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Optimum Design of RC Beams Strengthened with Externally Bonded FRP Using Pattern Search

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Keywords:
Cost; Coupled shear-flexure strengthening; FRP; Optimization; Pattern search (PS) technique; RC beams; Flexural strengthening; Shear strengthening.

Highlights:

- *The optimum design of RC beams strengthened with EB-FRP using the search approach is studied.*
- *Coupled shear-flexural strengthening is considered in the solution under the restrictions of ACI 440-2R-17.*
- *The pattern search approach produces good results in reducing the cost of the design variables.*

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Abstract: This paper discusses the efficiency of using the Pattern Search (PS) tool to optimize the design of RC beams strengthened with externally bonded (EB) fiber-reinforced polymer (FRP) materials. The study considers shear strengthening (three-sided or two-sided configurations), flexure strengthening, and coupled shear-flexure strengthening. The design process involved an objective function that represents the cost function of the optimum solution and design variables that define the optimum dimensions of the beam and FRP material, such as beam width, effective beam depth, FRP strip thickness, FRP strips spacing, and FRP strip width, according to the design constraints of the ACI 440-2R-17 code. To achieve the goal of this paper, different examples were conducted under shear, flexure, and coupled shear-flexure strengthening. In each case, the PS technique resulted in more compact designs with a faster convergence rate, reducing costs compared to the original dimensions of the design variables. In this context, an extensive parametric study was conducted to determine the ability to redesign the FRP material dimensions compared to previously published data. The results showed that the pattern search technique could save costs in designing RC beams strengthened with externally bonded FRP under similar loading conditions.

التصميم الامثل للعتبات الخرسانية المسلحة المقواة خارجياً بألياف الكاربون باستخدام طريقة البحث عن الانماط

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الخلاصة

يناقش هذا البحث كفاءة تقنية البحث عن الانماط لتحقيق التصميم الامثل للعتبات الخرسانية المسلحة المقواة خارجيًا بالياف الكاربون. اهتمت الدراسة في تقوية القص (تقوية ثلاثية الجوانب أو ثنائية الجوانب)، وتقوية الانثناء، وتقوية انحناء-القص المقترن. تضمنت عملية التصميم دالة الهدف والتي تمثل دالة التكلفة للحل الأمثل ومتغيرات التصميم التي تحدد الأبعاد المثلى للحزمة ومادة FRP (مثل عرض الحزمة وعمق الحزمة الفعال وسماكة شريط FRP وتباعد شرائح FRP وFRP عرض الشريط) وفقًا لقيود التصميم الخاصة برمز ACI 440-2R-17. تم تحقيق هذا الهدف بحل أمثلة مختلفة تحت تقوية القص، وتقوية الانثناء، وتقوية القص والانثناء المقترنة في كل حالة، ينتج عن تقنية PS تصميمات أكثر إحكامًا مع معدل تقارب أسرع يقلل التكاليف مقارنة بالأبعاد الأصلية لمتغيرات التصميم. في هذا السياق، أجريت دراسة بارامترية واسعة النطاق لتحديد القدرة على إعادة تصميم أبعاد مادة FRP مقارنة بالبيانات المنشورة سابقًا. تظهر النتائج أن تقنية البحث عن الأنماط يمكن أن توفر التكاليف في تصميم حزم RC المعززة خارجياً بالياف الكاربون في ظل ظروف تحميل مماثلة.

الكلمات الدالة: الكلفة، تقوية القص-الانحناء المقترن، الياف الكاربون، الامثلية، تقنية البحث عن الانماط، العتبات الخرسانية المسلحة، تقوية الانحناء، تقوية القص.

1.INTRODUCTION

Fiber Reinforced Polymer (FRP) materials have recently become popular building materials for strengthening and repairing structures [1-5]. FRP composites possess numerous advantages, such as high strength, lightweight, highly corrosion resistant, considerable creep strain, superior fatigue resistance, and easy installation. These characteristics have effectively resolved the challenges associated with traditional retrofitting and strengthening methods, like concrete and steel jacketing [6-9]. FRPs are commonly used as "externally bonded (EB)" systems to enhance the capacities of RC structural components, such as flexural, shear, torsional, and axial capacity. Additionally, these materials provide extra confinement, improving the stability and serviceability of structural sections [10, 11]. FRP materials are a commonly used method for increasing the shear strength of beams. These materials are applied to the tension side of structural members, such as slabs and beams, to provide additional reinforcement and enhance their flexural strength. Furthermore, columns can benefit from using FRP materials as they improve ductility, energy dissipation, and shear and axial strength [10, 12, 13]. Most studies have concentrated on flexural or shear failure [2, 14-18]. However, some research has shown that using FRP materials in a combined flexural and shear strengthening configuration can significantly enhance the strength and ductility of the RC beams [13, 19-22]. Studies on RC structures enhanced with EB-FRP have released numerous codes and design guides, e.g., ACI-440.2R-17 [23]. Some equations used for designing with ACI-440.2R-17 [23] give conservative results and lead to using more FRP material than needed. Therefore, more precise and effective design equations for FRP strengthening should be developed to improve the economic aspects of FRP strengthening projects [24, 25]. Cost and safety are important considerations in the design process of any structural engineering project, especially when using strengthening and retrofitting methods. So, the optimization technique is a fantastic tool for achieving this objective [26-28]. Currently, the conceptual and thorough design of structural systems is aided using advanced computer-aided tools and optimization techniques, where optimization achieves the best possible outcome under the given conditions. This work introduces an optimization approach for strengthening RC beams using externally bonded (EB) FRP composites. The optimization process aims to define the design parameters that can lead to the most cost-effective structural strengthening system using FRP fabrics within the restrictions set by ACI-440.2R-17 [23]. The Pattern Search Technique (PS) is used to complete the optimization function. It is a stochastic direct search optimization method that utilizes artificial intelligence methodology and nonlinear design constraints [28]. The objective function formulation includes the cost of all parts of the RC beams, i.e., concrete, steel bars reinforcement, and FRP material. To achieve optimum cost, various optimization techniques, such as genetic algorithms, have been used in previous studies to design RC beams strengthened for flexure and shear with FRP composites [26-28]. The present research aims to find the optimum design for RC beams strengthened with EB-FRP composite to maximize performance, maintain dimensional accuracy, and minimize costs using the PS technique under flexure and shear considerations under restrictions of the ACI-440.2R-17 code [23].

2.SOLUTION METHODOLOGY

This study calculated the minimum cost of an RC beam strengthened with EB-FRP materials. This cost includes the cost of the concrete, the steel reinforcing rebars, and the FRP materials, depending on their volume and the local market price. An objective function will represent different parameters using the pattern search technique to achieve this goal, depending on the following steps:

1- Building a group of points termed a mesh around the specific point can be calculated from a previous iteration step or the user-supplied initial point to start.
2- A multiple scalar group of vectors known as a pattern is added to the present point to create the mesh, which is then created by looking through a collection of points (the mesh) around the current point of parameters for a point with a lower objective function value.
3- Once a point with a lower objective function value is found, the algorithm labels it as the current point. The next iteration will use a larger mesh size due to the expansion factor.
4- The iteration is considered unsuccessful if the algorithm does not identify a point that enhances the objective function.
5- In the subsequent iteration, the present points remain unchanged; however, the mesh size decreases because of the contraction factor.

In this regard, the PS method determines which places to seek at each iteration by examining a set of vectors called a pattern. The pattern is specified by the objective function's number of independent variables, N, and the positive basis set. The maximum basis contains 2N vectors; $v_1 = [1 \quad 0 \quad 0]$, $v_2 = [0 \quad 1 \quad 0]$, $v_3 = [0 \quad 0 \quad 1]$, $v_4 = [-1 \quad 0 \quad 0]$, $v_5 = [0 \quad -1 \quad 0]$, and $v_6 = [0 \quad 0 \quad 1]$, and the minimum basis contains $N+1$ vectors $v_1 = [1 \quad 0 \quad 0]$, $v_2 = [0 \quad 1 \quad 0]$, $v_3 = [0 \quad 0 \quad 1]$. At each iteration, the PS method scans a collection of points called a mesh surrounding the present point (the point computed in the prior step) for a point that enhances the objective function value [29-31]. The mesh is formed as follows [32, 33]:

1- Generate a group of vectors $\{d_i\}$ by multiplying each pattern vector v_i by a scalar m, denoted by the mesh size.
2- Adding the current position to the $\{d_i\}$.

The PS algorithm terminates when any of the following cases happen [32]:

1- The size of the mesh is less than the tolerance.
2- Iterations have reached the maximum allowed number.
3- The overall number of estimations of objective functions has been reached.
4- The time restriction has elapsed.
5- The point identified and the mesh size must be within a specified tolerance for the two successive iterations to be considered close enough.
6- The changes in the objective function and the mesh size after two consecutive iterations are smaller than the function tolerance.

2.1. *Objective Function*

An objective function is a mathematical expression that defines one or more quantities that should be minimized or maximized. Optimization problems can be classified as having a single or several objective functions. Generally, distinct aims are incompatible. Variables that maximize one objective may be suboptimal for another [34]. Multi-objective problems could be reduced to single-objective problems by constructing a weighted mixture of different priorities or considering the objectives as constraints [34]. In the present research, the PS technique was used to find the optimal dimensions for FRP-reinforced beams in the cases of shear and bending, aiming to reduce the cost without exceeding the restrictions imposed by the ACI-440.2R-17 [23]. The objective function $f(x)$ for cost optimization (minimization) of a strengthened beam with FRP is [20, 28]:

$$\boldsymbol{f(x) = V_c\, C_c + V_s C_s + V_f\, C_f} \qquad (1)$$

subjected to:

$$\boldsymbol{G_i(x) \le 0,\ \ i = 1, 2, 3, \dots, p} \qquad (2)$$

$$\boldsymbol{H_j(x)\ ,\ \ j = 1, 2, 3, \dots, m} \qquad (3)$$

where:

$f(x)$ is the objective function.

$G_i(x)$ and $H_j(x)$ are the set of inequality and inequality constraints, respectively.

p and m are the number of inequality and inequality constraints, respectively.

C_c, C_s, and C_f are the unit costs of concrete, steel rebar reinforcement, and FRP sheets, respectively.

V_c, V_s, and V_f are the concrete volume (m3), steel rebar reinforcement volume (m3), and FRP volume (m3), respectively.

C_c, C_s, and C_f are the cost of concrete, steel rebar reinforcement, and FRP in dollars, respectively.

2.2. *Design Variables*

A study will be conducted to find the minimum cost of EB-FRP strengthening of RC beams in flexure and shear by examining multiple design variables. The design variables include:

A. For flexural strength, Fig. 1:

1- b_w and d are the beam cross-section's width and the effective depth, respectively.
2- The reinforcement ratio of the conventional steel bar reinforcement (ρ), ($\rho = \frac{A_s}{b_w\, d}$).
3- The width (w_{ff}) and the flexure thickness (t_{ff}) represent the cross-sectional dimensions of FRP composites.

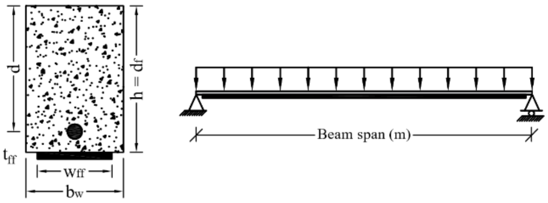
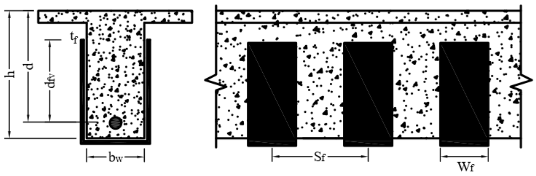


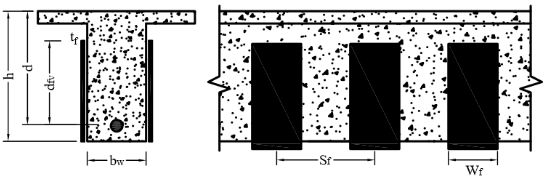
Fig. 1 Flexural Design Variables for FRP Strengthening System

B. Shear design variables, Fig. 2:

1- The spacing between FRP laminates (S_{fs}).
2- The thickness of the FRP laminates (t_{fs}).



(a) U-Configuration Wrapping (Three-sided).



(b) Two-Sided Wrapping.

Fig. 2 Design Variables for Shear in the FRP Strengthening System.

The objective function volumes can be normalized based on design variables:

$$V_{c,flex.} = b_w h(d + d^-) \times 1 \quad (4)$$

$$V_{s,flex.} = \rho b_w d \quad (5)$$

$$V_{f,flex.} = W_{ff} t_{ff} \quad (6)$$

$$V_{f,shear} = W_{fs} t_{fs} S_{fs} \quad (7)$$

where:
h is the total depth (mm).
d is the total effective depth (mm).
$d^-$is the concrete cover (mm).

2.3. Design Constraints

A set of constraints, such as stress on the member, imposes specific values on the unknown while eliminating others. Some conditions must be met for the plan to be practical. For this case, the design constraints is based on the ACI-440.2R-17 [23] to ensure that the design variables are limited within a robust solution while being cost-effective and able to withstand applied loads like moments and shear forces.

2.3.1. Design Constraints in Flexure

The flexural strengthening design constraints of an RC beam with FRP laminates are divided into geometrical and material properties constraints, defined as lower and upper limits for each one:

1- Beam width, b_w.
2- Effective depth, d.
3- Steel reinforcement ratio, ρ.

The limits are represented by $\rho_{min.}$ and $\rho_{max.}$ according to ACI-318-19 [35];

$$\rho_{min.} = maximum \begin{bmatrix} 1.4/f_y \\ \sqrt{f'_c}/4 f_y \end{bmatrix} \quad (8)$$

$$\rho_{max.} = 0.85 f'_c \beta_1 (3/f_y) \quad (9)$$

where:
f'_c is the concrete compressive strength of a standard cylinder at 28 days, MPa. f_y is the steel rebar reinforcement yield strength, MPa. β_1 is the ratio of the depth of an equivalent rectangular stress block to the depth of the neutral axis of the RC beam and can be calculated as follows [35]:

$$\beta_1 = \begin{bmatrix} 0.85 \; for \; f'_c \leq 28 \, MPa \\ 0.85 - 0.05/7 \, (f'_c - 28) for \; f'_c > 28 \, MPa \end{bmatrix} \quad (10)$$

FRP laminates limitations, width, W_{ff} and thickness, t_{ff}, follow the manufacturer's worksheet. The upper width of the FRP laminate is equal to the beam width b_w. While the upper limit of the thickness takes four layers of FRP laminates [20, 21, 36].

2.3.2. Flexure Strength Considerations

In the strength design approach, the design flexural strength φM_n of a member exceeds its required moment strength M_u, which is indicated as:

$$\varphi M_n \geq M_u \quad (11)$$

The flexural strength is reported based on the classical design principles for reinforced concrete as indicated in the specifications of ACI-440.2R-17 [23] and ACI-318-19 [35], as well as the contribution of the FRP strengthening [37]. Then, the design strength will be as follows:

$$\varphi M_n = \varphi[M_{ns} + \psi_f M_{nf}] \quad (12)$$

$$M_{ns} = A_s f_s [d - \beta_1 c/2] \quad (13)$$

$$M_{nf} = A_f f_{fe} [d_f - \beta_1 c/2] \quad (14)$$

where
φ is the reduction factor of strength, 0.9. The ACI-318-19 [35] Code defines members that meet tensile strain (ε_t) > 0.5 as tension-controlled. The corresponding strength reduction factor is ϕ = 0.9.
$M_n$is the nominal moment strength (N.mm).
M_u is the factored moment (N.mm).
ψ_f is the reduction factor of FRP strength, 0.85.
M_{ns} is the conventional steel reinforcement impact on the nominal flexural strength (N.mm).
M_{nf} is FRP laminates' impact on the nominal flexural strength (N.mm).
A_s is the conventional steel reinforcement area, $A_s = \rho \, b_w \, d$, mm^2.
f_s is the conventional steel reinforcement strength (MPa), where $f_s = f_y$ (steel yielding is followed by concrete crushing).
f_y is the conventional steel reinforcement yielding strength, MPa.

$$c = (A_s f_s + A_f f_{fe})/(0.85 f'_c \beta_1 b_w) \quad (15)$$

$$A_f = n t_{ff} W_{ff} \quad (16)$$

$$f_{fe} = E_f \varepsilon_{fd} \quad (17)$$

where

n is the number of FRP plies in flexure.

E_f is the tensile elastic modulus of FRP (MPa).

d_f is the effective depth of FRP of flexural reinforcement (mm).

$$\varepsilon_{fd} = 0.41\sqrt{f'_c/nE_f t_{ff}} \leq 0.9\varepsilon_{fu} \quad (18)$$

where ε_{fd} is the debonding strain of EB-FRP laminates (mm/mm).

$$\varepsilon_{fu} = C_E \varepsilon^*_{fu} \quad (19)$$

where ε_{fu} is the mean rupture strain of EB-FRP laminates (mm/mm).

C_E is the environmental reduction factor, 0.95.

ε^*_{fu} is the ultimate rupture strength of FRP laminates (mm/mm).

2.3.3. Design Constraints in Shear

The shear strengthening design constraints of an RC beam with FRP laminates are defined as:

1- Lower and upper limits of FRP laminate thickness, t_f (Fig. 2).

2- Lower and upper limits of FRP laminates spacing, S_f (Fig. 2).

3- FRP effective strain, ε_{fe}.

4- Steel reinforcement limit, ρ.

The effective strain, ε_{fe}, in the FRP material is the maximum value that can be developed and is governed by the failure mode of the FRP fabrics and the strengthened RC section [23]. In U-shape and two-sided shape configurations, the FRP system is de-laminated from the surface of the concrete before the aggregate interlock loss of the section, eliminating any concrete impact on the shear strength. Therefore, the bond stress becomes critical in locating the FRP laminate effective strain, ε_{fe} [23, 37, 38]. The effective stain, ε_{fe}, is computed using a bond-reduction coefficient k_v, as given by Eq. (20):

$$\varepsilon_{fe} = k_v \varepsilon_{fu} \leq 0.004 \quad (20)$$

$$k_v = \frac{k_1 k_2 L_e}{11900\varepsilon_{fu}} \leq 0.75 \quad (21)$$

$$k_1 = \left(\frac{f'_c}{27}\right)^{2/3} \quad (22)$$

$$k_2 = \begin{cases} \frac{d_{fv}-L_e}{d_{fv}} & \text{for U} - \text{warps type} \\ \frac{d_{fv}-2L_e}{d_{fv}} & \text{for 2} - \text{sides warps type} \end{cases} \quad (23)$$

$$L_e = \frac{23300}{(n_f t_f E_f)^{0.58}} \quad (24)$$

where,

ε_{fu} is the design rupture strain of FRP laminates (mm/mm) according to ACI-440.2R-17 [23],

n_f is the number of FRP plies for shear strengthening,

t_f is the thickness of FRP laminate for shear strengthening (mm)

E_f is FRP modulus of elasticity (MPa)

d_{fv} is the FRP effective depth for shear strengthening (mm)

L_e is the FRP laminates' active bond length (mm).

The reinforcement limit is determined by the contribution of steel reinforcement stirrups and FRP laminates. This contribution is always constrained by diagonal compression shear failure in the beam web, regardless of the quantity of shear reinforcement supplied [38]. The limits for total shear reinforcement are prescribed by Eq. (25) [23, 37, 39]:

$$V_s + V_f \leq 0.66\sqrt{f'_c} b_w d \quad (25)$$

where V_s is the nominal shear strength provided by conventional steel bar reinforcement (N).

V_f is the nominal shear strength provided by EB-FRP fabrics (N).

2.3.4. Shear Strength Considerations

The shear strength of an RC beam is increased by applying FRP laminates on the external side surfaces of the beam to be strengthened with different configurations, including completely warped, U-warp, and two-sided warp [36, 37]. In the present study, the U-shape and two-side shape were used as the warping configuration (Fig. 2). The design shear strength of an RC concrete beam should exceed the required shear strength [35]:

$$\varphi V_n \geq V_u \quad (26)$$

The nominal shear strength of an RC beam strengthened with EB-FRP laminates is a summation of the contribution of EB-FRP laminates, the reinforcing steel stirrups, and concrete:

$$\varphi V_n = \varphi(V_c + V_s + \psi_f V_f) \quad (27)$$

where

φ is the strength reduction factor and is considered to be 0.85 [35]

V_u is the required shear strength (N).

V_c, V_s, and V_f are the nominal shear strength of concrete, conventional steel bar reinforcement, and EB-FRP laminates, respectively (N).

ψ_f is the reduction factor applied to the shear contribution of the FRP reinforcement, 0.85 for U-warp and two-opposite-sides configurations [23].

$$V_f = A_{fv} f_{fe}(\cos\alpha + \sin\alpha) d_f / s_f \quad (28)$$

$$A_{fv} = 2 n_f t_f w_f \quad (29)$$

$$f_{fe} = \varepsilon_{fe} E_f \quad (30)$$

where

A_{fv} is the FRP shear reinforcement area with spacing s, (mm^2).

f_{fe} is the FRP effective stress at section failure (MPa).

α is the FRP reinforcement application direction relative to the longitudinal axis of the beam; in this study, it is 90^o.

d_{fv} is the effective depth of FRP reinforcement (mm).

s_f is the FRP reinforcement spacing, center-to-center (mm).

3.APPLICATIONS OF PATTERN SEARCH (PS) TECHNIQUE

Two examples, the first for flexural and the second for shear separately, have been provided to verify Pattern Search (PS) processing on the RC beams strengthened with EB-FRP laminates. Due to the newness of the technology and limited manufacturers, estimating the costs is not easy. Prices depend on the markets and how things are developed, which differ from one location to another [36]. Table 1 shows the cost of each item. According to the manufacturer datasheet, Table 2 displays the mechanical characteristics of the FRP laminates.

Table 1 Cost of Beam Items.

Item	Cost in dollars ($/m³)
C_c, Concrete	40
C_s, Conventional Steel bar reinforcement	5887.5
C_f, $E_f = 170000\ MPa$, FRP	29166.67

Table 2 FRP Mechanical Characteristics.

Properties	Value
t_{ff}, Ply thickness	1.2mm – 2.64mm
f_{fu}^{*}, Ultimate tensile strength	3100MPa
ε_{fu}^{*}, Rupture strain	0.019 mm/mm
E_f, Elastic modulus	170000MPa

3.1.Example of Flexure

Figure 1 shows that a simply supported beam was chosen to check the validation of the PS approach modeling to seek the optimum (minimum) cost of using the FRP laminates to strengthen the beam in flexure. Table 3 summarizes the material properties and applied loading.

Table 3 Material Characteristics and Applied Loading.

Properties	Value
f_c', Compressive strength of concrete at 28 days	34.5MPa
f_y, Yield strength of conventional steel bar reinforcement	414MPa
M_u, Ultimate moment capacity	304kN.m

Table 4 illustrates the lower and upper bounds for the design restrictions.

Table 4 Design Restriction in Flexure.

Properties	Lower limit	Upper limit
t_{ff}, FRP laminate thickness	1.2 mm	$10.4mm = 2.64 \times 4$
w_{ff}, FRP laminate width	50 mm	300 mm (b_w)
b_w, Beam width	200 mm	300 mm
d, Beam effective depth	250 mm	450 mm
ρ, Conventional Steel bar reinforcement ratio	$\rho_{min.}$	$\rho_{max.}$

Figure 3 clarifies that the solution needs four iterations to achieve the optimum design, where, after the third iteration, the solution remains mostly unchanged until it reaches the fourth iteration. In Fig. 4, it can be seen that the constraint violations reached zero by the second iteration, and the optimal solution is confirmed by the fourth iteration.

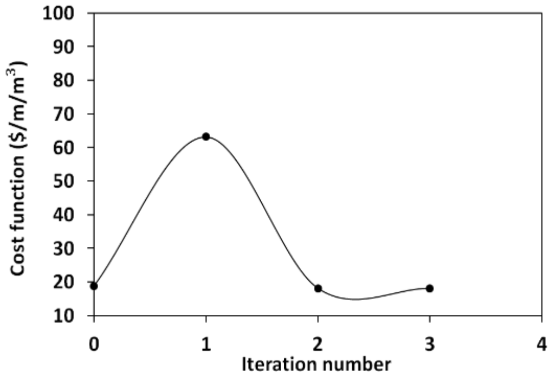


Fig. 3 Iteration Number Versus Cost Function in Flexural Design.

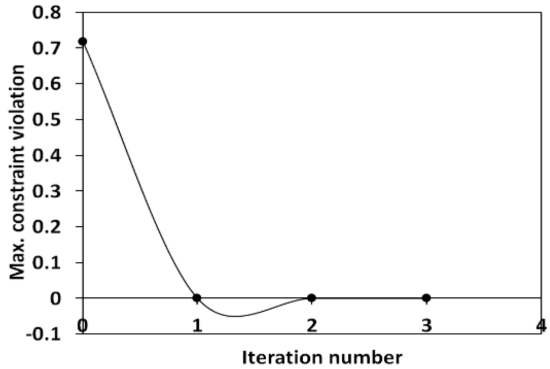


Fig. 4 Iterations Numbers Versus Max Constraints Violation in Flexural Design.

The same example has been solved according to the ACI 318-19 code to find the design cost and compare it to the results of the PS algorithm. The design variables were the effective section depth and the conventional steel reinforcement ratio. As shown in Table 5 and Fig. 5, the PS solution illustrated the best solution, giving the minimum FRP laminate thickness, t_{ff}, of 1.2 mm. The cost of the ACI solution procedures for design was considerably higher than the PS cost, even when beam dimensions were close to optimal values, as shown in Fig. 5. For instance, the ACI (1) solution resulted in a cost function of 42.55 ($/m3), which was 2.35 times higher than the PS cost, highlighting the effectiveness of the PS method in achieving the best design for FRP materials. It is worth noting that the optimum values that depend on the discrete values of the design variables that the local market can supply will not achieve an optimum solution or nearly one. So, choosing the continuous solution to represent these design variables will be preferred over the discrete solution since it can represent the exact optimum value or the one nearest to it. Therefore, after getting the optimum solution, the values were rounded to the closest practical values available in the local market.

3.2.Example of Shear

A T-section beam was selected to verify the validation of the PS simulation to seek the optimum design of using the FRP laminates to strengthen the beam in shear. The present example was adopted in [23, 37]. The live load of a building has been increased abruptly; therefore, the shear strength of a T-beam within this building was inadequate to resist the increased load, while the flexural strength was adequate. Table 6 summarizes the material properties and applied loading contributions. Table 7 defines the design constraints depending on shear simulation. The PS approach simulated U-wrapping and two-sided techniques to investigate the optimum solution of RC beam for shear design. The three-sided and two-sided techniques have been solved; see Fig. 6. A pattern search algorithm achieved the optimum design through five iterations for U-wrapping and two-sided techniques. The objective function was converged to the minimum value as indicated in Fig. 7. Figure 8 depicts the constraint violations through iteration number approach zero at the second iteration and confirms the optimum solution at the fifth iteration for both cases.

Table 5 Optimum Design in Flexure.

Solution Procedure	b_w (mm)	d (mm)	ρ	w_{ff} (mm)	t_{ff} (mm)	Cost Function
ACI(1)	200	400	0.024	90.49	10.37	42.55
ACI(2)	275	350	0.024	138.29	3.75	33.23
ACI(3)	300	390	0.021	50.00	1.20	21.62
PS	**205.42**	**449.98**	**0.022**	**54.65**	**1.23**	**18.11**
ACI(4)	250	440	0.020	50.00	1.20	19.43
ACI(5)	275	380	0.024	50.00	1.20	21.60
ACI(6)	200	340	0.024	274.13	9.73	90.76

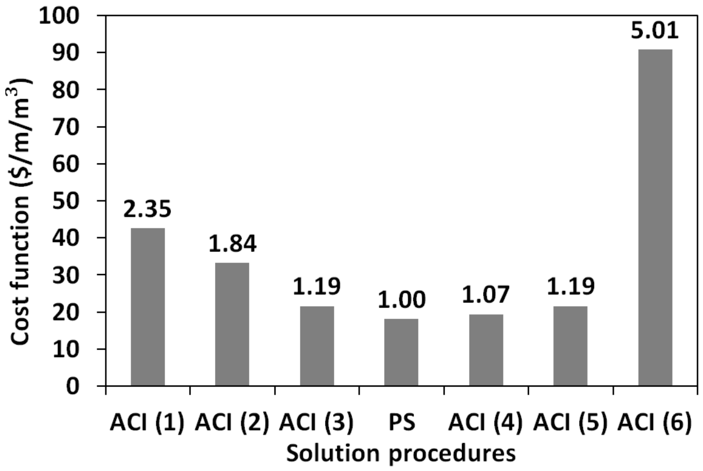


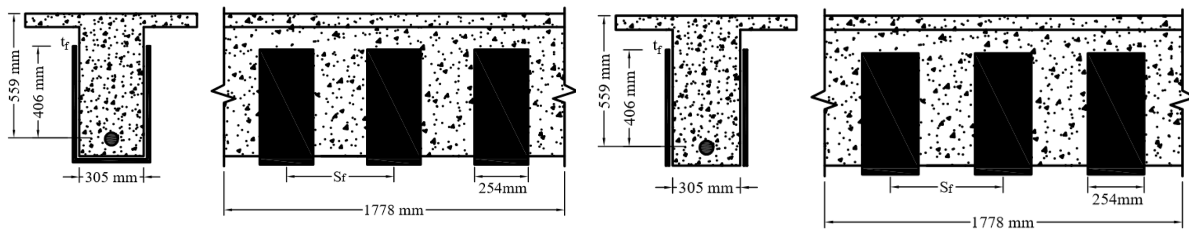
Fig. 5 Solution Procedures Versus Cost Function – Flexural Design.

Table 6 Data of Shear Example.

Properties	Value
f'_c, Compressive strength of concrete at 28 days	21MPa
V_{ud}, Ultimate required shear load	253.3kN
V_c, Nominal shear strength of concrete	196.6kN
V_s, Nominal shear strength of steel reinforcement	87.2kN
d, Beam effective depth	559mm
d_{fv}, FRP effective depth	406mm
Length of the FRP strip	1778mm
w_f, Width of the FRP strip	254mm

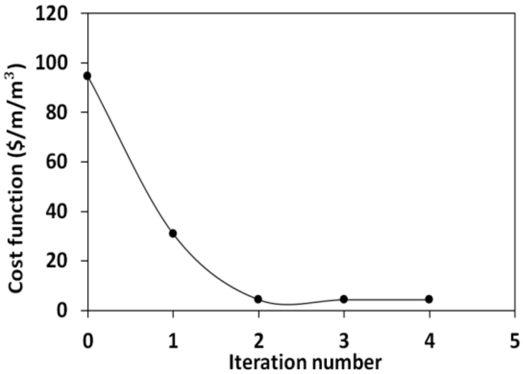
Table 7 Design Constraints for Shear.

Properties	Lower limit	Upper limit
t_f, Thickness of FRP laminate	0.1mm	$2.64 \times 4 = 10.4mm$
S_f, Spacing of FRP laminates	10mm	1220mm

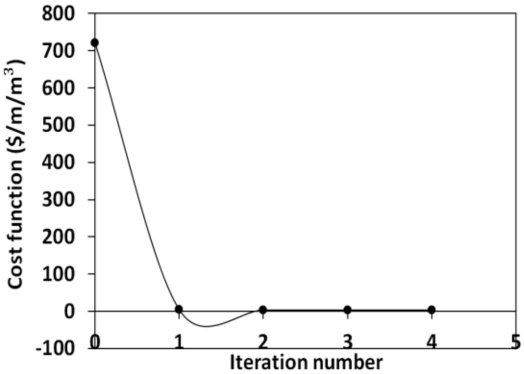


(a) Three-Sided Configuration. (b) Two-Sided Configuration.

Fig. 6 Wrapping Configurations for Shear Example.

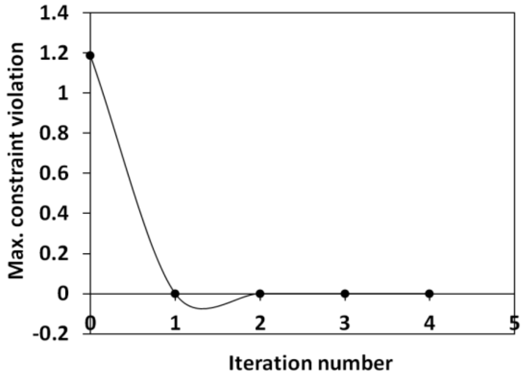


(a) U-Wrapping Technique.

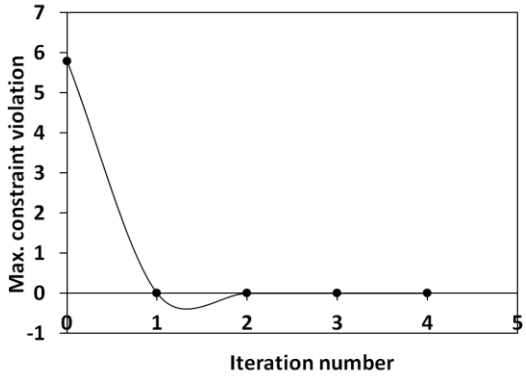


(b) Two-Sided Technique.

Fig. 7 Cost Function Scaling Through Iterations for Shear Design.



(a) U-Wrapping Technique.



(b) Two-Sided Technique.

Fig. 8 Max. Constraints Violation Through Iterations for Shear Design.

Again, the PS technique presented the least cost compared to ACI 318-19 code solutions for both cases. Table 8 and Fig. 9 demonstrate significant differences in the total design section costs. The cost function of the two-sided technique was about 12% lower than the three-sided technique due to the difference in the setup of the FRP sheets (as shown in Fig. 6). In turn, the U-wrapping shape provided a thickness of 0.137 mm, while the two-sided technique provided 0.191 mm to compensate for the difference in wrapping layout.

Table 8 Optimum Solution for RC Beam in Shear Design.

Solution procedure	t_f (mm)		S_f (mm)		Cost Function	
	U-wrap.	two-sided	U-wrap.	two-sided	U-wrap.	two-sided
ACI(1)	0.137	0.200	100	250	12.94	4.812
ACI(2)	0.237	0.100	400	150	5.6	4.01
ACI(3)	0.137	0.211	250	320	5.176	4
PS	**0.137**	**0.191**	**301.268**	**300.808**	**4.3**	**3.818**
ACI(4)	0.154	0.244	320	350	4.532	4.2
ACI(5)	0.182	0.306	350	400	5	4.6
ACI(6)	0.137	0.100	150	100	8.63	6

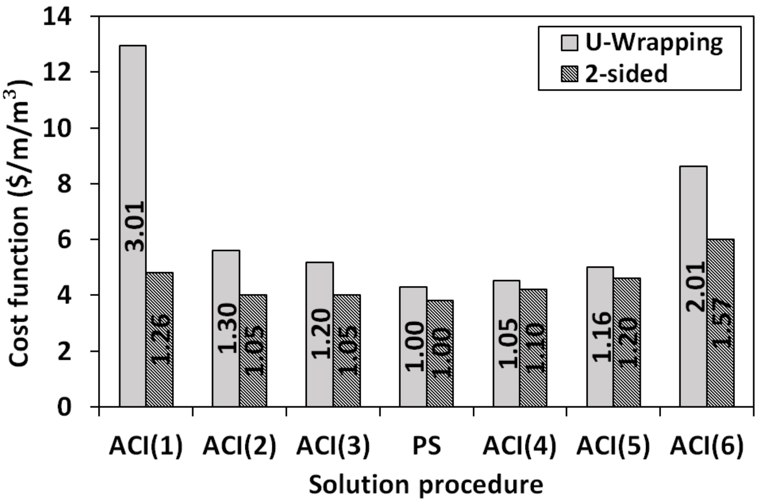


Fig. 9 Cost Function Versus Different Solution Procedures – Shear Design.

Figure 9 indicates that the PS technique provided the optimum design compared to the results of the ACI solution procedures for both shear configurations. In practice, the two-sided wrapping arrangement was less efficient than completely wrapping and U-wrapping in enhancing the shear strength of a beam due to the weakness of end anchoring in compression and tension zones [14].

3.3.RC Beam Strengthened with FRP Sheets under Coupled Shear and Flexure Considerations

The design approach of FRP materials for the flexural and shear strengthening of RC beams can be achieved by an optimization strategy of the FRP material sizes that must satisfy the restrictions recommended by the guidelines, such as ACI-440.2R-17 [23], to ensure the efficiency and safety of the method [20]. However, the following examples highlight the optimum design of the flexural or shear strengthening techniques together. However, the failure mode of the beam will change due to shear and flexural performance. So, coupled shear-flexural strengthening may resist the precocious failure mode due to the FRP strips. Also, in shear or flexure phases, it will be handled in addition to improving the strength capacity and ductility of the beam [12, 13]. Therefore, this section presents the PS technique for the combined flexural-shear strengthening of RC beams with FRP materials.

3.3.1.Example of Coupled Shear-Flexural

The previous examples have been combined to seek the optimum solution using the pattern search (PS) optimization procedure, considering all the previous constraints for the same RC beam under shear and flexure loads. Table 9 illustrates the material properties and applied loads.

Table 9 Data of Coupled Shear-Flexural Example

Properties	Value
f_c', Compressive strength of concrete at 28 days	21MPa
f_y, Yield strength of conventional steel bar reinforcement	414MPa
M_u, Ultimate required moment	304kN.m
V_{ud}, Ultimate required shear load	253.3kN
V_c, Nominal shear strength of concrete	196.6kN
V_s, Nominal shear strength of steel reinforcement	87.2kN

For shear, U-wrapping and two-sided configurations were considered independently and coupled with flexure to investigate the optimum designs and compare the outcomes. The optimum design was achieved through four iterations for both cases, Fig. 10, where the objective function converged to the minimum value, and the maximum constraints were close to zero, as shown in Fig. 11. The PS technique also provided a minimum cost function compared to the ACI 318-19 solution procedure for combined cases for both cases of combination, as shown in Fig. 12, Tables 10 and 11.

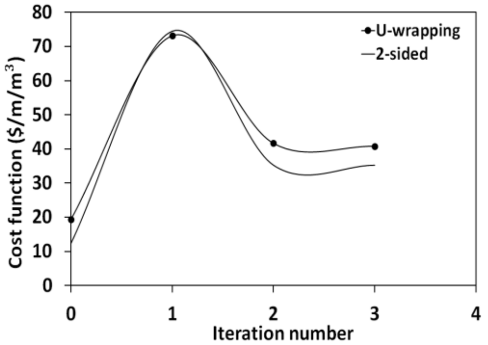


Fig. 10 Cost Function Versus Iterations for Coupled Case.

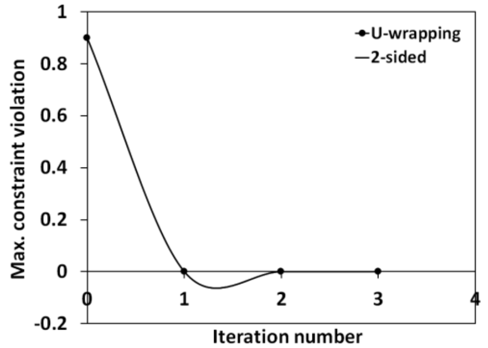


Fig. 11 Iterations Number Versus Max. Constraints Violation for Coupled Case.

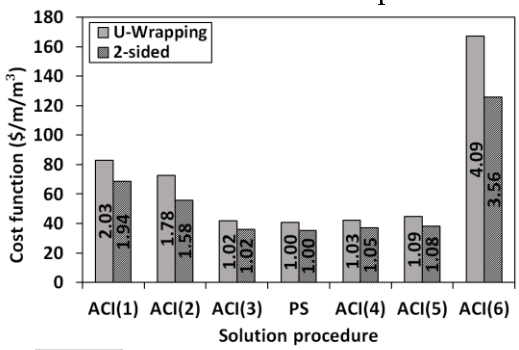


Fig. 12 Cost Function Versus Different Solution Procedures – Combined Case.

Tables 10 and 11 depict that the PS provided the minimum cost. However, the beam cross-section dimensions were not the minimum values ($260.746\,mm \times 449.844\,mm$), meaning the main factors affecting the cost were the FRP properties, $\boldsymbol{W_{ff}}$, $\boldsymbol{t_{ff}}$, **and** $\boldsymbol{t_f}$, which are directly proportional to the cost. In turn, the cost increased when the spacing between FRP strips, $\boldsymbol{S_f}$, decreased (Fig. 13). The U-wrapping technique provided a higher cost than two-sided bonding; however, it is preferred from the structural point of view [14], where the U-wrapping required more thickness in flexure compared to two-sided bonding (Fig. 13). Besides, U-wrapping needed lower thickness in shear strengthening than two-sided bonding. On the other hand, other FRP properties recorded unnoticeable differences (W_{ff} and S_f) (Fig. 13).

Table 10 Optimum Design for Coupled Case – U-wrapping.

Solution procedure	Flexure					Shear		Cost function
	b_w (mm)	d (mm)	ρ	W_{ff} (mm)	t_{ff} (mm)	S_f (mm)	t_f (mm)	
ACI(1)	275	350	0.024	85.761	8.681	103.818	2.276	83.02
ACI(2)	200	400	0.024	98.625	8.745	102.864	1.639	72.69
ACI(3)	250	440	0.0181	50.152	1.201	127.048	1.225	41.823
PS	**260.746**	**449.844**	**0.0161**	**50.089**	**1.659**	**137.315**	**1.200**	**40.827**
ACI(4)	200	440	0.0225	50.578	1.200	114.815	1.200	42.183
ACI(5)	300	400	0.0184	51.420	1.200	115.184	1.201	44.622
ACI(6)	250	315	0.0175	298.00	9.847	100.007	4.116	167

Table 11 Optimum Design for Coupled Case – Two-Sided.

Solution procedure	Flexure					Shear		Cost function
	b_w (mm)	d (mm)	ρ	W_{ff} (mm)	t_{ff} (mm)	S_f (mm)	t_f (mm)	
ACI(1)	275	350	0.024	85.152	8.805	100.027	2.449	68.57
ACI(2)	200	400	0.024	170.851	2.940	100.467	1.804	55.82
ACI(3)	250	440	0.0181	50.007	1.201	116.850	1.200	35.862
PS	**294.775**	**435.643**	**0.0153**	**50.475**	**1.201**	**139.333**	**1.348**	**35.275**
ACI(4)	300	400	0.0184	51.410	1.202	105.590	1.203	37
ACI(5)	200	440	0.0239	50.055	1.200	101.561	1.200	38.137
ACI(6)	250	315	0.024	249.723	9.001	100.518	4.727	125.72

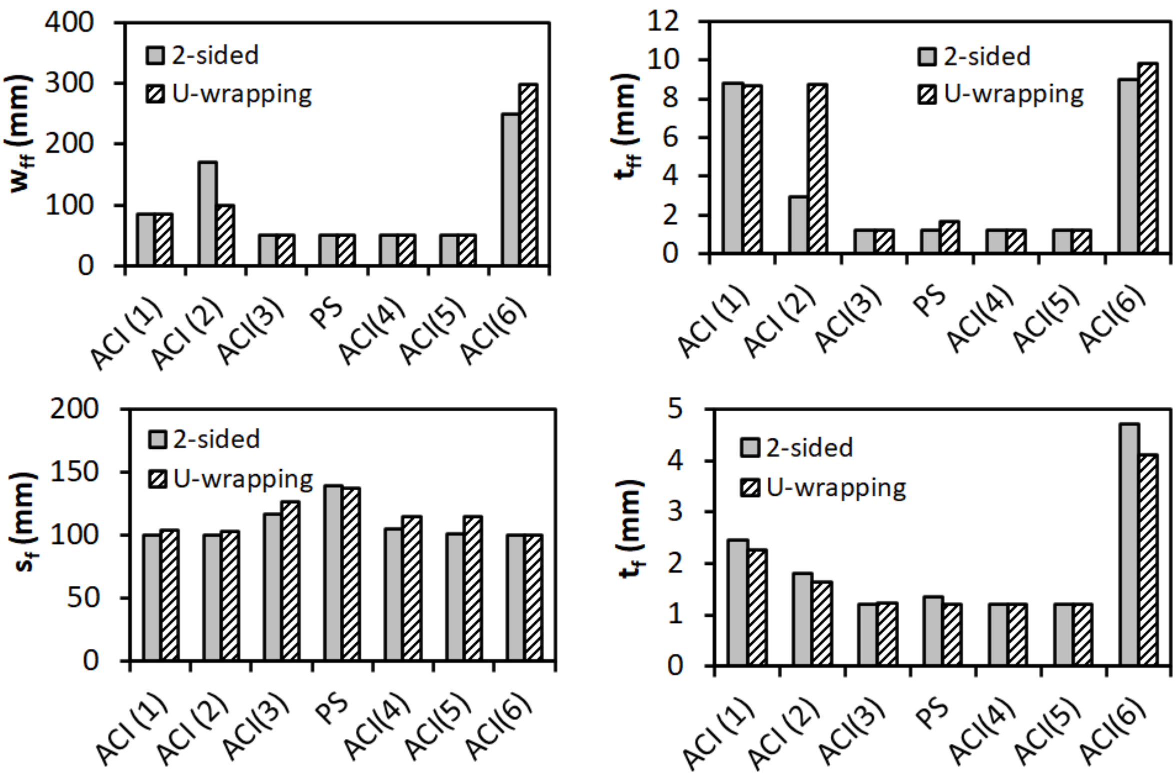


Fig. 13 Solution Procedures Versus FRP Properties.

4.A PARAMETRIC STUDY USING THE PS TECHNIQUE

In earlier sections, different samples were solved to calibrate the adequacy of the PS technique under considerations of shear and flexure separately and with the combined case. However, a parametric study could be conducted to apply the PS technique to optimize the cost of the experimental results. This parametric study seeks to check if it is possible to minimize the cost of the strengthening work using FRP plates with fewer quantities under the same loading conditions. This goal could be achieved by increasing the spacing between FRP plates and reducing the thickness of the used FRP plates.

4.1.Parametric Study Constraints

For this parametric study, only the FRP constraints were considered to meet the boundary conditions of the experimental data. The cost of the tested specimens was computed using the objective function (Eq. (1)) and compared to the PS results. Table 12 displays the database of previous works. The database of the parametric study included RC beams of various dimensions, reinforcement ratios, and FRP configurations. The beam specimens of the database failed in shear or flexure, as reported in Table 12.

Table 12 Database Details of the Parametric Study.

Symbol	Experimental						PS technique					$\frac{Cost_{PS}}{Cost_{Exp.}}$
	Flexure		Shear			$cost_{exp.}$ $/m/m^3$	Flexure		Shear		$Cost_{PS}$ $/m/m^3$	
	W_{ff} (mm)	t_{ff} (mm)	Warp	t_f (mm)	S_f (mm)		W_{ff} (mm)	t_{ff} (mm)	t_f (mm)	S_f (mm)		
Mofidi and Chaallal [24]												
S00.12R	\	\	U	0.11	115	0.848	\	\	0.1633	216	0.67	0.79
S0-0.17R1	\	\	U	0.22	175	2.438	\	\	0.2233	241	1.8	0.74
S0-0.20R2	\	\	U	0.11	50	1.46	\	\	0.1597	82	1.285	0.88
Abdel Hafez [40]												
B-2	\	\	U	0.12	120	0.63	\	\	0.1	120	0.525	0.83
B-5	\	\	U	0.12	60	1.05	\	\	0.1617	83	1.021	0.97
B-8	\	\	U	0.12	180	0.7	\	\	0.1456	265	0.57	0.83
Belarbi et al. [41]												
RC-8-S90-NA	\	\	U	0.16	381	3.178	\	\	0.1234	411	2.2	0.69
RC-8-S90-DMA	\	\	U	0.165	381	3.178	\	\	0.1459	394	2.72	0.85
RC-12-S90-HS-PC	\	\	U	0.165	381	3.178	\	\	0.1015	293	2.541	0.8
Alzate et al. [42]												
U90S5-a (L)	\	\	U	0.293	500	4.56	\	\	0.1421	254	4.354	0.95
U90S5-b (L)	\	\	U	0.293	500	4.56	\	\	0.1456	308	3.68	0.81
Bae S-WS-W and Belarbi A (2013) [34]												
S-Str	\	\	U	0.165	229	1.5	\	\	0.1002	163	1.27	0.85
L-Str	\	\	U	0.165	381	5.742	\	\	0.144	404	4.72	0.82
Triantafillou and Plevris [43]												
B4	63.2	0.65	\	\	\	1.78	144.090	0.167	\	\	1.287	0.72
B5	63.2	0.65	\	\	\	1.78	117.681	0.250	\	\	1.445	0.81
B6	63.3	0.9	\	\	\	2.25	165.217	0.210	\	\	1.6	0.71
B7	63.3	0.9	\	\	\	2.25	135.689	0.312	\	\	1.82	0.81
Altin et al. [44]												
Specimen 2	\	\	U	0.12	125	0.966	\	\	0.1116	129	0.867	0.9
Specimen 3	\	\	U	0.12	150	0.81	\	\	0.1097	166	0.66	0.83
Rahimi and Hutchinson [45]												
A6	150	7.2	\	\	\	33.62	104.755	8.736	\	\	28.8	0.85
A8	150	3.2	\	\	\	16.12	160.037	2.368	\	\	13.17	0.82
A10	150	3.2	\	\	\	16.12	134.696	3.342	\	\	15.25	0.94
A11	150	3.2	\	\	\	16.12	166.630	2.184	\	\	12.7	0.79
B3	150	0.8	\	\	\	5.62	289.786	0.288	\	\	4.55	0.81
B4	150	0.8	\	\	\	5.62	282.618	0.303	\	\	4.6	0.82
Quantrill et al. [46]												
B2	80	1.2	\	\	\	3.7	125.312	0.694	\	\	3.4	0.93
B3	30	1.2	\	\	\	2	50	0.689	\	\	1.9	0.98
B4	60	1.6	\	\	\	3.7	126.478	0.689	\	\	3.44	0.93
B6	80	1.2	\	\	\	3.7	256.354	0.289	\	\	3.06	0.83
A1b	80	1.2	\	\	\	3.7	92.657	0.910	\	\	3.35	0.91
A1c	80	1.2	\	\	\	3.7	92.607	0.911	\	\	3.36	0.91
A2b	80	1.2	\	\	\	3.7	157.872	0.546	\	\	3.41	0.93
A2c	80	1.2	\	\	\	3.7	156.73	0.554	\	\	3.43	0.93

4.2.Parametric Study Outcomes

The main goal of this parametric study is to apply the PS approach to check if the cost of strengthening configurations made of FRP materials may be reduced. Equation (1) was used to determine the cost of each specimen and compared it to that calculated using the PS method. According to the study, adjusting the thicknesses of the strips, spacing between strips, and width of the strips for shear and flexure failure modes in tested beam conditions could reduce the cost of the strengthening, as demonstrated in Table 12. For instance, Mofidi and Chaallal [24] tested specimen S00.12R under shear consideration using an FRP strip that had a thickness (t_f) of 0.11 mm and a spacing (S_f) between strips of 115 mm. The original cost function was 0.848 $/m/m^3. However, after applying the PS technique to optimize the specimen, the cost of the strips decreased to 0.67 $/m/m^3, resulting in an 18% cost savings. They accomplished this reduction by increasing the thickness to 0.163 mm and the distance between strips to 216 mm. As a result, the PS results demonstrated that changing the thickness, width, and spacing between the FRP elements effectively reduced costs.

5.SUMMARY AND CONCLUSIONS

EB-FRP composites were used to reinforce or retrofit beams, which enhanced shear and flexure strengths. The optimization strategy is to minimize the costs of strengthening, which can be high due to the expensive FRP material. The effectiveness of the PS technique was demonstrated by the simulation of several RC beams that failed under shear, flexure, and coupled shear-flexure. Additionally, a parametric study was conducted to compare the cost of the tested beams and check the possibility of minimizing the costs of the strengthening process using the PS technique. The following conclusions can be drawn:

1- The PS technique yielded better outcomes (lower costs) than the costs of the specimens solved according to the ACI 318-19 code for the same conditions.
2- The strengthening in shear was accomplished using U-wrapping (three-sided) and two-sided wrapping configurations. Because of the differences in the FRP sheet arrangement, the two-sided approach had a cost function approximately 12% lower than the three-sided technique. However, in practice, U-warping proved superior to two-sided design configuration due to the weakness of end anchoring in compression and tension zones for two-sided configuration.
3- The coupled shear-flexural case was also solved. Based on the results, it was found that using the PS approach was more effective in achieving the optimum cost for the beams compared to those designed in accordance with the ACI 318-19 code and under identical conditions.
4- A parametric study was performed based on a database of beams compiled from previous works. The results indicated the ability to reduce the costs of these beams under identical conditions using the PS approach by calibrating the dimensions of the FRP composites.

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