P- ISSN 1991-8941 E-ISSN 2706-6703 Journal of University of Anbar for Pure Science (JUAPS) Open Access
2025,(19), (01):219– 225

Proton–Pb Collision Utilizing the Woods–Saxon Density Shape Within the Optical Glauber Model

Hajer Abbas Ali *, Akram Mohammed Ali

Department of Physics, College of Science, University of Anbar, Anbar, Iraq
*Email: haj21s2005@uoanbar.edu.iq, dr.akram@uoanbar.edu.iq

ARTICLE INFO

Received: 22 / 08 /2024
Accepted: 15/ 10 /2024
Available online: 20/ 06/2025

DOI: 10.37652/juaps.2024.152975.1312

Keywords:
Optical Glauber model, CM energy, collision probability, overlap function, geometric cross section.

Copyright©Authors, 2025, College of Sciences, University of Anbar. This is an open-access article under the CC BY 4.0 license (http://creativecommons.org/licenses/by/4.0/).

ABSTRACT

This study investigates the effect of changing kinetic energy on proton–lead (p–Pb) collisions using the Woods–Saxon model of nucleus density distribution. The foundation of the model is density distribution, which steadily decreases from the center to the periphery. The results for p–Pb collisions at center-of-mass energies $\sqrt{s_{NN}}$= 0.2, 5.44 TeV are first discussed. The optical Glauber method is considered in the evaluation, and the nucleus is perceived as a continuous distribution of matter devoid of boundaries or density changes. The whole procedure of interaction is carried out analytically. Many super-relativistic p–Pb collisions can be understood on the basis of geometry using a harmonic oscillator potential within the Woods–Saxon density formula for lead nuclei, in which the thickness function increases after the collision. This result agrees with the experimental data. $T_{p\text{–}Pb}$ of the overlapping function at impact parameter (b) is large within the small values of (b) in the geometric case and then decreases at large (b). The numbers of nucleons that the participant (N_{par}) and binary collision (N_{coll}) found in the zone of peripheral collisions decrease with increasing effect parameter (b) owing to the influence of deviation fluctuations. The average values for participant $<N_{part}>$ and binary collision $<N_{coll}>$ at 0.2TeV are 12.3 and 15.3 at constant impact parameter b=2 fm, respectively.

Introduction

A systematic investigation of quark–gluon plasma (QGP) cannot be conducted in a lab because of the thermodynamic conditions required, unless large-scale experiments that can accelerate particle beams to relativistic speeds before hitting them are conducted. An exception is hadronic collision, a complicated process in which QGP represents only a small portion of the physics involved. Heavy ion collisions (HICs) are currently the only known method for producing QGP. Diverse stages of collisions present different features, but all are defined by Lorentz-invariant proper time, $\tau \equiv \sqrt{t^2 - z^2}$, where $\tau = 0$ is the collision time [1]. In this short time of high-energy hadron collision, the spatial and quantum configuration of the hadron components (quarks and gluons) remains unchanged, whereas some physical properties of the parton system (e.g., total transverse area) [2, 3] change. As a result of changes in the internal structure of hadrons, elastic elongation occurs over time [4–6]. Figure (1) [7] shows the contributions of the components of a proton projected onto a nucleus at a tube, which represents the transverse projection of the proton, with the affected nucleons inside the target nucleus (red balls). This contribution analogy allows us to understand the experimental data, given that they play an important role in interpreting these multiple contributions when nuclear collisions occur [8] and in investigating collision-induced diffraction [9–11].

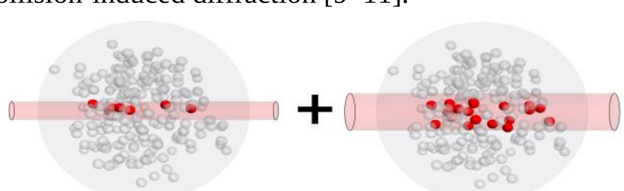

Fig. (1): Geometric representation of a proton–nucleus collision at a fixed target level. The red tubes represent the projection of the transverse proton onto the target nucleus. The red dots represent the nucleons that collide with the proton [7].

*Corresponding author at : Department of Physics, College of Science, University of Anbar, Anbar, Iraq
ORCID:https:// https://orcid.org/0000-0000-0000-0000 ,
Tel: +964 7704793770
Email: haj21s2005@uoanbar.edu.iq

Because they make it possible to distinguish between the impacts of the starting state already existent in cold nuclear matter and the effects of the final state of production of hot QCD matter, proton–lead (p–Pb) collisions are crucial for understanding and interpreting nucleus–nucleus data. Hence, proton–nucleus collisions at high energies offer a chance to investigate how the dynamics of soft and hard scattering processes and eventual generation of particles are affected by a larger nuclear target.

For investigating the role of nuclear effects in p–Pb collisions, describing p–A collisions, in which there is a reduction in proton components and thus a lower rate of nucleon–nucleon reactions due to the reduction in antagonistic events between the projectile and the target nucleus, is vital. Such collisions affect the number of nucleons hit in the target nucleus. p–Pb collisions are an integral part of the nuclear program at the Large Hadron Collider. Examining calculated theoretical results as a function of collision centrality is necessary to determine the involvement of nuclear processes in p–Pb collisions. Therefore, the main goal of this study is to ascertain how energy influences various endpoints, including reaction probability, nuclear production, and angular distribution, by utilizing various impact energies within the optical Glauber model. This work is an attempt to obtain predictions for P–Pb collisions at nucleon–nucleon center-of-mass system energies $\sqrt{s} = 0.2,\ 5.44\ \text{TeV}$. The hadron production rate is related to the nucleon–nucleon reaction rate, as experimentally proven [12] by changes in energy and momentum. For this purpose, our calculations will be based primarily on the Woods–Saxon density distribution, taking into account the σ_{NN} effect of the density of the two nuclear particles. The results will be compared with the first experimental results.

Materials and Methods

Theoretical aspects

Glauber assumed that the collision of two nuclei is not deflected at high energy, so they have a linear trajectory with the impact parameter (b) that determines the degree of centrality. The total number of contacts between participants' nucleons is represented by the number of collisions that occur during a particular event [**13**]. For spherical symmetric nuclei in collision with extremely high energies of heavy ions, the Fermi distribution, sometimes referred to as the Woods–Saxon density distribution, represents the number of nucleons per unit volume. It is typically used to parametrize the nucleon density, and it is employed as a nuclear profile and given as follows [**14**, **15**]:

$$\rho_A(r) = \rho_o \frac{1+\omega(r/R)^2}{1+\exp(\frac{r-R}{a})} \qquad (1)$$

The nucleon density at the nucleus's center is denoted by ρ_o (fm^{-3}), which provides the overall normalization. The definite radius of the nucleus is R (fm), which means that no nucleons exist outside at a distance larger than R. The skin depth or thickness is a (fm), and ω refers to spherical shape deviations. This formula is normalized to the number of nucleons $4\pi \int_0^\infty b^2 d\vec{b}\ \rho_\circ(\vec{b}) = A$, which we will used in our calculations [**16**].

When two nuclei (A and B) collide at relativistic speeds, two flux tubes consider displacement $\vec{s}$ and $\vec{s}-\vec{b}$, concerning the center of the target and projectile, respectively. The optical calculations depend on the thickness function that describes the transverse nucleon density and represents the integrated density along the incident beam's longitudinal z-axis direction. It indicates the quantity of NN collisions that the nucleon experiences at impact parameter (b) when traveling within a nucleus [**17**], i.e.,

$$T_A(\vec{b}) = \int dz\, \rho_A(\vec{b}, z) \qquad (2)$$

The probability per unit volume $\rho_{A,B}(\vec{s}, z_{A,B})$ of two ions, known as the nuclear overlap function, is normalized to unity to find the nucleon at the position $(\vec{s}, z_{A,B})$. Integrating it over the two transverse dimensions gives [**18**]

$$T_{AB}(\vec{b}) = \int T_A(\vec{s}) T_B(\vec{s}-\vec{b}) d^2 s \qquad (3)$$

which is normalized by $\int d^2 b T_{AB}(\vec{b}) = AB$ overall (b), and $\vec{s}$ and $\vec{b}$ are perpendicular to the direction of the beam z-axis. This thickness function is purely a geometric factor that indicates the occurrence of interactions, $T_{AB}\sigma_{NN}^{inel}$.

Nucleons that have collided with another nucleon at least once are considered participants. The average number of participants in d–A collisions at an impact parameter *b* can be computed using the following formula [**18**, **19**]:

$$N_{partAB}(b)=\int d^2sT_A(\vec{s})\left\{1-\left[1-\frac{\sigma_{NN}T_B(\vec{s}-\vec{b})}{B}\right]^B\right\}+\int d^2sT_B(\vec{s})\left\{1-\left[1-\frac{\sigma_{NN}T_A(\vec{s}+\vec{b})}{A}\right]^A\right\} \quad (4)$$

The total number of participants is then determined as follows [19]:

$$N_{partAB}(b)=\int d^2sT_A(\vec{s})\left\{1-e^{-\sigma_{NN}T_A(\vec{s}-\vec{b})}\right\}+\int d^2sT_B(\vec{s})\left\{1-e^{-\sigma_{NN}T_B(\vec{s}+\vec{b})}\right\} \quad (5)$$

In the optical Glauber model, local density fluctuations are ignored, i.e., each projectile nucleon acts as a flux tube when interacting with the approaching target. The nucleus is viewed as a continuous distribution of matter devoid of boundaries or density changes (smooth density). The total cross section for such d–Au collisions, where at least one NN collision takes place, is known as the total geometric cross section and given as

$$\sigma_{geo}=\int d^2b\left[1-e^{-T_{AB}(b)\sigma_{NN}}\right] \quad (6)$$

Results and Discussion

The Woods–Saxon density parameters used in the calculations are as follows: $\rho o = 0.17\ fm^{-3}$, R= 1.12 $A^{1/3}$–0.86$A^{-1/3}$ = 6.4909 *fm*, a = 0.54 *fm* [20], and ω=0. The results are compared with those with experimental values of R=6.36 *fm* and a=0.535 *fm* [15]. To estimate the overlapping variables, we conduct tests at different high energies, √s = 0.2, 5.44 TeV, and utilize inelastic nucleon–nucleon cross sections of σ_{NN} =37, 68 *mb* obtained by algorithmic interpolation of cross-section measurements concerning collision energies [16]. Through the optical Glauber approach, with the impact parameter adopted as (2 *fm*), the estimation of geometrical quantities (N_{coll}, T_{AB}) and others is usually realized by utilizing the “overlap” program written in Fortran 90.

The nuclear density as a function of nuclear radius for p–Pb collision according to the Woods–Saxon distribution at 0.2 TeV and 37 mb is shown in Figure (2-a). The Woods–Saxon distribution is a mathematical model that helps understand how this density varies within the nucleus. It is a widely used approximation for the actual, more complex nuclear density profile. This figure is consistent with the expected distribution of nuclear density, which is highest at the nucleus’s center. This high density is attributed to the strong nuclear gravitational forces that bind the particles, and it gradually decreases (this decrease is not abrupt but smooth, following the bell-shaped curve of the model) and then tails off as it moves farther away from the center. The diffuseness parameter controls the “diffuseness” of the nuclear edge; smaller “*a*” indicates a sharper drop-off in density at the edge, whereas a larger value creates a smoother transition. In the outer region, the nuclear forces of attraction weaken, causing the particles to diverge from one another.

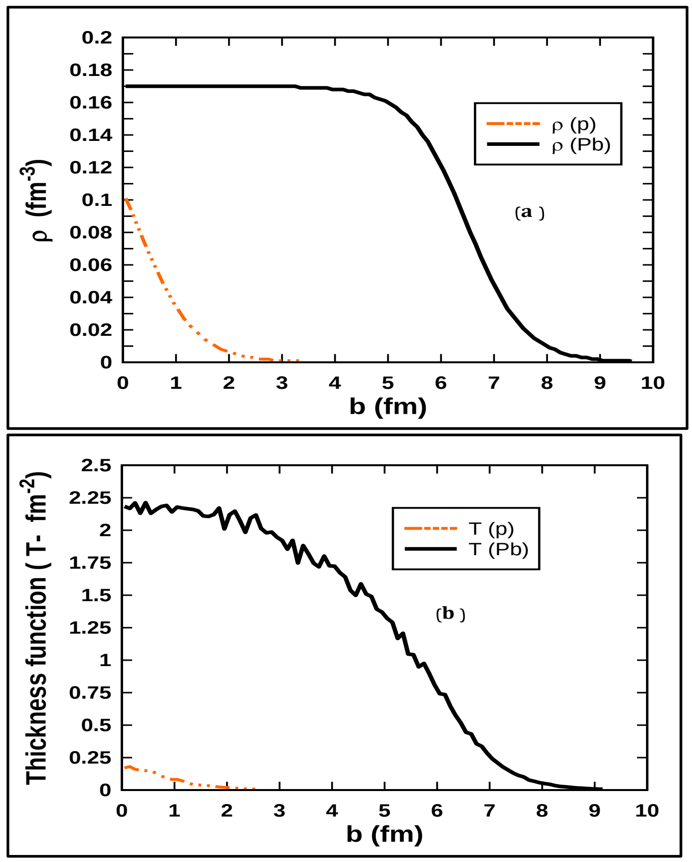

Fig. (2): (a) Nuclear density for p–Pb collision according to the Woods–Saxon distribution at 0.2 TeV and 37 mb. (b) Thickness density function for p–Pb collision.

Figure (2-b) shows the thickness density function (or collision function) for p–Pb collision at an energy of 0.2 TeV. The thickness density function is an important tool for understanding nuclear collisions. It can be used to calculate the probability of the occurrence of different reaction channels, such as the production of new particles. The thickness function usually exhibits behavior similar to the bell of a Gaussian curve. The function is valued high at small (b) values, indicating a high probability of interaction when the nuclei are close. As the value of (b) increases, the value of the function decreases, implying a decrease in the probability of the reaction.

From Figure (2-b), the thickness function is highest at around 2 *fm* and tapers off to zero at around 10 *fm*. That is, collisions are most likely to happen when the impact parameter is around 2 *fm*. This result makes sense intuitively because this is the closest two nuclei can be without actually overlapping. The fact that the thickness function does not go to zero at exactly zero impact parameter reflects that protons and lead nuclei have a finite size. As this function is normalized to one, the integral of the thickness function over all possible impact parameters is equal to one.

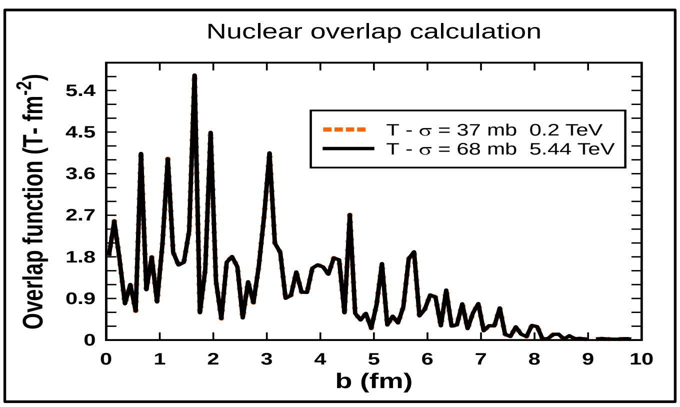

Fig. (3): Nuclear overlap function T_{pPb} (b) for p–Pb collisions at different inelastic cross sections. Overall, from Figure (3), the model can calculate the overlap function to have the collision probability depending on the nucleon density and reaction energy of P–Pb collisions at two energies of 0.2 and 5.44 TeV, at which the same results are obtained. b = 2 *fm* seems to indicate the impact parameter for a head-on collision, in which the centers of the nuclei collide directly.

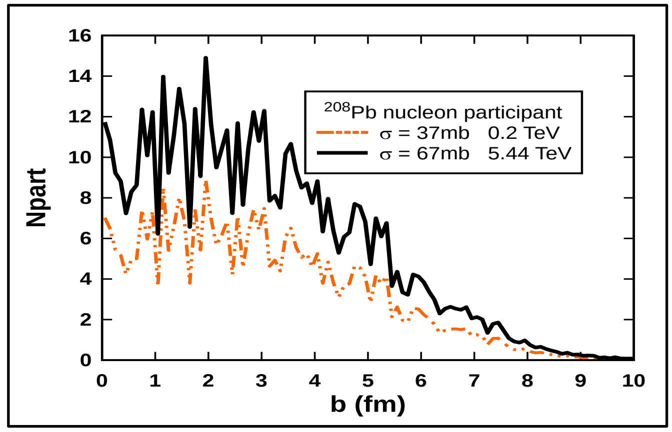

Fig. (4): Number of nucleons that participate in the collisions at 0.2 and 5.44 TeV.

Figure (4) shows the number of participant nucleons at chosen energies. N_{par} refers to the number of nucleons that participate in the collision. Different lines in the figure correspond to different collision energies and inelastic cross sections. The number of participant nucleons generally increases with decreasing impact parameter (b). A smaller impact parameter means more head-on collisions, which will involve more nucleons from the projectile colliding with nucleons from the target ^{208}Pb nucleus. When a collision occurs, (Pb) nucleons collide with the projectile (p) within the reaction region, sharing more nucleons. Pb nucleon sharing lines tend to rise more sharply with a lower impact coefficient compared with extruded sharing lines. The reason is that the reaction region inside the Pb nucleus allows more nucleons to participate in direct collisions (small impact factor). In general, the effect of impact energy on the number of participating Pb nucleons is less pronounced than its effect on the number of ejected nucleons

Nonetheless, the number of participant nucleons increases with increasing collision energy, i.e., more collisions will happen. Higher-energy collisions can cause more nucleon–nucleon interactions to occur within the nucleus. The shaded area around each curve represents the uncertainty in the measurement of the number of participant nucleons. Our results reveal 300-time collisions of the projectile at 0.2 TeV and more than 800-time collisions at 5.44 TeV. The differences in the size and nucleon number in lead results in more nucleons being participants in the collision. Large-sized lead nucleons interact with one another frequently, creating a "reaction zone" within the nucleus.

The process of determining the number of particles does not allow comparison with the experimental one, given that the count is only for charged particles (protons). Moreover, the detectors used in the experiment only detect a certain fraction of the contributing nucleons.

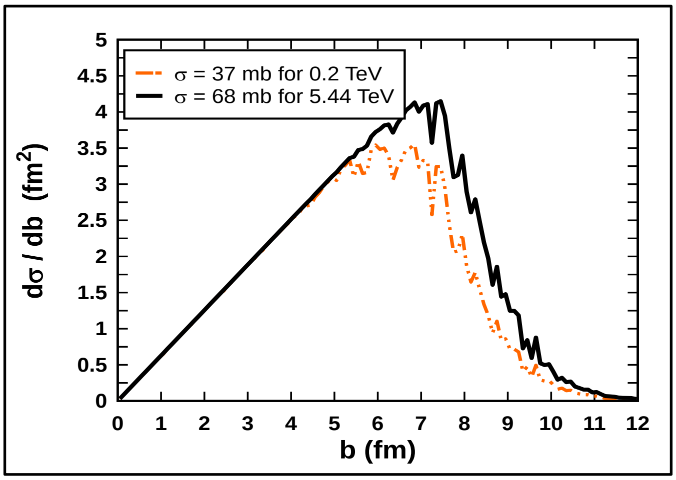

Fig. (5): DCS concerning impact parameter (b).

To describe the probability of particles scattering elastically at a certain angle, we calculate the elastic differential cross section (DCS), as shown in Figure (5). DCS means that the colliding particles retain their internal structure after the collision.

In the context of particle collisions with a nucleus, DCS depends on various factors, including the energy of the incident particle, the type of particle and nucleus involved, and the scattering angle. The energy dependence of the cross section often exhibits resonant behavior, with peaks and valleys corresponding to specific energy levels in the Pb nucleus. These resonances provide valuable information about the electronic structure and energy levels of this nucleus.

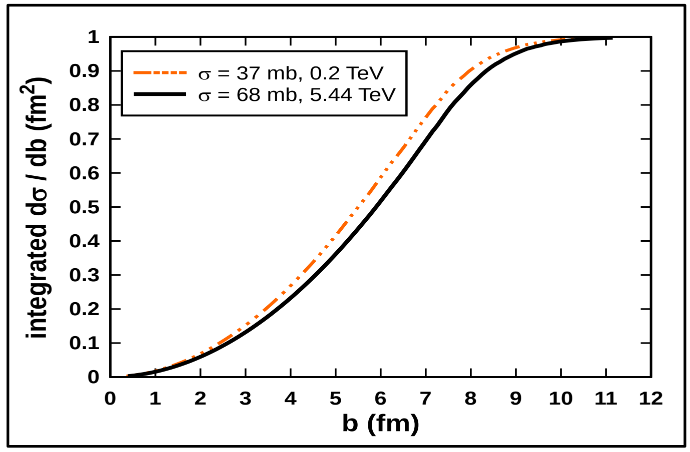

Fig. (6): Normalized integral cross section for collisions in different energy ranges.

The normalized integral cross section for collisions in two energy ranges shown in Figure (6) indicates the probability of a collision occurring, given a certain impact parameter. A larger impact parameter means that the collision is more peripheral. The fact that the cross section is normalized implies that the total area under the curve is equal to one. The data points plotted in the figure show the integral cross section at different collision energies. For example, the data point at around 2 *fm* at a collision energy of 0.2 TeV presents that the normalized integral cross section is about 0.2 when the impact parameter is 2 *fm*. The normalized integral cross section is higher for lower impact parameters (more head-on collisions) and lower collision energies. This trend is not always the case, and the shape of the curve depends on the specific particles involved in the collision.

Conclusions

The purely geometrical Glauber approach, which originates from the quantum mechanical model for p–Pb scattering, offers a consistent description of p–A collisions. A crucial part of HICs is p–Pb collisions. Given that nuclei are extended objects, the impact parameter (b) of the collision, i.e., the distance between the centers of the two colliding nuclei in a plane transverse to the beam axis, determines the volume of the interaction zone. In heavy ion physics, the idea of the collision's centrality, which is determined by comparing data with collision simulations and is closely related to the impact parameter, is typically introduced. The number of participating nucleons expresses the volume of the initial overlap region. A nucleon that experiences one or more binary collisions with nucleons from the other nucleus is referred to as a participating nucleon of that nucleus.

References

[1] Krane, K. S (1987). Introductory. Nuclear Physics. John Wiley & Sons, Inc., 2nd edition.

[2] Alvioli, M. and Strikman, M (2013). Color fluctuation effects in proton-nucleus collisions," Phys. Lett. B722, 347–354, arXiv:1301.0728 [hep-ph].

[3] M. Alvioli, B. A. Cole, L. Frankfurt, D. V. Perepelitsa, and M. Strikman, (2016). "Evidence for x-dependent proton color fluctuations in pA collisions at the CERN Large Hadron Collider," Phys. Rev. C93, 011902, arXiv:1409.7381 [hep-ph]

.[4] I. Ya. Pomeranchuk and E. L. Feinberg, Dokl. Akad.Nauk SSSR, 93, 439 (1953).

[5] E. L. Feinberg and I. Pomeranchuk, "High energy inelastic diffraction phenomena," Il Nuovo Cimento 3, 652–671(1956).

[6] L. Frankfurt and M. Strikman, "Diffractive phenomena in high energy processes," in 100 Years of Subatomic Physics, edited by Ernest M. Henley and Stephen D. Ellis(2013) pp. 363–423, arXiv:1304.4308 [hep-ph].

[7] M. Alvioli, L. Frankfurt, D.V. Perepelitsa, and M. Strikman," Global analysis of color fluctuation e_ects in proton- and deuteron-nucleus collisions at RHIC and the LHC", Phys. Rev. D 98, 071502 (2018).

[8] H. Heiselberg, G. Baym, B. Blaettel, L. L. Frankfurt, and M. Strikman, "Color transparency, color opacity, and fluctuations in nuclear collisions," Phys. Rev. Lett. 67,2946–2949 (1991).

[9] L. Frankfurt, G. A. Miller, and M. Strikman, "Coherent nuclear diffractive production of mini-jets: Illuminating color transparency," Phys. Lett. B304, 1–7 (1993),arXiv:hep-ph/9305228 [hep-ph].

[10] L. Frankfurt, G. A. Miller, and M. Strikman, "Coherent QCD phenomena in the coherent pion nucleon and pion nucleus production of two jets at high relative momenta, "Phys. Rev. D65, 094015 (2002), arXiv:hep-ph/0010297[hep-ph].

[11] E. M. Aitala et al. (E791), "Observation of color transparency in diffractive dissociation of pions," Phys. Rev.Lett. 86, 4773–4777 (2001), arXiv:hep-ex/0010044 [hep-ex].

[12] G. Aad et al. (ATLAS), "Measurement of the dependence of transverse energy production at large pseudorapidity on the hard-scattering kinematics of proton-proton collisions at $\sqrt{s}$ = 2.76 TeV with ATLAS," Phys. Lett. B756,10–28 (2016), arXiv:1512.00197 [hep-ex].

[13]R. S. Bhalerao, "Relativistic heavy-ion collisions," CERN-2014-001 and KEK-Proceedings-2013-8, pp. 219-239, arXiv:1404.3294, 2014.

[14]B. Landolt, "Numerical data and functional relationships in science and technology, Magnetic and Another Properties of Oxides and Related Compounds:. Pt. A Garnets and perovscites, 1978.

[15]H. De Vries, C. De Jager, and C. De Vries, "Nuclear charge-density-distribution parameters from elastic electron scattering," Atomic data and nuclear data tables, 36, 3, 495-536, 1987.

[16]A. M. Ali and H. M. Mohammed, "Determination of Differential Cross-Section of (n+ 89Y) Elastic and Inelastic Scattering using Eikonal Approximation," Iraqi Journal of Science, 63, 6 2514-2522, 2022.

[17]M. Nasim, Md. Nasim ; Shusu Shi ; Sandeep Chatterjee ; Subhash Singha ; Victor Roy"Collectivity in High Energy Heavy-Ion Collisions.", Advances in High Energy Physics, (2017/12) 1-2-012, 2017.

[18]A. Białłas, M. Bleszyński, and W. Czyż, "Multiplicity distributions in nucleus-nucleus collisions at high energies," Nuclear Physics B, 111, 3, 461-476, 1976.

[19]D. Kharzeev, C. Lourenco, M. Nardi, and H. Satz, "A Quantitative analysis of charmonium suppression in nuclear collisions," Zeitschrift für Physik C Particles and Fields, 74, 307-318, 1997.

[20]W. Collaboration and M. Aggarwal, "Scaling of particle and transverse energy production in Pb+ Pb collisions at 158 GeV," The European Physical Journal C-Particles and Fields, 18, 4, 651-663, 2001.

P- ISSN 1991-8941 E-ISSN 2706-6703 *Journal of University of Anbar for Pure Science (JUAPS)* Open Access
2025,(19), (01):219– 225

تصادم البروتون–الرصاص باستخدام شكل كثافة وودز– ساكسون ضمن نموذج جلوبر البصري

هاجر عباس علي ، اكرم محمد علي

قسم الفيزياء ، كلية العلوم ، جامعة الانبار

الخلاصة:

تبحث هذه الورقة في تأثير الطاقة الحركية المتغيرة على التصادم بين البروتون ونواة **(Pb)** باستخدام نموذج وودز–ساكسون لتوزيع كثافة النواة. أساس النموذج هو توزيع الكثافة الذي يتناقص باطراد من المركز إلى المحيط. نوقشت النتائج الأولى لتصادمات **P–Pb** عند طاقة مركز الكتلة **sNN** √ =0.2 و5.44 **TeV**. أُخذت طريقة كلاوبر الضوئية في الاعتبار في تقييم النتائج، ويُنظر إلى النواة على أنها توزيع مستمر للمادة خالية من الحدود أو تغيرات الكثافة. يتم تنفيذ الإجراء الكامل للتفاعل بشكل تحليلي، ويمكن فهم العديد من تصادمات النسبية الفائقة **P–Pb** على أساس الهندسة. تمت دراسة توزيع الكثافة لكل نواة باستخدام جهد المذبذب التوافقي ضمن معادلة الكثافة وودز–ساكسون لنواة الرصاص حيث زادت دالة السُمك بعد الاصطدام، وهذه النتيجة تتفق مع البيانات التجريبية، بينما وجدت أن دالة التداخل كبيرة ضمن معلمة التأثير**(b)** الصغير في الحالة الهندسية ثم انخفضت بالنسبة لمعلمة التأثير الكبير. بالإضافة إلى ذلك، تم حساب أعداد النيوكليونات المشاركة في التصادم **(Npar)** والنيوكليونات المساهمة في التصادمات الثنائية **(Ncoll)** في منطقة التصادمات الطرفية، حيث وجد انها تتناقص مع زيادة معامل التأثير بسبب تأثير تقلبات الانحراف.القيمة المتوسطة بالنسبة للمشارك **<Npart>** والاصطدامات الثنائية **<Ncoll>** عند 0.2 **TeV** هي 12.3 و15.3 عند معلمة التأثير الثابت **b=2 fm**.