A New Approach For Determining A Redundant Constraint In Linear Programming Problems

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Abstract :
Linear Programming is a mathematical technique to help plan and make decisions relative to the trade-offs necessary to allocate resources, will find the minimum or maximum value of the objective and Guarantees the optimal solution to the model formulated. Linear programming is a widely used mathematical modeling technique to determine the optimum allocation of scarce resources among competing demands, but in the formulation stage in linear programming
Problems (LPP)may occurRedundancy that is belonging tothe tackyorigin data or to keeping away from the risk of neglecting some relevant constraints while modeling a problem. The existence of redundant constraints is common situation that occurs in large LP formulation.This paper presents a new approach for selecting a constraint in linear programming problems to identify the redundant constraints

المستخلص:
تعتبر البرمجة الخطية احدى الاساليب التي تساعد في التخطيط واتخاذ القرارات المتعلقة في تخصيص الموارد والتي بدورها تجد القيمة العظمى او الادنى لدالة الهدف الخاضعة لعديد من القيود, ويضمن الحل الامثل للنموذج المصاغ.للبرمجة الخطية استخدامات واسعة للاساليب الرياضية التي تحدد التخصيص الامثل للموارد النادرة, ولكن ما يحدث في البعض من تلك النماذج ما يعرف بالقيود الفائضة ويعود ذلك الى سوء مصدر البيانات او لتجنب المخاطرة في اهمال بعض القيود – بالنسبة لصائغ النموذج الرياضي– , ان ظاهره الفائض غالبا ما تحدث في نماذج البرمجة الخطية الكبيره, في هذا البحث تم اقتراح اسلوب رياضي يحدد القيد الفائض في نماذج البرمجة الخطية.

Keywords: linear programming problems, classification of constraints.

1.Introduction

In general, when we want to defend the redundant, it neglected from the system without occur consequences change this system. We may describe redundancy as an event allows decreasing of a system to the simplest case have the same properties. Mathematical programming problems consist of an objective function which is to be maximized or minimized subject to a set of constraints.

The mathematical programming problem is solved by using one of solution methods. The solutions are thenshows the execution in coinciding real life situation. We can see there are three points at which redundant might occur [8]:

(1) Redundant in the problem context: some aspect of the problem may be neglect without changing the solution results.
(2) Redundant in the methodology context: some as of the problem which affects the method used may be neglect without changing the solution results.
(3) Redundant in the solution context: some aspect of the problem may be neglect with affecting the solution.

The first type of redundant may be labeled as absolute redundant. The second and the third types of redundant should be labeled as relative redundant.

Redundant may occur in the formulation phase because of difficulties inherent in the formulation process, especially in the large system. It is possible for problem to become so large that formulator loses sight of the entire problem. Regardless of the size of the problem, redundant constraints may cause degeneracy in linear programming problems. Degeneracy in turn may result in degenerate pivot steps, that is steps in which the objective function value does not improve.Apart from computational difficulties caused, redundant tends to conceal certain information and possibilities. The knowledge something is redundant might lead to different decisions. For example, in a production planning problem if a capacity constraint is redundant, it generally indicates excess capacity, which could be used in some other way.

If we want to deal with the redundant, we have three clear options whose costs can be objective calculated (i) do nothing, (ii) identify redundant, (iii) identify and remove redundant. The options are listed in increasing order of cost, but any more precise determination of the costs requires a more precise description of the mathematical programming problem and the kind of redundant to be identified.

The remove of redundant in (iii) is not necessarily as trivial an operation as it seems. For example, if a constraint in linear programming problem determined to be redundant and its associated slack variable happens to be non-basic in the current extreme point solution, we have to perform a simplex iteration to be able to remove the redundant constraint.

Telgen[1] first defined strongly and weakly redundant constraints. Boneh[2] fist defined absolute and relative redundancy.Boneh and caron[3] first discussed weakly andstrongly redundant constraints.

Many researchers have proposed different algorithms to identify the redundanciesand removed them to get a reduced model for linear programming.Thompson [4] use techniques for removing nonbinding constraints.Paulrajet al [5]proposed a heuristic method to identify redundant constraintsby using the intercept matrix of constraints of a linear programming problem. Paulraj and Sumathi[6] introduced acomparative study of redundant constraints identification methods in linear programming problems.Sumathi and. Paulraj[7] submitted a new approach proposed for reducing time and more datamanipulation by selecting a restrictive constraint in linear programming problemsto identify the redundant constraints.

2.Definition and classification of Redundant Constraint

A redundant constraint is a constraint that can be removed froma system of linear constraintswithout changing the feasible region [6].

Consider the following system of m nonnegative linear inequality constraints and n variables $m \geq n$:

$$AX \leq b \ , \ X \geq 0 \qquad (2.1)$$

where $A \in R^{m+n}, b \in R^{m}, X \in R^{n}$.

Let $A_i X \le b_i$ be the ith constraint of the system (2.1) and let $S = \{X \in R^n / A_i X \le b_i \ , X \ge 0\}$, be the feasible region associated with system (2.1).

Let $S_k = \{X \in R^n / A_i X \le b_i \ , X \ge 0, i \ne k\}$ be the feasible region associated with thesystem of equations $A_i X \le b_i , i = \overline{1, m}, i \ne k$. The k thconstraint $A_k X \le b_i , (1 \le k \le m)$ is redundant for the system (2.1)if and only if $S = S_K$.

Definition1 Redundant constraints can be classified as weakly and strongly redundantconstraints.

Weakly Redundant Constraints

The constraint $A_i X \le b_i$ is weakly redundant if it is redundant and $A_i X = b_i$ for some $X \in S$.

Strongly Redundant Constraints

The constraint $A_i X \le b_i$ is strongly redundant if it is redundant and $A_i X < b_i$ for all $X \in S$.

Binding Constraint

Binding constraint is the one which passes through the optimal solution point. It is also calleda relevant constraint.

Nonbinding Constraint

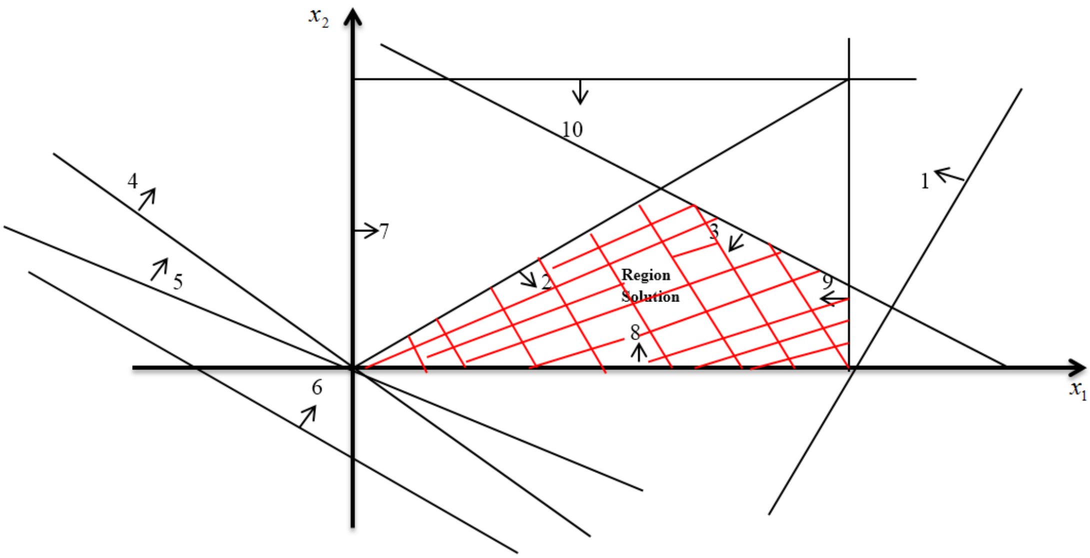
Nonbinding constraint is the one which does not pass through the optimal solution point.But it can determine the boundary of the feasible region.

Example 2.1:

Consider the set of constraints

Constraint	**Number of constraint**
$x_1 - x_2 \le 8$	1
$-x_1 + x_2 \le 0$	2
$x_1 + x_2 \le 12$	3
$-2x_1 - x_2 \le 0$	4
$-x_1 - x_2 \le 0$	5
$-x_1 - x_2 \le 4$	6
$-x_1 \quad \le 0$	7
$x_1 \quad \le 8$	8
$-x_2 \le 0$	9
$x_2 \le 8$	10

The figure (1) showing redundant and necessary constraints



The constraints (2,3,8,9) are necessary and the constraints (1,4,5,6,7,10) are redundant. Furthermore the constraints (1, 4, 5, 7) are weakly redundant and the constraints (6, 10) are strongly redundant constraints.

From figure1, we see that weakly redundant constraints touch the feasible region. and that strongly redundant constraints do not touch the feasible region.

Proposed Approach

In this section, a new approach is suggested to determine the redundant constraint. The steps of the proposedapproach are as follows.

We choice the traditional linear programming problem describes the basic idea of the approach.Initially let us introduce the notations:

$$Max \quad \sum_{j=1}^{n} c_j x_j \tag{2.2}$$

s.t

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \, , i = \overline{1,m} \tag{2.3}$$

$x \ge 0; x \in R^n$.

$A_i = (a_{i1}, a_{i2}, \dots, a_{in})^T$ - The coefficient vector of the left hand side of constraint number i.

$\overline{A}_i = (a_{i1}, a_{i2}, \dots, a_{in}, b_i)^T$ - The extended vector, contains on additional coefficients

b_i, The right hand side of the inequality (constraint) number i., $i = \overline{1, m}$.

Our approach depends on the grouping of two constraints by using the relationship between them, to illustrate this we using the following definitions.

Definition2. Two constraints systems linear inequality (2.3) with numbers $i \, , \, j, 1 \le i, j \le m$, Called γ -weakly relation, if the values

$$v_{ij} = (A_i, A_j)/(\|A_i\|\|A_j\|),\ 1 \le i, j \le m, \tag{2.4}$$

Rightly relationship

$v_{ij} \le \gamma,\ 0 \le \gamma < 1.$

where

$\|A_i\| = \sqrt{a_1^2 + a_2^2 + ... + a_m^2}$, $\|A_j\| = \sqrt{a_1^2 + a_2^2 + ... + a_m^2}$ and

$(A_i, A_j) = \sum_{s=1}^{n} a_{is} a_{js}$ is scalar product of vectors A_i, A_j, $1 \le i, j \le m$.

The value of parameter γ allowsto us to specific the level of interconnection of two inequalities. Easy to know the value of v_{ij} coincides with the value of the cosine of the angle between the gradients A_i, A_j, $1 \le i, j \le m$,($v_{ij} = cos\,\varphi_{ij} = cos\,\varphi(A_i, A_j)$),and for minor values of γ two inequalities with numbers i and j, $1 \le i, j \le m$,differ significantly.

Definition3. Two constraints systems linear inequality (2.3) with numbers $i,\ j, 1 \le i, j \le m,$

Called γ -strongly relation, if the values v_{ij} in equation (2.4) rightly relationship

$v_{ij} \ge \gamma,\ 0 < \gamma \le 1.$

Definition4. Two constraints systems linear inequality (2.3) with numbers $i,\ j, 1 \le i, j \le m,$

Called $\bar{\gamma}$ -weakly relation, if the values

$$\bar{v}_{ij} = (\bar{A}_i, \bar{A}_i)/(\|\bar{A}_i\|\|\bar{A}_j\|),\ 1 \le i, j \le m, \tag{2.5}$$

Rightlyrelationship $\bar{v}_{ij} \le \gamma, 0 \le \gamma < 1.$

Definition5. Two constraints systems linear inequality (2.3) with numbers $i,\ j, 1 \le i, j \le m,$

Called $\bar{\gamma}$ -strongly relation, if the values $\bar{v}_{ij}$ in equation (2.5) rightly relationship

$\bar{v}_{ij} \ge \gamma,\ 0 < \gamma \le 1.$

Obviously, the concept $\bar{\gamma}$ -strongly relation and $\bar{\gamma}$ weakly relation constraints as in the case γ -weakly relation and γ -strongly relation constraints related to the value of the angle between the vectors($\bar{v}_{ij} = cos\,\bar{\varphi}_{ij} = cos\,\bar{\varphi}(\bar{A}_i, \bar{A}_j)$),obtained for extended vectors $\bar{A}_i, \bar{A}_j$, $1 \le i, j \le m$.

The main idea of our proposal to determine the redundant constraint we must examine each constraint with all residual constraints which one of them have the big number of corresponding to the condition $\bar{v}_{ij} < v_{ij}$ then the constraint consider redundant.

3. Numerical Examples

This section illustrates the proposed approach by solving varioussize LP problems

Example 3.1:

Consider the following linear programming problem

$$\max z = 3x_1 + 4x_2$$
$$s.t$$
$$\begin{aligned} x_1 + 3x_2 &\le 15 \\ 2x_1 + x_2 &\le 10 \\ 2x_1 + 3x_2 &\le 18 \\ x_1 + x_2 &\le 7 \\ 4x_1 + 5x_2 &\le 40 \end{aligned}$$

$$x_i \ge 0 ,\ i = \overline{1,2}$$

To apply this proposal we follow the following steps:

Step1: calculate the value of v_{ij} for each constraint i with the constraint j by the equation (2.4) as the results in table1.

Step2:calculate the value of $\overline{v_{ij}}$ for each constraint i with the constraint j by the equation (2.5) see table1.

Step3: compare each value of v_{ij} with the analogue value $\bar{v}_{ij}$, if value $\overline{v_{ij}} < v_{ij}$ for every constraint i, then this constraint is redundant, also the same mater for constraint j, notice in table1 the value of $\bar{v}_{14} < v_{14}$, $\bar{v}_{24} < v_{24}$ and $\bar{v}_{34} < v_{34}$, thereforeconstraint number 4 is redundant .

Table (1) show the values of v_{ij} , $\bar{v}_{ij}$, associated with example 3.1

v_{12}	v_{13}	v_{14}	v_{15}	v_{23}	v_{24}	v_{25}	v_{34}	v_{35}	v_{45}
0.752	0.965	1.052	0.94	0.924	1.2	0.966	1.7	0.995	0.991
$\bar{v}_{12}$	$\bar{v}_{13}$	$\bar{v}_{14}$	$\bar{v}_{15}$	$\bar{v}_{23}$	$\bar{v}_{24}$	$\bar{v}_{25}$	$\bar{v}_{34}$	$\bar{v}_{35}$	$\bar{v}_{45}$
0.98	0.998	0.995	0.997	0.994	0.997	0.994	0.999	0.998	1.004

Example3.2:

Considerthe following linear programming problem

$$\max z = 5x_1 + 6x_2 + 3x_3$$
$$s.t$$
$$\begin{aligned} 5x_1 + 5x_2 + 3x_3 &\le 50 \\ 2x_1 + 2x_2 + x_3 &\le 40 \\ 7x_1 + 6x_2 + 3x_3 &\le 30 \\ 5x_1 + 5x_2 + 5x_3 &\le 35 \\ 12x_1 + 6x_2 + 9x_3 &\le 40 \\ 4x_1 + x_2 + 2x_3 &\le 20 \\ x_1 &\le 4 \end{aligned}$$

$$x_i \ge 0 ,\ i = \overline{1,3}$$

We use the same procedure in example1, obtain the table 2.

From table2 we see that $\bar{v}_{12} < v_{12}$ and $\bar{v}_{13} < v_{13}$,thereforeconstraint number1 is redundant.

Table (2) show the values of v_{ij} , $\bar{v}_{ij}$, associated with example 3.2

v_{12}	v_{13}	v_{14}	v_{15}	v_{16}	v_{17}	v_{23}	v_{24}	v_{25}	v_{26}	v_{27}
0.9981	0.994	0.977	0.943	0.881	0.65	0.971	0.962	0.923	0.87	0.667
$\bar{v}_{12}$	$\bar{v}_{13}$	$\bar{v}_{14}$	$\bar{v}_{15}$	$\bar{v}_{16}$	$\bar{v}_{17}$	$\bar{v}_{23}$	$\bar{v}_{24}$	$\bar{v}_{25}$	$\bar{v}_{26}$	$\bar{v}_{27}$
0.997	0.987	0.995	0.998	0.993	0.982	0.973	0.985	0.994	0.987	0.98
v_{34}	v_{35}	v_{36}	v_{37}	v_{45}	v_{46}	v_{47}	v_{56}	v_{57}	v_{67}	
0.95	0.94	0.889	0.722	0.964	0.882	0.577	0.973	0.743	0.87	
$\bar{v}_{34}$	$\bar{v}_{35}$	$\bar{v}_{36}$	$\bar{v}_{37}$	$\bar{v}_{45}$	$\bar{v}_{46}$	$\bar{v}_{47}$	$\bar{v}_{56}$	$\bar{v}_{57}$	$\bar{v}_{67}$	
0.994	0.998	0.989	0.977	0.995	0.992	0.975	0.998	0.987	0.993	

Conclusion

In this paper, a new approach is used to identify the redundant constraints. The proposed method depend on the compare between two constraints in determine the redundant constraint.

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