709	1715	1287	715	286	78	13	1		
14	6	12	4	62	308	931	1946	2975	3424	3002	2002	1001	364	91	14	1	
15	6	13	4	68	371	1239	2877	4921	6399	6426	5004	3003	1365	455	105	15	1

Table 2. d(G_n, i) The number of dominating sets of G_n with cardinality i sech that G_n be K_4-gluing of K_6 and K_m

In the following theorem, we obtain some properties of $d(G_n\,, i)$: G_n be k_4-gluing of K_6 and K_m

Theorem 3 The following properties hold of $d(G_n\,, i)$, for every $n \in Z^+$:

(i) $d(G_n, 1) = 4\ \forall n \geq 7$.

(ii) $d(G_n, 2) = d(G_{n-1}, 2) + 6$.

(iii) $d(G_n, 3) = d(G_{n-1}, 3) + d(G_{n-1}, 2) + 1$.

(iv) $d(G_n, i) = d(G_{n-1}, i) + d(G_{n-1}, i-1)\ \forall i \geq 4$.

(v)$\gamma(G_n) = 1\ \forall n \geq 7$

(vi)$d(G_6, i) = d(K_6, i)$

Proof. The proof is similar to the proof of (Theorem 2)■

In the following theorem, we obtain some properties of $d(G_n\,, i)$: G_n be k_r-gluing of K_t and K_m sech that (t) is constant $\forall\ 1 \leq m \leq n-t+r$.

Theorem 4 The following properties hold of $d(G_n\,, i)$, for every $n \in Z^+$:

(i) $d(G_n, 1) = r\ \forall n \geq t$.

(ii) $d(G_n, i) = d(G_{n-1}, i) + d(G_{n-1}, i-1) + \binom{t-r}{i-1}\ \forall n > t$.

(iii) $d(G_n, i) = d(G_{n-1}, i) + d(G_{n-1}, i-1)\ \forall i-1 > t-1$.

(iv) $(G_n, n-1) = n$

(v) $d(G_n, n) = 1$.

(vi)$\gamma(G_n) = 1\ \forall n > t$

(vii)$d(G_n, i) = d(K_n, i) \forall n = t$

Proof. The proof is similar to the proof of (Theorem 2) and (Theorem 3) $\forall\ (i), (iv), (v), (vi) and\ (vii)$

(ii) By Theorem 1we have $d(G_n, i) = \binom{n}{i} - \binom{n-t}{i} - \binom{t-r}{i} = \frac{n}{n-1}\binom{n-1}{i} - \frac{n-t}{n-i-t}\binom{n-t-1}{i} - \binom{t-r}{i}$

and we have $d(G_{n-1},i)+ d(G_{n-1},i-1)+\binom{t-r}{i-1}=\binom{n-1}{i}-\binom{n-t-1}{i}-\binom{t-r}{i}+\binom{n-1}{i-1}-\binom{n-t-1}{i-1}-\binom{t-r}{i-1}+\binom{t-r}{i-1}=\binom{n-1}{i}-\binom{n-t-1}{i}+\frac{i}{n-i}\binom{n-1}{i}-\frac{n-t}{n-i-t}\binom{n-t-1}{i}-\binom{t-r}{i}=\left(1+\frac{i}{n-i}\right)\binom{n-1}{i}-\left(1+\frac{i}{n-i-t}\right)\binom{n-t-1}{i}-\binom{t-r}{i}=\frac{n}{n-1}\binom{n-1}{i}-\frac{n-t}{n-i-t}\binom{n-t-1}{i}-\binom{t-r}{i}=d(G_n,i)$

(iii) We have $i-1>t-1$ then $\binom{t-r}{i-1}=0$ (by Lemma 1), therefore $d(G_n,i)=d(G_{n-1},i)+d(G_{n-1},i-1)$by (ii). ■

3 Domination Polynomial of K_r-gluing of Graphs

In this section we introduce and investigate the domination polynomial of K_r-gluing of K_m and K_t sech that (t) is constant $\forall\ 1 \le m \le n-t+r$.

Defnition.

Let let G_n^i be the family of dominating sets of a graph G_n (K_r-gluing of K_m and K_t) with cardinality i, and let $d(G_n\ ,i)=|G_n^i|$, and since $\gamma(\ G_n)=1$. Then the domination polynomial
$D(G_n,x)$ of G_n is defned as

$$D(G_n,x)=\sum_{i=1}^{n} d(G_n\ ,i)x^i\ \forall\ n\ >\ t$$

In the following corollary, we obtain some properties of $D(G_n,x)$: G_n be K_r-gluing of K_m and K_t sech that (t) is constant $\forall\ 1 \le m \le n-t+r$.

Corollary 1

The following properties hold forall $D(G_n,x)\ \forall\ n\ >\ t$

(i) $D(G_n,x)=\sum_{i=1}^{n}\binom{n}{i}x^i-\sum_{i=1}^{m-r}\binom{m-r}{i}x^i-\sum_{i=1}^{t-r}\binom{t-r}{i}x^i$

(ii) $D(G_n,x)=D(G_{n-1},x)+xD(G_{n-1},x)+\sum_{i=1}^{t-r}\binom{t-r}{i}x^{i+1}$

Proof.

(i) From defnition of the domination polynomial and Theorem 4, we have
$D(G_n,x)=\sum_{i=1}^{n} d(G_n\ ,i)x^i=\sum_{i=1}^{n}\ [\binom{n}{i}-\binom{m-r}{i}-\binom{t-r}{i}]x^i$
$=\sum_{i=1}^{n}\binom{n}{i}x^i-\sum_{i=1}^{n}\binom{m-r}{i}x^i-\sum_{i=1}^{n}\binom{t-r}{i}x^i$
$=\sum_{i=1}^{n}\binom{n}{i}x^i-\sum_{i=1}^{m-r}\binom{m-r}{i}x^i-\sum_{i=1}^{t-r}\binom{t-r}{i}x^i$ (by Lemma1) $\binom{n}{i}=0$ if i > n

(ii) From de_nition of the domination polynomial and Theorem 4, we have
$D(G_n,x)=\sum_{i=1}^{n} d(G_n\ ,i)x^i=\sum_{i=1}^{n}[d(G_{n-1}\ ,i)+d(G_{n-1}\ ,i-1)+\binom{t-r}{i-1}]x^i=\sum_{i=1}^{n} d(G_{n-1}\ ,i)x^i+\sum_{i=1}^{n} d(G_{n-1}\ ,i-1)x^i+\sum_{i=1}^{n}\binom{t-r}{i-1}x^i$, we have $d(G_n\ ,i)=0$ if $i\ >\ n\ or\ i\ =\ 0$ (Lemma1), then
$\sum_{i=1}^{n} d(G_{n-1}\ ,i)x^i=\sum_{i=1}^{n-1} d(G_{n-1}\ ,i)x^i=D(G_{n-1},x)$

and $\sum_{i=1}^{n} d(G_{n-1}\ ,i-1)x^{i} = x\sum_{i=1}^{n} d(G_{n-1}\ ,i-1)x^{i-1} =$ $x[\sum_{i=1}^{n-1} d(G_{n-1}\ ,i)x^{i}]$=$x\ D(G_{n-1},x)$ and $\sum_{i=1}^{n}\binom{t-r}{i-1}x^{i} = \sum_{i=1}^{n}\binom{t-r}{i}x^{i+1}$ then $D(G_n,x)$ = $D(G_{n-1},x)$+ $xD(G_{n-1},x)$+$\sum_{i=1}^{t-r}\binom{t-r}{i}x^{i+1}$.■

Example 1 .

Let G_8 be K_3-gluing of two complete graphs K_5 and K_6 with order 8, we can get on $D(G_8,x)$ without the table. We have

$D(G_8,x)$= $\sum_{i=1}^{8}\binom{8}{i}x^{i} - \sum_{i=1}^{3}\binom{3}{i}x^{i} - \sum_{i=1}^{2}\binom{2}{i}x^{i}$ =$(8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8) - (3x + 3x^3 + x^3) - (2x + x^2) = 3x + 24x^2 + 55x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$

(by Corollary 1).

(see Fig-1).

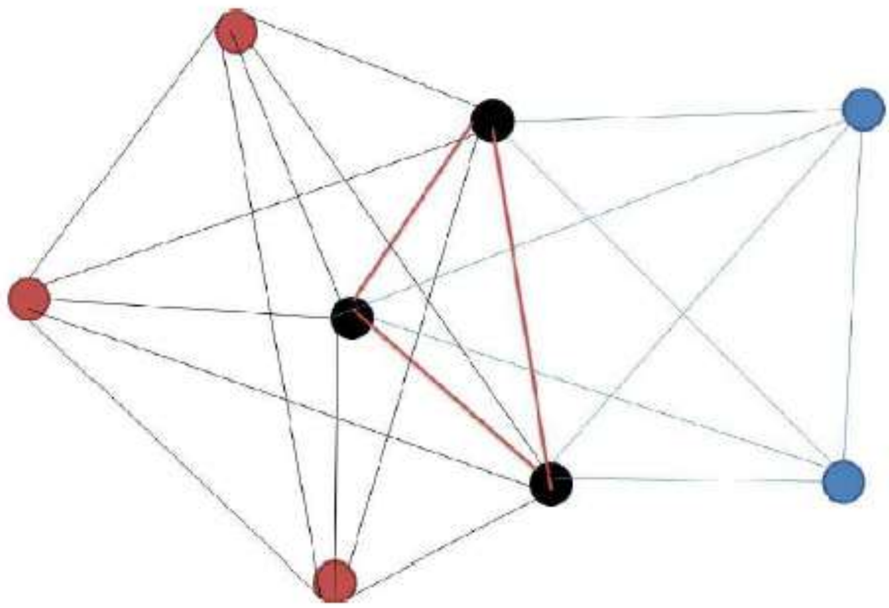

Fig-1:

G_8 be K_3-gluing of two complete graphs K_5 and K_6

References

[1] Alikhani, S.Y. H. Pengb, Dominating Sets and Domination Polynomial of Cycles, Global Journal of Pure and Applied Mathematics, 42: 151-162, (2008) .

[2] Alikhani, S.Y.H. Peng, Dominating Sets and Domination Polynomial of Certain Graphs, II, Opuscula Mathematica 30 (1): 37-51, (2010).

[3] Alikhani, S. ,On the domination polynomials of non P4-free graphs, Iranian Journal of Mathematical Sciences and Informatics Vol. 8, No. 2 (2013), pp 49-55

[4] M. Dod, T. Kotek, J. Preen, P. Tittmann, Bipartition Polynomials, the Ising Model, and Domination in Graphs, Discussiones Mathematicae Graph Theory. Volume 35, Issue 2, Pages 335-353, ISSN (Online) 2083-5892, DOI: 10.7151/dmgt.1808, April (2015).

[5] F.M. Dong, K.M. Koh and K.L. Teo, Chromatic Polynomials and Chromaticity of graphs, Uk offce: 57 Shelton Sueet, Covent London WC2H 9HE, (2005).

[6] M. R. Garey and D. S. Johnson, Computers and Intractability: A Guide to the Theorey of NP-Completness. Freeman, New York, (1979).

[7] S. Sh. Kahat, A. M. Khalaf, R. Hasni, Dominating Sets and Domination Polynomials of Stars, Australian Journal of Basic and Applied Sciences, 8(6), Pages: 383-386, (June 2014).

[8] S. Sh. Kahat, A. M. Khalaf, Dominating Sets and Domination polynomial of Complete Graphs with Missing Edges, Journal of Kufa for Mathematics and Computer Vol.2, No.1, 64-68, (may 2014).

[9] S. Sh. Kahat, A. M. Khalaf, R. Hasni, Dominating Sets and Domination Polynomial of Wheels, Asian Journal of Applied Sciences, Volume 02 - Issue 03, 287-290, June (2014).

[10] S. Sh. Kahat, A. M. Khalaf, Dominating Sets and Domination polynomial of kr-gluing of Graphs, to appear in Journal of Dirasat Tarbawiya, by No. 267 in 24/5/(2015)

[11] T.W. Haynes, S.T. Hedetniemi, P.J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, NewYork, (1998).

[12] T. Kotek, J. Preen, P. Tittmann, Domination Polynomials of Graph Products, arXiv.org (math) arXiv:1305.1475v2, (Submitted on 7 May 2013 (v1), last revised 23 Dec (2013) (this version, v2)).