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Study Effect of Majorana Coefficient on the Energy Levels of $^{138-144,148}$Gd Even-Even Isotopes Using IBM-2

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Abstract: The mixed symmetry states property (MSS) for even-even $^{138-144}$ and ^{148} Gd isotopes was studied using IBM-2. This model discovered and emphasised various empirically questionable states and provided good agreement for the excited energy levels for the energy bands ground, g-band and acceptable for the excitations – β and γ bands. The effect of the Majorana parameter was studied as a function of changing the mixing symmetry of the levels using three different values (0.005, 0.02, and 0.04), and the value of the Majorana parameter was almost constant. By increasing the Majorana parameter to the values (0.02 and 0.04), the mixing levels began to appear clearly, especially the levels. Energy levels and energy ratios showed that ^{138}Gd and ^{140}Gd are between rotational SU(3) and gamma-unstable O(6) symmetries, and ^{142}Gd, ^{144}Gd, and ^{148}Gd are transitional nuclei among gamma-unstable O(6) to vibrational SU(5) symmetries. The results of theoretical and practical energy levels were in good agreement.

Keywords: IBM-2, Energy levels, MSS, Gd isotope, SU(3), SU(5), O(6).

1.Introduction

2. The study of nuclear structure is a fundamental area of nuclear physics, aiming to understand the properties and behaviour of atomic nuclei. Among the many elements investigated, gadolinium (Gd) holds particular significance due to its neutron-rich isotopes, which exhibit diverse structural properties. The isotopes ^{138}Gd, ^{140}Gd, ^{142}Gd, ^{144}Gd, and ^{148}Gd are of special interest because of their role in nuclear deformation, shape coexistence,

and collective excitations. The theoretical framework often used to describe these properties is the Interacting Boson Model-2 (IBM-2), which effectively treats the nucleus as a system of interacting bosons representing nucleon pairs.IBM-2 extends the capabilities of IBM-1 by differentiating between neutron and proton bosons, allowing for a more detailed representation of nuclear dynamics [1]. The gadolinium isotopes under consideration belong to the rare-earth region, where complex nuclear structures emerge due to strong correlations between nucleons.The nuclear shell model suggests that these isotopes exhibit properties associated with shape transitions, from spherical to deformed structures, depending on the neutron-to-proton ratio.The IBM-2 model provides a powerful tool to analyse these transitions by considering neutron and proton bosons separately, leading to a better understanding of phase coexistence and symmetry-breaking phenomena in medium-to-heavy nuclei [2].^{138}Gd is near closed shells, exhibiting limited collectivity, while ^{140}Gd and ^{142}Gd show increasing quadrupole collectivity, transitioning to deformation [3]. ^{144}Gd and ^{148}Gd show increased deformation due to extra neutrons, enhancing quadrupole correlations and transitioning from rotational to vibrational motion [4].IBM-2 describes shape mixing in gadolinium isotopes, with ^{148}Gd showing strong spherical-deformed interactions, affecting level crossings and transition strengths [5].IBM-2 effectively describes structural evolution in Gd isotopes, capturing shape transitions and collectivity, with future studies refining nuclear structure understanding [6].

Theoretical Part

The IBM-2 represents a significant advancement in the evolution of the IBM. This method provides a comprehensive basis for collective nuclear states, as delineated by the IBM-1, with the potential that the IBM-2 may be constructed from the shell model. This advancement, grounded in the concept of generalized fermion seniority [7,8], was suggested by Arima et al. [9-14]. The model provides a clear physical description of bosons as correlated pairs of particles with spin and parity $J^{\pi} = 0^{+}$ and 2^{+}. The Hamiltonian operator in the IBM-2 comprises three components: first for proton bosons, second for neutron bosons, and a third for the proton-neutron interaction [15-17].

$$H = H_{\pi} + H_{v} + V_{\pi v} \tag{1}$$

A clear schematic Hamiltonian based on microscopic principles is shown as [15-17]:

$$H = \varepsilon(n_{dv} + n_{d\pi}) + \kappa Q_{v} Q_{\pi} + V_{vv} + V_{\pi\pi} + M_{v\pi} \tag{2}$$

In the first term presumed that ε_{π} and ε_{v} represent the energies of the proton and neutron, respectively, the second term refers to the quadrupole – quadrupole interaction between proton and neutron with strength κ, where the quadruple operatorQ_{ρ} can be written as

$$Q_{\rho} = \left(d_{\rho}^{\dagger} s_{\rho} + s_{\rho}^{\dagger} d_{\rho}\right)_{\rho}^{2} + \chi_{\rho}\left(d_{\rho}^{\dagger} d_{\rho}\right)_{\rho}^{2} \qquad \rho = \pi \text{ or } v \tag{3}$$

The $V_{\pi\pi}$ and V_{vv}, which refer to the interaction between like – boson, are sometimes present to improve the fit to experimental energy spectra and they are given by

$$V_{\rho} = \sum_{L=0.2.4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_{L}^{\rho} \left[\left(d_{\rho}^{\dagger} d_{\rho}^{\dagger}\right)^{(L)} \cdot \left(d_{\rho} d_{\rho}\right)^{(L)}\right]^{(0)} \tag{4}$$

The Majorana operator $M_{\pi v}$, included in the last component of equation (2) to eliminate proton-neutron mixed symmetry states, is included. This word can be expressed as follows:

$$M_{v\pi} = \zeta_{2}\left(s_{v}^{\dagger} d_{\pi}^{\dagger} - d_{v}^{\dagger} s_{\pi}^{\dagger}\right)^{(2)} \left(s_{v} d_{\pi} - d_{v} s_{\pi}\right)^{(2)} + \sum_{k=1.3} \zeta_{k}\left(d_{v}^{\dagger} d_{\pi}^{\dagger}\right)^{(k)} - \left(d_{v} d_{\pi}\right)^{(k)} \tag{5}$$

If empirical evidence substantiates the presence of the 'mixing symmetrical condition' the Majorana factor will be adjusted to realign the positions of these levels within the continuum. Consequently, a system composed of neutron and proton bosons is analyzed. In the IBM-2, the microscopic interpretation of the boson count $N=N_\pi+N_\nu$ determines the total numbers of bosons, N, formerly regarded as a parameter in the IBM-1. Energy levels may be derived by diagonalising Hamiltonian equation (2). and the parameters $\varepsilon,\kappa,\chi_\pi,\chi_\nu$ and C_L with experimenting to identify the optimal correspondence with the observed spectrum. A certain type of boson can generate spectra analogous to those of the IBM [18]. When protons and neutrons are out of stage in the quantum state, the MSS occurs [19]. Two wave functions, one for the proton and one for the neutron, were blended to create these states [20]. The F-turn shape dictates the blended symmetry states, where the nucleons' isospin quantum number is comparable to the F-turn formalism. $F_z = +1/2, -1/2$ for protons and neutrons independently, and $F = 1/2$ and z-projection for proton and neutron bosons. The blended symmetry states are defined by declining F-turn esteem $(F = F_{max}, F = F_{max} - 1, F = F_{max} - 2, \dots, F_{min} = |N\pi - N\nu|/2)$ while the most extreme F-turn for a framework consisting of N_π proton boson and N_v neutron boson is $F = F_{max} = (N\pi + N\nu)/2$. The generation and obliteration administrators F_+ and F_- can also be described in the F-turn space by [21,22];

$$F_+ = s_\pi^\dagger s_\nu + \left(d_\pi^\dagger . d_\nu\right) \quad (6)$$

$$F_- = s_\nu^\dagger s_\pi + \left(d_\nu^\dagger . d_\pi\right) \quad (7)$$

The projection operator F_z is given by;

$$F_z = \frac{N_\pi - N_\nu}{2} \quad (8)$$

The increasing action of the F-turn raising administrator F_+ can transform a state composed of N_π proton bosons and N_ν neutron bosons with F-turn quantum number $F = F_{max}$ into an express that, in a sense, consists of proton bosons. Since the raising administrator does not alter the aggregate F-turn quantum number, this state still has an aggregate F-turn quantum number of $F = F_{max}$. With only proton bosons, this new state is obviously unaffected by a pairwise exchange of proton and neutron names. Thus, Full Symmetry States (FSS) are IBM-2 states where $F = F_{max}$. These states are very similar to the mostly symmetric IBM-1 states. Sets (at least one) of proton and neutron bosons are against symmetric under a pairwise trade of protons and neutrons names in all other states with F-turn quantum numbers $F < F_{max}$. [23] They are known as mixed-symmetry states (MSS). is the standard F-turn choice run. The quality of M1 progress between low-lying aggregate states and F-turn blending can be used when. In addition, M1 progress gathered between two absolutely symmetric states is forbidden, so M1 change occurs between low-lying aggregate states. These states must contain segments with a blended symmetry state [20, 24].

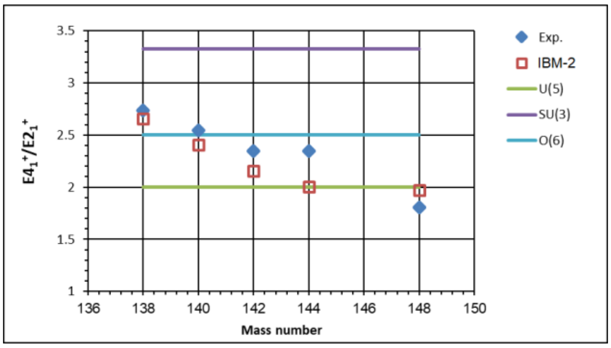
3. Calculations and Results

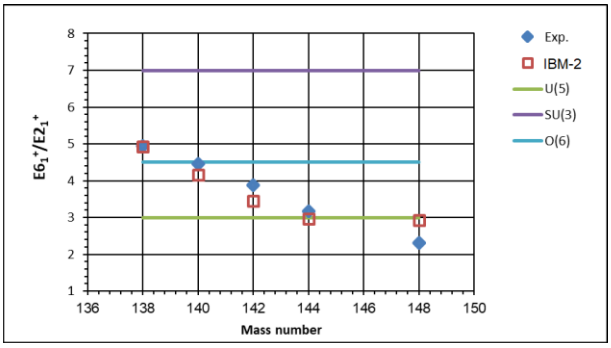
The nuclear structure of gadolinium isotopes $^{138-144}$ and ^{148}Gd are an essential topic of study within the framework of the Interacting Boson Model (IBM). This model provides a powerful approach to describing medium and heavy nuclei by considering the interaction between paired nucleons (protons and neutrons), which form bosons with specific angular momentum states (s-bosons with angular momentum 0 and d-bosons with angular momentum 2) [25, 26].

By calculating the many variables shown in the Hamiltonian operator equations (1) and (2). The software package Neutron Proton Boson code for has been used to calculate energy levels for 138-144 and 148Gd isotopes by estimating a set of parameters described in the Hamiltonian operator [27, 28], equation (1) as shown in Table 1. The first test of the dynamic symmetries has been shown through theoretical and experimental energy levels and after a comparison with the standard values for the energy ratios [29-33]. The calculation of experimental energy ratios of ( ), ( ) and ( ) ratios for allied $^{142, 144}$, and ^{148} Gd isotopes as a function of mass numbers has been indicated in Figure 1. This leads to guessing the nearest dynamic symmetries corresponding to the characteristics of one of the dynamic symmetries or may possess transitional features between two or more symmetries. The levels of the calculated energy compared with the experimental data [29-33] of $^{142, 144}$, and ^{148} Gd isotopes have been shown in Figure 2. The results were expected to be acceptable according to the values of the correlation coefficient between the practical and calculated values, shown in Figure 3.

Table 1: The IBM-2 Hamiltonian parameters for studied isotopes.

The parameters	**Isotopes**				
	^{138}Gd	**^{140}Gd**	**^{142}Gd**	**^{144}Gd**	**^{148}Gd**
N_π	7	7	7	7	7
N_ν	4	3	2	1	1
ε_d	0.4	0.5	0.5	0.92	0.8
κ	-0.2	-0.04	-00.4	-0.1	-0.01
χ_π	-0.9	-0.9	-0.9	-0.9	-0.9
χ_ν	0.89	-0.8	0.2	-0.1	0.03
ζ_2	0.01	0.02	0.02	0.1	0.1
$\zeta_{1,3}$	-0.02	-0.02	-0.02	-0.02	-0.02
C_π^L	(0.1,0.1,0.01)	(0.0,-0.1,0.08)	(-2.5,-1.5,0.05)	(-1.2,0.0,-0.10)	(2.6,2.02,-0.06)
C_ν^L	(0.1,0.1,0.01)	(0.0,0.01,0.8)	(2.3,0.2,0.07)	(0.3,0.4,-0.10)	(0.0,2.2,-0.03)





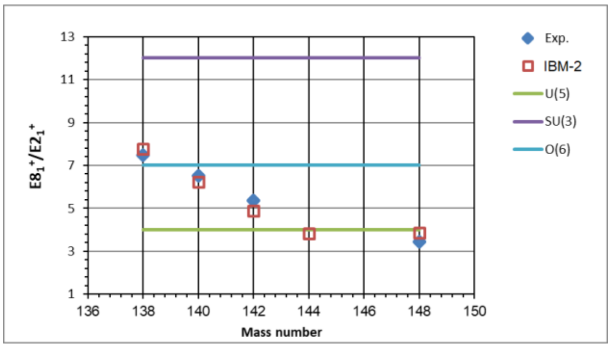
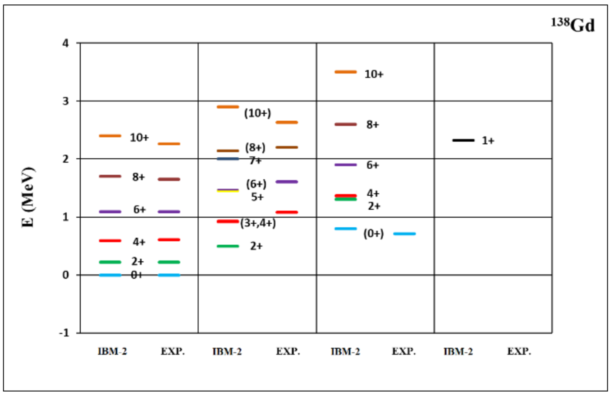
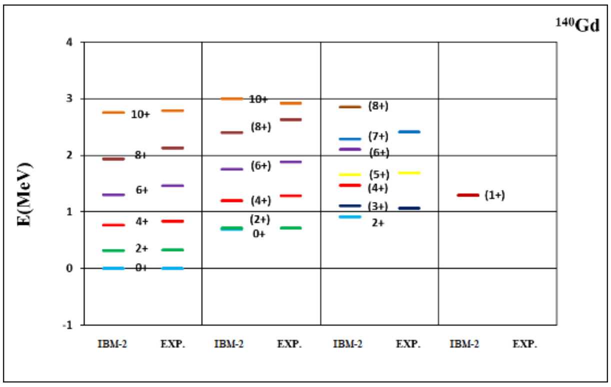
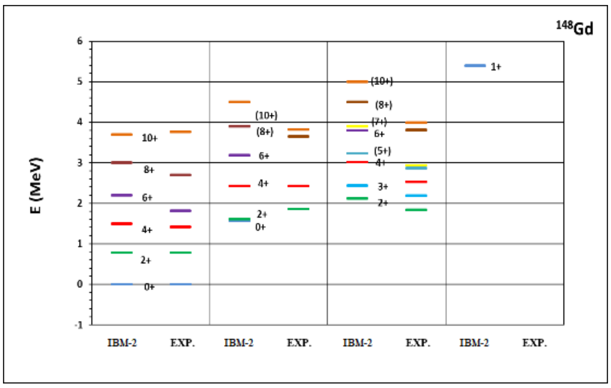
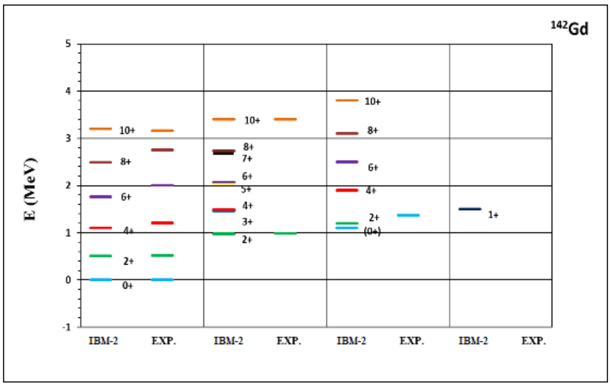


Fig. (1): The theoretical, and experimental [29–33] energy ratios for the isotopes $^{138-144,148}$Gd as a function of mass numbers. These are $(E4_1^+/E2_1^+)$, $(E6_1^+/E2_1^+)$ and $(E8_1^+/E2_1^+)$, respectively.









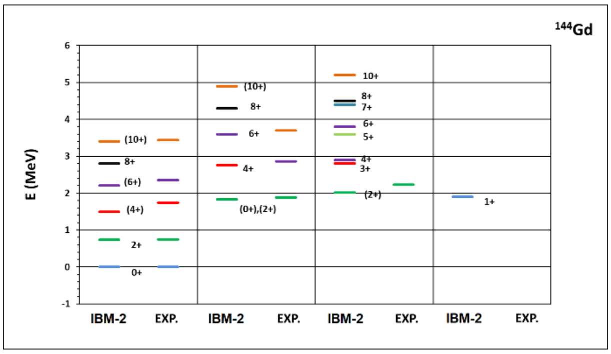


Fig. (2): Comparing of the estimated and experimental states [29-33] for the isotopes $^{138-144,148}$Gd.

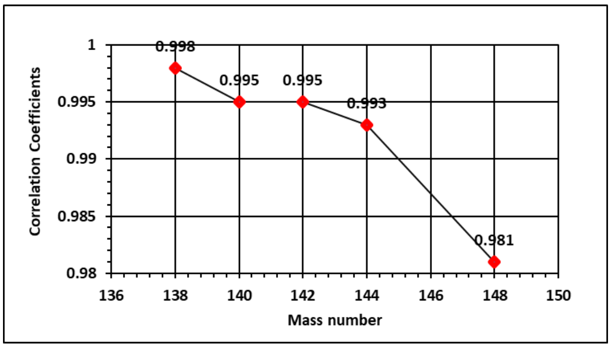
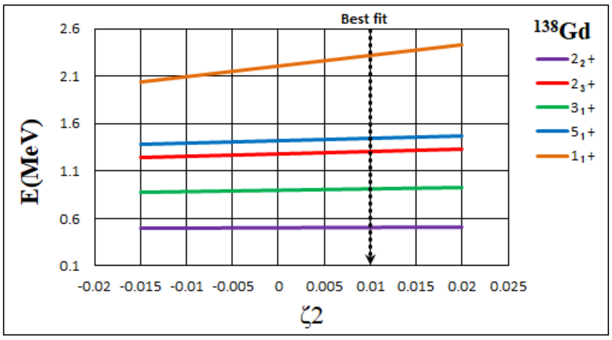


Fig. (3): Correlation coefficients between experimental and theoretical values of $^{138-144,148}$Gd isotopes.

The presence of some evidences indicating cases of mixing symmetry ,the effect of the Majorana coefficient (ζ_2) on the excitation energy level calculated by fixing the values of the coefficient ($\zeta_{1,3}$) for all isotopes $^{138\text{-}144,148}$Gd and changing the values of ζ_2 around the best match with the practical values by three values (0.005), (0.02) and (0.04) as shown in Figure 4, 5, and 6.



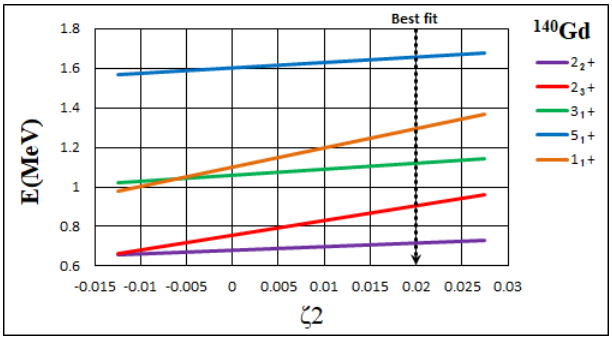
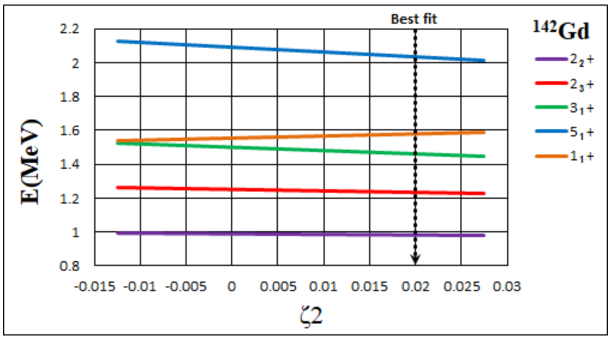
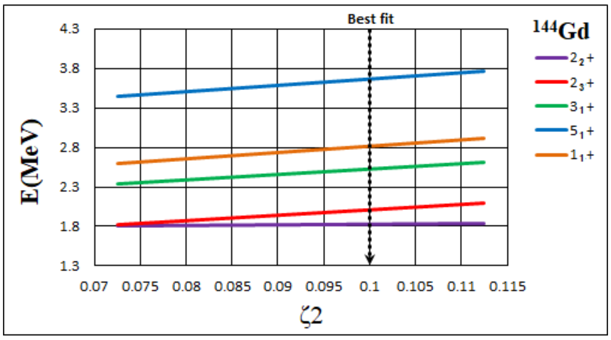
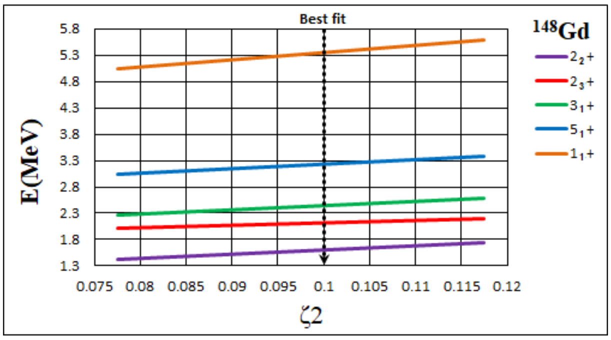
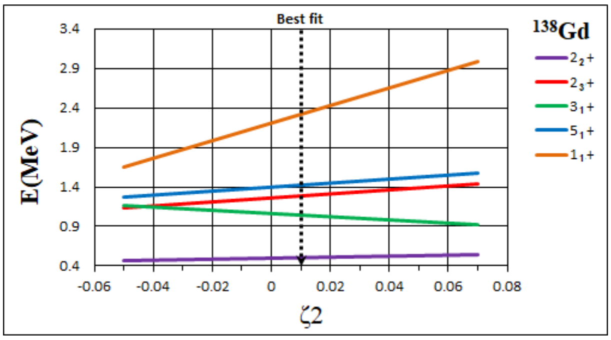


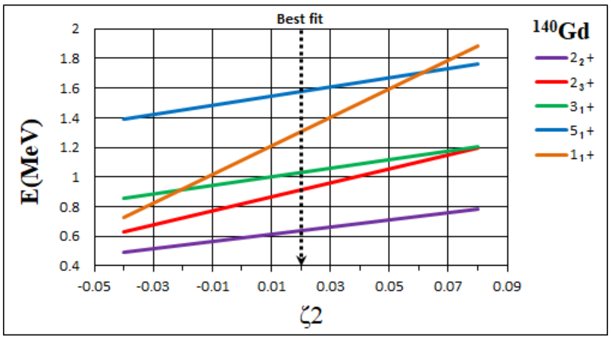
Fig. (4): Mixed symmetry states of the ($2_2^+, 2_3^+, 1_1^+, 3_1^+$ and 5_1^+) levels as a function of Majorana parameter at the value (0.005) of the even-even isotopes ($^{138\text{-}144}$Gd,^{148}Gd)











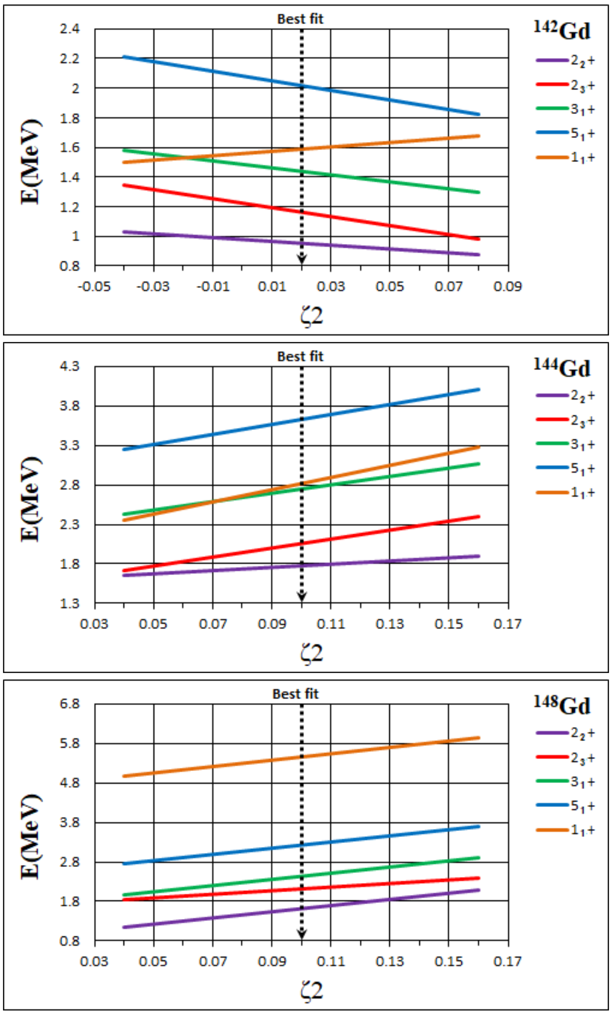
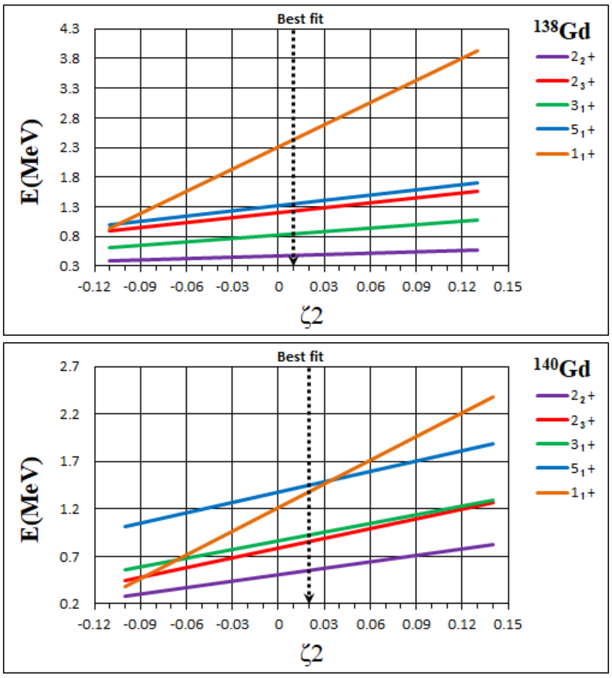


Fig. (5): Mixed symmetry states of the $(2_2^+, 2_3^+, 1_1^+, 3_1^+ \ and \ 5_1^+)$ levels as a function of Majorana parameter at the value (0.02) of the even-even isotopes ($^{138-144}$Gd, ^{148}Gd).



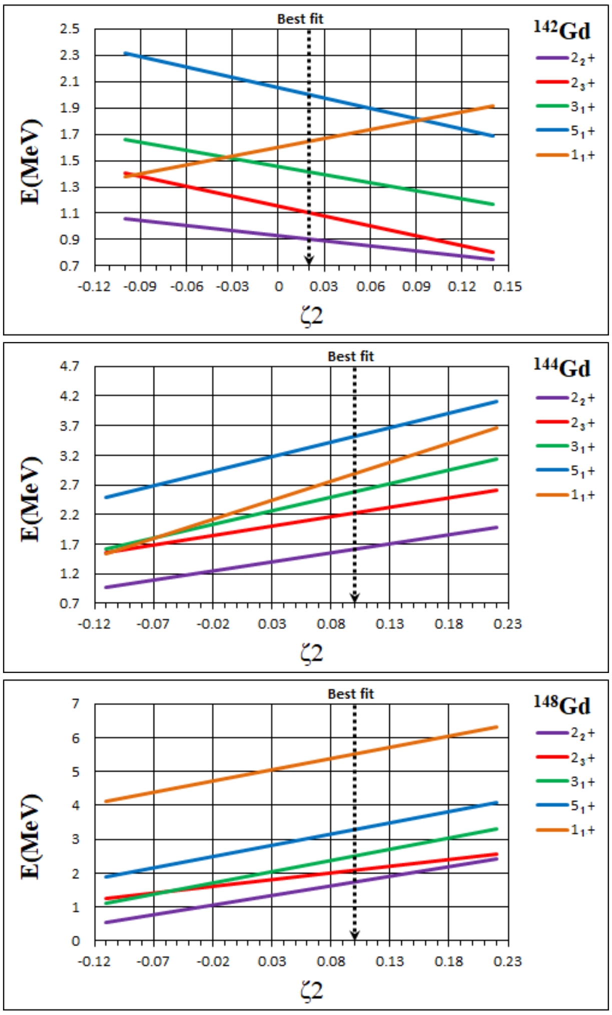


Fig. (6): Mixed symmetry states of the $(2_2^+, 2_3^+, 1_1^+, 3_1^+ \; and \; 5_1^+)$ levels as a function of Majorana parameter at the value (0.04) of the even-even isotopes ($^{138-144}$Gd,^{148}Gd).

DISCUSSION AND CONCLUSIONS

In the current work, energy levels were calculated, energy ratios and dynamic symmetries for $^{138-144}$ and ^{148}Gd isotopes, the ground band, which includes the levels, the beta band, which includes the levels, and the gamma band, which is represented by the levels, showing good agreement between IBM-2 predictions and experimental values. Dynamic symmetry and energy ratios have been examined in the second IBM, finding that the behaviour of energy level ratios is almost similar for all isotopes. The effect of the Majorana parameter was studied as a function of changing the mixing symmetry of the levels using three different values (0.005, 0.02, and 0.04), and the value of the Majorana parameter was almost constant. The effect of increasing on mixing symmetry states in $^{138-144}$ and ^{148}Gd isotopes is different from nucleus to nucleus and from state to state; it was found that the state was the lowest mixing symmetry state. It is often constant for all isotopes when the value of the parameter changes by an amount of 0.005; on the contrary, at that level, the mixing of symmetry was clear for all isotopes, while the change in the energy for the levels was slow when the value of the parameter changes by an amount of 0.02. The effect of increasing on mixing symmetry states in $^{138-144}$ and ^{148}Gd isotopes was clear at all levels, especially for levels. The change of level by mixing symmetry when increasing by 0.04 was slightly exaggerated, so the best value for the change of the Majorona parameter according to this study was 0.02, where the mixing value of the levels was shown in its expected form.

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