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حل نظام معادلات فريدهولم الكسرية التكاملية التفاضلية غير الخطية باستخدام طريقة التحليل المجالي وتعديل طريقة التكرار المتغير لـHE

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في هذه الورقة البحثية، نقارن بين طريقة تفكيك المجال A (ADM)، وتعديل طريقة التكرار المتغير لـ He (MVIM)، وذلك للحصول على حل تقريبي لنظام معادلات تكاملية تفاضلية كسرية غير خطية. ونقدم بعض الأمثلة للتحقق من دقة الطرق.تعديل طريقة التكرار المتغير لـ He.

الكلمات المفتاحية: معادلات تكاملية تفاضلية كسرية، مشتقة كابوتو، نظام معادلات تكاملية تفاضلية كسرية غير خطية، طريقة تفكيك المجال A،

Solving System of Nonlinear Fredholm Fractional Integro-Differential Equations by Using A domain decomposition method and the modification of He's Variational iteration method

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Abstract
This work presents a comparison between the A domain decomposition technique (ADM) The process of adaption of He's method. iterative procedure based on variation (MVIM) to get a close answer to a system of nonlinear equations Integrated and differential equations with fractions. We give some examples to check how well the methods work.

Keywords: Caputo derivative, fractional integro-differential equations, and system of nonlinear fractional integro-differential equations, A domain decomposition method, modification of He's variational iteration method.

1. Introduction

In recent years, the fractional calculus has been used increasingly in different areas of applied science, has turn out that many phenomena in physics, engineering, chemistry, and other sciences can be described very successfully by models using mathematical tool from fractional calculus,([3],[4]). Some important problems in science and engineering can usually be reduced to a system of fractional integro-differential equations. since few of these equations can be solved explicitly, it is often necessary to develop the numerical integration and interpolation, ([1],[6],[14]). The our propose of this paper we study A domain decomposition method (ADM), and the modification of He's variational iteration method for approximating the solution of system of nonlinear fractional integro-differential equations. We will consider fractional order integro-differential equations of the form:

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$$D^{\alpha i} y_i(x) = g_i(x) + \int_0^1 \sum_{i=1}^{m} k_{ij}\,(x,t) F_i[y_i(t)]dt\,, \quad i = 1,2,\ldots,r \qquad (1.1)$$

with initial values $y_i(0) = c_i$;
where the functions $g_i(x), k_{ij}(x,t)$ are known functions and for $x, t \in [0,1], \alpha_i$ are a numerical parameters,
and $y_i(x)$ are the unknown functions, $D^{\alpha i}$ i is the partial derivative of Caputo and $F_i[y_i(t)]$are a nonlinear
continuous functions.

2. Basic definitions

This part talks about some basic terms and features of the fractional calculus theory that are used in this work,([2],[5]).

Definition: 2.1

"A real function y(x), x > 0, is said to be in the space $C_\mu, \mu \in R$, if there exists a real number $P > \mu$, such that $y(x) = x^\rho y_1(x), where\ y_1(x) \in C[0,1). Clearly\ C_\mu \subset C_\beta\ if\ \beta \le \mu$."

Definition: 2.2

A function y(x), x > 0, is said to be in the space $C_\mu^m\,, m \in N \cup \{0\}\,, if\ y^{(m)} \in C_\mu$.

Definition: 2.3

The left sided Riemann-Liouville fractional integral operator $\alpha \ge 0\,, of\ a\ function,\ y \in C_\mu, \mu \ge -1$, is defined as :

$$I^\alpha y(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1}\, y(t)dt, \quad \alpha > 0, \quad x > 0\,. \qquad (2,1)$$

$$I^0 y(x) = y(x) \quad ,(I^0 = I\ idendity\ operator). \qquad (2:2)$$

Definition: 2.4

Let $y \in C_{-1}^m,\ m \in N \cup 0\,,$ that the Caputo's fractional derivative of $y(x)$ is defined as.

$$D^{\alpha}y(x) = \begin{cases} J^{m-\alpha}y^{m}(x)\,, & m-1<\alpha\leq m\ ,\ m\in N \\ \frac{D^{m}y(x)}{Dx^{m}} \quad , & \alpha = m. \end{cases}$$

(2.3)

Hence, we have the following properties:

$$I^{\alpha}I^{\beta}y(x) = I^{\alpha+\beta}y(x)\,,\quad for\ all\ \ \alpha,\beta\geq 0,\ y\in C_{\mu}\,,\mu>0\,.$$

(2.4)

$$I^{\alpha}x^{\gamma} = \frac{\Gamma(\gamma+1)}{\Gamma(\gamma+1-\alpha)}x^{\gamma-\alpha}\,,$$

(2.5)

for $x>0\,,\alpha\geq 0\geq,\gamma>-1$.

$$I^{\alpha}D^{\alpha}y(x) = y(x)-\sum_{k=0}^{m-1}y^{(k)}(0^{+})\frac{x^{k}}{k!},\ x>0\ .$$

(2.6)

Furthermore, Caputo fractional differentiation is a linear process that functions similarly to inter-order differentiation.
D^α [λ y(x) +μ g(x)] = λ D^α y(x) + μ D^α g(x), with λ and μ constants.

3. "Numerical solution a system of nonlinear Fredholm fractional integro-differential equations"

This section includes a domain decomposition method and a modified version of He's variational iteration approach.
are applied for solving system of nonlinear fractional integro-differential equations.

A domain decomposition method

Consider equations (1.1) where $D^{\alpha i}$are the operator defined by (2.3), operating with $I^{\alpha i}$ on both sides
of equations (1.1) we obtain:

$y_i(x) =$

$y_i(0) - \sum_{i=0}^{m-1} y_i^{(k)}(0^+)\frac{x^k}{k_!} + I^{\alpha i}\left[g_i(x) + \int_0^1 \sum_{j=1}^{m} k_{ij}(x,t)\, F_i[y_i(t)]dt\right],\ i = 1,2,\dots,r$ (3.1)

we employ A domain decomposition method to solve the system of equation (3.1) by the series,([7],[12]),

$$y_i(x) = \sum_{m=0}^{\infty} y_{i,m}\,(x)\,, \quad (3.2)$$

and nonlinear functions $\boldsymbol{F_i}$ are decomposed as:

$$F_i[y_i(t)] = \sum_{n=0}^{\infty} A_{i,n,} \quad (3.3)$$

Where $A_{i,n}$ are the A domain polynomials given by:

$$A_{i,n} = \frac{1}{n!}\left[\frac{d^m}{d\lambda^n} F_i\left(\sum_{i=0}^{\infty} \lambda^i y_i\right)\right] \lambda = 0, \quad n = 0,1,2,\dots \quad i = 1,2,\dots,r \quad (3.4)$$

The components y_{i0}, y_{i1}, y_{i2} , ... are determined recursively by:

$$y_{i,0}(x) = y_i(x) + I^{\alpha i}\left(g_i(x)\right), \quad (3.5)$$

$$y_{i,k+1}(x) = I^{\alpha i}\left(g_i(x)\right) + I^{\alpha i}\left[\int_0^1 \sum_{j=1}^{m} k_{ij}(x,t) A_{i,k}(t)\, dt\right],\ k = 0,1,\dots,\quad i = 0,1,\dots,r \quad (3.6)$$

Having defined the components $y_{i,0}, y_{i,1}, y_{i,2},$ the solution y_i in series from defined by (3.2) follows immediately.

"**The Modified of He's Variational iteration Method**"

First, we'll talk about a safe change to the (VIM) that can be used to solve a set of nonlinear fractional integro-differential equations.

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We are going to look at the following set of fractional functional equations,

$$\mathrm{L}y_i(x) + Ry_i(x) + Ny_i(x) = g_i(x)\ ,\ i = 0,1,2,\dots,r \quad (3.7)$$

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g_i is the source term; R is a linear differential operator; L_x=D^αi; N is the set of nonlinear terms. We obtain by applying the inverse L_x^(-1) to both sides of (3.7) (2.6):

$$y_i(x) = f_i(x) - L_x^{-1}[Ry_i(x)] - L_x^{-1}[Ny_i(x)]\,,\ i = 1,2,..,r$$
(3.8)

L_x^(-1)=I^αi, and L_x^(-1) [g_i (x)]=f_i (x). We can use both (1.1) and the general idea of He's method to come up with a fixed fraction for (3.7), which says,

$$y_{i,n+1}(x) = y_{i,n}(x) + \int_0^x \lambda_i(t)\big[Ly_iN(t) + R\tilde{y}_iN(t) + N\tilde{y}_{i,n}(t) - g_i(t)\big]dt\,,$$ (3:9)

y_(i,n) is the nth number, and λ is a Lagrange factor that can be found best using variational theory [10].
To find a close answer to (1.1), let ỳ_(i,n) represent a restricted variation and εỳ_in=0, = 0. To solve equations (3.9), use a Lagrange factor that you get from integrating by parts.
With the help of Two variables, y and the Lagrange multiplier _0 (x), you can get closer and closer to the answer y_i (x) with y_(i,n) (x), where n>0. You can get the correct answer by using,

$$\lim_{n\to\infty} y_{i,n}(x) = y_i(x),\ i = 1,2,\dots,r\,.$$
(3.10)

Because of this, we have the following solution for variational iteration: (3.8),

$$\begin{cases} y_{i,0}(x)\ \textit{is an arbitrary initial guess}\,, \\ y_{i,n+1}(x) = f_i(x) - L_x^{-1}\big[Ry_{i,n}(x)\big] - L_x^{-1}\big[Ny_{i,n}(x)\big]. \end{cases}$$
(3.11)

Now, let's say that the function fi(x) of the iterative relation (3.11) can be split into two parts, which we call f_(i,0) (x) and f_(i,1) (x),

$$f_i(x) = f_{i,0}(x) + f_{i,1}(x)\,.$$
(3.12)

We come up with the following variational iteration formula based on assumption (3.12) and equation (3.6),

$$\begin{cases} y_{i,0}(x) = f_{i,0}(x)\,, \\ y_{i,1}(x) = f_i(x) - L_x^{-1}[Rf_{i,0}(x)] - L_x^{-1}[Nf_{i,0}(x)], \\ y_{i,n+1}(x) = f_i(x) - L_x^{-1}[Ry_{i,n}(x)] - L_x^{-1}[Ny_{i,n}(x)]. \end{cases} \quad (3.13)$$

4.Numerical Examples

In this section we present some numerical examples, of a system of nonlinear fractional integro-differential equations by using A domain decomposition method and the modified of He's Variational iteration method an[d compare the results.

Example 4.1

Consider the following system of nonlinear fractional integro-differential equation:

$$\begin{cases} D^{1/2}y_1(x) = g_1(x) + \int_0^1 x\,t[y_1(t)y_2(x)]dt, \\ D^{1/2}y_2(x) = g_2(x) + \int_0^1 \frac{x}{t^2}[y_1^2(t)y_2(t)]dt\,, \end{cases} \quad (4.1)$$

where $g_1(x) = \frac{8}{3}\frac{x^{3/2}}{\sqrt{\pi}} - \frac{x}{7}\,, g_2(x) = \frac{16}{5}\frac{x^{5/2}}{\sqrt{\pi}} - \frac{x}{6},$ with the initial condition $y_1(0) = 0, y_2(0)$, and the exact solutions are $y_1(x) = x^2\,, y_2(x) = x^3$.

The solution according to(ADM)

Equation (4.1) requires us to use operator on both sides. I^(1⁄2) to get to

$$\begin{cases} y_1(x) = y_1(0) + I^{1/2}\left[\frac{8}{3}\frac{x^{3/2}}{\sqrt{\pi}} - \frac{x}{7}\right] + I^{1/2}\left[\int_0^1 x\,t[y_1(t)y_2(t)]dt\right], \\ y_2(x) = y_2(0) + I^{1/2}[\frac{16}{5}\frac{x^{5/2}}{\sqrt{\pi}} - \frac{x}{6}] + I^{1/2}[\int_0^1 \frac{x}{t^2}[y_1^2(t)y_2(t)]dt, \end{cases}$$

$$\begin{cases} y_{1,0}(x) = I^{1/2}\left[\frac{8}{3}\frac{x^{3/2}}{\sqrt{\pi}} - \frac{x}{7}\right], \\ y_{2,0}(x) = I^{\,1/2}\left[\frac{16}{5}\frac{x^{5/2}}{\sqrt{\pi}} - \frac{x}{6}\right], \end{cases}$$

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$$\begin{cases} y_{1,0}(x) = x^2 - 0.1074646826x^{3/2}\,, \\ y_{2,0(x)} = x^3 - 0.1253754630x^{3/2}\,, \end{cases}$$

(4.2)

Now , we using equation (3.6) obtain to

$$\begin{cases} y_{1,1}(x) = I^{1/2}(\int_0^1 xt\left[A_{1,0}(t)\right]dt\,, \\ y_{2,1}(x) = I^{1/2}\left(\int_0^1 \frac{x}{t^2}\left[A_{2,0}(t)\right]dt\right), \end{cases}$$

$$\begin{cases} y_{1,1}(x) = 0.07990674461x^{3/2} \\ y_{2,1}(x) = 0.08151421943x^{3/2}, \end{cases}$$

(4.3)

The same way we find

$$\begin{cases} y_{1,2}(x) = 0.002184970692x^{3/2}\,, \\ y_{2,2}(x) = 0.003007566127x^{3/2}, \end{cases}$$

(4.4)

substituting from (4.2),(4.3),and (4.3) into equation (3.2) we obtain

$$\begin{cases} y_1(x) \cong x^2 - 0.02537296729x^{3/2}, \\ y_2(x) \cong x^3 - 0.4085367747x^{3/2}, \end{cases}$$

(4.5)

Equations (4.5) implies the approximate solution of (4.1).

"**The solution according to(MVIM)**"

"On both sides of equation (4.1), we put operator I^(1⁄2) to get to"

$$\begin{cases} y_1(x) = y_1(0) + I^{1/2}\left[\frac{8}{3}\frac{x^{3/2}}{\sqrt{\pi}} - \frac{x}{7}\right] + I^{1/2}\left[\int_0^1 xt\,[y_1(t)y_2(x)]dt\right], \\ y_2(x) = y_2(0) + I^{1/2}\left[\frac{16}{5}\frac{x^{5/2}}{\sqrt{\pi}} - \frac{x}{6}\right] + I^{1/2}\left[\int_0^1 \frac{x}{t^2}[y_1^2(t)y_2(t)]dt\right], \end{cases}$$

From the first VIM (3.8) and the repetitive method (3.11) that goes with it, we get:

$$\begin{cases} f_1(x) = f_{1,0}(x) + f_{1,1}(x) = I^{1/2}\left[\frac{8}{3}\frac{x^{3/2}}{\sqrt{\pi}} - \frac{x}{7}\right], \\ f_2(x) = f_{2,0}(x) + f_{2,1}(x) = I^{1/2}\left[\frac{16}{5}\frac{x^{5/2}}{\sqrt{\pi}} - \frac{x}{6}\right], \end{cases}$$

$$\begin{cases} f_1(x) = x^2 - 0.1074646826x^{3/2}, \\ f_2(x) = x^3 - 0.1253754630x^{3/2}, \end{cases}$$

by assuming

$$\begin{cases} f_{1,0}(x) = x^2, \;\; f_{1,1}(x) = -0.1074646826x^{3/2}, \\ f_{2,1}(x) = x^3\,, \;\; f_{2,1}(x) = -0.1253754630x^{3/2}\,, \end{cases}$$

Along with the start of initial

$$\begin{cases} y_{1,0}(x) = f_{1,0}(x) = x^2, \\ y_{2,0}(x) = f_{2,0}(x) = x^3, \end{cases}$$

$$\begin{cases} y_{1,1}(x) = x^2 - 0.1074646826x^{3/2} \; + I^{1/2}[\int_0^1 x\,t\left[\left(y_{1,0}(t)y_{2,0}(t)\right)\right]dt\right], \\ y_{2,1}(x) = x^3 - 0.1253754630x^{3/2} + I^{1/2}\left[\int_0^1 \frac{x}{t^2} \quad [y_1^2(t)y_{2,0}(t)]dt\right], \end{cases}$$

(4.6)

$$\begin{cases} y_{1,1}(x) = x^2, \\ y_{2,1}(x) = x^3, \end{cases}$$

$$\begin{cases} y_{1,n+1}(x) = x^2 - 0.1074646826x^{3/2} + L_x^{-1}\left[\left(y_{1,n}(x), y_{2,n}(x)\right)\right] = x^2, \;\; n \geq 1. \\ y_{2,n+2}(x) = x^3 - 0.1253754630x^{3/2} + \;\; L_x^{-1}[(y_{1,n}(x), y_{2,n}(x))] = x^3, \;\; n \geq 1. \end{cases}$$

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"In similarly view equation (4.6) it is obtained":

$$\begin{cases} y_1(x) = x^2 \,, \\ y_2(x) = x^3 \,, \end{cases}$$

"where those are the exact answers to equation (4.1)".
The numbers from Example 4.1 are shown in Table 1 and Figures 1 and 2.

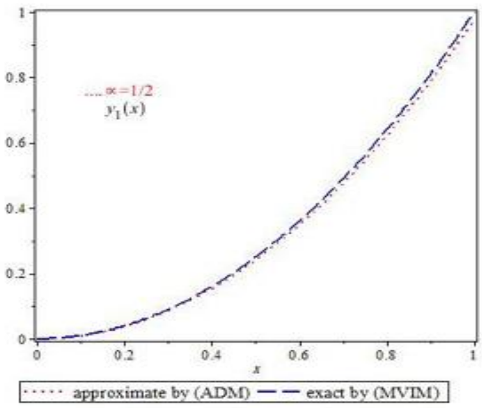

Figure 1: Numerical result of example 1

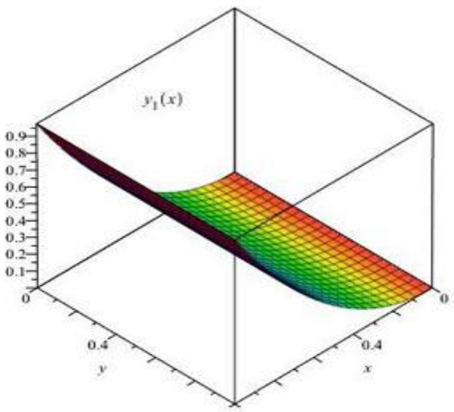

Figure 2: Approximate solution of example 1

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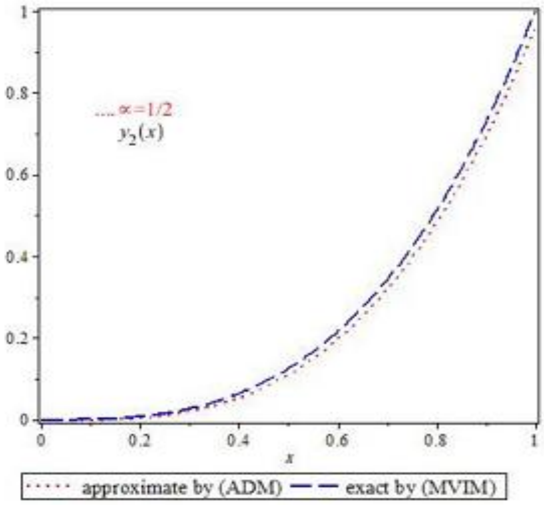

Figure 3: Numerical result of example 1

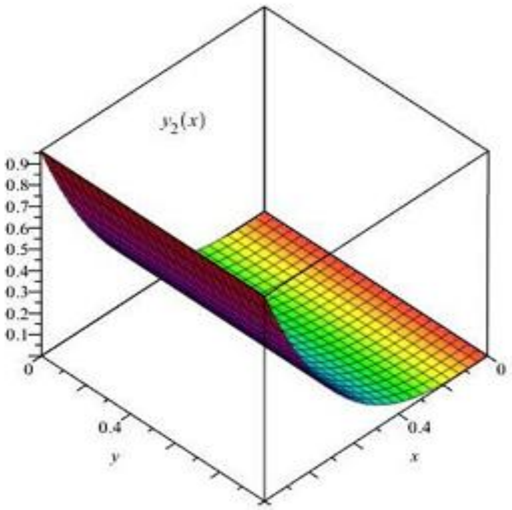

Figure 4: Approximate solution of example 1

x	$exact = (MVIM)$	$Approximant by (ADM)$	$Error of (ADM)$
0.1	0.01	0.0009197636324	0.000802363676
0.2	0.04	0.03773057281	0.00226942719
0.3	0.09	0.08583079604	0.00416920396
0.4	0.16	0.1535810906	0.0064189094
0.5	0.25	0.2410293014	0.0089706986
0.6	0.36	0.3482077104	0.0117922896
0.7	0.49	0.4751400168	0.0148599832
0.8	0.64	0.6218445825	0.0181554175
0.9	0.81	0.7883361807	0.0216638193
1	1.00	0.9746270327	0.0253729673

Table 1. Show how much mistake there is in example 4.1:[$y_2(x) = x^3$]..

x	$exact = (MVIM)$	$Approximant\ by (ADM)$	$Error\ of (ADM)$
0.1	0.001	0.000291906716	0.001291906716
0.2	0.008	0.004345936002	0.003654063998
0.3	0.027	0.02028705579	0.00671294421
0.4	0.064	0.05366474627	0.01033525373

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Iraqi Journal of Humanitarian, Social and Scientific Research
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0.5	0.125	0.1105560438	0.0144439562
0.6	0.216	0.1970129265	0.0189870735
0.7	0.343	0.3190735528	0.0239264472
0.8	0.512	0.4827674880	0.0292325120
0.9	0.729	0.6941185187	0.0348814813
1	1000	0.9591463225	0.0408536775

Table 2. Show how much mistake there is in example 4.1[$y_2(x) = x^3$].

Example 4.2

Take a look at this set of nonlinear fractional integro-differential equations:

$$\begin{cases} D^{3/4}y_1(x) = g_1(x) + \int_0^1 x\, t[y_1^2(t) + y_2^2(t)]dt, \\ D^{3/4}y_2(x) = g_2(x) + \int_0^1 x\sqrt{t}[y_1^2(t)y_2^2(t)]dt, \end{cases}$$

(4.7)

Where $g_1(x) = \frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{7}{24}x$, $g_2(x) = \frac{16}{5}\frac{x^{5/2}\sqrt{2}\Gamma(3/4)}{\sqrt{\pi}} - \frac{2}{23}x$, with the initial condition $y_1(x) = 0$, $y_2(x) = 0$, and the exact solutions are $y_1(x) = x^3$, $y_2(x) = x^2$.

The solution according to (ADMM)

Putting operator I^(3⁄4) on both sides of equation (4.7) gives us

$$\begin{cases} y_1(x) = y_1(0) + I^{3/4}\left[\frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{7}{24}x\right] + I^{3/4}\left[\int_0^1 x\sqrt{t}[y_1^2(t) + y_2^2(t)]dt\right], \\ y_2(x) = y_2(0) + I^{3/4}\left[\frac{16}{5}\frac{x^{5/2}\sqrt{2}\Gamma(3/4)}{\sqrt{\pi}} - \frac{2}{23}x\right] + I^{3/4}[\int_0^1 \frac{x}{t^2}[y_1^2(t)y_2^2(t)]dt , \end{cases}$$

$$\begin{cases} y_1(x) = I^{3/4}\left[\frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{7}{24}x\right], \\ y_2(x) = I^{3/4}\left[\frac{16}{5}\frac{x^{5/2}\sqrt{2}\Gamma(3/4)}{\sqrt{\pi}} - \frac{2}{23}x\right], \end{cases}$$

$$\begin{cases} y_{1,0}(x) = x^3 - 0.1813442087x^{7/4}\,, \\ y_{2,0}(x) = x^2 - 0.05406535416x^{7/4} \end{cases}$$

(4.8)

Now , we using equation (3.6) obtain to

$$\begin{cases} y_{1,1}(x) = I^{3/4}\left(\int_0^1 x\,t[A_{1,0}(t)]dt\right), \\ y_{2,1}(x) = I^{3/4}\left(\int_0^1 x\sqrt{t}[A_{2,0}(t)]dt\right), \end{cases}$$

$$\begin{cases} y_{1,1}(x) = 0.1402922716x^{7/4}, \\ y_{2,1}(x) = 0.03065131084x^{7/4}, \end{cases}$$

(4.9)

The same way we find

$$\begin{cases} y_{1,2}(x) = 0.05151853355x^{7/4}, \\ y_{2,2}(x) = 0.001165525150x^{7/4}, \end{cases}$$

(4.10)

Substituting (4.8) , (4.9) , and (4.10) into equation (3.2) we obtain

$$\begin{cases} y_1(x) \cong x^2 - 0.02537296729x^{3/2}, \\ y_2(x) \cong x^3 - 0.04085367747x^{3/2}, \end{cases}$$

(4.11)

Equations (4.11) implies the approximate solution of the system (4.8) .

"**The solution according to (MVIM)**"

"We put operator I^(3⁄4) on both sides of equation (4.7) to get to "

$$\begin{cases} y_1(x) = y_1(0) + I^{3/4}\left[\frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{7}{24}x - \frac{x}{7}\right] + I^{3/4}\left[\int_0^1 x\sqrt{t}[y_1^2(t) + y_2^2(t)]dt\right], \\ y_2(x) = y_2(0) + I^{3/4}[\frac{16}{5}\frac{x^{5/2}\sqrt{2}\Gamma(3/4)}{\sqrt{\pi}} - \frac{2}{23}x] + I^{3/4}[\int_0^1 \frac{x}{t^2}[y_1^2(t)y_2^2(t)]dt, \end{cases}$$

Based on the first VIM (3.8) and the method that repeats itself (3.11) that goes with it, we get:

$$\begin{cases} f_1(x) = f_{1,0}(x) + f_{1,1}(x) = I^{3/4}\left[\frac{64}{15}\frac{x^{9/4}\sqrt{2}\Gamma(3/4)}{\pi} - \frac{7}{24}x\right], \\ f_2(x) = f_{2.0}(x) + f_{2,1}(x) = I^{3/4}\left[\frac{16}{5}\frac{x^{5/2}\sqrt{2}\Gamma(3/4)}{\sqrt{\pi}} - \frac{2}{23}x\right], \end{cases}$$

$$\begin{cases} f_1(x) = x^3 - 0.181342087x^{7/4}, \\ f_2(2) = x^2 - 0.05406535416x^{7/4}, \end{cases}$$

By assuming

$$\begin{cases} f_{1,0}(x) = x^3, \ \ f_{1,1}(x) = -0.1813442087x^{7/4}, \\ f_{2,1}(x) = x^2, \ \ f_{2,1}(x) = -0.05406535416x^{7/4}, \end{cases}$$

With starting of initial approximate

$$\begin{cases} y_{1,0}(x) = f_{1,0}(x) = x^3, \\ y_{2,0}(x) = f_{2,0}(x) = x^2, \end{cases}$$

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$$\begin{cases} y_{1,1}(x) = x^3 - 0.1813442087x^{7/4} + I^{3/4}\left[\int_0^1 x\, t\left[y_{1,0}^2(t) + y_{2,0}^2(t)\right]\right], \\ y_{2,1}(x) = x^2 - 0.05406535416x^{7/4} + I^{3/4}\left[\int_0^1 x\, \sqrt{t}\left[y_{1,0}^2(t) y_{2,0}^2(t)\right]\right], \end{cases}$$
(4.12)

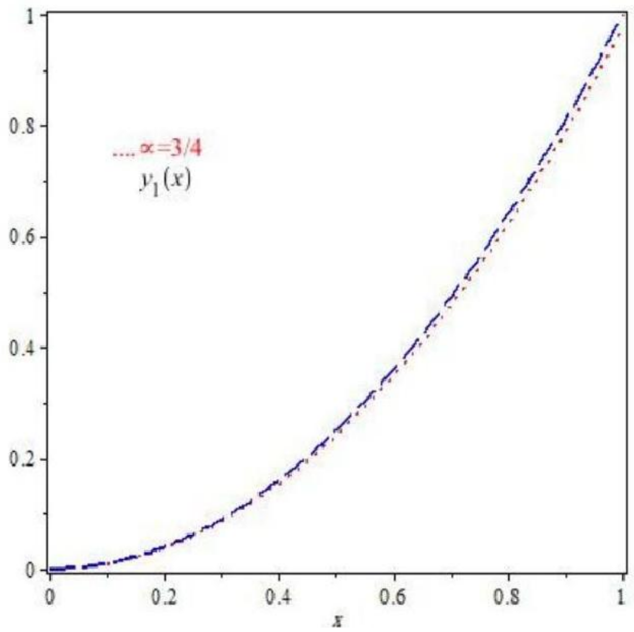
$$\begin{cases} y_{1,1}(x) = x^3, \\ y_{2,1}(x) = x^2, \end{cases}$$

$$\begin{cases} y_{1,n+1}(x) = x^3 - 0.1813442087x^{7/4} + L_x^{-1}\left[\left(y_{1,n}(x), y_{2,n}(x)\right)\right] = x^3, \quad n \geq 1\,. \\ y_{2,n+}(x) = x^2 - 0.05406535416x^{7/4} + L_x^{-1}\left[\left(y_{1,n}(x), y_{2,n}(x)\right)\right] = x^2, \quad n \geq 1\,. \end{cases}$$
(4.13)

In the same way, look at equation (4.16) and get:

$$\begin{cases} y_1(x) = x^3, \\ y_2(x) = x^2, \end{cases}$$

Where those are the exact solution of equation (4.7).
Tables 3, and 4 Figures 3and 4 shown the number that comes up in example 4.2



Approximate by (ADM) _ _ exact by (MVIM)

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Figure (5) Numerical result of example 2

x $(MVIM)$	$exact =$ $Approximate\ by\ (ADM)$	$Errorof\,(ADM)$
0.1 0.000186125329	1.001	0.001186125329
0.2 0.000626048487	0.008	0.08862604847
0.3 0.00127282200	0.027	0.02827282200
0.4 0.00210576771	0.064	0.6610576771
0.5 0.0031117377	0.125	0.1281117377
0.6 0.0042812458	0.216	0.2202812458
0.7 0.0056069547	0.343	0.34860069547
0.8 0.0070829301	0.512	0.5190829301
0.9 0.0087042202	0.729	0.7377042202
1 0.010466596	1.000	1.010466596

Table 2 . Show how much mistake there is in example 4.2 : $[y_1(x) = x^3]$.

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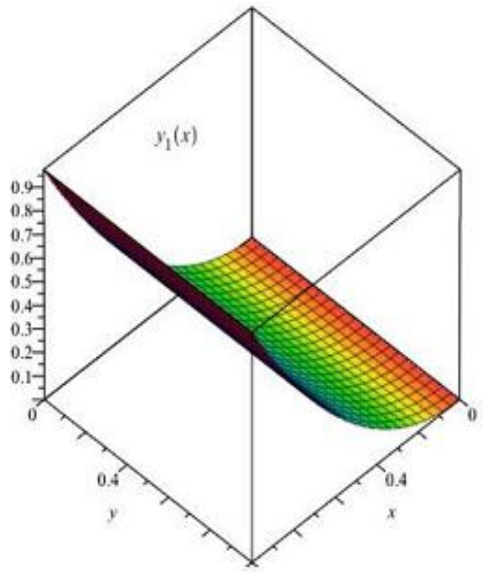

Figure 6: Approximate solution of example 2

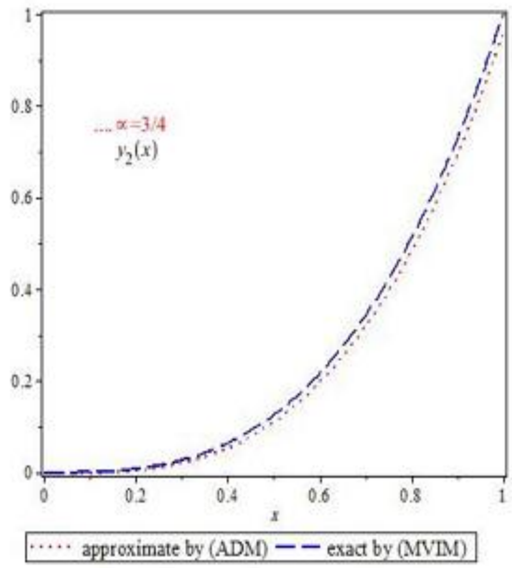

Figure 7: Numerical result of example 2

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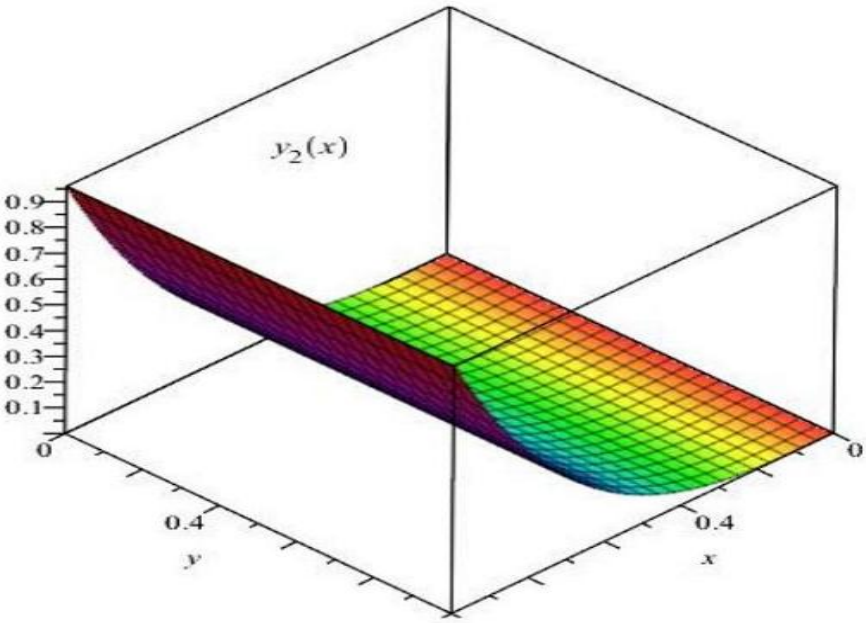

Figure 8: Approximate solution of example_2

x $exact = (MVIM)$	$Approximant\ by\ (ADM)$	$Error\ of\ (ADM)$
0.1 0.000395640818	0.01	0.00960259182
0.2 0.00133077178	0.04	0.3866922822
0.3 0.00270559811	0.09	0.98729440189
0.4 0.0044761649	0.16	0.1555238351
0.5 0.0066145240	0.25	0.2433854760
0.6 0. 0091005110	0.36	0.3508994890

0.7	0.49	0.4780814709	0.0119185291
0.8	0.64	0.6249440360	0.0150559640
0.9	0.81	0.7914977101	0.0185022899
1	1.00	0.9777514818	0.0222485182

Table 2. Show how much mistake there is in example 4.2: $[y_1(x) = x^2]$

5. Conclusion

Based on the results above, we can say that the changes to He's variation iteration method (MVIM) work better.
The results you get by using Maple 16 are better than those you get with the ADM method.

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