On βk Generalized & αk Generalized in Intuitionistic Topological Spaces

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Abstract
In this paper, we introduce some two weaks called βk −generalized and αk −generalized by using βk −intuitionistic open and αk − intuitionistic open and investigate some properties . Finaly we study the relations between this concepts .

Keywords: generalizations, BK

حول التعميمات من النوع βk وαk في الفضاءات التبولوجية الحدسية

هند فاضل عباس

المستخلص
في هذا البحث قدمنا شكلين ضعيفين هما المعمم βk والمعمم αk باستخدام المجموعة المفتوحة الحدسية βk والمجموعة المفتوحة الحدسية αk واستقصينا بعض خواصها . واخيرا درسنا العلاقات بين هذه المفاهيم.

كلمات مفتاحية : تعميمات ، BK

1 -Introduction

In this sections we give some definitions which are needed in this paper .

Definition 1.1 [1] Let $M \subseteq X \neq \emptyset$ and. The Intuitionistic set $\widetilde{M}$ (IS, for short) is the form $\widetilde{M} = \langle x, M_1, M_2 \rangle$ and $M_1 , M_2 \subseteq X$ with condition $M_1 \cap M_2 = \emptyset$ **.** The set M_1 is called "the set of members" of $\widetilde{M}$ and M_2 is called " the set of non-members" of $\widetilde{M}$.

Definition 1. 2 [1] Let $X \neq \emptyset$, and let $\widetilde{M} = \langle x, M_1, M_2 \rangle$, $\widetilde{N} = \langle x, N_1, N_2 \rangle$ are two Intuitionistic sets respectively. Also, let$\{ \widetilde{M}_s ;\ s \in S\}$ be a collection of "Intuitionistic sets in X" , and $\widetilde{M}_i = \langle x, M_s^{(1)}, M_s^{(2)} \rangle$, the following is valid .

1) $\widetilde{M} \subseteq \widetilde{N}$ iff $M_1 \subseteq N_1$ and $N_2 \subseteq M_2$,

2) $\widetilde{M} = \widetilde{N}$ iff $\widetilde{M} \subseteq \widetilde{N}$ and $\widetilde{N} \subseteq \widetilde{M}$,

3) The complement of $\widetilde{M}$ is denoted by $\overline{\widetilde{M}}$ and defined by $\overline{\widetilde{M}} = \langle x, M_2, M_1 \rangle$,

4) $\cup \widetilde{M}_i = \langle x, \cup M_s^{(1)}, \cap M_s^{(2)} \rangle$, $\cap \widetilde{M}_i = \langle x, \cap M_s^{(1)}, \cup M_s^{(2)} \rangle$,

5) $\dot{\emptyset} = \langle x, \emptyset, X \rangle$, $\dot{X} = \langle x, X, \emptyset \rangle$**.**

Definition 1.3 [2] Let $X \neq \emptyset$, $w \in X$ and let $\widetilde{M} = \langle x, M_1, M_2 \rangle$ be an Intuitionistic set the Intuitionistic point (IP f, for briefily) "Is $\dot{w}$ " defined by $\dot{w}= \langle x, \{w\}, \{w\}^c \rangle$ in X. Also

a Vanishing Intuitionistic point defined by Is $\ddot{w} = \langle x, \emptyset, \{w\}^c \rangle$ in X . The Is $\dot{w}$ is said

belong in $\widetilde{M}$ ($\dot{w} \in M$, for brief) iff $w \in M_1$, also Is $\ddot{p}$ contained in $\widetilde{M}$ ($\ddot{w} \in \widetilde{M}$, for short) iff $w \notin M_2$.

Definition 1.4 [3] Let $X \neq \emptyset$. An Intuitionistic topology (ITS, for short) on X is

a collection μ of an "Intuitionistic sets " in X satisfying :

(1) $\dot{\emptyset}, \dot{X} \in \mu$.

(2) μ is closed under finite intersections.

(3) μ is closed under arbitrary unions.

Each element in μ is called "Intuitionistic open set " and denoted by "IOS"

The complement of an " Intuitionistic open set" is called "Intuitionistic closed set"

denoted by "ICS" .

Definition 1.5 [3] Let (X, μ) be an ITS and let $\widetilde{M} = \langle x, M_1, M_2 \rangle \subseteq X$. Then:

$int(\widetilde{M}) = \cup\{ \widetilde{V} : \widetilde{V} \subseteq \widetilde{M}, V \in \mu \}$,

$cl(\widetilde{M}) = \cap\{ \tilde{J} : \widetilde{M} \subseteq \tilde{J}, \bar{\tilde{J}} \in \mu \}$.

Definition 1.6. [3]

Let (X,μ) be an ITS. IS $\widetilde{M}$ of X is said to be

1. IαOS if $\widetilde{M} \subseteq Iint(Icl(Iint(\widetilde{M})))$,
2. IβOS if $\widetilde{M} \subseteq Icl(Iint(Icl(\widetilde{M})))$.

The family of all intuitionistic α-open and β-open sets of (X, μ) are denoted by "IαOS(X)"

and "IβOS(X)" respectively. Also the complement of all intuitionistic α-open and β-open sets of (X, μ) are denoted by "IαCS(X)" and "IβCS(X)" respectively.

Section 2 βk generalized & αk generalized Closed Sets

In this section, we give define and study some properties of βk −generalized & αk −generalized closed sets.

Definition 2.1 Let (X, μ) be an ITS and IS $\widetilde{M} \subseteq X$ is said to be βk intuitionistic open if
there exists intuitionistic closed set $\tilde{F}$ s.t, $\tilde{F} \subseteq \widetilde{M}$ A. The collections of all βk intuitionistic open
sets in intuitionistic topological space (X, μ) denoted by βkIO .

Definition 2.2 Let (X, μ) be an ITS and IS $\widetilde{M} \subseteq X$ is said to be $\beta k -$ generalized intuitionistic closed (for short , βkgIC) set if $\beta kcl(\widetilde{M}) \subseteq \widetilde{Q}$ whenever $\widetilde{M} \subseteq \widetilde{Q}$ and $\widetilde{Q}$ is βk is IOS in (X, μ) . Also IS $\widetilde{M} \subseteq X$
is βk − generalized intuitionistic open (for short , βkgIO) if $\widetilde{M}$ is βkgIC in (X, μ)

.

Definition 2.3 Let (X, μ) be an ITS and IS $\widetilde{M} \subseteq X$ is said to be αk intuitionistic open if
there exists intuitionistic closed set $\tilde{F}$ s.t, $\tilde{F} \subseteq \widetilde{M}$. The collections of all αk intuitionistic open
sets in intuitionistic topological space (X, μ) denoted by αkIO .

Definition 2.4 Let (X, μ) be an ITS and IS $\widetilde{M} \subseteq X$ is said to be $\alpha k -$ generalized intuitionistic closed (for short , αkgIC) set if $\alpha kcl(\widetilde{M}) \subseteq \widetilde{Q}$ whenever $\widetilde{M} \subseteq \widetilde{Q}$ and $\widetilde{Q}$ is αk is IOS in (X, μ) . Also IS $\widetilde{M} \subseteq X$
is αk − generalized intuitionistic open (for short , αkgIO) if $\widetilde{M}$ is αkgIC in (X, μ)

.

Proposition 2.5 If $\widetilde{M}$ is βkIO and βkgIc then $\widetilde{M}$ is βkIc.
Proof. Suppose that $\widetilde{M}$ is βkIO and βkgIc. Since $\widetilde{M}$ is βkIO and $\widetilde{M} \subseteq \widetilde{M}$, we have $\beta kcl(\widetilde{M}) \subseteq \widetilde{M}$, also $\widetilde{M} \subseteq \beta kcl(\widetilde{M})$, thus $\beta kcl(\widetilde{M}) = \widetilde{M}$. Therefore $\widetilde{M}$ is βkIc .

Proposition 2.6 If $\widetilde{M}$ is αkIO and αkgIC then $\widetilde{M}$ is αkIc.
Proof: it's obvious .

Proposition 2.7 If IS $\widetilde{M}$ of X is βkgIc and $\widetilde{M} \subseteq \widetilde{N} \subseteq \beta kcl(\widetilde{M})$, then $\widetilde{N}$ is βkgIC set in X .
Proof. Let $\widetilde{M}$ be βkgIc set s.t, $\widetilde{M} \subseteq \widetilde{N} \subseteq \beta kcl(\widetilde{M})$. Let $\widetilde{W}$ be βkIO of X s.t, $\widetilde{N} \subseteq$ of X s.t, $\widetilde{N}$ _ $\widetilde{W}$.
Since $\widetilde{M}$ is $\beta kcl(\widetilde{M})$, so $\beta kcl(\widetilde{M}) \subseteq \widetilde{W}$. Now $\beta kcl(\widetilde{M}) \subseteq \beta kcl(\widetilde{N}) \subseteq \beta kcl(\beta kcl(\widetilde{M})) = \beta kcl(\widetilde{M}) \subseteq \widetilde{W}$.
Thus $\beta kcl(\widetilde{N}) \subseteq \widetilde{W}$, where $\widetilde{W}$ is βkIO. Therefore $\widetilde{N}$ is $\beta kcl(\widetilde{M})$ set in X .

Proposition 2.8 If IS $\widetilde{M}$ of X is αkgIC and $\widetilde{M} \subseteq \widetilde{N} \subseteq \alpha kcl(\widetilde{M})$, then $\widetilde{N}$ is αkgIC set in X .

Proof. Let $\widetilde{M}$ be αkgIc set s.t, $\widetilde{M} \subseteq \widetilde{N} \subseteq \alpha kcl(\widetilde{M})$. Let $\widetilde{W}$ be αkIO of X s.t, $\widetilde{N} \subseteq$ of X s.t, $\widetilde{N}$ _ $\widetilde{W}$.

Since $\widetilde{M}$ is $\alpha kcl(\widetilde{M})$, so $\alpha kcl(\widetilde{M}) \subseteq \widetilde{W}$. Now $\alpha kcl(\widetilde{M}) \subseteq \alpha kcl(\widetilde{N}) \subseteq \alpha kcl(\alpha kcl(\widetilde{M})) = \alpha kcl(\widetilde{M}) \subseteq \widetilde{W}$.

Thus $\alpha kcl(\widetilde{N}) \subseteq \widetilde{W}$, where $\widetilde{W}$ is αkIO. Therefore $\widetilde{N}$ is $\alpha kcl(\widetilde{M})$ set in X .

Proposition 2.9 The intersection of a βkgIc set and a βkIc set is βkgIc.

Proof. Let $\widetilde{M}$ be βkgIc and $\tilde{F}$ be βkIc. Assume that $\widetilde{N}$ is βkIc set s.t, $\widetilde{M} \cap \tilde{F} \subseteq \widetilde{N}$, Let $\widetilde{W} = X \setminus \tilde{F}$. Thus $\widetilde{M} \subseteq \widetilde{N} \cup \widetilde{W}$, since $\widetilde{W}$ is βkIO, then $\widetilde{N} \cup \widetilde{W}$ is βkIO and since

$\widetilde{M}$ is βkgIc, then $\beta kIc(\widetilde{M}) \subseteq \widetilde{N} \cup \widetilde{W}$. So that $\beta kIc\left(\widetilde{M} \cap \tilde{F}\right) \subseteq \beta kIc\left(\widetilde{M}\right) \cap \beta kIc\left(\tilde{F}\right) = \beta kIc\left(\widetilde{M}\right) \cap \tilde{F}$

$\subseteq \left(\widetilde{N} \cap \widetilde{W}\right) \cap \tilde{F} = \left(\widetilde{N} \cap \tilde{F}\right) \cup \left(\widetilde{W} \cap \tilde{F}\right) = \left(\widetilde{N} \cap \tilde{F}\right) \cup \widetilde{\emptyset} \subseteq \widetilde{N}$.

Proposition 2.10 The intersection of a αkgIc set and a αkIc set is αkgIc.

Proof: it's obvious .

Proposition 2.11 Let (X, μ) be an ITS. Then $\widetilde{M}$ is βkgIc iff $\beta kcl(\widetilde{M}) \setminus \widetilde{M}$ does not contain

any non-empty αkcl set.

Proof. Suppose that $\widetilde{M}$ is βkgIc set in X. Let $\tilde{F}$ be βkcl set s.t, $\tilde{F} \subseteq \beta kcl(\widetilde{M}) \setminus \widetilde{M}$ and $\tilde{F} \neq \widetilde{\emptyset}$. Then $\tilde{F} \subseteq X \setminus \widetilde{M}$ this implies $\widetilde{M} \subseteq X \setminus \tilde{F}$. Since $\widetilde{M}$ is βkgIc and $X \setminus \tilde{F}$ is βkIO , thus $\beta kcl(\widetilde{M}) \subseteq X \setminus \tilde{F}$, that is $\tilde{F} \subseteq \beta kcl(\widetilde{M})$. Hence $\tilde{F} \subseteq \beta kcl(\widetilde{M}) \cap (X \setminus \beta kcl(\widetilde{M})) = \widetilde{\emptyset}$.

So that $F = \widetilde{\emptyset}$ and it a contradiction. Therefore $\beta kcl(\widetilde{M}) \setminus \widetilde{M}$ does not contain any non-empty αkcl

set in X .

conversely . Let $\widetilde{M} \subseteq \widetilde{W}$, where $\widetilde{W}$ is βkIO in X . If $\beta kcl(\widetilde{M})$ is not contained in $\widetilde{W}$,

then $\beta kcl(\widetilde{M}) \cap X \setminus \widetilde{W} \neq \widetilde{\emptyset}$. Now, since $\beta kcl(\widetilde{M}) \cap X \setminus \widetilde{W} \subseteq \beta kcl(\widetilde{M}) \setminus \widetilde{M}$ and $\beta kcl(\widetilde{M}) \cap X \setminus \widetilde{W}$ is a non-empty αkcl set, this is a contradiction and so $\widetilde{M}$ is βkgIc .

Proposition 2.12 Let (X, μ) be an ITS. Then $\widetilde{M}$ is βkgIc iff $\beta kcl(\widetilde{M}) \setminus \widetilde{M}$ does not contain

any non-empty αkcl set.

Proof. It's obvious .

Proposition 2.13 Let (X, μ) be an ITS. Then $\widetilde{M}$ is βkgIc, then the following are equivalent:

1- $\widetilde{M}$ is βkIc.

2- βkcl($\widetilde{M}$) \ $\widetilde{M}$ is βkIc .

Proof. (1)) ⟹(2). If $\widetilde{M}$ is a βkgIc set which is also βkIc, then βkcl($\widetilde{M}$) \ $\widetilde{M}$ = $\widetilde{\emptyset}$.

Thus $\widetilde{M}$ is a βkIc.

(2)) ⟹ (1). Let βkcl($\widetilde{M}$) \ $\widetilde{M}$ be a βkIc set and $\widetilde{M}$ be βkgIc. Then by Proposition 2.11 ,

βkcl($\widetilde{M}$) \ $\widetilde{M}$ does not contain any non-empty βkIc subset. Since βkcl($\widetilde{M}$) \ $\widetilde{M}$ is βkIc and

βkcl($\widetilde{M}$) \ $\widetilde{M}$ = $\widetilde{\emptyset}$, this shows that $\widetilde{M}$ is βkIc.

Proposition 2.14 Let (X, μ) be an ITS. Then $\widetilde{M}$ is αkgIc, then the following are equivalent:

1- $\widetilde{M}$ is αkIc.

2- αkcl($\widetilde{M}$) \ $\widetilde{M}$ is αkIc .

Proof. It's obvious .

Proposition 2.15 Let (X, μ) be an ITS. Then the following are equivalent:

1- each subset of X is βkgIc.

2- βkOl($\widetilde{M}$) = βkcl($\widetilde{M}$) .

Proof. (1)) ⟹ (2). Let $\widetilde{M}$ ∈ βkOl($\widetilde{M}$).Then by hypothesis, $\widetilde{M}$ is βkgIc ,so βkcl($\widetilde{M}$) ⊆ $\widetilde{M}$,hence βkcl($\widetilde{M}$) ⊆ $\widetilde{M}$, therefore $\widetilde{M}$ ∈ βkOl($\widetilde{M}$). Also let $\widetilde{N}$ ∈ βkOl($\widetilde{N}$).

Then X \ $\widetilde{N}$ ∈ βkOl($\widetilde{M}$), so by hypothesis X \ $\widetilde{N}$ is βkgIc and then X \ $\widetilde{N}$ ∈ βkOl($\widetilde{M}$),

thus $\widetilde{N}$ ∈ βkOl($\widetilde{M}$) . Therefore βkOl($\widetilde{M}$) = βkcl($\widetilde{M}$) .

(2)) ⟹ (1). If $\widetilde{N}$ is a subset of (X, μ) s.t, $\widetilde{N}$ ⊆ $\widetilde{M}$ where $\widetilde{N}$ ∈ βkOl($\widetilde{N}$), then $\widetilde{N}$ ∈ βkcl($\widetilde{N}$), and so βkcl($\widetilde{N}$) ⊆ $\widetilde{N}$ .Therefore $\widetilde{N}$ is βkgIc.

Proposition 2.16 Let (X, μ) be an ITS. Then the following are equivalent:

1- each subset of X is αkgIc.

2- αkOl($\widetilde{M}$) = αkcl($\widetilde{M}$) .

Proof. It's obvious .

Proposition 2.17 Let (X, μ) be an ITS. Then the following are equivalent:

1- each subset of X is αkgIc.

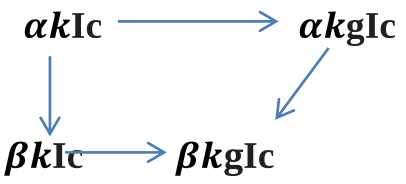
2- αkOl($\widetilde{M}$) ⊆ αkcl($\widetilde{M}$) .

Proof. It's clear .

Section 3 The relations between βk generalized & αk generalized Closed Sets

We give this important proposition .

Proposition 3.1. Let (X, μ) be an ITS. Then The implication among some types are given by the
following diagram .



Proof. It is clear that every αkIc set is αkgIc. Also every βkIc set is βkgIc .

αkgIc $\longrightarrow$ βkgIc

Let (X, μ) be an ITS. Since every αIc is βIc also every αgIc is βgIc . Therefore

αkgIc is βkgIc .

Remark 3.2 by transitivity we get αkIc $\longrightarrow$ βkgIc

But the converse is not true in general as it is shown in the following example.

Example 3.3. Let $J = \{c, d, n\}$ with topology $\mu = \{\dot{J}, \dot{\emptyset}, \widetilde{M}, \widetilde{N}\}$, where $\widetilde{M} = \langle j, \{c\}, \{d, n\}\rangle$, $\widetilde{N} = \langle j, \{c\}, \emptyset\rangle$. Then

1- (J, μ) is βkgIc , but not βkIc .

2- (J, μ) is βkgIc , but not αkgIc .

3-(J, μ) is βkgIc , but not αkIc .

Example 3.4. Let $J = \{c, d, n\}$ with topology $\mu = \{\dot{J}, \dot{\emptyset}, \widetilde{M}\}$, where $\widetilde{M} = \langle j, \{c\}, \{d, n\}\rangle$ .Then (J, μ) is βkIc , but not αkIc .

Example 3.4. Let $J = \{c, d, n\}$ with topology $\mu = \{\dot{J}, \dot{\emptyset}, \widetilde{M}\}$, where $\widetilde{M} = \langle j, \{c\}, \{d, n\}\rangle$ .Then (J, μ) is βkIc , but not αkIc .

Remark 3-5. αkgIc and βkIc are independent notions .The following two examples
shows this two cases .

Example 3.6. Let $E = \{w, r, i\}$ with topology $\mu = \{\dot{E}, \dot{\emptyset}, \widetilde{K}, \tilde{S}, \widetilde{G}, \tilde{Z}, \tilde{I}\}$, where $\widetilde{K} = \langle e, \{w\}, \{r, i\}\rangle$, $\tilde{S} = \langle e, \{w\}, \emptyset\rangle$, $\widetilde{G} = \langle e, \{w, r\}, \{i\}\rangle$, $\tilde{Z} = \langle e, \{w\}, \{r\}\rangle$, $\tilde{I} = \langle e, \{w, r\}, \emptyset\rangle$ and

$D = \{2,4,6\}$. Then (E, μ) is βkIc but not αkgIc .

Example 3.7. Let E = {o, p, u } with topology μ = {Ė, Ø̇, W̃ , R̃, K̃, P̃, T̃, H̃, Z̃} , where

W̃= ⟨ e, {o}, {p, u}⟩ , R̃ = ⟨e, {o}, {p}⟩, K̃ = ⟨ e, {o}, Ø⟩, P̃ = ⟨ e, {o, p}, Ø⟩, T̃ = ⟨ e, Ø, Ø⟩ ,

H̃ = ⟨ e, Ø, {p, u}⟩,Z̃ = ⟨ e, Ø, {p}⟩ . Then (E , μ) is αkgIc but not βkIc .

Proposition 3.8 Let (X, μ) be an ITS. Then M̃ is αkgIc, then the following are equivalent:

1- M̃ is αkIc.

2- βkcl(M̃) \ M̃ is βkIc .

Proof: by using the proposition 3-1 we get the result .

Proposition 3.9 Let (X, μ) be an ITS. Then the following are equivalent:

1- each subset of X is βkgIc.

2- αkOl(M̃) ⊆ βkcl(M̃) .

Proof: by using the proposition 3-1 we get the result .

Proposition 3.10 Let (X, μ) be an ITS. Then the following are equivalent:

1- each subset of X is αkgIc.

2- αkOl(M̃) ⊆ αkcl(M̃) ⊆ βkcl(M̃) .

Proof. It's clear .

Proposition 3.11 If IS M̃ of X is αkgIc and M̃ ⊆ Ñ ⊆ αkcl(M̃) , then Ñ is βkgIC set in X .

Proof: by using the proposition 3-1 we get the result .

Proposition 3.12 If IS M̃ of X is αkIc and M̃ ⊆ Ñ ⊆ αkgcl(M̃) , then Ñ is αkgIC set in X .

Proof: by using the proposition 3-1 we get the result .

Proposition 3.13 If IS M̃ of X is αkIc and M̃ ⊆ Ñ ⊆ βkcl(M̃) , then Ñ is βkgIC set in X .

Proof: by using the proposition 3-1 we get the result .

Proposition 3.14 The intersection of a αkgIc set and a αkIc set is αkgIc.
Proof: it's obvious .

Proposition 3.15 If M̃ is αkIO and αkgIc then M̃ is βkIc.

Proof. Suppose that M̃ is αkIO and αkgIc. Since M̃ is αkIO and M̃ ⊆ M̃, we have αkcl(M̃) ⊆ βkcl(M̃) ⊆ M̃, also M̃ ⊆ αkcl(M̃) ⊆ βkcl(M̃) , thus βkcl(M̃) = M̃. Therefore M̃ is βkIc .

Proposition 3.16 If M̃ is αkIO and βkgIC then M̃ is βkIc.

Proof: It's clear .

Proposition 3.17 Let (X, μ) be an ITS. Then the following are equivalent:

1- each αkgIc is βkgIC.

2- $\alpha kOl(\widetilde{M}) \subseteq \beta kcl(\widetilde{M})$.

Proof: by using the proposition 3-1 we get the result .

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