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Two-row resolution of the Weyl module in case partition (12,6)

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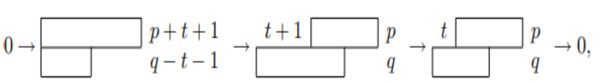
ABSTRACT

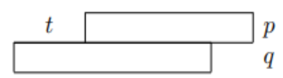
Let $\mathcal{F}$ be a free R-module, where R is a commutative ring with 1 and D_nF be the nth degree divided power. The place polarization technique is a combinatorial approach for computing the complex elements, in which $\partial_{21}^{(k)}$ is a location polarization that occurs from place one to place two. The induction argument on the amount of overlaps between the two rows provides a description of the result that we want and identifies whether the resolution is a Koszul-like complex (also known as "arithmetic Koszul Complex"). The Weyl module is given by $K_{\lambda/\mu}\mathcal{F} = Im(\, d'_{\lambda/\mu})$, where $d'_{\lambda/\mu}$and $d'_{\lambda/\mu} : \square_{\lambda/\mu}\mathcal{F} \longrightarrow \wedge_{\tilde{\lambda}/\tilde{\mu}}\mathcal{F}$ is the Weyl map, from which the term "Weyl module" is derived. In this paper, we investigate the two-row Weyl module resolution for the partition (12,6) using homological diagram, contracting homotopy, and place polarization.

INTRODUCTION

Akin and Buchsbaum (or basic representations) tackled the problem of resolving Schur modules in terms of direct sums of the tensor products of exterior powers in the early 1980s [1],[2]. Applying the two-row Schur module "basic precise sequence" (more on this will be discussed in a later section), we have:



, we will review the "substantial" module theory that may be performed using letter-place techniques. Specifically, we define the equivariant filtration on a two-rowed skew shape using the letter-place basis. This results in the Pieri decomposition of the relevant Weyl module [3],[4]. Assume that we have the following two-row, skew shape:

(A) 

As previously mentioned [3], this is the result of $D_p \otimes D_q$ by the Weyl map, and the letter-place basis for $D_p \otimes D_q$ is the set of all double standard tableaux $\left\{\begin{pmatrix} r_1 \\ r_2 \end{pmatrix}\middle| \begin{matrix} 1^{(p)}2^{(l)} \\ 2^{(q-l)} \end{matrix}\right\}$, with $q \leq p + l$, and where w and $w'0$ are words in the letter alphabet (In this case, just the numbers 1 and 2 in their usual sequence make up the place alphabet).

$$\sum \square_{p+k} \otimes \square_{q-k} \xrightarrow{\ \square\ } \square_p \otimes \square_q$$

Additionally, the maps are explained as follows using letter-place:

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix}\begin{vmatrix} 1^{(p+k)} \\ 2^{(q-k)} \end{vmatrix} \xrightarrow{\partial_{21}^{(k)}} \begin{pmatrix} r_1 \\ r_2 \end{pmatrix}\begin{vmatrix} 1^{(p)}2^{(k)} \\ 2^{(q-k)} \end{vmatrix} \longmapsto \sum_r \left(\begin{matrix} r_1 \\ r_2 \end{matrix}\middle| \begin{matrix} (t+1)'(t+2)' \ldots..(p+t)' \\ 1'2'3' \ldots q' \end{matrix}\right), \qquad (2)$$

where $w \otimes w' \in \square_{p+k} \otimes \square_{q-k}$, $\square = \sum_{k=t+1}^{q} \partial_{21}^{(k)}$ is the box map,

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and $d'_{\lambda/\mu}=\partial_{1'2}\,\partial_{(p+t)'1}$ is the arrangement of polarized places, starting from positive locations $\{1,2\}$ and ending at negative locations $\{1',2',\ldots,(p+t)'\}$. Additionally, as (2) illustrates, □ delivers a component $x\otimes y$ of □$_{p+k}\otimes$□$_{q-k}$ to $\sum x_p\otimes x'_k y$, where $\sum x_p\otimes x'_k$ in which the element of the diagonal of x in □$_p\otimes$□$_q$ divides the power element $z_{21}^{(k)}$ of the degree k of the free generator. Here, $(\mathcal{Z}_{21})$ acts on □$_{p+k}\otimes$□$_{q-k}$ through the place polarization of degree k from place (1) to place (2) This algebra is "graded" with identity. A = □ $(\mathcal{Z}_{21})$ acts on the graded module $\mathcal{M}=$ □$_{p+k}\otimes$□$_{q-k}=\sum\mathcal{M}_{q-k}$. Given that $w=z_{21}^{(k)}\in A$ and $v\in$ □$_{\beta 1}\otimes$□$_{\beta 2}$, $\mathcal{M}$ is a graded A-module. Thus, we have:

$r(u)=z_{21}^{(k)}(u)=\partial_{21}^{(k)}(u)$.

If we take (t^+) graded strand of degree q, we have:

$$\mathcal{M}_\bullet: 0\to\mathcal{M}_{q-t}\xrightarrow{\partial_{21}}\ldots\to\mathcal{M}_e\xrightarrow{\partial_s}\mathcal{M}_1\xrightarrow{\partial_s}\mathcal{M}_0,$$

Bar $(\mathcal{M}, A, ;\bullet)$ of the normalized Bar complex, where $S=\{x\}$. Here are some illustrations of key basic notions that we require in our work.

The following is the definition of the maps {□$_i$} [2]:

$$□_0: □_p\otimes□_q \longrightarrow \sum_{k>0} z^{(t+k)}\,x\; □_{p+t+k}\otimes□_{q-t-k}$$

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix}\!\left|\begin{matrix}1^{(p)}2^{(k)}\\ 2^{(q-k)}\end{matrix}\right. \longrightarrow \left\{\begin{matrix} z_{21}^{(k)}x \left|\begin{pmatrix} r_1 \\ r_2\end{pmatrix}\!\left|\begin{matrix}1^{(p+k)}\\ 2^{(q-k)}\end{matrix}\right.\right. & ; if\ k\le t \\ 0 & ; if\ k>t \end{matrix}\right\}.$$

Additionally, with the higher dimensions, we have:

$$□_{t-1}: □_{p+t+|K|}\otimes□_{q-t-|K|} \quad \sum k_i>0\; z_{21}^{(t+k_1)}x\,z_{21}^{(k_2)}x\ldots z_{21}^{(k_{t-1})}x\; z_{21}^{(t+k_1)}x\,z_{21}^{(k_2)}\ldots z_{21}^{(k_{t-1})}\,x\; □_{p+t+|K|}\otimes□_{q-t-|K|}$$

$$\to z_{21}^{(t+k_1)}x\,z_{21}^{(k_2)}x\ldots z_{21}^{(k_{t-1})}x\begin{pmatrix} r_1 \\ r_2\end{pmatrix}\!\left|\begin{matrix}1^{(p+t+k)}2^{(u)}\\ 2^{(q-t-k)}\end{matrix}\right.\to$$

$$\left\{\begin{matrix} z_{21}^{(t+k_1)}x\,z_{21}^{(k_2)}x\ldots z_{21}^{(k_{t-1})}x z_{21}^{(u)}x\begin{pmatrix} r_1 \\ r_2\end{pmatrix}\!\left|\begin{matrix}1^{(p+t+k)}2^{(u)}\\ 2^{(q-t-k)}\end{matrix}\right. & ; if\ u>0 \\ 0 & ; if u=0, \end{matrix}\right.$$

where the resolution's modules define the following terms:

$(\mathcal{M}_i)$ for $(i=0,1,\ldots.\,q-t)$, with $\mathcal{M}_0=$ □$_p\otimes$□$_q$,
$\mathcal{M}_i=z_{21}^{(t+k_1)}x\,z_{21}^{(k_2)}x\ldots z_{21}^{(k_i)}x$ □$_{p+t+|K|}\otimes$□$_{q-t-|K|}$; $for\ i\ge 1$ [2].

A previous study examined the Weyl module resolution for the two-rowed skew shape problem (p^+,t,q)/(t,0) [4]. However, in the situation of skew shape (12,6), another investigation [5] demonstrated the terms and the accuracy of the Weyl resolution.

2.1 Weyl module resolution of the case partition (12,6)

The terms of the sequence of the characteristic free resolution are given below.

$M_0=D_{12}\otimes D_6$

$M_1=Z_{21}^{(1)}x\,D_{13}\otimes D_5\oplus Z_{21}^{(2)}x\,D_{14}\otimes D_4\oplus Z_{21}^{(3)}x\,D_{15}\otimes D_3\oplus Z_{21}^{(4)}x\,D_{16}\otimes D_2\oplus Z_{21}^{(5)}x\,D_{17}\otimes D_1\oplus Z_{21}^{(6)}x\,D_{18}\otimes D_0$

$M_2=Z_{21}^{(1)}xZ_{21}^{(1)}x\,D_{14}\otimes D_4\oplus$
$Z_{21}^{(2)}xZ_{21}^{(1)}x\,D_{15}\otimes D_3\oplus Z_{21}^{(1)}xZ_{21}^{(2)}x\,D_{15}\otimes D_3\oplus$
$Z_{21}^{(3)}xZ_{21}^{(1)}x\,D_{16}\otimes D_2\oplus Z_{21}^{(1)}xZ_{21}^{(3)}x\,D_{16}\otimes D_2\oplus$
$Z_{21}^{(2)}xZ_{21}^{(2)}x\,D_{16}\otimes D_2\oplus Z_{21}^{(4)}xZ_{21}^{(1)}x\,D_{17}\otimes D_1\oplus$
$Z_{21}^{(1)}xZ_{21}^{(4)}x\,D_{17}\otimes D_1\oplus Z_{21}^{(2)}xZ_{21}^{(3)}x\,D_{17}\otimes D_1\oplus$
$Z_{21}^{(3)}xZ_{21}^{(2)}x\,D_{17}\otimes D_1\oplus Z_{21}^{(5)}xZ_{21}^{(1)}x\,D_{18}\otimes D_0\oplus$
$Z_{21}^{(1)}xZ_{21}^{(5)}x\,D_{18}\otimes D_0\oplus Z_{21}^{(2)}xZ_{21}^{(4)}x\,D_{18}\otimes D_0\oplus$
$Z_{21}^{(4)}xZ_{21}^{(2)}x\,D_{18}\otimes D_0\oplus Z_{21}^{(3)}xZ_{21}^{(3)}x\,D_{18}\otimes D_0$

$M_3=Z_{21}^{(1)}xZ_{21}^{(1)}x\,Z_{21}^{(1)}xD_{15}\otimes D_3\oplus$
$Z_{21}^{(2)}xZ_{21}^{(1)}x\,Z_{21}^{(1)}xD_{16}\otimes D_2\oplus$
$Z_{21}^{(1)}xZ_{21}^{(2)}x\,Z_{21}^{(1)}xD_{16}\otimes D_2\oplus$
$Z_{21}^{(1)}xZ_{21}^{(1)}x\,Z_{21}^{(2)}xD_{16}\otimes D_2\oplus$
$Z_{21}^{(3)}xZ_{21}^{(1)}x\,Z_{21}^{(1)}xD_{17}\otimes D_1\oplus$
$Z_{21}^{(1)}xZ_{21}^{(3)}x\,Z_{21}^{(1)}xD_{17}\otimes D_1\oplus$
$Z_{21}^{(1)}xZ_{21}^{(1)}x\,Z_{21}^{(3)}xD_{17}\otimes D_1\oplus$
$Z_{21}^{(2)}xZ_{21}^{(2)}x\,Z_{21}^{(1)}xD_{17}\otimes D_1\oplus$
$Z_{21}^{(1)}xZ_{21}^{(2)}x\,Z_{21}^{(2)}xD_{17}\otimes D_1\oplus$
$Z_{21}^{(2)}xZ_{21}^{(1)}x\,Z_{21}^{(2)}xD_{17}\otimes D_1\oplus$
$Z_{21}^{(4)}xZ_{21}^{(1)}x\,Z_{21}^{(1)}xD_{18}\otimes D_1\oplus$
$Z_{21}^{(1)}xZ_{21}^{(4)}x\,Z_{21}^{(1)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(1)}xZ_{21}^{(1)}x\,Z_{21}^{(4)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(3)}xZ_{21}^{(2)}x\,Z_{21}^{(1)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(3)}xZ_{21}^{(1)}x\,Z_{21}^{(2)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(2)}xZ_{21}^{(3)}x\,Z_{21}^{(1)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(2)}xZ_{21}^{(1)}x\,Z_{21}^{(3)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(1)}xZ_{21}^{(2)}x\,Z_{21}^{(3)}xD_{18}\otimes D_0\oplus$
$Z_{21}^{(1)}xZ_{21}^{(3)}x\,Z_{21}^{(2)}xD_{18}\otimes D_0\oplus Z_{21}^{(2)}xZ_{21}^{(2)}x\,Z_{21}^{(2)}xD_{18}\otimes D_0$

$M_4 = Z_{21}^{(1)} x Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{16} \otimes D_2 \oplus$
$Z_{21}^{(2)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x Z_{21}^{(1)} x D_{17} \otimes D_1 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(2)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{17} \otimes D_1 \oplus$
$Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(2)} x Z_{21}^{(1)} x D_{17} \otimes D_1 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(2)} x D_{17} \otimes D_1 \oplus$
$Z_{21}^{(3)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(3)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(3)} x Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(3)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(2)} x\, Z_{21}^{(2)} x\, Z_{21}^{(1)} x Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(2)} x Z_{21}^{(1)} x\, Z_{21}^{(2)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(2)} x Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(2)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(2)} x\, Z_{21}^{(2)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(1)} x\, Z_{21}^{(2)} x\, Z_{21}^{(2)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x Z_{21}^{(2)} x\, Z_{21}^{(1)} x\, Z_{21}^{(2)} x D_{18} \otimes D_0$

$M_5 = Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} D_{17} \otimes D_1 \oplus$
$Z_{21}^{(2)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x\, Z_{21}^{(2)} x\, Z_{21}^{(1)} x Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(2)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x Z_{21}^{(2)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0 \oplus$
$Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(2)} x D_{18} \otimes D_0$

$M_6 = Z_{21}^{(1)} x Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x\, Z_{21}^{(1)} x D_{18} \otimes D_0$

2.2 The Weyl resolution exactness in the partition case (12,6)

This section explains the building of a contracting homotopies $\{\square_i\}$, where i=1, 2,….5. We define the $\square_i$ map by:

$\square_0 : D_{12} \otimes D_6 \to \sum_{k>0} Z_{21}^{(k)}\, x\, D_{12+k} \otimes D_{6-k}$

$$\square_0\left(\left(\begin{array}{c|c} r_1 & 1^{(12)} \quad 2^{(k)} \\ r_2 & 2^{(6-k)} \end{array}\right)\right) = \begin{cases} 0 & ;\ if\ k = 0 \\ Z_{21}^{(k)} x \left(\begin{array}{c|c} r_1 & 1^{(12+k)} \\ r_2 & 2^{(6-k)} \end{array}\right) & ; if\ k = 1,2,3,4,5,6 \end{cases}$$

$\square_1 : \sum_{k>0} Z_{21}^{(k)}\, x\, D_{12+k} \otimes D_{6-k} \to Z_{21}^{(k_1)} x Z_{21}^{(k_2)} x D_{12+k} \otimes D_{6-k}$

$$\square_1\left(Z_{21}^{(k)} x \left(\begin{array}{c|c} r_1 & 1^{(12+k)} \quad 2^{(u)} \\ r_2 & 2^{(6-k-u)} \end{array}\right)\right) = \begin{cases} 0 & ;\ if\ u = 0 \\ Z_{21}^{(k)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|c} r_1 & 1^{(12+k+u)} \\ r_2 & 2^{(6-k-u)} \end{array}\right) & ; if\ u = 1,2,3,4,5 \end{cases}$$

$\square_2 : \sum_{k_i>0} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, D_{12+|k|} \otimes D_{6-|k|} \to Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x D_{12+|k|} \otimes D_{6-|k|}$

such that

$$\square_2\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right) = \begin{cases} 0 & ; if\ u = 0 \\ Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) & ; if\ u = 1,2,3,4 \end{cases}$$

where $|k| = k_1 + k_2$.

$\square_3 : \sum_{k_i>0} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)}\, x\, D_{12+|k|} \otimes D_{6-|k|} \to Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, D_{12+|k|} \otimes D_{6-|k|}$

such that

$$\square_3\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right) = \begin{cases} 0 & ; if\ u = 0 \\ Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) & ; if\ u = 1,2,3 \end{cases}$$

where $|k| = k_1 + k_2 + k_3$.

$\square_4 : \sum_{k_i>0} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)}\, x\, Z_{21}^{(k_4)} x\, D_{12+|k|} \otimes D_{6-|k|} \to Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, D_{12+|k|} \otimes D_{6-|k|}$

such that

$$\square_4\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right) = \begin{cases} 0 & ; if\ u = 0 \\ Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) & ; if\ u = 1,2 \end{cases}$$

where $|k| = k_1 + k_2 + k_3, +k_4$.

Finally, we define

$\square_5 : \sum_{k_i>0} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)}\, x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, D_{12+|k|} \otimes D_{6-|k|} \to Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(k_6)} x\ D_{12+|k|} \otimes D_{6-|k|}$

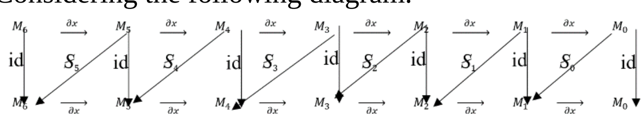
such that

$$\square_5\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x Z_{21}^{(k_5)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right) = \begin{cases} 0 & ; if\ u = 0 \\ Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|c} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) & ; if\ u = 1 \end{cases}$$

where $|k| = k_1 + k_2 + k_3, +k_4 + k_5$.

2.3 The exactness of the Weyl module resolution

Considering the following diagram:



,

we need to prove that

$\{\square_0, \square_1, \square_2, \square_3, \square_4, \square_5\}$ is a contracting homotopy, i.e.,

$\square_0\ \partial_x + \partial_x \square_1 = id_{M1}$
$\square_1\ \partial_x + \partial_x \square_2 = id_{M2}$
$\square_2\ \partial_x + \partial_x \square_3 = id_{M3}$
$\square_3\ \partial_x + \partial_x \square_4 = id_{M4}$
$\square_4\ \partial_x + \partial_x \square_5 = id_{M5}$

Now

$$\square_0 \partial_x\left(\ Z_{21}^{(k)} x \left(\begin{array}{c|c} r_1 & 1^{(12+k)} \quad 2^{(u)} \\ r_2 & 2^{(6-k-u)} \end{array}\right)\right) = \square_0 \partial_{21}^{(k)} \left(\begin{array}{c|c} r_1 & 1^{(12+k)} \quad 2^{(u)} \\ r_2 & 2^{(6-k-u)} \end{array}\right)$$

$$= \binom{k+u}{u} Z_{21}^{(k+u)} x \left(\begin{array}{c|c} r_1 & 1^{(12+k+u)} \\ r_2 & 2^{(6-k-u)} \end{array}\right),$$

and

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$$\partial x\square_1\left(Z_{21}^{(k)}x\left(\begin{array}{c|l} r_1 & 1^{(12+k)}\ \ 2^{(u)} \\ r_2 & 2^{(6-k-u)}\end{array}\right)\right)=$$

$$\partial_x\left(Z_{21}^{(k)}xZ_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+k+u)} \\ r_2 & 2^{(6-k-u)}\end{array}\right)\right)$$

$$=-\binom{k+u}{u}Z_{21}^{(k+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+k+u)} \\ r_2 & 2^{(6-k-u)}\end{array}\right)+Z_{21}^{(k)}x\left(\begin{array}{c|l} r_1 & 1^{(12+k)}\ \ 2^{(u)} \\ r_2 & 2^{(6-k-u)}\end{array}\right)$$

$\therefore\ \square_0\partial_x+\partial_x\square_1=id_{M1}$

$$\square_1\partial_x\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\square_1\left(-\binom{|k|}{k_2}Z_{21}^{(|k|)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)+\right.$$

$$\left.Z_{21}^{(k_1+1)}x\,\partial_{21}^{(k_2)}\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=-\binom{|k|}{k_2}Z_{21}^{(|k|)}xZ_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)+$$

$$\binom{k_2+u}{u}Z_{21}^{(k_1)}xZ_{21}^{(k_2+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right),$$

and

$$\partial_x\square_2\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\partial_x\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\binom{|k|}{k_2}Z_{21}^{(|k|)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_2+u}{u}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

where $|k|=k_1+k_2$

$\therefore\ \square_1\partial_x+\partial_x\square_2=id_{M2}$

now

$$\square_2\partial_x\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\square_2\left(\left(\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}xZ_{21}^{(k_3)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)\right.$$

$$-\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$\left.+\,Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,\partial_{21}^{(k_3)}\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+\binom{k_3+u}{u}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_2+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right),$$

and

$$\partial_x\square_3\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\partial_x\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+\binom{k_2+k_3}{k_2}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_3+u}{u}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,\partial_{21}^{(u)}\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_3+u}{u}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

where $|K|=k_1+k_2+k_3$.

$\therefore\ \square_2\partial_x+\partial_x\square_3=id_{M3}$

Now, we have to prove that

$$\square_3\partial_x\left(Z_{L1}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\square_3\left(-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$+\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\,Z_{21}^{(k_4)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_3+k_4}{k_4}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3+k_4)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,\partial_{21}^{(k_4)}\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$=-\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4)}Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\,Z_{21}^{(k_4)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_3+k_4}{k_4}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3+k_4)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+\binom{k_4+u}{u}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4+u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right),$$

and

$$\partial_x\square_4\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|)}\ \ 2^{(u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\partial_x\left(Z_{21}^{(k_1)}x\,Z_{21}^{(k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)\right)$$

$$=\binom{k_1+k_2}{k_2}Z_{21}^{(k_1+k_2)}x\,Z_{21}^{(k_3)}x\,Z_{21}^{(k_4)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$-\binom{k_2+k_3}{k_3}Z_{21}^{(k_1)}x\,Z_{21}^{(k_2+k_3)}x\,Z_{21}^{(k_4)}x\,Z_{21}^{(u)}x\left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)}\end{array}\right)$$

$$+\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3+k_4)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_4+u}{u} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4+u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, \partial_{21}^{(u)} \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$=\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2+k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3+k_4)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_4+u}{u} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4+u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

where $|K| = k_1 + k_2 + k_3 + k_4$.

$\therefore\ \square_3\partial_x + \partial_x\square_4 = id_{M4}$

$$\square_4\partial_x\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right)$$

$=$

$$\square_4\left(\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) -\right.$$

$$\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2+k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) +$$

$$\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3+k_4)} x\, Z_{21}^{(k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) -$$

$$\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4+k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right) +$$

$$\left.Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, \partial_{21}^{(k_5)} \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right)$$

$$=\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} x\ Z_{21}^{(k_2+k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3+k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4+k_{54})} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_5+u}{u} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5+u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

and

$$\partial_x\square_5\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right)=$$

$$\partial_x\left(Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)\right)=$$

$$-\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2+k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3+k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_4+k_5}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4+k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_5+u}{u} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5+u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, \partial_{21}^{(k_5)} \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$=$

$$-\binom{k_1+k_2}{k_2} Z_{21}^{(k_1+k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_2+k_3}{k_3} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2+k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_3+k_4}{k_4} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3+k_4)} x\, Z_{21}^{(k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+\binom{k_4+k_5}{k_5} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4+k_5)} x\, Z_{21}^{(u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$-\binom{k_5+u}{u} Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5+u)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|+u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

$$+Z_{21}^{(k_1)} x\, Z_{21}^{(k_2)} x\, Z_{21}^{(k_3)} x\, Z_{21}^{(k_4)} x\, Z_{21}^{(k_5)} x \left(\begin{array}{c|l} r_1 & 1^{(12+|k|)} \quad 2^{(u)} \\ r_2 & 2^{(6-|k|-u)} \end{array}\right)$$

where $|K| = k_1 + k_2 + k_3 + k_4 + k_5$.

$\therefore\ \square_4\partial_x + \partial_x\square_5 = id_{M5}$

$\therefore \{\square_0, \square_1, \square_2, \square_3, \square_4, \square_5\}$ is the contracting homotopy

$0 \quad \longrightarrow M_6 \longrightarrow\ M_5 \longrightarrow M_4 \longrightarrow M_3 \longrightarrow M_2 \longrightarrow M_1 \longrightarrow M_0.$

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تحلل صفين لمقاس وايل في حالة التجزئة (12,6)

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خلاصة

لتكن F مقاس-R الحر، حيث R حلقة ابدالية تحوي 1 و FD_n قوى القسمة من الدرجة n، تقنية استقطاب المكان هي طريقة اندماجية لحساب العناصر المعقدة. $\partial_{21}^{(k)}$ هو الاستقطاب الموقعي الذي يحدث من المكان الأول إلى المكان الثاني. الفكرة الاستقرائية حول مقدار التدخلات بين الصفين تعطينا وصفا للنتيجة التي نريدها وتعطينا اجابة عن نوع حل يشبه معقد Koszul (اطلقنا عليه معقد Koszul الحسابي). مقاس وايل $\mathcal{F} = Im\ K_{\lambda/\mu}$ حيث $(d'_{\lambda/\mu})$ $d'_{\lambda/\mu}$ وان $d'_{\lambda/\mu}\ \mathcal{F}\ \wedge_{\tilde{\lambda}/\tilde{\mu}} \longrightarrow \mathcal{F}_{\lambda/\mu}$ ⍰: تطبيق وايل ومنه سيتم اشتقاق مقاس وايل.

في هذا البحث سوف نركز على دراسة تحلل مقاس وايل في حالة صفين للتجزئة (12,6) باستخدام المخطط المتماثل، التوافق الهوموتوبي والاستقطاب المكاني.