Artin Exponent of $SL(2, \mathbb{Z}_{2p})$

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المستخلص

لتكن G الزمرة الخطية الخاصة المنتهية $SL(2,\mathbb{Z}_{2p})$. في هذا البحث قمنا بايجاد رتبة وأس وعدد صفوف التكافؤ وأس ارتن لهذه الزمرة . عندما p=5,7,11 وجدنا بأن رتبة G تساوي $[6p.(p^2-1)]$, أس G يساوي $[\frac{1}{2}p(p^2-1)]$, , عدد صفوف التكافؤ يساوي $(3p+12)$ وأس ارتن يساوي p^2-1 .

Abstract

Let G be the finite special linear group $SL(2,\mathbb{Z}_{2p})$. In this work we determined the order of this group, exponent of G, number of conjugacy classes and Artin exponent $A(G)$.
For $p = 5,7$ *and* 11, We found that the order of G is $[6p.(p^2-1)]$, exponent of G is $[\frac{1}{2}p(p^2-1)]$, the number of conjugacy classes is $(3p+12)$ and $A(G) = p^2 - 1$.

Introduction

Let f be an integral valued class function on a finite group G , Artin induction Theorem[8] states that $|G|f$ is an integral linear combination of characters of G induced from characters of linear representations of cyclic subgroups of G.
In (1968), Lam [8] proved a sharp form of Artins Theorem, he determined the least positive integer, $A(G)$, such that $A(G)\chi$ is an integral linear combination of induced principal characters of cyclic subgroups for all rational valued characters χ of G.

This is a continuation of the papers [1] and [2]. In [1], the authors found that $A\left(SL\left(2,\mathbb{Z}_{2^k}\right)\right) = 2^{3(k-1)}$ for $k = 2,3,4,5$ and in [2], the authors found that $A\left(SL\left(2,\mathbb{Z}_{3.2^k}\right)\right) = 3.2^{3k}$ for $k = 1,2,3,$ where $SL(2,\mathbb{Z}_n)$ is the special linear group over the ring $\mathbb{Z}_n$.

In this work, the group G under consideration is $SL(2,\mathbb{Z}_{2p})$, where $p = 5,7$ *and* 11 The main results will be stated in section 2, as follows : in Theorem(2.5) we found that
$A(G) = p^2 - 1$, in Theorem(2.6) we found that $|G| = 6p(p^2-1)$, in Theorem(2.7) we found that $exp(G) = \frac{1}{2}p(p^2-1)$ and in Theorem(2.8) we found that the number of conjugacy classes is $(3p+12)$.

§.1 Basic Definitions and Examples

In this section we shall set up the basic notations and definitions for later work.

Definition(1.1), [9]: The set of all nxn non-singular matrices over a ring R ,which form a group under the set operation of matrix multiplication. This group is called ***The General Linear Group*** of degree n over R, and denoted by GL(n,R).

<u>Definition(1.2), [7]:</u> Let V be a vector space over any field F, GL(V) denotes the group of all linear isomorphism of V onto itself.

<u>Definition(1.3), [3]</u>: ***A representation*** of a group G is a homomorphism T : G ⟶GL(V).

<u>Definition(1.4), [3]</u>: ***A matrix representation*** of a group G is a homomorphism T : G ⟶GL(n,F), where n is called the degree of the matrix representation.

<u>Definition(1.5), [3]</u>: A representation T : $G \longrightarrow GL(1,\mathbb{C})$ such that T(x)=1 , $\forall\ x \in G$, it is called the ***linear representation*** or ***principle representation*** of G.

<u>Definition(1.6), [4]</u>: ***A class function*** on a group G is a function f:G⟶ $\mathbb{C}$ which is constant on conjugacy classes ,that is, $f(x^{-1}yx) = f(y) \quad \forall\ x, y \in G$.

If all value of f are in $\mathbb{Z}$, then it is called $\mathbb{Z}-$ valued class function.

<u>Definition(1.7), [5]:</u> Let T be a matrix representation of a finite group G over a field F, the ***character*** $\boldsymbol{\chi}$ of T is the mapping $\chi : G \longrightarrow F$ defined by $\chi(g) = tr\big(T(g)\big)$, $\forall\ g \in G$, where $tr\big(T(g)\big)$ refers to the trace of the matrix $T(g)$.

Clearly, $\chi(1) = n$, which is called the degree of χ ,also character of degree 1 is called linear character.

<u>Definition(1.8), [5]</u>: The function $\mathbf{1_G}$ with constant value 1 on G, is a linear character, it is called the ***principle*** or ***unit*** or ***trivial character*** of G.

<u>Lemma(1.9), [5]:</u> Characters of a group G are class functions on G.

<u>Definition(1.10), [5]</u>: Let H be a subgroup of a group G and ϕ be a class function of H, then $\phi\uparrow^G$, the ***induced class function*** on G is given by :

$$\phi\uparrow^G(g) = \frac{1}{|H|}\sum_{x\in G}\phi^{\circ}(xgx^{-1})$$

Where $\begin{cases}\phi^{\circ}(h) = \phi(h) & if\ \ h \in H\\ \phi^{\circ}(h) = 0 & if\ h \notin H\end{cases}$

Clearly $\phi\uparrow^G$ is a class function on G and $\phi\uparrow^G(1) = [G{:}H]\phi(1)$.

Another useful formula for computing $\phi\uparrow^G(y)$ explicitly is to choose representatives $x_1, x_2, \dots, x_m$ for the m classes of H contained in the conjugacy class $C(y)$ in G which is given by $\phi\uparrow^G(y) = \frac{|C_G(y)|}{|C_H(x_i)|}\sum_{i=1}^{m}\phi(x_i)$ ....(1-1)

Where $\phi\uparrow^G(y) = 0$ if $H \cap C(y) = \emptyset$. This formula is immediate from the definition of $\phi\uparrow^G$ since as x runs over G , $xyx^{-1} = x_i$ for exactly $|C_G(y)|$ values of x .

Proposition(1.11), [5]: Let H be a subgroup of G, and ϕ to be a character of H, then $\phi\uparrow^{G}$ is a character.

Definition(1.12), [8]: The character induced from the unit character of a cyclic subgroups of G is called ***Artin character***, and denoted by $\phi(x)$.

Example(1.13): The three conjugacy classes of the symmetric group S_3 are

$C(1)=(1)$, $C(12)=\{(12),(13),(23)\}$ and $C(123)=\{(123),(132)\}$, We calculate the Artin characters (induced characters) of S_3 from the unit characters of the cyclic subgroups H_i , i=1,2,3 by using formula (1-1)

The orders of the three classes are $|C(1)|=1$, $|C(12)|=3$, $|C(123)|=2$

and the orders of the centralizers are $|C_{S_3}(1)|=6$, $|C_{S_3}(12)|=2$, $|C_{S_3}(123)|=3$

Thus

1) (1^3) : $1_{H_1}\uparrow^{S_3}(1)=\frac{6}{1}\sum 1=6$, $1_{H_1}\uparrow^{S_3}(12)=0$ and $1_{H_1}\uparrow^{S_3}(123)=0$

$\phi_1(x)=(6\quad 0\quad 0)$ Since, $(1)\notin C(12)$ and $(1)\notin C(123)$.

2) (12): $1_{H_2}\uparrow^{S_3}(1)=\frac{6}{2}\sum 1=3$, $1_{H_2}\uparrow^{S_3}(12)=\frac{2}{2}\sum 1=1$, and $1_{H_2}\uparrow^{S_3}(123)=0$

$\phi_2(x)=(3\quad 1\quad 0)$ Since, $\langle(12)\rangle\cap C(123)=\emptyset$.

3) (123): $1_{H_3}\uparrow^{S_3}(1)=\frac{6}{3}\sum 1=2$, $1_{H_3}\uparrow^{S_3}(12)=0$ and $1_{H_3}\uparrow^{S_3}(123)=\frac{3}{3}\sum 1+1=2$

$\phi_3(x)=(2\quad 0\quad 2)$ Since, $\langle(123)\rangle\cap C(12)=\emptyset$.

$C(x)$	(1^3)	(12)	(123)
$\lvert C(x)\rvert$	1	3	2
$\lvert C_{S_3}(x)\rvert$	6	2	3
ϕ_1	6	0	0
ϕ_2	3	1	0
ϕ_3	2	0	2

Table(1-1) Artin characters of S_3 .

Lemma(1.14), [5]: Let χ be a rational valued character of G , then, $\forall g \in G$, $\chi(g) \in \mathbb{Z}$.

Lemma(1.15), [5]: Let χ be a rational valued character of G, and let $x, y \in G$ with $\langle x \rangle = \langle y \rangle$, Then $\chi(x) = \chi(y)$.

Definition(1.16), [8]: The ***Artin exponent***, A(G), of a group G is the smallest positive integer A(G) such that A(G) ϕ is an integer linear combination of the induced principle characters of the cyclic subgroups of G, for all rational valued characters ϕ of G.

Remark(1.17), [8]: Let $H_1 = \{1\}, H_2, \dots, H_q$ be the full set of nonconjugate cyclic subgroups of G. We write 1_j, for the principle character on H_j and denote the Artin character (induced character) on G by ϕ_j , which is the character afforded by the rational representation of G and it is clearly depends only on the conjugacy class of the cyclic subgroup H_j.

Definition(1.18), [8]: Let G be a finite group, an integer $m \in \mathbb{Z}$ is said to be an Artin exponent for G if, given any rational character χ on G such that :

$$m\chi = \sum_{k=1}^{q} a_k \phi_k$$

is solvable for integer unknowns $a_k \in \mathbb{Z}$ and for any given rational character χ on G.

Remark(1.19), [8]: All Artin exponents form an ideal in the integers and [G:1] is in this ideal.We pick the (unique) positive generator A(G) for this ideal and we shall call it the Artin exponent of G, A(G) divides $|G|$.

Proposition(1.20), [8]: Let 1_G denote the principal character of G and $d \in \mathbb{Z}$, then d is an Artin exponent of G if it has the following property:
There exist (unique) integers $a_k \in \mathbb{Z}$ such that $d.1_G = \sum_{k=1}^{q} a_k \phi_k$
Where $\phi_1, \phi_2, \dots, \phi_q$ are the Artin characters.
If, $a_1, a_2, \dots, a_q$ have no common factor, then $d = A(G)$ and conversely.

Proposition(1.21), [8]: Let G be an arbitrary finite group, and $H = \{H_1, H_2, \dots, H_q\}$ be a full set of non conjugate cyclic subgroups of G, then A(G) is the smallest positive integer m such that:

$$m.1_G = \sum_{H_k \in H} a_k . 1_{H_k} \uparrow^G \quad \text{.............................(1-2)}$$

With each $a_k \in \mathbb{Z}$.

Remark(1.22), [8]:

1) If m is a positive integer , and (1-2) holds for some set of integers $\{a_k\}$ with greatest common divisor=1, then necessarily $m = A(G)$.

2) Given a group G, We can compute the characters $\{1_{H_k}\uparrow^G\}$ explicitly, and then use proposition(1.21) to determine A(G).

<u>Theorem(1.23), [8]:</u> $A(G) = 1$ iff G is cyclic.

<u>Remark(1.24), [8]:</u> A(G) gives an interesting numerical measure of the deviation of G from being a cyclic group. The invariant A(G) is, therefore, merely a measure of noncyclicity.

<u>Example(1.25):</u> Consider G=S_3 ,Let $H = \{H_1, H_2, H_3\}$ with H_i cyclic subgroups of order i.

According to example(1.13) and its table, if we multiply ϕ_1 by -1 , ϕ_2 by 2 , and ϕ_3 by 1,

then we have : $2\,.\,1_{S_3} = -\left(1_{H_1}\uparrow^{S_3}\right) + 2\left(1_{H_2}\uparrow^{S_3}\right) + (1_{H_3}\uparrow^{S_3})$

and therefore A(S_3)=2.

<u>Definition(1.26), [4]:</u> Let G be a group, then the ***exponent*** of G is the least common multiple of the orders of its elements, and denoted by ***exp(G)*** .

§2 Artin Exponent of $SL(2,\mathbb{Z}_{2p})$

In this section we will find the order of the group $SL(2,\mathbb{Z}_{2p})$, exponent, Artin characters and Artin exponent for this group when p=5,7 and 11 .

<u>(2.1) The Special Linear Group [9]:</u> Let R be commutative ring with 1 , $GL(n,R)$ has a very important subgroup

$$SL(n,R) = \{\, x \in M(n,R) | \det(x) = 1 \,\}$$

Consisting of matrices with determinant 1. The group $SL(n,R)$ is called the ***special linear group* of degree n over R.**

In this work, we interested in the finite special linear groups $SL(2,R)$ in case of $R = \mathbb{Z}_{2p}$, the ring of integers modulo $(2p)$ where p =5,7 and 11 .

To find Artin exponent of $\boldsymbol{SL(2,\mathbb{Z}_{2p})}$**,** We construct a powerful programs to compute the elements of $\boldsymbol{SL}(2,\mathbb{Z}_{2p})$, its order, all conjugacy classes, cyclic subgroups, Artin characters, and then Artin exponent A(G), All programs have been written in ***Mathcad Professional 2001i*** .

Let C_i be the classes of the group $\boldsymbol{SL}(2,\mathbb{Z}_{2p})$, $|C_i|$ be the number of elements in C_i, x_i be a representative of a class C_i (we take the first element in a class C_i as a representative), $o(x)$ be the order of the element x in a group G, Since the elements in the same class have the same order. Thus, We denote $o(x_i)$ to be the order of the elements in a class C_i , H_i be the cyclic subgroup generated by x_i , i.e , $H_i = \langle x_i \rangle$, $C_G(x_i)$ be the centralizer of x_i in G, and ϕ_i be the Artin character (induced character) of G from the unit characters of the cyclic subgroup H_i .

(2.2) Artin Exponent of $SL(2,\mathbb{Z}_{2.5})$: Let $G = SL(2,\mathbb{Z}_{2.5})$ **,** by using our programs, we

found that $|\boldsymbol{G}| = \boldsymbol{720}$, This group has 27 conjugacy classes : C_i , $i = 0,1,\dots,26$.

We write the representative x_i for each class C_i :

$$x_0 = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},\ x_1 = \begin{bmatrix}9 & 0\\0 & 9\end{bmatrix},\ x_2 = \begin{bmatrix}4 & 5\\5 & 4\end{bmatrix},\ x_3 = \begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}, x_4 = \begin{bmatrix}1 & 5\\5 & 6\end{bmatrix}\ , x_5 = \begin{bmatrix}1 & 2\\6 & 3\end{bmatrix},$$

$$x_6 = \begin{bmatrix}0 & 1\\9 & 9\end{bmatrix}\ x_7 = \begin{bmatrix}1 & 2\\4 & 9\end{bmatrix},\ x_8 = \begin{bmatrix}0 & 1\\9 & 0\end{bmatrix},\ x_9 = \begin{bmatrix}1 & 0\\2 & 1\end{bmatrix},\ x_{10} = \begin{bmatrix}1 & 0\\4 & 1\end{bmatrix},\ x_{11} = \begin{bmatrix}4 & 5\\5 & 9\end{bmatrix},$$

$$x_{12} = \begin{bmatrix}1 & 2\\2 & 5\end{bmatrix},\ x_{13} = \begin{bmatrix}0 & 1\\9 & 1\end{bmatrix},\ x_{14} = \begin{bmatrix}0 & 1\\9 & 4\end{bmatrix},\ x_{15} = \begin{bmatrix}0 & 1\\9 & 6\end{bmatrix},\ x_{16} = \begin{bmatrix}1 & 2\\8 & 7\end{bmatrix},$$

$$x_{17} = \begin{bmatrix}1 & 4\\4 & 7\end{bmatrix},\ x_{18} = \begin{bmatrix}0 & 3\\3 & 2\end{bmatrix}, x_{19} = \begin{bmatrix}0 & 3\\3 & 8\end{bmatrix},\ x_{20} = \begin{bmatrix}0 & 1\\9 & 2\end{bmatrix}\ ,\ x_{21} = \begin{bmatrix}0 & 1\\9 & 8\end{bmatrix}$$

$$, x_{22} = \begin{bmatrix}0 & 1\\9 & 5\end{bmatrix}, x_{23} = \begin{bmatrix}0 & 1\\9 & 7\end{bmatrix}, x_{24} = \begin{bmatrix}0 & 3\\3 & 7\end{bmatrix}, x_{25} = \begin{bmatrix}0 & 1\\9 & 3\end{bmatrix},\ x_{26} = \begin{bmatrix}0 & 3\\3 & 3\end{bmatrix}$$

The order of the elements in the group $G = SL(2,\mathbb{Z}_{2.5})$ are :

x_i	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}
$o(x_i)$	1	2	2	2	3	3	3	4	4	5	5	6	6	6	6	6

x_i	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}
$o(x_i)$	10	10	10	10	10	10	12	15	15	30	30

Then $\boldsymbol{exp(G) = 60}$.

Since, conjugate cyclic subgroups give the same Artin characters, thus we need to find the intersection of the non conjugate cyclic subgroups with conjugacy classes of the group $G = SL(2,\mathbb{Z}_{2.5})$

In the following table, we means by (1e) set contain only one element, (2e) set contain two elements and (4e) set contain four elements

∩	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}	C_{16}	C_{17}	C_{18}	C_{19}	C_{20}	C_{21}	C_{22}	C_{23}	C_{24}	C_{25}	C_{26}
H_0	1e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_1	1e	1e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_2	1e	∅	1e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_3	1e	∅	∅	1e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_4	1e	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_5	1e	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_6	1e	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_7	1e	1e	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_8	1e	1e	∅	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_9	1e	∅	∅	∅	∅	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{11}	1e	1e	∅	∅	2e	∅	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{12}	1e	1e	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{13}	1e	1e	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{14}	1e	∅	∅	1e	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{15}	1e	∅	1e	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{16}	1e	1e	∅	∅	∅	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅
H_{18}	1e	∅	∅	1e	∅	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	∅	∅	2e	∅	2e	∅	∅	∅	∅	∅	∅
H_{19}	1e	∅	1e	∅	∅	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	∅	∅	∅	2e	∅	2e	∅	∅	∅	∅	∅
H_{22}	1e	1e	∅	∅	2e	∅	∅	2e	∅	∅	∅	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	4e	∅	∅	∅	∅
H_{23}	1e	∅	∅	∅	2e	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	∅	4e	4e	∅	∅
H_{25}	1e	1e	∅	∅	2e	∅	∅	∅	∅	2e	2e	2e	∅	∅	∅	∅	2e	2e	∅	∅	∅	∅	∅	4e	4e	4e	4e

Table (2.1) Intersection of cyclic subgroups H_i with conjugacy classes C_j

By using formula (1.1), the Artin characters table for $G = SL(2,\mathbb{Z}_{2.5})$ is :

C_i	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{18}	x_{19}	x_{22}	x_{23}	x_{25}
$\|C_i\|$	1	1	3	3	2	20	40	30	90	12	2	20	40	60	60	12	36	36	60	24	24
$\|C_G(x_i)\|$	720	720	240	240	360	36	18	24	8	60	360	36	18	12	12	60	20	20	12	30	30
ϕ_0	720	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_1	360	360	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_2	360	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_3	360	0	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_4	240	0	0	0	240	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_5	240	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_6	240	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_7	180	180	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_8	180	180	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_9	144	0	0	0	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0
ϕ_{11}	120	120	0	0	120	0	0	0	0	0	120	0	0	0	0	0	0	0	0	0	0
ϕ_{12}	120	120	0	0	0	12	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0
ϕ_{13}	120	120	0	0	0	0	6	0	0	0	0	0	6	0	0	0	0	0	0	0	0
ϕ_{14}	120	0	0	40	0	12	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0
ϕ_{15}	120	0	40	0	0	12	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0
ϕ_{16}	72	72	0	0	0	0	0	0	0	12	0	0	0	0	0	12	0	0	0	0	0
ϕ_{18}	72	0	0	24	0	0	0	0	0	12	0	0	0	0	0	0	4	0	0	0	0
ϕ_{19}	72	0	24	0	0	0	0	0	0	12	0	0	0	0	0	0	0	4	0	0	0
ϕ_{22}	60	60	0	0	60	0	0	4	0	0	60	0	0	0	0	0	0	0	4	0	0
ϕ_{23}	48	0	0	0	48	0	0	0	0	8	0	0	0	0	0	0	0	0	0	8	0
ϕ_{25}	24	24	0	0	24	0	0	0	0	4	24	0	0	0	0	4	0	0	0	4	4

Table(2-2) Artin Characters table of the group $G = SL(2,\mathbb{Z}_{2.5})$

Now, from the Artin characters table :

$1/8\ \phi_0 =$	**90**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-5/24\ \phi_1 =$	**-75**	**-75**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-1/8\ \phi_2 =$	**-45**	0	**-15**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-1/8\ \phi_3 =$	**-45**	0	0	**-15**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$-1/4\ \phi_5 =$	**-60**	0	0	0	0	**-6**	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$1/4\ \phi_8 =$	**45**	**45**	0	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0	0	0	0	0
$-1/4\ \phi_9 =$	**-36**	0	0	0	0	0	0	0	0	**-6**	0	0	0	0	0	0	0	0	0	0	0
$-1/6\ \phi_{11} =$	**-20**	**-20**	0	0	**-20**	0	0	0	0	0	**-20**	0	0	0	0	0	0	0	0	0	0
$1/12\ \phi_{12} =$	**10**	**10**	0	0	0	**1**	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0	0
$1/6\ \phi_{13} =$	**20**	**20**	0	0	0	0	**1**	0	0	0	0	0	**1**	0	0	0	0	0	0	0	0
$1/4\ \phi_{14} =$	**30**	0	0	**10**	0	**3**	0	0	0	0	0	0	0	**1**	0	0	0	0	0	0	0
$1/4\ \phi_{15} =$	**30**	0	**10**	0	0	**3**	0	0	0	0	0	0	0	0	**1**	0	0	0	0	0	0
$1/4\ \phi_{18} =$	**18**	0	0	**6**	0	0	0	0	0	**3**	0	0	0	0	0	0	**1**	0	0	0	0
$1/4\ \phi_{19} =$	**18**	0	**6**	0	0	0	0	0	0	**3**	0	0	0	0	0	0	0	**1**	0	0	0
$1/4\ \phi_{22} =$	**15**	**15**	0	0	**15**	0	0	**1**	0	0	**15**	0	0	0	0	0	0	0	**1**	0	0
$1/4\ \phi_{25} =$	**6**	**6**	0	0	**6**	0	0	0	0	**1**	**6**	0	0	0	0	**1**	0	0	0	**1**	**1**
Summation	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

In other words,

$$\mathbf{24}\,.\,\mathbf{1}_G = 3\boldsymbol{\phi}_0 + (-5)\boldsymbol{\phi}_1 + (-\mathbf{3})\boldsymbol{\phi}_2 + (-\mathbf{3})\,\boldsymbol{\phi}_3 + (-6)\,\boldsymbol{\phi}_5 + 6\,\boldsymbol{\phi}_8 + (-6)\,\boldsymbol{\phi}_9 + (-\mathbf{4})\,\boldsymbol{\phi}_{11} + \mathbf{2}\,\boldsymbol{\phi}_{12} + \mathbf{4}\,\boldsymbol{\phi}_{13} + 6\,\boldsymbol{\phi}_{14} + 6\,\boldsymbol{\phi}_{15} + 6\,\boldsymbol{\phi}_{18} + \mathbf{6}\,\boldsymbol{\phi}_{19} + 6\,\boldsymbol{\phi}_{22} + \mathbf{6}\,\boldsymbol{\phi}_{25}$$

Therefore, from this equations we get, the Artin exponent of $G = SL(2, \mathbb{Z}_{2.5})$ is equal to 24,

$$\boldsymbol{A}\big(\boldsymbol{SL}(\mathbf{2}, \mathbb{Z}_{\mathbf{2.5}})\big) = \mathbf{24} = \mathbf{5^2 - 1}$$

(2.3) Artin Exponent of $\boldsymbol{SL}(\mathbf{2}, \mathbb{Z}_{\mathbf{2.7}})$: Let $G = SL(2, \mathbb{Z}_{2.7})$ **,** by using our programs, we found

that $|G| = 2016$, This group has 33 conjugacy classes : C_i , $i = 0,1, \dots, 32$.

The representatives for each class are :

$$\text{Set\_of\_Representative} = \begin{bmatrix} \begin{pmatrix}1&0\\0&1\end{pmatrix} & \begin{pmatrix}13&0\\0&13\end{pmatrix} & \begin{pmatrix}6&7\\7&6\end{pmatrix} & \begin{pmatrix}1&0\\7&1\end{pmatrix} & \begin{pmatrix}1&7\\7&8\end{pmatrix} & \begin{pmatrix}1&2\\2&5\end{pmatrix} & \begin{pmatrix}0&1\\13&13\end{pmatrix} & \begin{pmatrix}1&2\\6&13\end{pmatrix} & \begin{pmatrix}0&1\\13&0\end{pmatrix} & \begin{pmatrix}6&7\\7&13\end{pmatrix} & \begin{pmatrix}1&2\\10&7\end{pmatrix} \\ \begin{pmatrix}0&1\\13&1\end{pmatrix} & \begin{pmatrix}0&1\\13&6\end{pmatrix} & \begin{pmatrix}0&1\\13&8\end{pmatrix} & \begin{pmatrix}1&0\\2&1\end{pmatrix} & \begin{pmatrix}1&0\\6&1\end{pmatrix} & \begin{pmatrix}1&2\\4&9\end{pmatrix} & \begin{pmatrix}1&2\\8&3\end{pmatrix} & \begin{pmatrix}0&1\\13&4\end{pmatrix} & \begin{pmatrix}0&1\\13&10\end{pmatrix} & \begin{pmatrix}0&1\\13&7\end{pmatrix} & \begin{pmatrix}1&2\\12&11\end{pmatrix} \\ \begin{pmatrix}1&6\\4&11\end{pmatrix} & \begin{pmatrix}0&3\\9&2\end{pmatrix} & \begin{pmatrix}0&3\\9&12\end{pmatrix} & \begin{pmatrix}0&1\\13&2\end{pmatrix} & \begin{pmatrix}0&1\\13&12\end{pmatrix} & \begin{pmatrix}0&1\\13&9\end{pmatrix} & \begin{pmatrix}0&3\\9&9\end{pmatrix} & \begin{pmatrix}0&1\\13&3\end{pmatrix} & \begin{pmatrix}0&1\\13&11\end{pmatrix} & \begin{pmatrix}0&1\\13&5\end{pmatrix} & \begin{pmatrix}0&3\\9&5\end{pmatrix} \end{bmatrix}$$

The order of the elements in the group $G = SL(2, \mathbb{Z}_{2.7})$ are :

x_i	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
$o(x_i)$	1	2	2	2	3	3	3	4	4	6	6	6	6	6	7	7	8

x_i	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}	x_{32}
$o(x_i)$	8	8	8	12	14	14	14	14	14	14	21	21	24	24	42	42

Then $\boldsymbol{exp}(\boldsymbol{G}) = \mathbf{168}$.

By using formula (1.1), the Artin characters table for $G = SL(2,\mathbb{Z}_{2.7})$ is :

C_i	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{16}	x_{18}	x_{20}	x_{21}	x_{23}	x_{24}	x_{27}	x_{29}	x_{31}
$\|C_i\|$	1	1	3	3	2	56	112	42	126	2	56	112	168	168	24	42	126	84	24	72	72	48	84	48
$\|C_G(x_i)\|$	2016	2016	672	672	1008	36	18	48	16	1008	36	18	12	12	84	48	16	24	84	28	28	42	24	42
ϕ_0	2016	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_1	1008	1008	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_2	1008	0	336	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_3	1008	0	0	336	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_4	672	0	0	0	672	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_5	672	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_6	672	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_7	504	504	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_8	504	504	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_9	336	336	0	0	336	0	0	0	0	336	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{10}	336	336	0	0	0	12	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{11}	336	336	0	0	0	0	6	0	0	0	0	6	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{12}	336	0	0	112	0	12	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0
ϕ_{13}	336	0	112	0	0	12	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
ϕ_{14}	288	0	0	0	0	0	0	0	0	0	0	0	0	0	36	0	0	0	0	0	0	0	0	0
ϕ_{16}	252	252	0	0	0	0	0	12	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0
ϕ_{18}	252	252	0	0	0	0	0	12	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0
ϕ_{20}	168	168	0	0	168	0	0	8	0	168	0	0	0	0	0	0	0	8	0	0	0	0	0	0
ϕ_{21}	144	144	0	0	0	0	0	0	0	0	0	0	0	0	18	0	0	0	18	0	0	0	0	0
ϕ_{23}	144	0	0	48	0	0	0	0	0	0	0	0	0	0	18	0	0	0	0	6	0	0	0	0
ϕ_{24}	144	0	48	0	0	0	0	0	0	0	0	0	0	0	18	0	0	0	0	0	6	0	0	0
ϕ_{27}	96	0	0	0	96	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	12	0	0
ϕ_{29}	84	84	0	0	84	0	0	4	0	84	0	0	0	0	0	4	0	4	0	0	0	0	4	0
ϕ_{31}	48	48	0	0	48	0	0	0	0	48	0	0	0	0	6	0	0	0	6	0	0	6	0	6

From The Artin characters table, We find that :

$$\mathbf{48\,.\,1_G = 5\phi_0 - 7\phi_1 - 5\phi_2 - 5\phi_3 - 12\phi_5 - 6\phi_7 + 6\phi_8 - 4\phi_9 + 4\phi_{10} + 8\phi_{11} + 12\phi_{12} + 12\phi_{13} - 8\phi_{14} + 12\phi_{18} + 8\phi_{23} + 8\phi_{24} + 12\phi_{29} + 8\phi_{31}}$$

Therefore, from this equation we get, the Artin exponent of $G = SL(2, \mathbb{Z}_{2.7})$ is equal to 48,

$$\boldsymbol{A(SL(2, \mathbb{Z}_{2.7})) = 48 = 7^2 - 1\,.}$$

(2.4) Artin Exponent of $\boldsymbol{SL(2, \mathbb{Z}_{2.11})}$: Let $G = SL(2, \mathbb{Z}_{2.11})$ **,** by using our programs, we find
that $|G| = 7920$, This group has (45) conjugacy classes : C_i $, i = 0,1, \dots, 44$.
The representatives for all classes are :

$$\text{Set\_of\_Representative} = \begin{bmatrix} \begin{pmatrix}1&0\\0&1\end{pmatrix} & \begin{pmatrix}21&0\\0&21\end{pmatrix} & \begin{pmatrix}10&11\\11&10\end{pmatrix} & \begin{pmatrix}1&0\\11&1\end{pmatrix} & \begin{pmatrix}1&11\\11&12\end{pmatrix} & \begin{pmatrix}1&2\\4&9\end{pmatrix} & \begin{pmatrix}0&1\\21&21\end{pmatrix} & \begin{pmatrix}1&2\\10&21\end{pmatrix} & \begin{pmatrix}0&1\\21&0\end{pmatrix} \\ \begin{pmatrix}1&2\\6&13\end{pmatrix} & \begin{pmatrix}1&2\\8&17\end{pmatrix} & \begin{pmatrix}10&11\\11&21\end{pmatrix} & \begin{pmatrix}1&2\\16&11\end{pmatrix} & \begin{pmatrix}0&1\\21&1\end{pmatrix} & \begin{pmatrix}0&1\\21&10\end{pmatrix} & \begin{pmatrix}0&1\\21&12\end{pmatrix} & \begin{pmatrix}1&2\\12&3\end{pmatrix} & \begin{pmatrix}1&2\\14&7\end{pmatrix} \\ \begin{pmatrix}0&1\\21&14\end{pmatrix} & \begin{pmatrix}0&1\\21&18\end{pmatrix} & \begin{pmatrix}0&1\\21&4\end{pmatrix} & \begin{pmatrix}0&1\\21&8\end{pmatrix} & \begin{pmatrix}1&0\\2&1\end{pmatrix} & \begin{pmatrix}1&0\\4&1\end{pmatrix} & \begin{pmatrix}1&2\\2&5\end{pmatrix} & \begin{pmatrix}1&2\\18&15\end{pmatrix} & \begin{pmatrix}0&1\\21&17\end{pmatrix} \\ \begin{pmatrix}0&1\\21&5\end{pmatrix} & \begin{pmatrix}0&1\\21&11\end{pmatrix} & \begin{pmatrix}0&1\\21&16\end{pmatrix} & \begin{pmatrix}0&1\\21&6\end{pmatrix} & \begin{pmatrix}0&1\\21&3\end{pmatrix} & \begin{pmatrix}0&1\\21&7\end{pmatrix} & \begin{pmatrix}1&2\\20&19\end{pmatrix} & \begin{pmatrix}1&4\\10&19\end{pmatrix} & \begin{pmatrix}0&7\\3&2\end{pmatrix} \\ \begin{pmatrix}0&7\\3&20\end{pmatrix} & \begin{pmatrix}0&1\\21&2\end{pmatrix} & \begin{pmatrix}0&1\\21&20\end{pmatrix} & \begin{pmatrix}0&1\\21&15\end{pmatrix} & \begin{pmatrix}0&1\\21&19\end{pmatrix} & \begin{pmatrix}0&1\\21&13\end{pmatrix} & \begin{pmatrix}0&7\\3&13\end{pmatrix} & \begin{pmatrix}0&1\\21&9\end{pmatrix} & \begin{pmatrix}0&7\\3&9\end{pmatrix} \end{bmatrix}$$

The order of the elements in the group $G = SL(2, \mathbb{Z}_{2.11})$ are :

x_i	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}
$o(x_i)$	1	2	2	2	3	3	3	4	4	5	5	6	6	6	6	6	10	10	10	10	10	10	11

x_i	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}	x_{30}	x_{31}	x_{32}	x_{33}	x_{34}	x_{35}	x_{36}	x_{37}	x_{38}	x_{39}	x_{40}	x_{41}	x_{42}	x_{43}	x_{44}
$o(x_i)$	11	12	12	12	12	12	12	12	15	15	22	22	22	22	22	22	30	30	33	33	66	66

Then $\boldsymbol{exp(G)} = 660$

By using formula (1.1), the Artin characters table for $G = SL(2, \mathbb{Z}_{2.11})$ is :

C_i	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{18}	x_{20}	x_{22}	x_{24}	x_{26}	x_{28}	x_{29}	x_{31}	x_{33}	x_{35}	x_{36}	x_{39}	x_{41}	x_{43}
$\lvert C_i\rvert$	1	1	3	3	2	110	220	110	330	132	2	110	220	330	330	132	396	396	60	110	220	220	330	264	60	180	180	264	120	120
$\lvert C_G(x_i)\rvert$	7920	7920	2640	2640	3960	72	36	72	24	60	3960	72	36	24	24	60	20	20	132	72	36	36	24	30	132	44	44	30	66	66
ϕ_0	7920	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_1	3960	3960	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_2	3960	0	1320	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_3	3960	0	0	1320	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_4	2640	0	0	0	2640	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_5	2640	0	0	0	0	48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_6	2640	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_7	1980	1980	0	0	0	0	0	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_8	1980	1980	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_9	1584	0	0	0	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{11}	1320	1320	0	0	1320	0	0	0	0	0	1320	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{12}	1320	1320	0	0	0	24	0	0	0	0	0	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{13}	1320	1320	0	0	0	0	12	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{14}	1320	0	0	440	0	24	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{15}	1320	0	440	0	0	24	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{16}	792	792	0	0	0	0	0	0	0	12	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{18}	792	0	0	264	0	0	0	0	0	12	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{20}	792	0	264	0	0	0	0	0	0	12	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0
ϕ_{22}	720	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	60	0	0	0	0	0	0	0	0	0	0	0
ϕ_{24}	660	660	0	0	0	12	0	12	0	0	0	12	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0	0	0
ϕ_{26}	660	660	0	0	0	0	6	12	0	0	0	0	6	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0	0	0
ϕ_{28}	660	660	0	0	660	0	0	12	0	0	660	0	0	0	0	0	0	0	0	0	0	12	0	0	0	0	0	0	0	0
ϕ_{29}	660	660	0	0	0	12	0	0	4	0	0	12	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0
ϕ_{31}	528	0	0	0	528	0	0	0	0	8	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	0	0	0	0	0
ϕ_{33}	360	360	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	0	0	0	0	0	30	0	0	0	0	0
ϕ_{35}	360	0	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	0	0	0	0	0	0	10	0	0	0	0
ϕ_{36}	360	0	120	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	30	0	0	0	0	0	0	0	10	0	0	0
ϕ_{39}	264	264	0	0	264	0	0	0	0	4	264	0	0	0	0	4	0	0	0	0	0	0	0	4	0	0	0	4	0	0
ϕ_{41}	240	0	0	0	240	0	0	0	0	0	0	0	0	0	0	0	0	0	20	0	0	0	0	0	0	0	0	0	20	0
ϕ_{43}	120	120	0	0	120	0	0	0	0	0	120	0	0	0	0	0	0	0	10	0	0	0	0	0	10	0	0	0	10	10

From the values of Artin characters, ϕ_i's, We get :

$$\begin{aligned}\mathbf{120\,.\,1_G} &= \mathbf{12\phi_0 - 12\phi_2 - 12\phi_3 - 15\phi_5 - 10\phi_7 - 30\phi_9 - 12\phi_{11} - 15\phi_{12}} \\ &\quad \mathbf{+ 15\phi_{14} + 15\phi_{15} + 30\phi_{18} + 30\phi_{20} - 12\phi_{22} + 10\phi_{24} + 20\phi_{26}} \\ &\quad \mathbf{+ 10\phi_{28} + 30\phi_{29} + 12\phi_{35} + 12\phi_{36} + 30\phi_{39} + 12\phi_{43}}\end{aligned}$$

Therefore, from this equation we get, the Artin exponent of $G = SL(2,\mathbb{Z}_{\mathbf{2.11}})$ is equal to 120,

$$\boldsymbol{A(SL(2,\mathbb{Z}_{2.11})) = 120 = 11^2 - 1}\ .$$

From sections (2.2),(2.3) and (2.4) ,We deduce the following propositions:

<u>Proposition (2.5)</u>: For $p = 5{,}7\ and\ 11$; The Artin exponent of the group $\boldsymbol{SL(2,\mathbb{Z}_{2p})}$ is

$$\boldsymbol{A\left(SL(2,\mathbb{Z}_{2p})\right)} = p^2 - 1\ .$$

<u>Proposition (2.6)</u>: For $p = 5{,}7\ and\ 11$; The order of the group $\boldsymbol{SL(2,\mathbb{Z}_{2p})}$ is equal to

$$\left|\boldsymbol{SL(2,\mathbb{Z}_{2p})}\right| = 6p(p^2 - 1)\ .$$

<u>Proposition (2.7)</u>: For $p = 5{,}7\ and\ 11$; The exponent of the group $\boldsymbol{SL(2,\mathbb{Z}_{2p})}$ is equal to

$$\boldsymbol{exp\left(SL(2,\mathbb{Z}_{2p})\right)} = \frac{\mathbf{1}}{\mathbf{2}}p(p^2 - 1)\ .$$

<u>Proposition (2.8)</u>: For $p = 5{,}7\ and\ 11$; The number of conjugacy classes of the group $\boldsymbol{SL(2,\mathbb{Z}_{2p})}$ is equal to $(3p + 12)$.

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