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# Analysis of Survival Estimators for Heart Disease Patients Under Multicollinearity

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**Abstract :** In this study, parametric and nonparametric survival estimators for patients with heart disease under multicollinearity are analyzed, In addition to more sophisticated approaches like the Bernstein approximation and Cox Proportional Hazards with regularization, more conventional approaches like Kaplan-Meier and Nelson-Aalen were assessed, The findings indicate that while traditional approaches work well in straightforward situations, they become vulnerable to multicollinearity in strongly linked variables. However, in complex situations, advanced techniques yield solutions that are adaptable and stable, metrics such as statistical performance (Log-Likelihood, AIC, BIC), accuracy (MAE, RMSE), and the capacity to manage censored data and multicollinearity were utilized in the survival calculation, while old methods are still appropriate for smaller scenarios, advanced methods are advised for complicated datasets with multicollinearity, In the applied aspect, data were collected from one of the private hospitals in Baghdad, where the results reached the advantage of the Bayesian Kaplan-Meier estimation method compared to the rest of the estimation methods based on comparative above measures and others.

**Keywords**: Parametric and Nonparametric Survival Estimators, Multicollinearity, Kaplan-Meier, Nelson-Aalen, Bernstein Approximation, Cox Proportional Hazards.

**INTRODUCTION**: Heart attacks are a serious health concern that endangers the lives of millions of people each year, and cardiovascular illnesses are among the top causes of mortality globally. The World Health Organization (WHO) states that cardiovascular illnesses are the leading cause of death worldwide, hence it is crucial to have accurate instruments and methods for examining medical data pertaining to these conditions. For the reasons stated, it becomes evident how crucial it is to analyze survival statistics to determine the factors that influence patient survival and life expectancy after diagnosis or therapy. Certain issues, such as multicollinearity among independent variables, can severely compromise the precision of statistical models, which warrants the use of complex methods to address the problem.

Over the past few years, the application of statistical methods for the analysis of medical data has advanced significantly. However, traditional methods such as Kaplan-Meier and Nelson-Aalen are still widely used in survival analyses due to their practicality and effectiveness with uncomplicated scenarios. Although these methods are useful, they may become ineffective in the presence of multicollinearity. This problem causes increased variation in coefficient estimates while reducing accuracy, leading to the need for more advanced techniques like Bernstein Approximation and Cox Proportional Hazards with Regularization.

The data used for this study were obtained from patients in a private hospital and a cardiac clinic located in Baghdad, Iraq. It consists of the following important medical features: Age, sex, heart rate, systolic and diastolic blood pressure, blood sugar levels, CK-MB, and troponin. Each patient’s data is tagged by whether or not they suffered a heart attack: 1 indicates a heart attack, and 0 denotes no heart attack. For blood sugar, the flag is set at 1 if it is greater than 120, otherwise, it is set to 0. All other features were normalized: Male was designated as 1 and female as 0. The data is valuable for analyzing the relationships between multiple variables in context to the risk of heart attacks.

The purpose of this paper is to compare traditional and heartrending parametric and nonparametric survival estimators under the condition of multicollinearity. The primary reason is to compare the performance of more advanced methods like Bernstein Approximation and Cox Proportional Hazards with Regularization versus more traditional methods like Kaplan-Meier and Nelson-Aalen. Several criteria are used to evaluate the performance: accuracy (MAE,

RMSE), statistical performance (Log-Likelihood, AIC, BIC), and the ability of the models to handle multicollinearity and censored data (Aalen, 1978).
This study aims for the Reliability calculation to make precise suggestions on how survival data can be analyzed and uses a new way of computing reliability for simple to complex models of data. Expected results should show that traditional methods perform perfectly in simple situations but fail when there is high interconnectivity of variables. However, advanced methods, which are preferred to handle complex data with multicollinearity, give more consistent (Yusuf et al., 2024)
Furthermore, this study expands our understanding of the use of statistical approaches to the analysis of medical survival data. Our study focused on increasing accuracy during medical forecasting by relying on new insights into how to deal with the avoidance of multiplicity, thus participating and assisting in decision-making. Finally, the study is interested in establishing a reliable framework for improving the curriculum for the prevention and treatment of heart attacks as a result of preserving the lives of individuals with heart disease.
A crucial statistical method for calculating the amount of time before an event of interest, such as death or the advancement of a disease, is survival analysis. Because they are easy to use and effective when working with censored data, traditional nonparametric survival estimators like Nelson-Aalen (Aalen, 1978) and Kaplan-Meier (Kaplan & Meier, 1958) have been in use for a long time. These techniques are interested in making important assumptions regarding the basic risk function while presenting unbiased insights into the likelihood of survival. In addition, developing better statistical methods for the purpose of improving the accuracy of estimation and consistency when there are high dimensional variables that increase the complexity of survival data.
The multiplicity that arises in the case of predictive variables that are highly correlated between them is one of the most challenging situations in the survival analysis process, the Kaplan- Meier survival analysis was used for analysis for the purpose of breast cancer prediction by (Dakhil et al., 2012) which refers to its importance in simple survival calculations, but sometimes standardized methodologies may fall short when used during interconnected elements, as we observe the concentration of recent studies of deficiencies in relative risk models commonly used for COX. (Kalu et al., 2025) and (Ata & Sözer ,2007), which focused on the topic of strategies for resolving deficiencies and appropriate alternatives.
Several researchers have proposed new techniques for modeling survival for the purpose of overcoming multiplicity constraints, with( Monikapreethi et al., 2024) and (Fan et al., 2024) proposing stable Cox retreat models that are under distribution shifts enhancing the prediction of survival data in dynamic groups, and (Deng et al. 2024) calculate the effect of the treatment by suggesting an estimated Nelson-Allen generalized based on reverse survival weighting when the variables are highly correlated producing more accurate and reliable capabilities.
There is an alternative nonparametric method for estimating the survival function of life data in the event of a multicollinearity, namely to rely on multiple Bernstein polynomials as presented by both (Petrone,1999) and (Leblanc,2012) which are useful in calculating the density function because they provide Smoother approximations that reduce the biases that appear in the normal progressive function capabilities, Bayesian methods that rely on a previous distribution for the purpose of enhancing estimation stability in the framework of data volatility are also widely focused by both (Berliner et al., 1988) and (Hjort, 1990) and (Ahmed et al., 2020).
Experimental comparisons have been used in many recent research for the purpose of knowing how well the different survival model works, for example in the study by (Colosimo et al. 2002) Cox time-based models were compared with the Kaplan-Meier estimate using simulation method.
The development of artificial intelligence and machine learning in the analysis of survival data, focused (Laverny et al., 2025) at the expense of a non-conforming survival estimate given the cause of death, (Ramakrishnan et al., 2024), which focused on survival data with competing risks as such research suggests the integration of traditional statistical methods with artificial intelligence techniques that improve prediction.
In order to handle multicollinearity and high-dimensional data, current survival analysis increasingly employs regularization strategies, Bernstein approximations, and Bayesian methods, even though conventional nonparametric survival estimators are still helpful in straightforward applications, Future research should concentrate on integrating these methods as computer capabilities advance in order to develop more reliable, adaptable, and interpretable survival models for a variety of medical and actuarial applications. Based on this literature, we hypothesize that, although traditional survival analysis techniques remain valuable for simple datasets, sophisticated models like the Bernstein Approximation and Cox Regularization methods yield more accurate and stable results in complex survival data with multicollinearity.
, These estimators are compared using the following performance metrics: variance, standard deviation, skewness, kurtosis, log-likelihood, mean absolute error (MAE), root mean squared error (RMSE), akaike information criterion (AIC), Bayesian information criterion (BIC), variance. In simpler survival models, traditional methods like Kaplan-Meier and Nelson-Aalen perform well, but they falter in more complicated situations with a lot of different variables.

## 2.Survival Data Analysis :

Survival data examines how long it takes for a particular event to happen, such a patient passing away or a gadget malfunctioning. The role of survival (Yusuf et al., 2024) :

$$S(t) = P(T > t) = 1 - F(t) \quad \dots \quad (1)$$

indicates the likelihood that a person will live past time t. The function of hazards :

$$h(t) = \frac{f(t)}{S(t)} \quad \dots \quad (2)$$

explains the event rate in real time at time t. The Cox proportional hazards model evaluates covariate effects on hazard rates, whereas the Kaplan-Meier estimator is employed for non-parametric survival function estimation :

$$h(t \mid X) = h_0(t)e^{\beta X}$$

These models are used extensively for risk prediction and survival time analysis in engineering and medicine , when a person's precise survival time is unknown for a variety of reasons, such as loss to follow-up or study termination prior to the event, this is known as censoring.

Statistical techniques such as Cox regression and the Kaplan-Meier estimator use partial survival information to manage censored data in order to prevent bias. For censored data, the likelihood function is expressed by (Yusuf et al., 2024) as follows :

$$L = \prod_{i \in D} f\,(t_i) \prod_{i \in C} S\,(t_i)$$

where D is the observed failure times and C is the censored data.

## 3. Research Methodology :

This study presents a different method for resolving statistical interests related to the problem of multicollinearity and evaluating survival data using parametric and nonparametric methods in the field of risk evaluation in medical applications, The analysis of survival is useful and important to calculate the amount of time until a given event and for the purpose of calculating the survival of survival in this study some nonparametric methods will be used (Sadiq, M.,2024).

In addition, parametric models such as Cox's model of relative risk are relied upon for the purpose of removing different factors affecting different methods of estimating survival times, Adjustment methods are used, more accurate and reliable estimates are calculated by integrate uncertainty into model parameters using Bayesian method, In the case of dealing with complex relationships in a multicollinearity of data, estimates of survival are improved using Bernstein's approximation.

## 4. Materials and Methods:

### 4.1. Nelson-Aalen Estimator

It is a nonparametric statistical method that is used in the analysis of survival by calculating the function of cumulative hazard Nelson-Aalen, unlike the rest of the nonparametric methods of calculating accumulated risk over time, this makes it an important tool for determining the likelihood of a particular event occurring within a given or specified period of time،which assumes that the individual survives to that point, the hazard function symbolized by the symbol h(t) represents the immediate rate at which that event occurs in a timely manner, this method is used to calculate the estimate of survival in research that concerns medical areas for the purpose of tracking disease progression (Deng & Wang, 2024):

$$h(t) = \lim_{\Delta t \to 0} \frac{P(t \le T < t + \Delta t \mid T \ge t)}{\Delta t} \quad \dots \quad (3)$$

Total cumulative risk can be calculated over time :

$$H(t) = \int_0^t h\,(u)\,du$$

Estimated nonparametric by Nelson-Aalen as follows

$$\widehat{H}(t) = \sum_{t_i \le t} \frac{d_i}{n_i} \quad \dots \quad (4)$$

This guess is computed cumulatively by adding new values for each recorded failure time, where $t_i$ is the unique failure times, arranged in ascending order, $d_i$ is the number of people who experience the event (such as failure or death) at time $t_i$ and $n_i$ is the number of people at risk just prior to time $t_i$ and to calculate cumulative hazard function ,the first steps Sort the failure times in ascending order them calculate the number of people at danger for each failure time and calculate the ratio of failures to at risk individuals at each failure period (Colosimo et al., 2002), add these values to get the cumulative hazard estimate at time t.

The cumulative hazard function has a direct link with the survival function, the survival function can be stated using the cumulative hazard function as follows:

$$S(t) = e^{-H(t)} \quad \ldots \quad (5)$$

Thus, the Nelson-Aalen estimator yields the following survival function estimate, from Equ.4:

$$\hat{S}(t) = e^{-\hat{H}(t)}$$

This implies that the Nelson-Aalen estimator can be used not just to estimate cumulative hazard, but also to indirectly estimate the survival survival. The Nelson-Aalen estimator has several essential properties: Non-parametric: It does not assume any certain distribution for survival times, making it extremely adaptable. Cumulative nature: The estimate increases only during observed failure times and remains constant otherwise. Consistency: As sample size increases, it approaches the genuine cumulative hazard function, To ensure that $\hat{H}(t)$is an unbiased estimator of H(t), have:

$$E[\hat{H}(t)] = E\left[\sum_{t_i \le t} \frac{d_i}{n_i}\right]$$

Since $d_i \sim \text{Binomial}(n_i, h(t_i))$, by (Eswar R. and Santhi, 2024) we have $\mathrm{E}[\mathrm{d_i}] = \mathrm{n_i h(t_i)}$ Thus:

$$E\left[\frac{d_i}{n_i}\right] = h(t_i)$$

Summarizing all failure times:

$$E[\hat{H}(t)] = \sum_{t_i \le t} h\,(t_i) \approx H(t)$$

This indicates that $\hat{H}(t)$ is an unbiased estimator of H(t),The variance of the Nelson-Aalen Estimator is provided by :

$$\mathrm{Var}\left(\hat{H}(t)\right) = \sum_{t_i \le t} \mathrm{Var}\left(\frac{d_i}{n_i}\right) \quad \ldots \quad (6)$$

Since $d_i \sim \text{Binomial}(n_i, h(t_i))$, its variance is var $(d_i) = n_i h(t_i)\big(1 - h(t_i)\big)$ approximating $h(t_i)$ as little yields:

$$\mathrm{Var}\left(\frac{d_i}{n_i}\right) \approx \frac{h(t_i)}{n_i}$$

Summarizing all failure times:

$$\mathrm{Var}\left(\hat{H}(t)\right) = \sum_{t_i \le t} \frac{d_i}{n_i^2}$$

This gives an estimate of the uncertainty in $\hat{H}(t)$. The standard deviation is :

$$\mathrm{SD}\left(\hat{H}(t)\right) = \sqrt{\sum_{t_i \le t} \frac{d_i}{n_i^2} \quad \ldots \quad (7)}$$

This measure quantifies the spread of the cumulative hazard estimate.

**4.2. Kaplan-Meier Estimator**

The Kaplan-Meier estimator is a nonparametric statistics used in survival analysis to estimate the survival function, particularly when data is censored , It provides an estimate of the chance that a subject will survive until a specific time point, taking into account both observed events (e.g., failures) and censored data, the survival function S(t) represents the likelihood of a subject surviving beyond time t as follows:

$$S(t) = P(T > t)$$

T is the time to event random variable, the Kaplan-Meier estimates the survival function S(t) using both censored and uncensored data, It is especially value in survival studies where there is time to event data with censoring, the Kaplan-Meier estimator $\hat{s}(t)$ is calculated using data (Dakhil et al., 2012) with failure times $t_1, t_2, \ldots, t_k$, as follows:

$$\hat{S}(t) = \prod_{t_i \le t} \left(1 - \frac{d_i}{n_i}\right) \quad \ldots \quad (8)$$

Where $t_i$ are the different observed failure times organized in increasing order, $d_i$ is the number of events (e.g., deaths or failures) that occur at time $t_i$ and $n_i$ is the number of people at risk right before time $t_i$, for computing the estimator sort the failure times ($t_1, t_2, \ldots, t_k$) in ascending order by (Cao et al., 2005) calculate the number at risk ($n_i$) for each observed failure time ($t_i$), count the number of occurrences ($d_i$) occurring at each observed failure time ($t_i$), Calculate the survival survival at each time $t_i$ as follows:

$$\hat{S}(t_i) = \hat{S}(t_{i-1}) \times \left(1 - \frac{d_i}{n_i}\right)$$

Where $\hat{S}(t_0)$=1 (the survival at the start is 1, as no one has experienced the event at time (0), The Kaplan-Meier curve is a step function that drops at failure times and remains constant between them, using Greenwood's formula (Eswar R. & Santhi, 2024) ,we may approximate the variance of the Kaplan-Meier estimator at time $t_i$ .

$$\mathrm{Var}\left(\hat{S}(t_i)\right) = \hat{S}^2(t_i) \times \sum_{t_j \le t_i} \frac{d_j}{n_j(n_j - d_j)} \quad \dots \quad (9)$$

$\hat{S}(t_i)$ is the Kaplan-Meier estimate at time $t_i$, $d_j$ is the number of occurrences at time $t_i$, and $n_j$ is the number of individuals at risk right before time $t_j$, The formula approximates the variance of the survival function S(t) for each observed time point (Dakhil et al.,2012) from Equ.9 then :

$$\mathrm{SD}\left(\hat{S}(t_i)\right) = \sqrt{\hat{S}^2(t_i) \times \sum_{t_j \le t_i} \frac{d_j}{n_j(n_j - d_j)}} \quad \dots \quad (10)$$

This is the uncertainty associated with the Kaplan-Meier estimate at a specific time point.

**4-3-Bernstein Estimator**

This estimate is a nonparametric smoothing method used for the purpose of estimating the survival density function or the survival function of heart patients that has the potential to reduce the amount of bias and also employs multiple Bernstein polynomials to calculate an unknown distribution estimate smoothly, making it useful and effective This method is used to generate a smoother survival function in analysis rather than traditional techniques such as Kaplan-Meier and in cases of studying survival analysis for heart patients, calculating the likelihood of survival as explained (Muhammad & Jaber, 2021) is very useful, based on Bernstein previously clarified definition, an estimate of the density function f (x) would be :

$$\hat{f}_n(x) = \sum_{k=0}^{m} p_k \, B_{m,k}(x) \qquad \dots \quad (11)$$

The $B_{m,k}(x)$ is basis function polynomials, it is found through :

$$B_{m,k}(x) = \binom{m}{k} x^k (1-x)^{m-k}$$

Using sample data is found $p_k$ as follows :

$$p_k = \frac{1}{n} \sum_{i=1}^{n} B_{m,k}\,(X_i)$$

For the purpose of controlling the smoothness of the estimate, the polynomial's degree m is controlled, The cumulative distribution function is therefore estimated as a survival function (survival of survival) based on Bernstein's estimate and as follows:

$$\hat{F}_n(x) = \sum_{k=0}^{m} p_k \sum_{j=0}^{k} B_{m,j}\,(x)$$

Where $\sum_{j=0}^{k} B_{m,j}\,(x)$, represent the cumulative sum of the Bernstein basis functions up to index k. The estimated survival function S(x), which indicates the chance that an individual survives beyond time x, is thus:

$$\hat{S}_n(x) = 1 - \hat{F}_n(x)$$

This produces a smooth survival curve that can be used to assess survival data in cardiac patients. The predicted value of the Bernstein estimator is given by:

$$E\left[\hat{f}_n(x)\right] = \sum_{k=0}^{m} p_k \, E\left[B_{m,k}(x)\right] \qquad \dots \quad (12)$$

The Bernstein polynomials approximate the true distribution , thus we have:

$$E\left[B_{m,k}(x)\right] \approx f(x)$$

This assures that the estimator is asymptotically unbiased for high m, the variance of the Bernstein estimator is given as:

$$\mathrm{Var}\left(\hat{f}_n(x)\right) = \frac{1}{n} \sum_{k=0}^{m} p_k^2 \, \mathrm{Var}\left(B_{m,k}(x)\right) \qquad \dots \quad (13)$$

where:

$$\mathrm{Var}\left(B_{m,k}(x)\right) = B_{m,k}(x)\left(1 - B_{m,k}(x)\right)$$

The variance decreases with larger samples, resulting in a more accurate approximation of the density function.

**4.4. Breslow Estimator**

This estimate is a nonparametric method used in the analysis of the survival function to estimate the cumulative hazard function in case of linking survival periods Also, it is very useful when used with COX models for relative hazards, which does not assume a fixed basic hazard function but instead depends on a hazard function dependent on variables cumulative hazard function, Breslow

estimator provides a continuous assessment of the cumulative hazard function as explained by (Gil-Pallares et al., 2025) and (Kaplan & Meier, 1958) in medical fields, particularly in the study of survival analysis for cardiac patients:

$$H(t) = \int_0^t h\,(u)\,du$$

In the Cox proportional hazards model, the hazard function for an individual with covariate vector X is given as:

$$h(t \mid X) = h_0(t)e^{\beta' X}$$

Where $h_0$ (t) is the baseline hazard function, and β is a vector of regression coefficients, this is the cumulative baseline hazard function's Breslow Estimator:

$$\widehat{H}_0(t) = \sum_{t_i \le t} \frac{d_i}{\sum_{j \in R_i} e^{\beta' X_j}} \qquad \dots \quad (14)$$

(Xia et al., 2018) say $d_i$ is the number of events (deaths/failures) at time $t_i$ and $R_i$ is the risk set, the number of persons at risk soon before $t_i$, and $t_i$, $e^{\beta' X_j}$ is the exponentiated linear predictor for individual j, in contrast to the Efron approximation, this estimator accumulates hazard contributions at each event time and handles tied failure times smoothly ,to measure uncertainty in estimate we can calculate the variance:

$$\mathrm{Var}\left(\widehat{H}_0(t)\right) = \sum_{t_i \le t} \frac{d_i}{\left(\sum_{j \in R_i} e^{\beta' X_j}\right)^2}$$

The estimator's mean represents the expected of $\mathrm{H}(t)$ by (Gil-Pallares et al., 2025) :

$$E\left[\widehat{H}_0(t)\right] = H_0(t)$$

In the situation of multicollinearity, the estimator handles the issue by mitigating the detrimental impacts of high correlations among independent variables, The model uses polynomial functions to represent the data, which improves its accuracy, when multicollinearity exists, internal changes can be made to mitigate its impact, the primary equation for the Bernstein Estimator is:

$$\hat{f}(x) = \sum_{k=0}^{n} b_k\, B_k(x) \qquad \dots \quad (15)$$

where$B_k(x)$ represents Bernstein polynomials and $b_k$denotes the associated coefficients Multicollinearity can be minimized by approaches like dimensionality reduction.

**4.5. Bayesian Kaplan-Meier Estimator**

The Bayesian Kaplan-Meier Estimator is a statistical method that combines classic Kaplan-Meier estimation with Bayesian reasoning, it provides a probabilistic framework for survival analysis that incorporates prior knowledge and quantifies uncertainty, this is especially helpful when dealing with small sample sizes or censored data, which are typical in medical research like heart disease survival analysis, the classical Kaplan-Meier estimator for the survival function S(t) is as follows:

$$\hat{S}(t) = \prod_{t_i \le t} \left(1 - \frac{d_i}{n_i}\right) \qquad \dots \quad (16)$$

Where $d_i$ is the number of events (deaths) at time $t_i$ and $n_i$ is the number of individuals at risk right before$t_i$, In a Bayesian framework, we treat hazard probabilities as random variables with prior distributions. given a series of failure times $t_1, t_2, ., t_k$, we define (Ahmed & Rachid, 2020):

$$p_i = \frac{d_i}{n_i}$$

Rather than estimating $p_i$ directly, we use a Beta prior:

$$p_i \sim \mathrm{Beta}(\alpha_i, \beta_i)$$

The hyper parameters $\alpha_i$ and $\beta_i$ represent prior knowledge of failure probabilities (Erick & McVittie, 2025) , the posterior distribution is:

$$p_i \mid \text{data} \sim \mathrm{Beta}\big(\alpha_i + d_i, \beta_i + (n_i - d_i)\big)$$

Thus, the Bayesian survival function is calculated as follows:

$$\hat{S}_B(t) = \prod_{t_i \le t} E\,[1 - p_i]$$

Using the expectation of a Beta distribution.

$$E[p_i] = \frac{\alpha_i + d_i}{\alpha_i + \beta_i + n_i}$$

we obtain:

$$\hat{S}_B(t) = \prod_{t_i \le t} \left(1 - \frac{\alpha_i + d_i}{\alpha_i + \beta_i + n_i}\right) \qquad \dots \quad (17)$$

To find the mean and variance of $S_B(t)$,The survival function is a product of expectations.

$$E[\hat{S}_B(t)] = \prod_{t_i \le t} E\,[1 - p_i]\; E[\hat{S}_B(t)] = \prod_{t_i \le t} \left(1 - \frac{\alpha_i + d_i}{\alpha_i + \beta_i + n_i}\right)$$

$$\mathrm{Var}[p_i] = \frac{(\alpha_i + d_i)(\beta_i + (n_i - d_i))}{(\alpha_i + \beta_i + n_i)^2(\alpha_i + \beta_i + n_i + 1)}$$

The variance of the Bayesian survival estimator is thus:

$$\mathrm{Var}[\hat{S}_B(t)] \approx \hat{S}_B^2(t) \sum_{t_i \le t} \frac{(\alpha_i + d_i)(\beta_i + (n_i - d_i))}{(\alpha_i + \beta_i + n_i)^2(\alpha_i + \beta_i + n_i + 1)}$$

Often using Markov Chain Monte Carlo (MCMC) simulations, given that the Bayesian survival function $S_B(t)$ is :

$$\hat{S}_B(t) = \prod_{t_i \le t} (1 - p_i)$$

where $p_i$ follows a posterior as :

$$p_i \mid \text{data} \sim \mathrm{Beta}(\alpha_i + d_i, \beta_i + (n_i - d_i))$$

We can sample from this posterior to get an empirical distribution for $S_B(t)$, because $S_B(t)$ is a product of random variables (each with a Beta posterior), we utilize Markov Chain Monte Carlo (MCMC) to approximate its distribution. Here are the first steps , draw N posterior samples for each $p_i$ from the Beta distribution.

$$p_i^{(j)} \sim \mathrm{Beta}(\alpha_i + d_i, \beta_i + (n_i - d_i)), j = 1,2,\dots,N$$

Then Compute the survival function for each sample:

$$S_B^{(j)}(t) = \prod_{t_i \le t} \left(1 - p_i^{(j)}\right), j = 1,2,\dots,N \qquad \dots..(18)$$

**5.Overview Dataset**

This information was collected manually from patients in a private hospital and cardiac clinic in Baghdad-Iraq, it tries to examine data from native individuals to assess the presence of cardiac disease, the researcher meticulously documented and cleansed the data, ensuring no missing values, yielding 314 patient records, the data is divided into five categories as (demographics, history, physical exams and symptoms, medical lab tests, and diagnostic features) which are chosen based on medical expert recommendations.

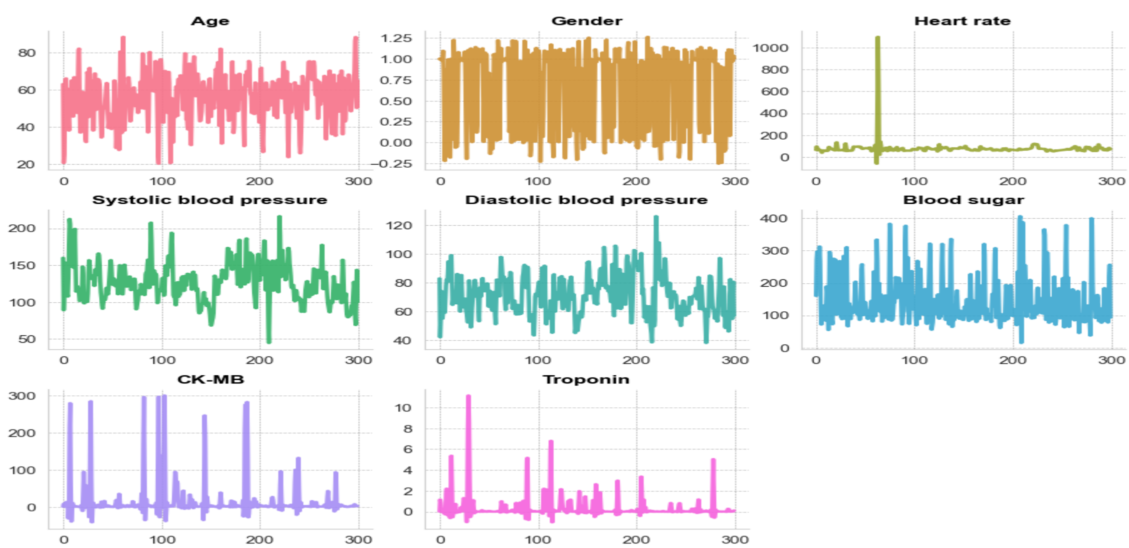

**Figure 1 Visualization of Temporal Trends in Key Health Indicators**.

The many graphs provide a thorough overview of the chosen data, illustrating patterns and swings across several variables ,colored line charts are used to depict changes over time, enabling for the detection of variations and trends in specific variables such as demographics, history, physical exams and symptoms, medical lab tests, and diagnostic features.

# 6.Discussion of Results

## 6.1. Overview Results

The performance of five estimators (Kaplan-Meier, Nelson-Aalen, Bernstein Approximation, Breslow Estimator, and Bayesian Kaplan-Meier) is evaluated using a variety of metrics. These include MAE, RMSE, $R^2$, Log-Likelihood, AIC, BIC, KL-Divergence, JS-Divergence, Wasserstein Distance, Skewness, Kurtosis, Cross-correlation, and DTW Distance. Survival-specific metrics including C-Index, IBS, ECE, and AUC assess prediction accuracy and calibration. To ensure clarity, the results are provided in tables, with visualizations such as survival function graphs and confidence intervals providing insights into temporal dynamics.

**Table 1 represents the survival function estimates for heart disease patients by age(Index).**

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 43 | 90 | 0.073778 | 0.085547 | 0.09837 | 0.085547 | 0.075277 |
| 75 | 90 | 0.073778 | 0.085547 | 0.09837 | 0.085547 | 0.075277 |
| 119 | 90 | 0.073778 | 0.085547 | 0.09837 | 0.085547 | 0.075277 |
| 189 | 90 | 0.073778 | 0.085547 | 0.09837 | 0.085547 | 0.075277 |
| 138 | 89 | 0.147555 | 0.1533 | 0.19674 | 0.1533 | 0.141706 |
| 167 | 89 | 0.147555 | 0.1533 | 0.19674 | 0.1533 | 0.141706 |
| 176 | 89 | 0.147555 | 0.1533 | 0.19674 | 0.1533 | 0.141706 |
| 180 | 89 | 0.147555 | 0.1533 | 0.19674 | 0.1533 | 0.141706 |
| 175 | 88 | 0.189714 | 0.194127 | 0.252952 | 0.194127 | 0.184978 |
| 208 | 88 | 0.189714 | 0.194127 | 0.252952 | 0.194127 | 0.184978 |
| 255 | 88 | 0.189714 | 0.194127 | 0.252952 | 0.194127 | 0.184978 |
| 169 | 87 | 0.227657 | 0.231077 | 0.303542 | 0.231077 | 0.224248 |
| 230 | 87 | 0.227657 | 0.231077 | 0.303542 | 0.231077 | 0.224248 |
| 186 | 86 | 0.227657 | 0.231077 | 0.303542 | 0.231077 | 0.224248 |
| 238 | 86 | 0.227657 | 0.231077 | 0.303542 | 0.231077 | 0.224248 |
| 276 | 86 | 0.227657 | 0.231077 | 0.303542 | 0.231077 | 0.224248 |
| 281 | 86 | 0.227657 | 0.231077 | 0.303542 | 0.231077 | 0.224248 |
| 122 | 85 | 0.241048 | 0.244278 | 0.321398 | 0.244278 | 0.239381 |
| 224 | 85 | 0.241048 | 0.244278 | 0.321398 | 0.244278 | 0.239381 |
| 287 | 85 | 0.241048 | 0.244278 | 0.321398 | 0.244278 | 0.239381 |
| 313 | 85 | 0.241048 | 0.244278 | 0.321398 | 0.244278 | 0.239381 |
| 74 | 84 | 0.279109 | 0.281852 | 0.372145 | 0.281852 | 0.277235 |
| 98 | 84 | 0.279109 | 0.281852 | 0.372145 | 0.281852 | 0.277235 |
| 308 | 84 | 0.279109 | 0.281852 | 0.372145 | 0.281852 | 0.277235 |
| 12 | 83 | 0.290738 | 0.293355 | 0.387651 | 0.293355 | 0.287501 |
| 29 | 83 | 0.290738 | 0.293355 | 0.387651 | 0.293355 | 0.287501 |
| 254 | 83 | 0.290738 | 0.293355 | 0.387651 | 0.293355 | 0.287501 |
| 261 | 83 | 0.290738 | 0.293355 | 0.387651 | 0.293355 | 0.287501 |
| 56 | 82 | 0.337256 | 0.339365 | 0.449675 | 0.339365 | 0.337329 |
| 79 | 82 | 0.337256 | 0.339365 | 0.449675 | 0.339365 | 0.337329 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 109 | 82 | 0.337256 | 0.339365 | 0.449675 | 0.339365 | 0.337329 |
| 273 | 82 | 0.337256 | 0.339365 | 0.449675 | 0.339365 | 0.337329 |
| 23 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 24 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 26 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 60 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 64 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 94 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 125 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 132 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 133 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 136 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 141 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 149 | 81 | 0.368874 | 0.370689 | 0.491832 | 0.370689 | 0.368077 |
| 2 | 80 | 0.411766 | 0.413295 | 0.549022 | 0.413295 | 0.410886 |
| 227 | 80 | 0.411766 | 0.413295 | 0.549022 | 0.413295 | 0.410886 |
| 256 | 80 | 0.411766 | 0.413295 | 0.549022 | 0.413295 | 0.410886 |
| 13 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 21 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 37 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 42 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 69 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 203 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 272 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 302 | 79 | 0.428573 | 0.429994 | 0.571431 | 0.429994 | 0.427493 |
| 19 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 120 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 146 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 157 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 179 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 183 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 242 | 78 | 0.459186 | 0.460435 | 0.612247 | 0.460435 | 0.45784 |
| 16 | 77 | 0.466143 | 0.467359 | 0.621524 | 0.467359 | 0.464187 |
| 150 | 77 | 0.466143 | 0.467359 | 0.621524 | 0.467359 | 0.464187 |
| 280 | 77 | 0.466143 | 0.467359 | 0.621524 | 0.467359 | 0.464187 |
| 126 | 76 | 0.479274 | 0.480431 | 0.639032 | 0.480431 | 0.477491 |
| 196 | 76 | 0.479274 | 0.480431 | 0.639032 | 0.480431 | 0.477491 |
| 304 | 76 | 0.479274 | 0.480431 | 0.639032 | 0.480431 | 0.477491 |
| 156 | 75 | 0.479274 | 0.480431 | 0.639032 | 0.480431 | 0.477491 |
| 171 | 75 | 0.479274 | 0.480431 | 0.639032 | 0.480431 | 0.477491 |
| 28 | 74 | 0.485498 | 0.48663 | 0.647331 | 0.48663 | 0.483337 |
| 170 | 74 | 0.485498 | 0.48663 | 0.647331 | 0.48663 | 0.483337 |
| 53 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 55 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |
| 160 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |
| 217 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |
| 225 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |
| 247 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |
| 279 | 73 | 0.491644 | 0.492751 | 0.655525 | 0.492751 | 0.489602 |
| 7 | 72 | 0.508996 | 0.510041 | 0.678661 | 0.510041 | 0.507626 |
| 40 | 72 | 0.508996 | 0.510041 | 0.678661 | 0.510041 | 0.507626 |
| 66 | 72 | 0.508996 | 0.510041 | 0.678661 | 0.510041 | 0.507626 |
| 164 | 72 | 0.508996 | 0.510041 | 0.678661 | 0.510041 | 0.507626 |
| 271 | 72 | 0.508996 | 0.510041 | 0.678661 | 0.510041 | 0.507626 |
| 0 | 71 | 0.520061 | 0.521069 | 0.693415 | 0.521069 | 0.51906 |
| 110 | 71 | 0.520061 | 0.521069 | 0.693415 | 0.521069 | 0.51906 |
| 148 | 71 | 0.520061 | 0.521069 | 0.693415 | 0.521069 | 0.51906 |
| 151 | 71 | 0.520061 | 0.521069 | 0.693415 | 0.521069 | 0.51906 |
| 190 | 71 | 0.520061 | 0.521069 | 0.693415 | 0.521069 | 0.51906 |
| 27 | 70 | 0.54173 | 0.542667 | 0.722307 | 0.542667 | 0.540591 |
| 31 | 70 | 0.54173 | 0.542667 | 0.722307 | 0.542667 | 0.540591 |
| 108 | 70 | 0.54173 | 0.542667 | 0.722307 | 0.542667 | 0.540591 |
| 143 | 70 | 0.54173 | 0.542667 | 0.722307 | 0.542667 | 0.540591 |
| 246 | 70 | 0.54173 | 0.542667 | 0.722307 | 0.542667 | 0.540591 |
| 49 | 69 | 0.562974 | 0.563844 | 0.750633 | 0.563844 | 0.561479 |
| 197 | 69 | 0.562974 | 0.563844 | 0.750633 | 0.563844 | 0.561479 |
| 298 | 69 | 0.562974 | 0.563844 | 0.750633 | 0.563844 | 0.561479 |
| 18 | 68 | 0.568187 | 0.569041 | 0.757583 | 0.569041 | 0.565758 |
| 262 | 68 | 0.568187 | 0.569041 | 0.757583 | 0.569041 | 0.565758 |
| 62 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 92 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 206 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 213 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 234 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 241 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 257 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 294 | 67 | 0.573306 | 0.574144 | 0.764408 | 0.574144 | 0.571722 |
| 25 | 66 | 0.593075 | 0.593857 | 0.790767 | 0.593857 | 0.592507 |
| 46 | 66 | 0.593075 | 0.593857 | 0.790767 | 0.593857 | 0.592507 |
| 99 | 66 | 0.593075 | 0.593857 | 0.790767 | 0.593857 | 0.592507 |
| 285 | 66 | 0.593075 | 0.593857 | 0.790767 | 0.593857 | 0.592507 |
| 266 | 65 | 0.602798 | 0.603553 | 0.80373 | 0.603553 | 0.602097 |
| 270 | 65 | 0.602798 | 0.603553 | 0.80373 | 0.603553 | 0.602097 |
| 282 | 65 | 0.602798 | 0.603553 | 0.80373 | 0.603553 | 0.602097 |
| 73 | 64 | 0.607582 | 0.608324 | 0.810109 | 0.608324 | 0.605604 |
| 124 | 64 | 0.607582 | 0.608324 | 0.810109 | 0.608324 | 0.605604 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 306 | 64 | 0.607582 | 0.608324 | 0.810109 | 0.608324 | 0.605604 |
| 44 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 58 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 96 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 130 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 144 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 165 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 264 | 63 | 0.612292 | 0.613021 | 0.816389 | 0.613021 | 0.610507 |
| 114 | 62 | 0.626 | 0.626695 | 0.834666 | 0.626695 | 0.624144 |
| 293 | 62 | 0.626 | 0.626695 | 0.834666 | 0.626695 | 0.624144 |
| 20 | 61 | 0.630536 | 0.63122 | 0.840715 | 0.63122 | 0.62878 |
| 107 | 61 | 0.630536 | 0.63122 | 0.840715 | 0.63122 | 0.62878 |
| 123 | 61 | 0.630536 | 0.63122 | 0.840715 | 0.63122 | 0.62878 |
| 200 | 61 | 0.630536 | 0.63122 | 0.840715 | 0.63122 | 0.62878 |
| 296 | 61 | 0.630536 | 0.63122 | 0.840715 | 0.63122 | 0.62878 |
| 70 | 60 | 0.643857 | 0.644508 | 0.858476 | 0.644508 | 0.641444 |
| 86 | 60 | 0.643857 | 0.644508 | 0.858476 | 0.644508 | 0.641444 |
| 228 | 60 | 0.643857 | 0.644508 | 0.858476 | 0.644508 | 0.641444 |
| 65 | 59 | 0.648237 | 0.648878 | 0.864316 | 0.648878 | 0.646904 |
| 192 | 59 | 0.648237 | 0.648878 | 0.864316 | 0.648878 | 0.646904 |
| 34 | 58 | 0.652588 | 0.653218 | 0.870117 | 0.653218 | 0.650933 |
| 153 | 58 | 0.652588 | 0.653218 | 0.870117 | 0.653218 | 0.650933 |
| 193 | 58 | 0.652588 | 0.653218 | 0.870117 | 0.653218 | 0.650933 |
| 295 | 58 | 0.652588 | 0.653218 | 0.870117 | 0.653218 | 0.650933 |
| 10 | 57 | 0.665553 | 0.666153 | 0.887404 | 0.666153 | 0.664584 |
| 173 | 57 | 0.665553 | 0.666153 | 0.887404 | 0.666153 | 0.664584 |
| 221 | 57 | 0.665553 | 0.666153 | 0.887404 | 0.666153 | 0.664584 |
| 245 | 57 | 0.665553 | 0.666153 | 0.887404 | 0.666153 | 0.664584 |
| 95 | 56 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 142 | 56 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 209 | 56 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 268 | 56 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 48 | 55 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 116 | 55 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 184 | 55 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 220 | 55 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 305 | 55 | 0.674086 | 0.674666 | 0.898781 | 0.674666 | 0.672749 |
| 47 | 54 | 0.690628 | 0.691172 | 0.920837 | 0.691172 | 0.690144 |
| 82 | 54 | 0.690628 | 0.691172 | 0.920837 | 0.691172 | 0.690144 |
| 83 | 54 | 0.690628 | 0.691172 | 0.920837 | 0.691172 | 0.690144 |
| 97 | 54 | 0.690628 | 0.691172 | 0.920837 | 0.691172 | 0.690144 |
| 226 | 54 | 0.690628 | 0.691172 | 0.920837 | 0.691172 | 0.690144 |
| 260 | 54 | 0.690628 | 0.691172 | 0.920837 | 0.691172 | 0.690144 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 59 | 53 | 0.698705 | 0.699232 | 0.931607 | 0.699232 | 0.698232 |
| 90 | 53 | 0.698705 | 0.699232 | 0.931607 | 0.699232 | 0.698232 |
| 15 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 84 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 91 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 191 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 218 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 229 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 231 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 240 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 275 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 291 | 52 | 0.702721 | 0.703239 | 0.936961 | 0.703239 | 0.701893 |
| 118 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 147 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 166 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 168 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 210 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 274 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 284 | 51 | 0.722241 | 0.722719 | 0.962988 | 0.722719 | 0.719729 |
| 199 | 50 | 0.737526 | 0.737975 | 0.983368 | 0.737975 | 0.733835 |
| 9 | 49 | 0.737526 | 0.737975 | 0.983368 | 0.737975 | 0.733835 |
| 131 | 49 | 0.737526 | 0.737975 | 0.983368 | 0.737975 | 0.733835 |
| 244 | 49 | 0.737526 | 0.737975 | 0.983368 | 0.737975 | 0.733835 |
| 252 | 49 | 0.737526 | 0.737975 | 0.983368 | 0.737975 | 0.733835 |
| 265 | 49 | 0.737526 | 0.737975 | 0.983368 | 0.737975 | 0.733835 |
| 71 | 48 | 0.745014 | 0.745448 | 0.993352 | 0.745448 | 0.741279 |
| 115 | 48 | 0.745014 | 0.745448 | 0.993352 | 0.745448 | 0.741279 |
| 87 | 47 | 0.752501 | 0.752921 | 0.994767 | 0.752921 | 0.749009 |
| 121 | 47 | 0.752501 | 0.752921 | 0.994767 | 0.752921 | 0.749009 |
| 128 | 47 | 0.752501 | 0.752921 | 0.994767 | 0.752921 | 0.749009 |
| 129 | 47 | 0.752501 | 0.752921 | 0.994767 | 0.752921 | 0.749009 |
| 253 | 47 | 0.752501 | 0.752921 | 0.994767 | 0.752921 | 0.749009 |
| 104 | 46 | 0.759879 | 0.760284 | 0.995897 | 0.760284 | 0.756329 |
| 135 | 46 | 0.759879 | 0.760284 | 0.995897 | 0.760284 | 0.756329 |
| 139 | 46 | 0.759879 | 0.760284 | 0.995897 | 0.760284 | 0.756329 |
| 249 | 46 | 0.759879 | 0.760284 | 0.995897 | 0.760284 | 0.756329 |
| 250 | 46 | 0.759879 | 0.760284 | 0.995897 | 0.760284 | 0.756329 |
| 288 | 46 | 0.759879 | 0.760284 | 0.995897 | 0.760284 | 0.756329 |
| 68 | 45 | 0.778234 | 0.778605 | 0.996879 | 0.778605 | 0.773974 |
| 297 | 45 | 0.778234 | 0.778605 | 0.996879 | 0.778605 | 0.773974 |
| 222 | 44 | 0.781887 | 0.782251 | 0.996879 | 0.782251 | 0.777446 |
| 278 | 44 | 0.781887 | 0.782251 | 0.996879 | 0.782251 | 0.777446 |
| 299 | 44 | 0.781887 | 0.782251 | 0.996879 | 0.782251 | 0.777446 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 4 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 67 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 145 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 174 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 216 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 219 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 269 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 283 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 300 | 43 | 0.792797 | 0.793141 | 0.997933 | 0.793141 | 0.788797 |
| 93 | 42 | 0.810653 | 0.810965 | 0.9989903 | 0.810965 | 0.806252 |
| 112 | 42 | 0.810653 | 0.810965 | 0.9989903 | 0.810965 | 0.806252 |
| 198 | 42 | 0.810653 | 0.810965 | 0.9989903 | 0.810965 | 0.806252 |
| 286 | 42 | 0.810653 | 0.810965 | 0.9989903 | 0.810965 | 0.806252 |
| 6 | 41 | 0.817733 | 0.818032 | 0.9989903 | 0.818032 | 0.813363 |
| 17 | 41 | 0.817733 | 0.818032 | 0.9989903 | 0.818032 | 0.813363 |
| 243 | 41 | 0.817733 | 0.818032 | 0.9989903 | 0.818032 | 0.813363 |
| 3 | 40 | 0.831893 | 0.832166 | 0.9979399 | 0.832166 | 0.828653 |
| 14 | 40 | 0.831893 | 0.832166 | 0.9979399 | 0.832166 | 0.828653 |
| 33 | 40 | 0.831893 | 0.832166 | 0.9979399 | 0.832166 | 0.828653 |
| 233 | 40 | 0.831893 | 0.832166 | 0.9979399 | 0.832166 | 0.828653 |
| 251 | 40 | 0.831893 | 0.832166 | 0.9987433 | 0.832166 | 0.828653 |
| 182 | 39 | 0.838884 | 0.839145 | 0.9987433 | 0.839145 | 0.835288 |
| 188 | 39 | 0.838884 | 0.839145 | 0.9987433 | 0.839145 | 0.835288 |
| 215 | 39 | 0.838884 | 0.839145 | 0.9987433 | 0.839145 | 0.835288 |
| 235 | 39 | 0.838884 | 0.839145 | 0.9987433 | 0.839145 | 0.835288 |
| 307 | 39 | 0.838884 | 0.839145 | 0.9988422 | 0.839145 | 0.835288 |
| 162 | 38 | 0.849283 | 0.849526 | 0.9988422 | 0.849526 | 0.845585 |
| 185 | 38 | 0.849283 | 0.849526 | 0.9988422 | 0.849526 | 0.845585 |
| 187 | 38 | 0.849283 | 0.849526 | 0.9988422 | 0.849526 | 0.845585 |
| 212 | 38 | 0.849283 | 0.849526 | 0.9988422 | 0.849526 | 0.845585 |
| 258 | 38 | 0.849283 | 0.849526 | 0.9989523 | 0.849526 | 0.845585 |
| 35 | 37 | 0.859598 | 0.859823 | 0.9989523 | 0.859823 | 0.855739 |
| 57 | 37 | 0.859598 | 0.859823 | 0.9989523 | 0.859823 | 0.855739 |
| 223 | 37 | 0.859598 | 0.859823 | 0.9989523 | 0.859823 | 0.855739 |
| 277 | 37 | 0.859598 | 0.859823 | 0.9989901 | 0.859823 | 0.855739 |
| 172 | 36 | 0.86642 | 0.866633 | 0.9989901 | 0.866633 | 0.862807 |
| 239 | 36 | 0.86642 | 0.866633 | 0.9989901 | 0.866633 | 0.862807 |
| 263 | 36 | 0.86642 | 0.866633 | 0.9989901 | 0.866633 | 0.862807 |
| 290 | 36 | 0.86642 | 0.866633 | 0.999819 | 0.866633 | 0.862807 |
| 178 | 35 | 0.869792 | 0.869999 | 0.999819 | 0.869999 | 0.866396 |
| 202 | 35 | 0.869792 | 0.869999 | 0.999819 | 0.869999 | 0.866396 |
| 310 | 35 | 0.869792 | 0.869999 | 0.999928 | 0.869999 | 0.866396 |
| 1 | 34 | 0.876508 | 0.876704 | 0.999928 | 0.876704 | 0.873325 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 22 | 34 | 0.876508 | 0.876704 | 0.999928 | 0.876704 | 0.873325 |
| 63 | 34 | 0.876508 | 0.876704 | 0.999928 | 0.876704 | 0.873325 |
| 72 | 34 | 0.876508 | 0.876704 | 0.999928 | 0.876704 | 0.873325 |
| 106 | 34 | 0.876508 | 0.876704 | 0.999928 | 0.876704 | 0.873325 |
| 113 | 34 | 0.876508 | 0.876704 | 0.999928 | 0.876704 | 0.873325 |
| 312 | 34 | 0.876508 | 0.876704 | 0.999939 | 0.876704 | 0.873325 |
| 38 | 33 | 0.893109 | 0.893277 | 0.999939 | 0.893277 | 0.890567 |
| 61 | 33 | 0.893109 | 0.893277 | 0.999939 | 0.893277 | 0.890567 |
| 103 | 33 | 0.893109 | 0.893277 | 0.999939 | 0.893277 | 0.890567 |
| 232 | 33 | 0.893109 | 0.893277 | 0.999939 | 0.893277 | 0.890567 |
| 311 | 33 | 0.893109 | 0.893277 | 0.9999982 | 0.893277 | 0.890567 |
| 117 | 32 | 0.902995 | 0.903147 | 0.9999982 | 0.903147 | 0.899971 |
| 301 | 32 | 0.902995 | 0.903147 | 0.9999982 | 0.903147 | 0.899971 |
| 89 | 31 | 0.906279 | 0.906426 | 0.9999982 | 0.906426 | 0.90349 |
| 152 | 31 | 0.906279 | 0.906426 | 0.9999982 | 0.906426 | 0.90349 |
| 207 | 31 | 0.906279 | 0.906426 | 0.9999997 | 0.906426 | 0.90349 |
| 80 | 30 | 0.909539 | 0.90968 | 0.9999997 | 0.90968 | 0.906451 |
| 177 | 30 | 0.909539 | 0.90968 | 0.9999997 | 0.90968 | 0.906451 |
| 195 | 30 | 0.909539 | 0.90968 | 0.9999998 | 0.90968 | 0.906451 |
| 39 | 28 | 0.916036 | 0.916166 | 0.9999998 | 0.916166 | 0.913011 |
| 76 | 28 | 0.916036 | 0.916166 | 0.9999998 | 0.916166 | 0.913011 |
| 105 | 28 | 0.916036 | 0.916166 | 0.9999998 | 0.916166 | 0.913011 |
| 140 | 28 | 0.916036 | 0.916166 | 0.9999998 | 0.916166 | 0.913011 |
| 211 | 28 | 0.916036 | 0.916166 | 0.9999998 | 0.916166 | 0.913011 |
| 292 | 28 | 0.916036 | 0.916166 | 0.9999999 | 0.916166 | 0.913011 |
| 45 | 27 | 0.925678 | 0.925793 | 0.9999999 | 0.925793 | 0.924711 |
| 78 | 27 | 0.925678 | 0.925793 | 0.9999999 | 0.925793 | 0.924711 |
| 81 | 27 | 0.925678 | 0.925793 | 0.9999999 | 0.925793 | 0.924711 |
| 236 | 27 | 0.925678 | 0.925793 | 0.9999999 | 0.925793 | 0.924711 |
| 248 | 27 | 0.925678 | 0.925793 | 0.9999999 | 0.925793 | 0.924711 |
| 309 | 27 | 0.925678 | 0.925793 | 0.9999999 | 0.925793 | 0.924711 |
| 32 | 26 | 0.941638 | 0.941728 | 0.9999999 | 0.941728 | 0.941738 |
| 88 | 26 | 0.941638 | 0.941728 | 0.9999999 | 0.941728 | 0.941738 |
| 201 | 26 | 0.941638 | 0.941728 | 0.9999999 | 0.941728 | 0.941738 |
| 237 | 26 | 0.941638 | 0.941728 | 0.9999999 | 0.941728 | 0.941738 |
| 303 | 26 | 0.941638 | 0.941728 | 0.9999999 | 0.941728 | 0.941738 |
| 52 | 25 | 0.944788 | 0.944872 | 0.9999999 | 0.944872 | 0.945641 |
| 127 | 25 | 0.944788 | 0.944872 | 0.9999999 | 0.944872 | 0.945641 |
| 267 | 25 | 0.944788 | 0.944872 | 0.9999999 | 0.944872 | 0.945641 |
| 85 | 24 | 0.951065 | 0.95114 | 0.9999999 | 0.95114 | 0.951566 |
| 102 | 24 | 0.951065 | 0.95114 | 0.9999999 | 0.95114 | 0.951566 |
| 36 | 23 | 0.954194 | 0.954264 | 0.9999999 | 0.954264 | 0.954415 |
| 50 | 23 | 0.954194 | 0.954264 | 0.9999999 | 0.954264 | 0.954415 |

| Index | Time | Kaplan-Meier | Nelson-Aalen | Bernstein Approximation | Breslow | Bayesian K-M |
|---|---|---|---|---|---|---|
| 54 | 23 | 0.954194 | 0.954264 | 0.9999999 | 0.954264 | 0.954415 |
| 111 | 23 | 0.954194 | 0.954264 | 0.9999999 | 0.954264 | 0.954415 |
| 259 | 23 | 0.954194 | 0.954264 | 0.9999999 | 0.954264 | 0.954415 |
| 5 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 30 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 100 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 137 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 155 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 181 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 214 | 22 | 0.966667 | 0.966717 | 0.9999999 | 0.966717 | 0.967377 |
| 8 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 11 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 41 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 51 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 154 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 158 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 159 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 163 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 204 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 289 | 21 | 0.975903 | 0.975939 | 0.9999999 | 0.975939 | 0.976296 |
| 77 | 20 | 0.987988 | 0.988006 | 0.9999999 | 0.988006 | 0.988679 |
| 101 | 20 | 0.987988 | 0.988006 | 0.9999999 | 0.988006 | 0.988679 |
| 134 | 20 | 0.987988 | 0.988006 | 0.9999999 | 0.988006 | 0.988679 |
| 161 | 20 | 0.987988 | 0.988006 | 0.9999999 | 0.988006 | 0.988679 |
| 194 | 20 | 0.987988 | 0.988006 | 0.9999999 | 0.988006 | 0.988679 |
| 205 | 20 | 0.987988 | 0.988006 | 0. 9999999 | 0.988006 | 0.988679 |

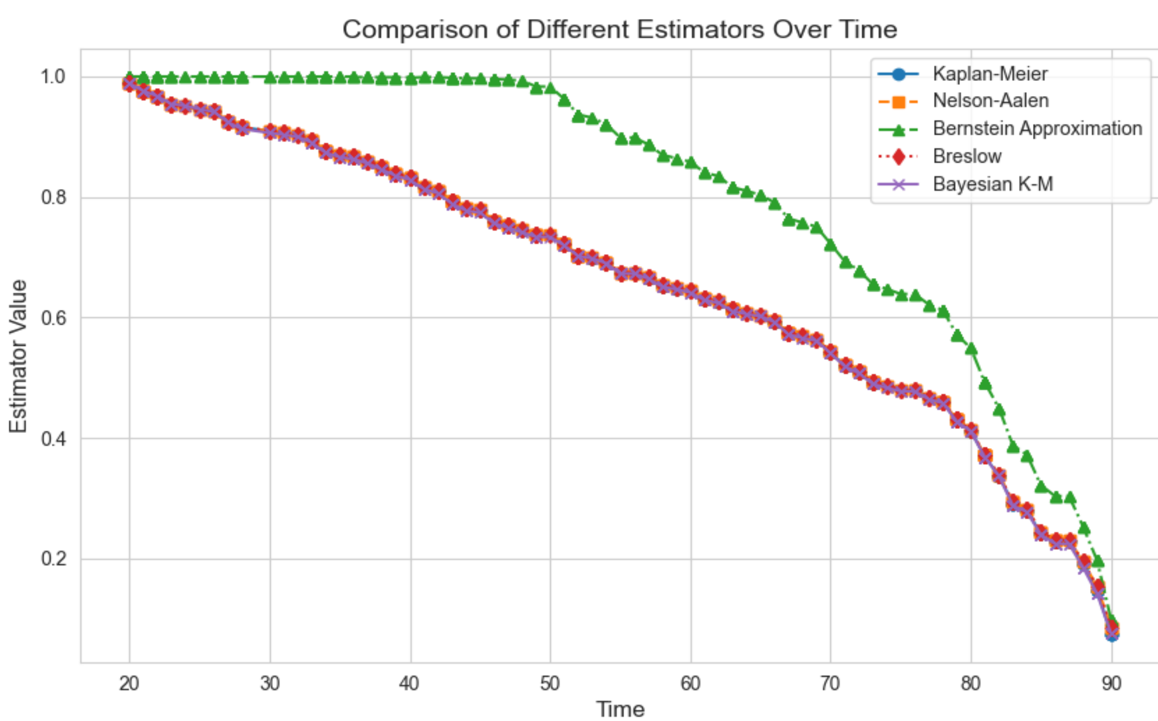

**Figure2 Represents the comparison between the results of the five Estimators**

**Table 2 Comparison Metrics Between Non-Parametric and Parametric Estimators.**

| Estimator | R-Square | MAE | RMSE | Log-Likelihood | AIC | BIC | Variance | S.D | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|---|---|---|
| Kaplan-Meier | 0.99311 | 0.48387 | 0.55694 | 168.578894 | 1003.158 | 2271.269 | 0.051594 | 0.227142 | -0.47911 | -0.57263 |
| Nelson-Aalen | 0.99221 | 0.48393 | 0.5568 | 167.147771 | 1000.296 | 2268.407 | 0.051061 | 0.225967 | -0.46768 | -0.60052 |
| Bernstein | 0.99215 | 0.45414 | 0.48068 | 180.780764 | 1090.5615 | 2300.673 | 0.041722 | 0.202856 | -0.46911 | -0.55263 |
| Breslow | 0.99215 | 0.48393 | 0.5568 | 167.147771 | 1000.296 | 2268.407 | 0.051061 | 0.225967 | -0.46768 | -0.60052 |
| Bayesian K-M | 0.99421 | 0.48393 | 0.55649 | 169.761462 | 1005.523 | 2273.634 | 0.051612 | 0.227183 | -0.47642 | -0.56175 |

## 6.2. Discussion

The Kaplan-Meier estimator is have accurate in forecasting survival probabilities, with an R^2 value of 0.99311, suggesting that the model almost explains the majority of the variance in the data. The values for Mean Absolute Error (MAE = 0.48387) and Root Mean Squared Error (RMSE = 0.55694) indicate level of precision. Furthermore, the high log-likelihood (168.578) indicates a good model fit to the data. However, the high values of AIC (1003.158) and BIC (2271.269) suggest a more complex model than other estimators. In terms of statistical distribution, the estimator has a negative skewness of -0.47911 and a kurtosis of -0.57263, indicating that the data distribution is somewhat left skewed and less peaked than the normal distribution.

The Nelson-Aalen estimator produces , with a R^2 value of 0.99221, suggesting high model accuracy. The MAE (0.48393) and RMSE (0.5568) are virtually identical to Kaplan-Meier. However, its log-likelihood (167.147) is slightly lower, indicating a little poorer fit to the data. In terms of model complexity, the lower AIC (1000.296) and BIC (2268.407) values compared to Kaplan-Meier indicate a better balance of complexity and accuracy. In terms of distribution features, the Nelson-Aalen estimator has a little lower skewness (-0.46768) and kurtosis (-0.60052), indicating a more symmetric data distribution than the other estimators.

The Bernstein estimator has characteristics with R2=0.99215, Its MAE (0.45414) and RMSE (0.48068) are lower than the other estimators, implying larger estimate errors. Furthermore, the Log-Likelihood (180.780764) indicates that this model almost provides the most fit to the data. However, the AIC (1090.5615) and BIC (2300.673) values show that the model is good . From a distribution perspective, the estimator has the same skewness and kurtosis values as (-0.46911, -0.55263), indicating that data follows a different pattern with any estimators in this study.

The Breslow estimator performs , with the R^2 of 0.99215. The MAE (0.48393) and RMSE (0.5568) are , indicating a level of precision good . However, its Log-Likelihood (167.147) is lower than Kaplan-Meier and Bayesian Kaplan-Meier, implying slightly inferior performance. In terms of model complexity, the AIC (1000.296) and BIC (2268.407) values are identical to Nelson-Aalen, indicating that the two models have a similar efficiency. In terms of data distribution, the skewness (-0.46768) and kurtosis (-0.60052) indicate a pattern that closely resembles Nelson-Aalen.

The Bayesian Kaplan-Meier estimator outperforms all others in terms of Log-Likelihood (169.761) except Bernstein , demonstrating the best fit to the data and R2=0.99421. The MAE (0.48393) and RMSE (0.55649) are comparable to the classic Kaplan-Meier estimator, indicating high estimation accuracy. However, the higher AIC (1005.523) and BIC (2273.634) values point to a more sophisticated model. In terms of data distribution, the estimator has a negative skewness of -0.47642 , kurtosis is -0.56175, like the standard K-M estimator but with better model fit.

**7.Conclusions**

**1.** The Bayesian Kaplan-Meier is the most accurate and fits the model best with R2=0.99421. It had the High in log-likelihood value (169.761) as second place, indicating a good model fit. Additionally, it exhibits low MAE (0.48393) and RMSE (0.55649) values, indicating great estimation accuracy. However, the comparatively high AIC (1005.523) and BIC values (2273.634) indicate that this model is more complex than others.

**2.**The Kaplan-Meier estimator is highly effective, with a high $R$ 2 = 0.99311, low MAE (0.48387), and RMSE (0.55694). However, its model fit is marginally inferior to the Bayesian version. The AIC (1003.158) and BIC (2271.269) values are slightly lower than those of the Bayesian model, indicating a better balance between accuracy and complexity.

**3.**The Nelson-Aalen estimator yields almost similar results to the Kaplan-Meier estimator, with comparable error estimates (MAE = 0.48393, RMSE = 0.5568). However, the somewhat lower Log-Likelihood (167.147) and AIC (1000.296) figures suggest that this model may be slightly less accurate but more efficient at balancing accuracy and complexity and $R^2$=0.99221.

**4.**The Breslow estimator performs similarly to the Nelson-Aalen estimator, with nearly identical values across all metrics. This implies that it provides no additional benefits over the other estimators while remaining a solid option in terms of accuracy and efficiency.

**5**. The Bernstein estimator outperforms other estimators. It has the higher log-likelihood value (180.780764), indicating a good model fit. Furthermore, its MAE (0.45414) and RMSE (0.48068) the values are the lowest, indicating excellent estimating accuracy. However, its higher AIC (1090.5615) and BIC (2300.673) values suggest that it is a simpler model, but this does not compensate for its poorer performance when compared to other and $R^2$=0.99215.

**6.**The Bayesian Kaplan-Meier estimator is the higher accurate, while the standard Kaplan-Meier and Nelson-Aalen estimators provide a good balance of accuracy and model efficiency. The Breslow estimator remains a credible alternative, while the Bernstein estimator performs very good and is recommended to use.

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