Print ISSN 2710-0952 Electronic ISSN 2790-1254

MESHFREE METHODS OF ELESITSTY PROPLEM

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Abstract

The goal of this thesis is to use the generated local radial basis function (RBF). Finite difference (FD) method which is RBF-FD to solve some problems in solids Mechanics. This method possesses some advantages such as ease of implementation, Ability to work on scattered data rather than a connected network, and flexibility. Regarding the geometry and dimensions of the problem domain. On the other hand, global RBF methods suffer from dense and unconditional finality systems while the final matrix of the RBF-FD method is sparse and well adapted

Keywords: Radial basis function (RBF), Finite difference (FD) method, Computational. stencil, Elasticity problems

الطرق الخالية من الشبكة لحل مسائل المرونة

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التخصص: الرياضيات

ملخص

الهدف من هذه الأطروحة هو استخدام دالة الأساس الشعاعي المحلية المولدة (RBF) طريقة الفروق المحدودة (FD) وهي RBF-FD لحل بعض المشاكل في ميكانيكا المواد الصلبة. تمتلك هذه الطريقة بعض المزايا مثل سهولة التنفيذ، والقدرة على العمل على بيانات متناثرة بدلاً من شبكة متصلة، والمرونة. فيما يتعلق بهندسة وأبعاد مجال المشكلة. من ناحية أخرى، تعاني طرق RBF العالمية من أنظمة نهائية كثيفة وغير مشروطة بينما تكون المصفوفة النهائية لطريقة RBF-FD متفرقة ومتكيفة بشكل جيد .

الكلمات المفتاحية: دالة الأساس الشعاعي (RBF)، طريقة الفروق المحدودة (FD)، مشاكل المرونة.

Introduction

In recent years, radial basis functions were first used by Hardy in 1971 to interpolate multivariate data. Then Kanza of interpolators of radial basis functions were designed to solve partial differential equations. Then, in 1993, Wu presented the advantages of these methods and in general the symmetric colocation method under radius basis functions [1]. In the 1990, in order to solve the problems of differential equations, Kanza (unsymmetric)[4,7] and symmetric [5,6]methods were designed. Finite difference method - Radial Baye function was designed by Feinberg and Flier in 2015 [9] .The matrices of the unsymmetric and asymmetric interpolation method inherit the radial Basis function. For this purpose, we intend to solve this problem by using local methods based on radial Basis functions.

before speaking :In this thesis, we investigated the solution of elasticity problems using the finite difference method of the radial basis function (RBF−FD) and we performed its numerical implementation in MATLAB software to solve some of these problems in two-dimensional space and obtained a good accuracy. The advantages of this method are ease of implementation and flexibility in choosing different basis functions, irregular points and complex calculation area to be. Also, the combination of this method with local stabilizing algorithms such as RBF-QR leads to acceptable numerical results, which can be considered as one of the important advantages of this method. We can also conclude that this method, as one of the meshless methods for solving elasticity problems, brings satisfactory results and it is possible to develop it for other solid mechanics problems such as dynamic elastic problem.

Meshfree method: the field of numerical analysis, meshfree methods are those that do not require connection between nodes of the simulation domain, i.e., a mesh, but are rather based on interaction of each node with all its neighbors. As a consequence, original extensive properties such as mass or kinetic energy are no longer assigned to mesh elements but rather to the single nodes. Meshfree methods enable the simulation of some otherwise difficult types of problems, at the cost of extra computing time and programming effort. The absence of a mesh allows Lagrange

simulations, in which the nodes can move according to the velocity field.

A radial basis function (RBF): is a real-valued function whose value depends only on the distance between the input point and some fixed point, either the origin so that, or some other fixed point, called a center, so that,

Any function that satisfies the property is called radial basis function.

conditional positive: Let R^d on q be a conditional positive semidefinite of order ϕ of even and continuous radial basis function and for each $z_1 z_2$, , ..., $z_n \in R^d$ and for each set of distinct points n∈ N we say that if for each that c ∈ R^n vector

apply

in the following side condition:

$\sum_{i=1}^{n} c_i$ p(z_i) =0, ∀p∈ P_{q-1}^{d}, to have

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_i c_j \emptyset(\|z_i - z_j\|) \geq 0 \quad . \qquad (1)$$

∅ is conditional positive definite from order q in R^d say that if Mathematical formula (1) $\forall c \in R^N \backslash [0]$ be positive.

Internalization of the radial basis function:

Let $\Omega \subseteq R^d$ and distinct scattered points X=$\{x_1, x_2, , \dots, x_n\} \subseteq \Omega$ and $f_{j=}$f(x_j) for 1≤ j≤ n are given . Internalization of the radial basis function f on X as: S(x) = $\sum_{j=1}^{n} \lambda_j \emptyset(\|x_i - x_j\|)$ It is written that in those coefficients$\lambda_j$with internal conditions [4,5] S $(x_i) = f_i$ for 1≤i≤N by solving Aλ=f be the result:

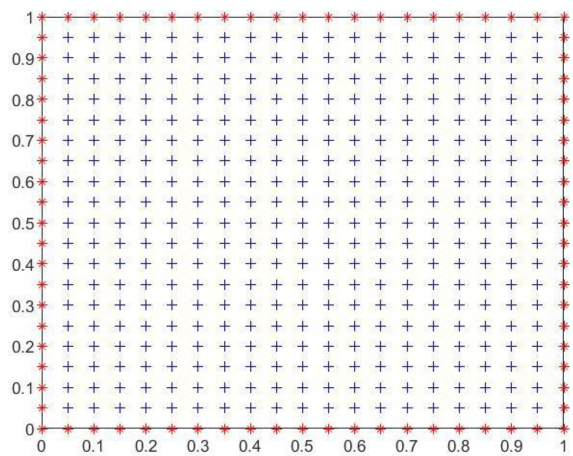
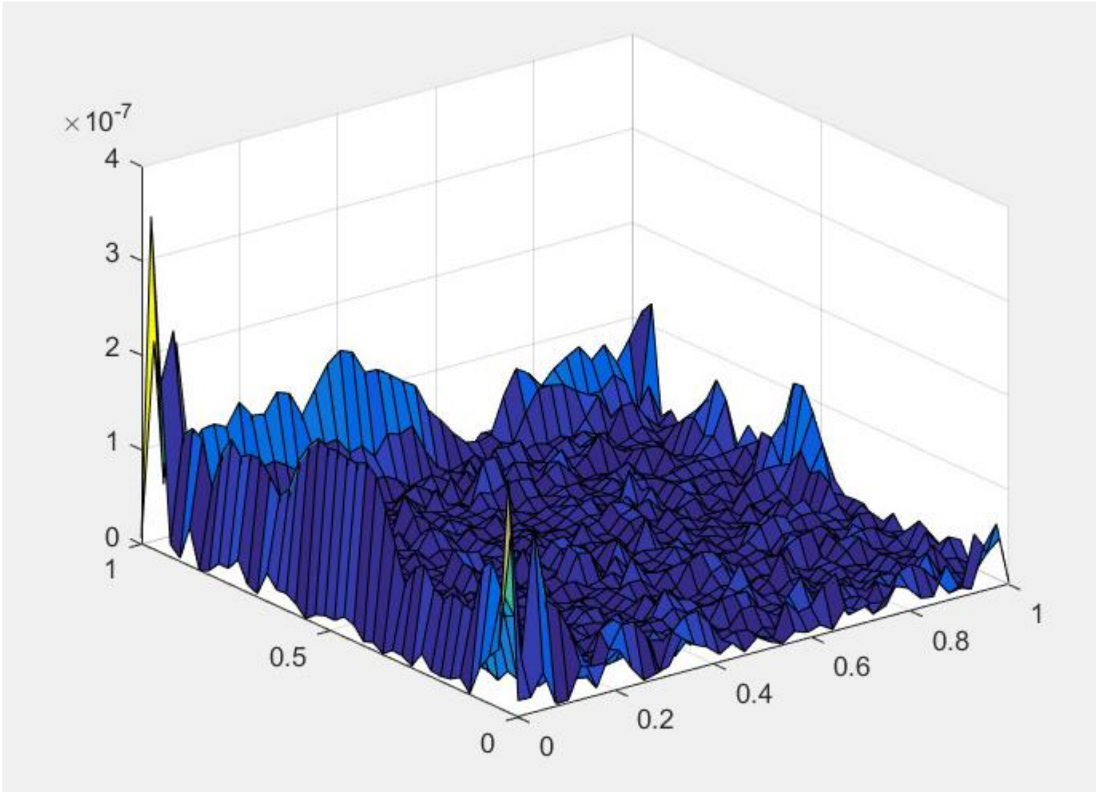


figure (1-1) (A square area containing regular points)



$$A= \begin{bmatrix} \emptyset(\|x_1-x_1\|) & \emptyset(\|x_1-x_2\|) & \cdots & \emptyset(\|x_1-x_N\|) \\ \emptyset(\|x_2-x_1\|) & \emptyset(\|x_2-x_2\|) & \cdots & \emptyset(\|x_2-x_N\|) \\ \vdots & \vdots & \cdots & \vdots \\ \emptyset(\|x_N-x_1\|) & \emptyset(\|x_N-x_2\|) & \cdots & \emptyset(\|x_N-x_N\|) \end{bmatrix},$$

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{bmatrix}, \quad f_x = \begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_N) \end{bmatrix}.$$

Generalization of internal problem:

$S(x) = \sum_{i=1}^{N} \alpha_i \, \emptyset(\|x - x_i\|) + \sum_{j=1}^{m} \beta_j \, \rho_{j(x)}$

Internal conditions: $S(x) = f(x_i)$, i=1,2,⋯,N.

Side conditions: $\sum_{i=1}^{N} \alpha_i \beta_j (x_i) = 0$, J = 1,⋯, m.

In this case, we reach to :

$\begin{bmatrix} A & P \\ P^T & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} F_X \\ 0 \end{bmatrix}$, S.T $P_{ij}=P_j(x_i)$ and $A_{ij}=\emptyset(\|x_i - x_j\|)$.

Solvability of linear interpolation and generalized interpolation:

- Differentiation of nodal points and their unique resolution.
- Positive definiteness (conditional, unconditional) of radial basis functions.

The above two conditions lead to the achievement of unique coefficients in linear systems of interpolation problems. And the interpolator function is obtained uniquely.

matrix of devices corresponding to global radial basis functions with the number of interpolation points, In these functions, the accuracy of ϵbecomes large, full and ill-positioned. In addition, by reducing the shape parameter increases, but the matrix of devices similar to them is in a bad position. And from a stage Later, it hinders the achievement of more accuracy. To overcome these problems of methods.

We use local and stable RBF-FD as described in the next section in particular, we focus on solving two-dimensional elasticity problems. It is worth noting that to overcome.

We can use stabilization algorithms such as RBF-QR and RBF-CP on the bad situation caused by the reduction of the shape parameter let's use.

Definition: Suppose L is differentiable with a linear operator, in this method the operator L can be approximated at an arbitrary point like

$x_c$located on the domain of the problem is defined as follows by using the data values of nodal points located in the stencil attributed to the point

x_c We define it as follows: $\text{Lu}|x_c \approx \sum_{i=1}^{N} w_i \, u_i$ (4)

To approximate the operator L, it is necessary to specify the weights in the above linear combination.

Finite difference method based on radial basis function:

In the classical finite difference method of weights with the effect of the fact that the linear combination (4) For polynomials to be exact up to high degrees, they are determined, now in the method RBF-FD, Weights with the effect of accuracy of linear combination(4) For the radial basis functions centered on points located in stencil x_c i.e., X_k,We calculate from the solution of the following linear device:

$$\begin{bmatrix} \emptyset(\|x_1 - x_1\|) & \emptyset(\|x_1 - x_2\|) & \cdots & \emptyset(\|x_1 - x_N\|) \\ \emptyset(\|x_2 - x_1\|) & \emptyset(\|x_2 - x_2\|) & \cdots & \emptyset(\|x_2 - x_N\|) \\ \vdots & \vdots & \cdots & \vdots \\ \emptyset(\|x_N - x_1\|) & \emptyset(\|x_N - x_2\|) & \cdots & \emptyset(\|x_N - x_N\|) \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix} = \begin{bmatrix} L\emptyset(\|x_c - x_1\| \\ L\emptyset(\|x_c - x_2\|) \\ \cdots \\ L\emptyset(\|x_c - x_N\|) \end{bmatrix}$$

In order to recover polynomials, add multivariable polynomials to the linear relationship 4and to calculate the weights, we solve the following linear system:

$$\begin{bmatrix} A\phi, k & p \\ p^t & 0 \end{bmatrix} \begin{bmatrix} w \\ v \end{bmatrix} = \begin{bmatrix} L_\emptyset \\ L_P \end{bmatrix} \qquad (5)$$

$P_m(R^d)$= span {p١, . . ., p_Q}... , ١⩽ i ⩽ Q , $x_j \in X_k$ · P = (p_i (x_j))

.

Q=$\binom{m+d}{d}$, p=(p_i(x))

- According to the way of choosing the points in the neighborhood of point X to form the corresponding stencil It (global and local), we divide this method into two types, global and local.
- Positive semi definiteness of radial basis functions of order 1 + m in the space of $R^d$and $P_m(R^d)$

- The unique solvability of points is a sufficient condition for the solvability of the linear device 5.

- In this method, we can control the ill-posed Ness of the coefficient’s matrix, caused by the selection of small values of the shape parameter

Apply RBF-QR[10]

Elasticity problem:

Elasticity: To the resistance of an object against changes in shape caused by the application of force and return to shape the initial size after removing the force is called "elasticity"[8,9]

Print ISSN 2710-0952 Electronic ISSN 2790-1254

Tension: In general, stress means the force exerted on a unit area, and its unit is in the SI system Pascal (newton per square). Force is a vector quantity, that is, it has size and is the direction in the definition of stress, depending on the direction in which the force is applied to the surface, types different tensions are created.

tensile stress: When the direction of the applied force is perpendicular to the desired surface and towards the outside of the part, tension is created It is called tensile stress. One of the important factors in calculating the strength of materials is their ability under It is tensile stress.

Compressive stress: When the direction of the incoming force is perpendicular to the desired surface and towards the inside of the piece, tension is created It is called compressive stress. Usually, the tolerance of parts under compressive stress is more than tensile stress.

Shear stress: Whenever the direction of the incoming force is parallel to the desired surface or in other words perpendicular to the normal vector of that surface is, shear stress is created in the object. The sign of shear stress in mechanics equations, tau τ (from Greek letters).

Stress-strain: They are among the most basic and important concepts in the resistance of materials. When the force When a structure is loaded with a member of it, tension and strain are created. The tension can be It is defined as the force acting on an object per unit area. According to this definition, the stress equation will be as follows:

$\sigma = F/A$

σ, F, A respectively as stress, force and cross-sectional area on which the force is applied ،we will consider it.

Body force: It is a force that acts on the entire volume of the body. Forces due to gravity, fields Electric and magnetic fields are examples of body forces. Body forces wit Contact forces are different from surface forces that are applied to the surface of an object. External forces and shear forces between objects as they are applied to the surface of an object are surface forces. All cohesive surface attractions and contact forces between Objects are also considered as surface forces.

Fictitious forces such as centrifugal force, Euler force and Coriolis effect are also examples they are from the forces of the body.

Young's modulus: E Young's modulus or modulus of elasticity, is a mechanical property that measures the tensile stiffness of a solid material. This relationship between tensile stress σ (force per unit area) and axial strain ϵ(proportional deformation) in the linear elastic region of one

It determines the substance and is determined using the following formula:

$E = \frac{\sigma}{\varepsilon}$ Young's moduli are usually so large that they are expressed not in pascals but in gigapascals (GPa).

Poisson's radius: In material science and solid mechanics, Poisson's ratio and to measure Poisson's effect means deformation expansion with contraction of the material is used in the direction perpendicular to the specific direction. The negative Poisson's ratio value is the ratio of transverse strain to axial strain. For small values of these changes ν, the amount of transverse elongation is greater than the amount of axial compression. Most materials they have easy ratio values between 0.0 and 0.5.

Introduction of two-dimensional elasticity problem:

The problem of two-dimensional elasticity in a bounded area$\Omega \subseteq R^2$

with a border $\gamma = \partial\Omega$ is defined as follows:

σij,j + bi =0 , in Ω. (6)

where σij, j is the stress tensor corresponding to the displacement field u = [u1, u2]T and the kinetic force b = [b1, b2]T.

σij are the components of the stress tensor matrix obtained from the following relationship: σ=D L u

where the derivative operator matrix L and the stress-strain matrix D for spherical isomorphic materials are respectively as:

$$L = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial Y} & \frac{\partial}{\partial x} \end{bmatrix}, D = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ \bar{v} & 1 & 0 \\ 0 & 0 & \frac{(1-\bar{v})}{2} \end{bmatrix}.$$

$$\acute{E} = \begin{cases} E, & plane\ stress \\ \frac{E}{1-\nu^2}, & plane\ strain \end{cases}$$

$$\bar{v} = \begin{cases} V, plane\ stress \\ \frac{V}{1-V}, plane\ strain \end{cases}$$

It is definable.

Therefore, the problem of two-dimensional elasticity with boundary conditions can be written as follows:

$$\acute{E}\frac{\partial^2 u_1}{\partial x^2} + \acute{E}\bar{v}\frac{\partial^2 u_2}{\partial x \partial y} + \acute{E}v\bar{v}\frac{\partial^2 u_1}{\partial y^2} + \acute{E}\bar{v}\frac{\partial^2 u_2}{\partial y \partial x} = -b_1$$

$$\acute{E}\tilde{\bar{v}}\frac{\partial^2 u_1}{\partial_x \partial_y} + \acute{E}\tilde{v}\frac{\partial^2 u_2}{\partial x^2} + \acute{E}\bar{v}\frac{\partial^2 u_1}{\partial x \partial y} + \acute{E}\frac{\partial^2 u_2}{\partial y^2} = -b_2 \qquad (7)$$

$$\acute{E} = \frac{\bar{E}}{1-\bar{v}^2}, \qquad \tilde{v} = \frac{1-\bar{v}}{2}.$$

$$Iu = \bar{u}, \qquad (8)$$

$$\frac{\partial u_1}{\partial x} n_1 + \bar{v}\frac{\partial u_2}{\partial y} + n_1 + \left(\frac{1-\bar{v}}{2}\right)\frac{\partial u_1}{\partial y} n_2 + \left(\frac{1-\bar{v}}{2}\right)\frac{\partial u_2}{\partial x} n_2 = \bar{t_1},$$

$$\left(\frac{1-\bar{v}}{2}\right)\frac{\partial u_1}{\partial y} n_1 + \left(\frac{1-\bar{v}}{2}\right)\frac{\partial u_2}{\partial x} n_1 + \bar{v}\frac{\partial u_1}{\partial x} n_2 + \frac{\partial u_2}{\partial y} n_2 = \bar{t}_2.$$

where n2 and n1 are the components of the normal vector on the boundary of the region Ω.

Implementation of RBF-FD method to solve

two-dimensional elasticity problem:

To apply the method, RBF-FD set Z = $\{z_1, z_2, \cdots, z_n\}$, and $U_z$space respectively as points for

We consider a node (approximation) and an approximation space, and using the displacement field at these points in the form of $[u_i(z_1), u_i(z_2), \cdots, u_i(z_n)]^T$ for i=1,2 ,

We expand the problem of elasticity. We assume that L is one of the operators used in the elasticity problem, that is:

$$L = \acute{E}\frac{\partial^2}{\partial x^2} + \acute{E}\bar{v}\frac{\partial^2}{\partial x \partial y} + \acute{E}v\bar{v}\frac{\partial^2}{\partial y^2} + \acute{E}\bar{v}\frac{\partial^2}{\partial y \partial x},$$

L = ΄E$\bar{\bar{v}}\frac{\partial^2}{\partial_x\partial_y}$ + ΄E$\tilde{v}\frac{\partial^2}{\partial x^2}$ + ΄E$\bar{v}\frac{\partial^2}{\partial x\partial y}$ + ΄E$\frac{\partial^2}{\partial y^2}$,

We assume for the boundary conditions:

B =$n_1\frac{\partial}{\partial x}+\bar{v}n_1\frac{\partial u_2}{\partial y}+\left(\frac{1-\bar{v}}{2}\right)n_2\ \frac{\partial}{\partial y}+\left(\frac{1-\bar{v}}{2}\right)n_2\frac{\partial}{\partial x}$,

B = $\left(\frac{1-\bar{v}}{2}\right)n_1\frac{\partial}{\partial y}\ +\left(\frac{1-\bar{v}}{2}\right)n_1\frac{\partial}{\partial x}+\bar{v}n_2\frac{\partial}{\partial x}\ +n_2\frac{\partial}{\partial y}$,

We put it in Dirikele B = I

Now sets X = $\{x_1, x_2, \cdots, x_{n_x}\}$ and Y = $\{y_1, y_2, \cdots, y_{n_y}\}$

Now set in order as test points We consider the interior and boundary and using the functions of the point finder δyk and δxj

For internal and boundary test points, we discretize equation (7) and (8) similarly to these points :

$L(u_i)(x_j) = \delta_{x_j} oL(u_i) = -b_i,$ *i = 1,2,*

$B(u_i)(y_k) = \delta_{y_k} o$ B$(u_i) = \begin{cases}\bar{t}_i\\ \bar{u}_i\end{cases}$ i = 1,2 (9)

The right side of relation (9) is based on the Neumann or Dirichlet boundary conditions of titi or ui, where i = 1, 2 Authorizes.

To reproduce the R BF−FD method A favorite point $x_k \in X$ *or* $y_k \in Y$ we will consider and points z_j In this neighborhood, we refer to the radius of the schema as a computational *stencil We assume*

Z_k = Z∩ B (x_k, δ) For interior points *or* $Z_k = Z\cap B\ (y_k, \delta)$ For border points. Now we get the weights related to the K row of the RBF-FD matrix as follows:

$L(u_i)(x_k) \approx \sum_{z_j\in Z_k} w_j{}^k u_i(z_j),$ i = 1,2 (10)

and $w_j{}^k$under the condition that the approximation (10) is exact for all radial basis functions made byZ_k points, in other words

$\sum_{z_j\in Z_k} w_j{}^k\emptyset(\|z_l - z_j\|)$ =L$\emptyset(\|z_l - x_k\|)$, $z_l \in Z_k$.

We get that in the representation of the matrix of the linear device we will have the following:

$A_L w^k = L\emptyset^k,$

$A_L = [\emptyset(\|z_l - z_j\|)]|z_l, z_j \in Z_k,$ (11)

$L\emptyset^k = [L\emptyset(\|x_k - z_j\|)]|z_j \in Z_k,$

$w^c = [w_1{}^k, w_2{}^k, \cdots, w_{n_k}{}^k]^T$.

where n_k *is the number of points* z_j *that are located in the molecule* z_k.

, if the radial basis function ϕ is positive, the mentioned linear system is monosyllable will be. If we want to get a more accurate answer, we can use the RBF-FD method Apply the condition of reproduction of polynomials. In this case, by choosing {p1, p2, ..., pQ} to A basic heading for the space of polynomials of certain degree and adding a linear combination of these. The bases in the molecules are similar to any arbitrary test point x_k assuming that the weights are accurate For both radial functions and polynomials, we will reach the following system of linear equations:

$$\begin{bmatrix} A_L & P \\ P^T & 0 \end{bmatrix} \begin{bmatrix} w^k \\ \beta^k \end{bmatrix} = \begin{bmatrix} L\emptyset^K \\ Lp^k \end{bmatrix} \qquad (12)$$

That*:*

$P = [p_k(z_j)]|z_j \in z_k \quad 1 \le k \le Q = \binom{q+1}{2}, z_j \in Z_k,$

$L\emptyset^K = [L\emptyset(\|z_j - z_k\|)]\ |z_j \in Z_k,$

$LP = [LP_k(x_k)]|1 \le k \le Q.$

If the radial basis function $\emptyset$ is conditional positive definite of q order. And if the points of the Z_k molecule are unique solvers on the polynomial space of $P_{q-1}(R^2)$multivariables, the mentioned device will be solvable and the coefficients of $w_j{}^k$will be uniquely obtained. Due to the fact that all points of approximation of Z in $Z_k$molecule are not forced, when we consider the weight vector w^kas a row of k matrix RBF-FD, corresponding to the points of Z that are not inZ_k, zero is added to it. let's do. We solve this process for all internal and boundary test points and the matrices D_B *and* $D_L$similar to the boundary and internal operators in the problem. We get the elasticity that the weight vectors such as to each test point, the lines of this forms the matrices, in this case we have:

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Iraqi Journal of Humanitarian, Social and Scientific Research
Print ISSN 2710-0952 Electronic ISSN 2790-1254

$$\begin{bmatrix} D_L \\ D_B \end{bmatrix} u_i = \begin{bmatrix} -b_i \\ g_i \end{bmatrix}, \qquad i = 1,2 \qquad (13)$$

Such that:

$$g_i = \begin{cases} \bar{t}_i, & Nowman\ condtions \\ \bar{u}_i, & \text{Derekleh condtions} \end{cases}$$

$$D_L = \begin{bmatrix} {w_1}^I \\ {w_2}^I \\ \vdots \\ {w_{nx}}^I \end{bmatrix}, \quad D_B = \begin{bmatrix} {w_1}^B \\ {w_2}^B \\ \vdots \\ {w_{ny}}^B \end{bmatrix}.$$

Here, n_x and n_y show the number of internal and boundary test points, respectively. After that the displacement field u = $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ at the z-points was obtained by solving the device (13), we can calculate the stress and strain by using the RBF-FD method again for any other desired point.

Chapter 2: Numerical results and discussions:

In order to obtain numerical results, we use Gaussian radial basis functions and consider the following hypotheses.

- Shape parameter $\epsilon = 0.1$ for Gaussian kernel.
- δ as the radius of each molecule in the form of a coefficient of the density distance h that is Ch = δ such that c$\geq$ 1.

Using RBF-QR stabilizer algorithm for Gaussian kernel, k=5 For the polynomial function r^k.

Example: We consider a retaining beam with the tensile force P loaded at its free end as shown in (2-1), where L is the length of the beam and D is its width.

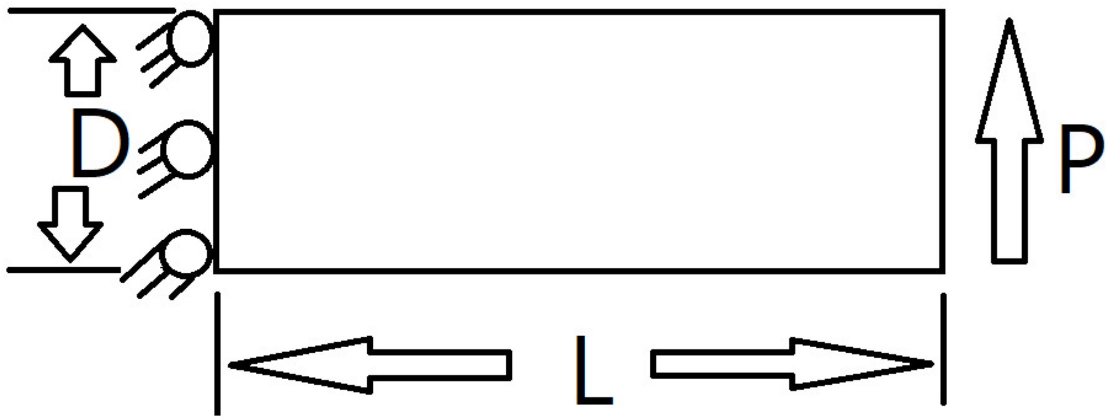

Figure (2-1):

The exact answer of this example in reference [10] is as follows:

U1 $= \frac{-P}{\bar{\sigma} EI}$ (y − $\frac{D}{2}$)(3 x(2L − x) + (2+ $\bar{v}$) y(y − D)),

u2 = $\bar{\sigma} EI$

$\frac{P}{\bar{\sigma} EI}(X^2$ (3L − x) + 3$\bar{v}$(L − x) (y − D) + $\frac{2+5\bar{v}}{4} D^2 X$).

Such that: I $=\frac{D^3}{12}$, X = (x,y) ∈ R^2.

The exact stresses associated with the displacement field are u=$[u_1, u_2]^T$ also in obtained as:

$\sigma_{1\,1} = \frac{-P}{I}$(L-x) (y- $\frac{D}{2}$) ,

$\sigma_{2\,2}$ = 0 ,

$\sigma_{1\,2} = \frac{-Py}{2I}$ (y- D) ,

For a numerical example L as the length of the beam, be in amount 8 and D as the width of that value 1We choose. to the tensile force P, Young's modulus and Poisson's radius are assigned values of 1.1 and 0.25 respectively.

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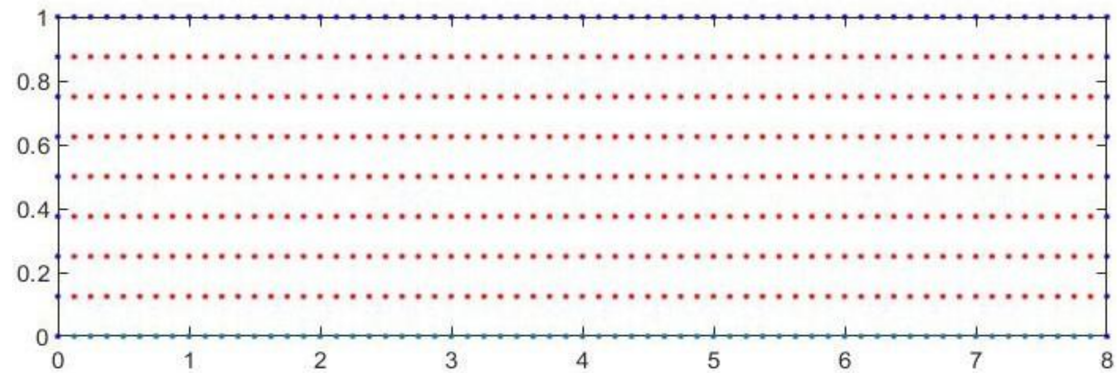

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(2-2): How to arrange the points on the beam

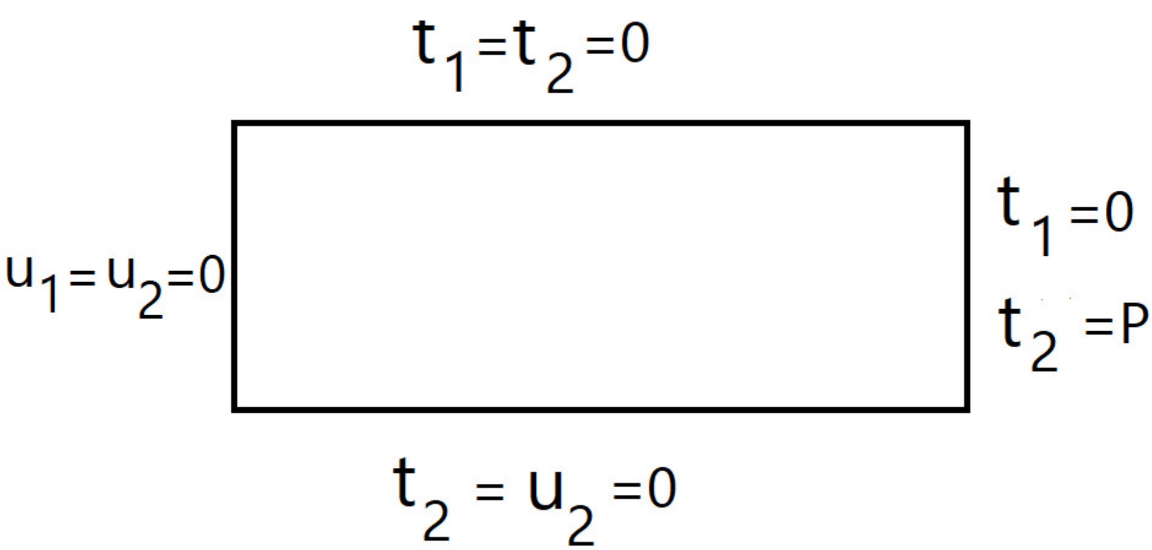

Figure (2-3): boundary conditions

Numberofpoints	Filldistance	error	orderofconvergence
[165]	2.5000e-01	4.4838e-03	0.0000e+00
[585]	1.2500e-01	1.5104e-04	4.8917e+00
[2193]	6.2500e-02	9.2345e-06	4.0318e+00
[8481]	3.1250e-02	5.6930e-07	4.0198e+00

Figure (2-4): Numerical results of RBF-FD method with Gaussian function.

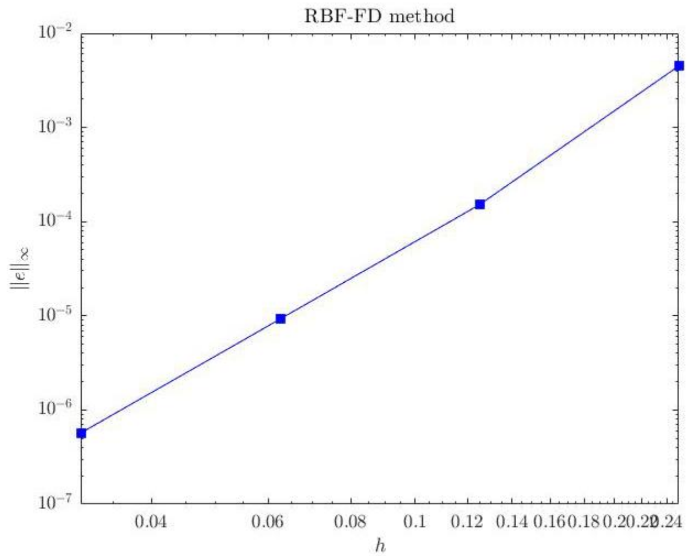

Figure (2-5): Gaussian RBF-FD method error for u function.

and it is displayed under power function r^5 in figure (2-6) and table (2-7).

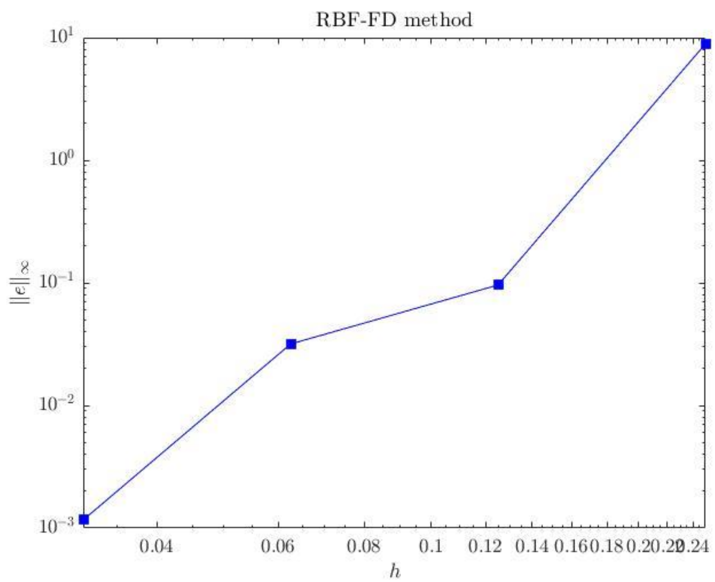

figure (2-6): The error of the RBF-FD method with multi-parallel kernel for function u.

Numberofpoints	Filldistance	error	orderofconvergence
[165]	2.5000e-01	8.8882e+00	0.0000e+00
[585]	1.2500e-01	9.6035e-02	6.5322e+00
[2193]	6.2500e-02	3.1726e-02	1.5979e+00
[8481]	3.1250e-02	1.1645e-03	4.7679e+00

table (2-7): Numerical results of RBF-FD method with multi-coordinate function.

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