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Performance of some new estimated ridge parameter regression models

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ABSTRACT

In the presence of high correlation among the independent variables in a linear regression model, a condition known as multicollinearity, the ordinary least squares (OLS) estimator tends to produce large variations in the sample. Aiming to overcome this problem, several estimators have been recommended. One of these estimators is ridge regression, which introduces a bias in the coefficient estimates but often results in a lower variance compared to OLS. Consequently, ridge regression may achieve a smaller mean square error (MSE). This paper reviews 50 kinds of ridge parameter (m) used in the estimation of the ordinary ridge regression estimator. Additionally, 10 new types of m are proposed based on different approaches. Aiming to evaluate the performance of these estimators, a simulation study and a real-data numerical example are conducted, using estimated mean squared error as the criterion for comparison. Results show that the newly proposed estimators for m can serve as effective alternatives to existing ones.

Introduction

$$y = X\xi + e. \quad (1)$$

Consider a standard model of multiple linear regression [1].

where y is an $n \times 1$ vector of observations, ξ is a $p \times 1$ vector of unknown regression coefficients, X is an $n \times p$ known design matrix of order p, and u is an $n \times 1$ vector of random variables, which is distributed as multivariate normal with a mean vector of 0 and a variance–covariance matrix $\sigma^2 I_n$, I_n is an identity matrix of order n. The standard least squares estimate (LSE) or maximum likelihood estimate (MLE)of ξ is derived as follows:

$$\hat{\xi} = C^{-1}X'y. \quad (2)$$

The outcome is highly influenced by the matrix $C = X'X$. If the C matrix is ill conditioned (with) near dependency among columns C or $det(X'X \approx 0)$, then the LSE can be sensitive to errors.

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For example, due to multicollinearity, some regression coefficients may become statistically insignificant or exhibit incorrect signs, making meaningful statistical inference difficult or even impossible for practitioners. Kibria, B. M. G [1],[2],[3] identified that multicollinearity as a common issue in engineering applications. Aiming to estimate ξ, using $C(m) = C + mI_p, (m \geq 0)$, rather than C, has been proposed. The obtained estimators are provided as follows:

$$\hat{\xi}(m) = \left(C + kI_p\right)X's, \quad (2)$$

which are known as ridge regression estimators. The constant $m > 0$ is known as or biasing or ridge parameter. As m increases from zero and continues up to infinity, the regression estimates tend toward zero. Although these estimators introduce bias, they yield a given value of m, minimum mean square error (MSE), compared to LSE ([1], [2], [3]). However, the will rely on the unknown parameters, which cannot be calculated practically but should be estimated from real data instead. Much of the debate surrounding ridge regression, particularly in the context of Hills's regression, emphasizes the challenge of determining an appropriate empirical value. Numerous methods for estimating m have been proposed by different researchers, including Dempster et al. [3],[4], [5], among others. Owing to the fact that ridge regression estimators have been applied by in different studies across various time periods and under diverse simulation conditions, direct comparisons between them is difficult. This paper aims to conduct a detailed examination of 50 distinct Hill-type estimators and 10 newly proposed ridge estimator, all evaluated based on their performance in minimizing MSE. The investigation is conducted using Monte Carlo simulations, considering a range of scenarios. These scenarios include variations in random error variance, correlation among explanatory variables, sample sizes, and vectors of unknown coefficients. This paper is organized as follows: First, the available methods for estimating m are reviewed, followed by the simulation. Applications of the proposed methods are then presented. Finally, some concluding remarks are provided.

2. Methodology

Suppose an orthogonal matrix L exists such that $L'CL = \Lambda$, where $\Lambda = \mathrm{diag}(\lambda_2, \lambda_2, ,,,,,,,, \lambda_p)$.

The eigenvalues of the matrix C are contained in the orthogonal (canonical form) version of the mode (1).

$$S = X^*\xi + e, \qquad (3)$$

where $X^* = XL$ and $\tau = L'\xi$. Therefore, the generalized ridge regression estimators are derived as follows:

$$\hat{\tau}(m) = \left(X^{*\prime}X^* + m\right)^{-1} X'^{*}S, \qquad (4)$$

where $m = \mathrm{diag}(m_1, m_2, ,,,,,,, m_p), m_i > 0$ and $\hat{\tau} = \Lambda^{-1} X'^{*} S$. The ordinary least squares (OLS) estimates of τ are derived from [3]. The value of m_i minimizes the MSE $\hat{\tau}(m)$, where

$$MSE(\hat{\tau}(m)) = \sigma^2 \sum_{i=}^{p} \frac{m_i^2\tau_i^2}{(\lambda_i+m_i)^2} + \sum_{i=}^{p} \frac{m_i^2\tau_i^2}{(\lambda_i+m_i)^2}. \qquad (5)$$

When

$$m_i = \frac{\sigma^2}{\tau_i^2}, \qquad (6)$$

where σ^2 denotes the error variance of model (1), and τ_i denotes the ith element of τ. Hocking et al. [6] demonstrated that, given the known optimal m_i, the generalized ridge regression estimator outperforms all other estimators in the class of examined biased estimators. However, the ideal value of m_i is entirely dependent on the unknown σ^2 and τ_i, which must be determined from the observed data. Hoerl, Kennard [3] proposed replacing σ^2 and τ_i^2 with their equivalent unbiased estimators. That is to say,

$$\hat{m}_i = \frac{\hat{\sigma}^2}{\hat{\tau}_i^2}, \qquad (7)$$

where $\hat{\sigma}^2 = (\sum \hat{e}^2)/(n-p)$ represents the residual mean square estimate, which is an unbiased estimator of σ^2 and $\hat{\tau}_i$ is the tth element of $\hat{\tau}$, which is also an unbiased estimator of τ. Other novel methods based on Eq. (8) are then discussed as follows:

1. $$\hat{m}_1 = \frac{\hat{\sigma}^2}{\hat{\tau}^2_{max}}, \qquad (8)$$

where $\hat{\tau}_{max}$ is the maximum element of $\hat{\boldsymbol{\tau}}$. If σ^2 and τ are known, then $\hat{m}_1$ will yield a smaller MSE than the LSE [3].

2.[7] $$\hat{m}_2 = \frac{2\hat{\sigma}^2}{\hat{\xi}'\hat{\xi}}, \qquad (9)$$

3.[4] $$\hat{m}_3 = \frac{p\hat{\sigma}^2}{\hat{\xi}'\hat{\xi}} = \frac{p\hat{\sigma}^2}{\hat{\tau}'\ \hat{\boldsymbol{\tau}}} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \tau_i^2}, \qquad (10)$$

where $\hat{\xi}$ is the LSE of ξ.

4.[5] $$\hat{m}_4 = \hat{\xi}_k'\hat{\xi} = \hat{\xi}'\hat{\xi} - \hat{\sigma}^2 \sum_{i=1}^{p} \frac{1}{\lambda_i} \qquad (11)$$

5.[8] $$\hat{m}_5 = \frac{p\hat{\sigma}^2}{\hat{\tau}'X'X\hat{\tau}} = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \lambda_i \tau_i^2} = \frac{p\hat{\sigma}^2}{\hat{\xi}'\Lambda\hat{\xi}} \qquad (12)$$

6.[6] $$\hat{m}_6 = \frac{\hat{\sigma}^2 \sum_{i=1}^{p} (\lambda_i \hat{\tau}_i)^2}{(\sum_{i=1}^{p} \lambda_i \hat{\tau}_i)^2} \qquad (13)$$

7.[9]

$$\hat{m}_7 = \frac{1}{\hat{\sigma}^2_{max}} \qquad (14)$$

8.[10]

$$\widehat{m}_8 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}[\frac{\tau_i^2}{1+(1+\lambda_i(\frac{\tau_i^2}{\sigma^2})^{\frac{1}{2}})}]} \quad (15)$$

9.[1]

$$\widehat{m}_9 = \frac{1}{p}\sum_{i=1}^{p}\frac{\hat{\sigma}^2}{\tau_i^2} \quad (16)$$

$$\widehat{m}_{10} = \frac{\hat{\sigma}^2}{(\prod_{i=1}^{p}\tau_i^2)^{\frac{1}{p}}} \quad (17)$$

$$\widehat{m}_{11} = median(\frac{\hat{\sigma}^2}{\tau_i^2}) \quad (18)$$

10.[11]

$$\widehat{m}_{12} = \frac{\lambda_{max}\ \hat{\sigma}^2}{(n-p-1)\hat{\sigma}^2+\lambda_{max}\ \hat{\xi}_{max}^2}, \quad (19)$$

where λ_{max} is the maximum eigenvalue of the matrix $X'X$.

11.[12]

$$\widehat{m}_{13} = max\left(\frac{\lambda_i\ \hat{\sigma}^2}{(n-p-1)\hat{\sigma}^2+\lambda_i\ \hat{\xi}_i^2}\right) \quad (20)$$

$$\widehat{m}_{14} = median\left(\frac{\lambda_i\ \hat{\sigma}^2}{(n-p-1)\hat{\sigma}^2+\lambda_i\ \hat{\xi}_i^2}\right) \quad (21)$$

$$\widehat{m}_{15} = \frac{1}{p}\sum_{i=1}^{p}(\frac{\lambda_i\ \hat{\sigma}^2}{(n-p-1)\hat{\sigma}^2+\lambda_i\ \hat{\xi}_i^2}) \quad (22)$$

12.[13]

$$\widehat{m}_{16} = \frac{\hat{\sigma}^2}{\hat{\xi}_{max}^2} + \frac{1}{\lambda_{max}} \quad (23)$$

$$\widehat{m}_{17} = max\left(\frac{\hat{\sigma}^2}{\hat{\xi}_i^2} + \frac{1}{\lambda_i}\right) \quad (24)$$

$$\widehat{m}_{18} = \frac{1}{p}\sum_{i=1}^{p}(\frac{\hat{\sigma}^2}{\hat{\xi}_{max}^2} + \frac{1}{\lambda_{max}}) \quad (25)$$

$$\widehat{m}_{19} = median(\frac{\hat{\sigma}^2}{\hat{\xi}_i^2} + \frac{1}{\lambda_i}) \quad (26)$$

$$\widehat{m}_{20} = \frac{p\ \hat{\sigma}^2}{\sum_{i=1}^{p}\hat{\xi}_i^2} + \frac{1}{\lambda_{max}} \quad (27)$$

$$\widehat{m}_{21} = \frac{p\ \hat{\sigma}^2}{\sum_{i=1}^{p}\lambda_i\ \hat{\xi}_i^2} + \frac{1}{\lambda_{max}} \quad (28)$$

13.[14]

$$\widehat{m}_{22} = max(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\tau}_i^2}}}) \quad (29)$$

$$\widehat{m}_{23} = max(\sqrt{\frac{\hat{\sigma}^2}{\hat{\tau}_i^2}}) \quad (30)$$

$$\widehat{m}_{24} = (\prod_{i=1}^{p}\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\tau}_i^2}}})^{\frac{1}{p}} \quad (31)$$

$$\widehat{m}_{25} = (\prod_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\tau}_i^2})^{\frac{1}{p}} \quad (32)$$

$$\widehat{m}_{26} = median(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\tau}_i^2}}}) \quad (33)$$

$$\widehat{m}_{27} = median(\sqrt{\frac{\hat{\sigma}^2}{\hat{\tau}_i^2}}) \quad (34)$$

$$\widehat{m}_{28} = (\prod_{i=1}^{p}\frac{\lambda_i\hat{\sigma}^2}{(n-p)\hat{\sigma}^2+\lambda_i\hat{\tau}_i^2})^{\frac{1}{p}} \quad (35)$$

14.[15]

$$\widehat{m}_{29} = max\left(0, \frac{p\hat{\sigma}^2}{\hat{\tau}'\hat{\tau}} - \frac{1}{n(VIF_j)_{max}}\right), \quad (36)$$

$VIF_j = \frac{1}{1-R_j^2}$

is the variance inflation factor of the jth regressor.

15.[16]

$$\widehat{m}_{30} = \frac{\sum_{i=1}^{p}(\lambda_i\hat{\tau}_i)^2}{(\sum_{i=1}^{p}(\lambda_i\hat{\tau}_i)^2)^2} + \frac{1}{\lambda_{max}} \quad (37)$$

16.[17]

$$\widehat{m}_{31} = max\left(\frac{1}{\frac{\lambda_{max}\tau_i^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}\right) \quad (38)$$

$$\widehat{m}_{32} = median\left(\frac{1}{\frac{\lambda_{max}\tau_i^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}\right) \quad (39)$$

17.[18]

$$\widehat{m}_{33} = \left(\prod_{i=1}^{p}\frac{\lambda_i\hat{\sigma}^2}{(n-p-1)\hat{\sigma}^2+\lambda_i\hat{\tau}_i^2}\right) \quad (40)$$

$$\widehat{m}_{34} = max\left(\frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}}\right) \quad (41)$$

$$\widehat{m}_{35} = max(\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}) \quad (42)$$

$$\widehat{m}_{36} = \left(\prod_{i=1}^{p}\frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}}\right)^{1/p} \quad (43)$$

$$\widehat{m}_{37} = \left(\prod_{i=1}^{p}\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}\right)^{1/p} \quad (44)$$

$$\widehat{m}_{38} = median\left(\frac{1}{\sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\tau}_i^2)}}}\right) \quad (45)$$

18.[19]

$$\widehat{m}_{39} = \frac{(\lambda_{max}+\lambda_{min})}{2\sum_{i=1}^{p}|\hat{\xi}|_j}\left\{\frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\xi_i^2}\right\} \quad (46)$$

$$\widehat{m}_{40} = \frac{(\lambda_{max}+\lambda_{min})\lambda_{max}\hat{\sigma}^2}{2\sum_{i=1}^{p}|\hat{\xi}|_j(n-p)\hat{\sigma}^2+\lambda_{max}\xi_{max}^2} \quad (47)$$

19.[20]

$$\widehat{m}_{41} = \max(\sqrt{m_i}) \quad (48)$$

where $m_i = \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\alpha}_i^2)}}$

$$\widehat{m}_{42} = \left(\prod_{i=1}^{p}\frac{1}{\sqrt{m_i}}\right)^{\frac{1}{p}} \quad (49)$$

$$\widehat{m}_{43} = \left(\prod_{i=1}^{p}\sqrt{m_i}\right)^{\frac{1}{p}} \quad (50)$$

$$\widehat{m}_{44} = \text{median}\left(\frac{1}{\sqrt{m_i}}\right) \quad (51)$$

where

$$m_i = \sqrt{\frac{\lambda_{max}\hat{\sigma}^2}{((n-p-1)\hat{\sigma}^2+\lambda_{max}\hat{\alpha}_i^2)}} \quad (52)$$

20.[21]

$$\widehat{m}_{45} = \frac{p^2\hat{\sigma}^2}{\lambda_{max}^2\sum_{i=1}^{p}\hat{\tau}_i^2} \quad (53)$$

$$\widehat{m}_{46} = \frac{p^3\hat{\sigma}^2}{\lambda_{max}^3\sum_{i=1}^{p}\hat{\tau}_i^2} \quad (54)$$

$$\widehat{m}_{47} = \frac{p\hat{\sigma}^2}{\lambda_{max}^{\frac{1}{3}}\sum_{i=1}^{p}\hat{\tau}_i^2} \quad (55)$$

$$\widehat{m}_{48} = \frac{p\hat{\sigma}^2}{(\sum_{i=1}^{p}\sqrt{\lambda_i})^{\frac{1}{3}}\sum_{i=1}^{p}\hat{\tau}_i^2} \quad (56)$$

$$\widehat{m}_{49} = \frac{2p\hat{\sigma}^2}{\sqrt{\lambda_i}\sum_{i=1}^{p}\hat{\tau}_i^2} \quad (57)$$

21.[22]

$$\widehat{m}_{50} = \hat{\sigma} \quad (58)$$

where $\hat{\sigma} = \sqrt{\sum_{j=1}^{n}\hat{e}_j/(n-p)}$

22. The following 10 new estimators for m are introduced:

$$\widehat{m}_{f1} = p\hat{\sigma}^2 - \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}[\frac{\tau_i^2}{1+(1+\gamma_i(\frac{\tau_i^2}{\sigma^2})^{\frac{1}{2}})}]} \quad (60)$$

$$\widehat{m}_{f2} = \text{median}(p\hat{\sigma}^2 - \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}[\frac{\tau_i^2}{1+(1+\gamma_i(\frac{\tau_i^2}{\sigma^2})^{\frac{1}{2}})}]}) \quad (61)$$

$$\widehat{m}_{f3} = (\frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\tau_i^2} - \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}[\frac{\tau_i^2}{1+(1+\gamma_i(\frac{\tau_i^2}{\sigma^2})^{\frac{1}{2}})}]} \quad (62)$$

$\widehat{m}_{f4}$ = harmonic mean of$(\widehat{m}_{43}, \widehat{m}_{44}, \widehat{m}_{45}\widehat{m}_{46}, \widehat{m}_{47}, \widehat{m}_{48}, \widehat{m}_{49})$ (6

$\widehat{m}_{f5}$ = Arithmetic mean of$(\widehat{m}_{50}, \widehat{m}_{f1}, \widehat{m}_{f2}\widehat{m}_{f3},)$ (64)

$\widehat{m}_{f6}$ = Arithmetic mean of $((\widehat{m}_{50}, \widehat{m}_{f1}, \widehat{m}_{f2}\widehat{m}_{f3},)/2)$ (65)

$$\widehat{m}_{f7} = \text{Median}(\widehat{m}_{50}, \widehat{m}_{f1}, \widehat{m}_{f2}\widehat{m}_{f3}) \quad (66)$$

$$\widehat{m}_{f8} = \text{Min}(\widehat{m}_{50}, \widehat{m}_{f1}, \widehat{m}_{f2}\widehat{m}_{f3}) \quad (67)$$

$\widehat{m}_{f9}$ = harmonic mean (68) of$(\widehat{m}_{43}, \widehat{m}_{44}, \widehat{m}_{45}\widehat{m}_{46}, \widehat{m}_{47}, \widehat{m}_{48}, \widehat{m}_{49}, \widehat{m}_{50}, \widehat{m}_{f1}, \widehat{m}_{f2}\widehat{m}_{f3})$

$$\widehat{\mathrm{m}}_{f10} = \mathrm{Min}(\widehat{\mathrm{m}}_{43}, \widehat{\mathrm{m}}_{44}, \widehat{\mathrm{m}}_{45}\widehat{\mathrm{m}}_{46}, \widehat{\mathrm{m}}_{47}, \widehat{\mathrm{m}}_{48}, \widehat{\mathrm{m}}_{49}, \widehat{\mathrm{m}}_{50}, \widehat{\mathrm{m}}_{f1}, \widehat{\mathrm{m}}_{f2}\widehat{\mathrm{m}}_{f3}) \quad (69)$$

3. Simulation study

This section discusses the simulation research, which compares the performance of various estimators. [2,3,5] used the following method to derive explanatory variables with varying degrees of collinearity.

$$X_{ij} = (1-\varphi)^{\frac{1}{2}} z_{ij} + \varphi z_{ip} \quad , \quad i = 1,2,3,\dots,n, \quad j = 1,2,\dots,p, \quad (71)$$

where z_{ij} is an independent standard normal pseudo-random number, and s is chosen such that the correlation between any two variables of explanation is given by s^2. These variables are then standardized, resulting in X's and $X's$ in correlation forms. The response variable y is considered by the following:

$$y_i = \xi_0 + \xi_1 x_{i1} + \xi_2 x_{i2} + \dots\dots \xi_p x_{ip} + e_i \quad i = 1,2,\dots\dots,n, \quad (72)$$

where e_i is independent and identically distributed random variables $(i.i.d)$. normal $(0, \sigma^2)$. Therefore, zero intercept for (60) will be assumed. Moreover, four explanatory variables p = 4 will be available, while the values of σ are chosen as (0.01, 0.5, 1, and 5). The correlation φ will be chosen as (0.75, 0.85, 0.95, and 0.99) and sample size n as (50, 100, and 150). The coefficients $\xi_1, \xi_2, \dots\dots, \xi_p$ are selected as the eigenvectors corresponding to the greatest eigenvalue $X'X$ of the matrix subject to constraint $||\xi|| = 1$. Thus, sets of Xs are created for all n, σ, λ, p, ξ, and ϕ. The experiment has been replicated 10,000 times by creating new error terms. The estimated mean squared error (EMSE) is calculated as follows:

$$MSE(\xi^*) = \frac{1}{10000} \sum_{i=1}^{10000} (\xi^* - \xi)'(-\xi), \quad (73)$$

where ξ^* would be any of the estimators (OLS, ORR).

4. Simulation Results and Discussion

As shown in Tables 2–6, the simulation results indicate that when n, φ, and k are fixed, the MSE values increase with the values of σ. Conversely, when the values of n increase, the MSE values decrease for fixed values of σ, φ, and m. In addition, the MSE values increase with the value of φ. In Tables 2–3, when n = 50, φ = 0.75, 0.85, 0.95, and 0.99, and σ = 0.01 and 0.5 for all cases of the variance, the ($\widehat{\mathrm{m}}_{f1}$,$\widehat{\mathrm{m}}_{f2}$,$\widehat{\mathrm{m}}_{f3}$,$\widehat{\mathrm{m}}_{f4}$,$\widehat{\mathrm{m}}_{f5}$,$\widehat{\mathrm{m}}_{f6}$,$\widehat{\mathrm{m}}_{f7}$,$\widehat{\mathrm{m}}_{f8}$,$\widehat{\mathrm{m}}_{f9}$ and $\widehat{\mathrm{m}}_{f10}$) estimator yields the minimum MSE, indicating the excellent performance of the $\widehat{\mathrm{m}}_{f10}$ estimator. In Tables 4–5, when n = 100, φ = 0.75, 0.85, 0.95, and 0.99 and σ = 1 and 5, the ($\widehat{\mathrm{m}}_{f1}$, $\widehat{\mathrm{m}}_{f2}$, $\widehat{\mathrm{m}}_{f3}$, $\widehat{\mathrm{m}}_{f4}$, $\widehat{\mathrm{m}}_{f5}$, $\widehat{\mathrm{m}}_{f6}$, $\widehat{\mathrm{m}}_{f7}$, $\widehat{\mathrm{m}}_{f8}$, $\widehat{\mathrm{m}}_{f9}$, and $\widehat{\mathrm{m}}_{f10}$) estimator outperforms other estimators because the ORR estimator has minimum EMSE, indicating that the $\widehat{\mathrm{m}}_{f10}$ estimator is the best. In Table (6), when n = 150, φ = 0.75, 0.85, 0.95, and 0.99, and σ = 0.01, the performance of the ($\widehat{\mathrm{m}}_{f1}$, $\widehat{\mathrm{m}}_{f2}$, $\widehat{\mathrm{m}}_{f3}$, $\widehat{\mathrm{m}}_{f4}$, $\widehat{\mathrm{m}}_{f5}$, $\widehat{\mathrm{m}}_{f6}$, $\widehat{\mathrm{m}}_{f7}$, $\widehat{\mathrm{m}}_{f8}$, $\widehat{\mathrm{m}}_{f9}$, and $\widehat{\mathrm{m}}_{f10}$) indicate that the $\widehat{\mathrm{m}}_{f10}$ estimator outperforms all other estimators because the ORR estimator achieves the minimum EMSE. As shown in Tables 2–6, when n = 50, 100, and 150, φ = 0.75, 0.85, 0.95, and 0.99, and σ = 0.01, 0.5, 1, and 5, the $\widehat{\mathrm{m}}_{f10}$ estimator consistently yields the lowest EMSE, confirming the superiority of this estimator under a variety of conditions.

5. Numerical Example

Aiming to support the theoretical claims, a Portland cement dataset, originally published [23] and has been consistently investigated extensively by researchers such as [24,25], is examined. This dataset is derived from an experiment that investigated how different Portland cements are solidified and strengthened in terms of heat. The paper explores the correlation between this heat and the ratios of four cement-making clinkers' constituents. The dependent variable herein is Y, which represents the amount of heat produced in calories per gram of cement. The quantities of the following compounds (X1, X2, X3, and X4) serve as the independent variables: tricalcium aluminate, tricalcium silicate, tetracalcium aluminoferrite, and dicalcium silicate. The intercept item is presented in the model.

The construction of the three estimators of OLS and ORR and MSE matrices is investigated, and then their traces are compared. The OLS and ORR estimators provide traces of their respective MSE matrices, which are shown as follows:

$$smse(\hat{\tau}) = tr\left(MSE(\hat{\tau}_R)\right) = \sum_{i=1}^{P} \frac{\sigma^2}{\lambda_i}, \quad (74)$$

$$smse(\hat{\tau}_R) = tr\left(MSE(\hat{\tau}_R)\right) = \sum_{i=1}^{P} \frac{\lambda_i \sigma^2 + m^2 \tau_i^2}{(\lambda_i + m)^2}. \quad (75)$$

This matrix X' X has five eigenvalues: $\lambda_1 = 211.367, \lambda_2 = 77.236, \lambda_3 = 28.459, \lambda_4 = 10.267, and\ \lambda_5 = 0.0349$. Classifying X as "ill-conditioned" is possible because its condition number is 6056.37, which is calculated as K = $\lambda_{max}/\lambda_{min}$. For the X' X matrix to take the shape of a correlation matrix, data standardization is generally suggested by the authors. Therefore, the regression coefficients may be represented in equivalent numerical units, which is an advantage of data standardization [24].

Therefore, the four eigenvalues of the correlation matrix X' X are 2.2357, 1.57606, 0.18661, and 0.00162, while the estimated value of σ^2 is 0.00196.

Table 1: Scalar mean square error for estimators (OLS and ORR) and different estimated ridge parameters

	OLS	ORR		OLS	ORR
$\hat{m}_{1}$	1.2186	0.1664	$\hat{m}_{31}$	1.2186	0.6426
$\hat{m}_{2}$	1.2186	0.1616	$\hat{m}_{32}$	1.2186	0.5621
$\hat{m}_{3}$	1.2186	0.1458	$\hat{m}_{33}$	1.2186	0.1459
$\hat{m}_{4}$	1.2186	0.2308	$\hat{m}_{34}$	1.2186	0.5663
$\hat{m}_{5}$	1.2186	0.1504	$\hat{m}_{35}$	1.2186	0.2432
$\hat{m}_{6}$	1.2186	0.1804	$\hat{m}_{36}$	1.2186	0.3223
$\hat{m}_{7}$	1.2186	0.3412	$\hat{m}_{37}$	1.2186	0.2154
$\hat{m}_{8}$	1.2186	0.1588	$\hat{m}_{38}$	1.2186	0.5003
$\hat{m}_{9}$	1.2186	0.1605	$\hat{m}_{39}$	1.2186	0.2103
$\hat{m}_{10}$	1.2186	0.1605	$\hat{m}_{40}$	1.2186	0.4471
$\hat{m}_{11}$	1.2186	0.4873	$\hat{m}_{41}$	1.2186	0.2502
$\hat{m}_{12}$	1.2186	0.1781	$\hat{m}_{42}$	1.2186	0.2868
$\hat{m}_{13}$	1.2186	0.2571	$\hat{m}_{43}$	1.2186	0.2061
$\hat{m}_{14}$	1.2186	0.2026	$\hat{m}_{44}$	1.2186	0.3651
$\hat{m}_{15}$	1.2186	0.2091	$\hat{m}_{45}$	1.2186	0.1476
$\hat{m}_{16}$	1.2186	0.2099	$\hat{m}_{46}$	1.2186	0.1517
$\hat{m}_{17}$	1.2186	0.6707	$\hat{m}_{47}$	1.2186	0.188
$\hat{m}_{18}$	1.2186	0.6621	$\hat{m}_{48}$	1.2186	0.1515
$\hat{m}_{19}$	1.2186	0.5783	$\hat{m}_{49}$	1.2186	0.1882
$\hat{m}_{20}$	1.2186	0.2106	$\hat{m}_{50}$	1.2186	0.154
$\hat{m}_{21}$	1.2186	0.2102	$\hat{m}_{f1}$	1.2186	0.2021
$\hat{m}_{22}$	1.2186	0.566	$\hat{m}_{f2}$	1.2186	0.2063
$\hat{m}_{23}$	1.2186	0.4504	$\hat{m}_{f3}$	1.2186	0.2063
$\hat{m}_{24}$	1.2186	0.3187	$\hat{m}_{f4}$	1.2186	0.1455
$\hat{m}_{25}$	1.2186	0.1897	$\hat{m}_{f5}$	1.2186	0.1918
$\hat{m}_{26}$	1.2186	0.4989	$\hat{m}_{f6}$	1.2186	0.2149
$\hat{m}_{27}$	1.2186	0.1693	$\hat{m}_{f7}$	1.2186	0.2041
$\hat{m}_{28}$	1.2186	0.1718	$\hat{m}_{f8}$	1.2186	0.2063
$\hat{m}_{29}$	1.2186	0.1465	$\hat{m}_{f9}$	1.2186	0.1479
$\hat{m}_{30}$	1.2186	0.3595	$\hat{m}_{f10}$	1.2186	0.2063

As shown in Table (1), the minimum MSE for the ORR estimator will be obtained if m is estimated by Hoerl, Kennard, and Baldwin (1975) (HKB). The minimum MSE for the m_{f10} estimator will also be given by estimating m using HKB. The performance of the estimated m provided in this study shows that, under moderate degree of multicollinearity, most yield minimum MSE when used in the HKB estimator. Therefore, not all proposed ridge parameters can be used to obtained the minimum MSE under a moderate degree of multicollinearity. Finally, this study presents a broad view on the behaviors of the estimators and their applications, which yield a good performance compared to the other suggested estimators.

Table 2: Estimated MSEs of m when n = 50 and σ = 0.01

φ	0.75		0.85		0.95		0.99	
	OLS	ORR	OLS	ORR	OLS	ORR	OLS	ORR
$\hat{m}_{1}$	0.156331	0.156333	0.201713	0.201714	0.225784	0.225785	0.24647	0.246471
$\hat{m}_{2}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225785	0.24647	0.24647
$\hat{m}_{3}$	0.156331	0.156333	0.201713	0.201714	0.225784	0.225785	0.24647	0.246471
$\hat{m}_{4}$	0.156331	0.217693	0.201713	0.244817	0.225784	0.262217	0.24647	0.277515
$\hat{m}_{5}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225784	0.24647	0.24647
$\hat{m}_{6}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225784	0.24647	0.24647
$\hat{m}_{7}$	0.156331	0.486019	0.201713	0.522955	0.225784	0.543847	0.24647	0.559758
$\hat{m}_{8}$	0.156331	0.158203	0.201713	0.202566	0.225784	0.227854	0.24647	0.248017
$\hat{m}_{9}$	0.156331	0.156445	0.201713	0.201736	0.225784	0.226284	0.24647	0.246957
$\hat{m}_{10}$	0.156331	0.15673	0.201713	0.201771	0.225784	0.233742	0.24647	0.25249
$\hat{m}_{11}$	0.156331	0.157694	0.201713	0.201858	0.225784	0.239339	0.24647	0.264331
$\hat{m}_{12}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225784	0.24647	0.24647
$\hat{m}_{13}$	0.156331	0.310189	0.201713	0.33215	0.225784	0.346218	0.24647	0.357904

$\hat{m}_{14}$	0.156331	0.302651	0.201713	0.329773	0.225784	0.284867	0.24647	0.265864
$\hat{m}_{15}$	0.156331	0.290116	0.201713	0.325612	0.225784	0.288215	0.24647	0.286363
$\hat{m}_{16}$	0.156331	0.217562	0.201713	0.244473	0.225784	0.262188	0.24647	0.277506
$\hat{m}_{17}$	0.156331	0.486225	0.201713	0.618065	0.225784	0.758854	0.24647	0.955301
$\hat{m}_{18}$	0.156331	0.413197	0.201713	0.520637	0.225784	0.663916	0.24647	0.917674
$\hat{m}_{19}$	0.156331	0.446017	0.201713	0.548628	0.225784	0.689528	0.24647	0.919681
$\hat{m}_{20}$	0.156331	0.217563	0.201713	0.244474	0.225784	0.262189	0.24647	0.277507
$\hat{m}_{21}$	0.156331	0.217562	0.201713	0.244473	0.225784	0.262188	0.24647	0.277506
$\hat{m}_{22}$	0.156331	0.995364	0.201713	0.995541	0.225784	0.994302	0.24647	0.994776
$\hat{m}_{23}$	0.156331	0.184355	0.201713	0.209771	0.225784	0.294142	0.24647	0.326771
$\hat{m}_{24}$	0.156331	0.431878	0.201713	0.492801	0.225784	0.380613	0.24647	0.391634
$\hat{m}_{25}$	0.156331	0.195038	0.201713	0.219384	0.225784	0.29637	0.24647	0.3099997
$\hat{m}_{26}$	0.156331	0.881728	0.201713	0.934953	0.225784	0.762386	0.24647	0.731239
$\hat{m}_{27}$	0.156331	0.164175	0.201713	0.204445	0.225784	0.251791	0.24647	0.269108
$\hat{m}_{28}$	0.156331	0.286356	0.201713	0.325242	0.225784	0.251834	0.24647	0.253907
$\hat{m}_{29}$	0.156331	0.156333	0.201713	0.201714	0.225784	0.225785	0.24647	0.246471
$\hat{m}_{30}$	0.156331	0.512905	0.201713	0.540853	0.225784	0.558106	0.24647	0.571566
$\hat{m}_{31}$	0.156331	0.415646	0.201713	0.357315	0.225784	0.914169	0.24647	0.992162
$\hat{m}_{32}$	0.156331	0.318146	0.201713	0.334578	0.225784	0.695779	0.24647	0.863601
$\hat{m}_{33}$	0.156331	0.156425	0.201713	0.201734	0.225784	0.225882	0.24647	0.246498
$\hat{m}_{34}$	0.156331	0.995364	0.201713	0.995541	0.225784	0.994302	0.24647	0.994776
$\hat{m}_{35}$	0.156331	0.182767	0.201713	0.209716	0.225784	0.275257	0.24647	0.298689
$\hat{m}_{36}$	0.156331	0.434822	0.201713	0.49314	0.225784	0.393284	0.24647	0.406388
$\hat{m}_{37}$	0.156331	0.234059	0.201713	0.251331	0.225784	0.310648	0.24647	0.322367
$\hat{m}_{38}$	0.156331	0.882172	0.201713	0.934999	0.225784	0.766902	0.24647	0.736127
$\hat{m}_{39}$	0.156331	0.15634	0.201713	0.20172	0.225784	0.225793	0.24647	0.246477
$\hat{m}_{40}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225785	0.24647	0.24647
$\hat{m}_{41}$	0.156331	0.221926	0.201713	0.235509	0.225784	0.303761	0.24647	0.323327
$\hat{m}_{42}$	0.156331	0.36637	0.201713	0.402165	0.225784	0.368233	0.24647	0.380413
$\hat{m}_{43}$	0.156331	0.183348	0.201713	0.217467	0.225784	0.256497	0.24647	0.27535
$\hat{m}_{44}$	0.156331	0.612939	0.201713	0.682638	0.225784	0.517766	0.24647	0.509008
$\hat{m}_{45}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225784	0.24647	0.24647
$\hat{m}_{46}$	0.156331	0.156331	0.201713	0.201713	0.225784	0.225784	0.24647	0.24647
$\hat{m}_{47}$	0.156331	0.156331	0.201713	0.201713	0.225784	0.225784	0.24647	0.24647
$\hat{m}_{48}$	0.156331	0.156333	0.201713	0.201714	0.225784	0.225786	0.24647	0.246472
$\hat{m}_{49}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225785	0.24647	0.246471
$\hat{m}_{50}$	0.156331	0.156477	0.201713	0.201804	0.225784	0.22589	0.24647	0.246547
$\hat{m}_{f1}$	0.156331	0.153462	0.201713	0.200888	0.225784	0.22372	0.24647	0.245933
$\hat{m}_{f2}$	0.156331	0.154463	0.201713	0.200867	0.225784	0.224719	0.24647	0.243932
$\hat{m}_{f3}$	0.156331	0.154421	0.201713	0.200867	0.225784	0.223819	0.24647	0.245632
$\hat{m}_{f4}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225784	0.24647	0.244447
$\hat{m}_{f5}$	0.156331	0.154965	0.201713	0.201101	0.225784	0.224261	0.24647	0.245222
$\hat{m}_{f6}$	0.156331	0.155648	0.201713	0.201406	0.225784	0.225022	0.24647	0.245961
$\hat{m}_{f7}$	0.156331	0.154461	0.201713	0.200868	0.225784	0.22372	0.24647	0.245933
$\hat{m}_{f8}$	0.156331	0.153561	0.201713	0.2005667	0.225784	0.223719	0.24647	0.245932
$\hat{m}_{f9}$	0.156331	0.156332	0.201713	0.201713	0.225784	0.225784	0.24647	0.2430647
$\hat{m}_{f10}$	0.156331	0.144561	0.201713	0.200567	0.225784	0.213719	0.24647	0.235932

Table 3: Estimated MSEs of m when n = 50 and $\sigma = 0.5$

φ	0.75		0.85		0.95		0.99	
	OLS	ORR	OLS	ORR	OLS	ORR	OLS	ORR
$\hat{m}_1$	0.188861	0.190983	0.257119	0.25604	0.268764	0.266208	0.296339	0.274538
$\hat{m}_2$	0.188861	0.190247	0.257119	0.256064	0.268764	0.266264	0.296339	0.276988
$\hat{m}_3$	0.188861	0.191642	0.257119	0.255419	0.268764	0.264414	0.296339	0.271673
$\hat{m}_4$	0.188861	0.239232	0.257119	0.276868	0.268764	0.27999	0.296339	0.285548
$\hat{m}_5$	0.188861	0.189828	0.257119	0.256437	0.268764	0.267197	0.296339	0.281894
$\hat{m}_6$	0.188861	0.189557	0.257119	0.256499	0.268764	0.267304	0.296339	0.282106
$\hat{m}_7$	0.188861	0.52686	0.257119	0.565939	0.268764	0.566398	0.296339	0.583141
$\hat{m}_8$	0.188861	0.295138	0.257119	0.291576	0.268764	0.277771	0.296339	0.293227
$\hat{m}_9$	0.188861	0.267963	0.257119	0.25708	0.268764	0.260605	0.296339	0.277149
$\hat{m}_{10}$	0.188861	0.465265	0.257119	0.26916	0.268764	0.261651	0.296339	0.304423
$\hat{m}_{11}$	0.188861	0.479526	0.257119	0.265346	0.268764	0.263284	0.296339	0.32313
$\hat{m}_{12}$	0.188861	0.189545	0.257119	0.256505	0.268764	0.267314	0.296339	0.282259
$\hat{m}_{13}$	0.188861	0.331746	0.257119	0.358511	0.268764	0.359783	0.296339	0.37098
$\hat{m}_{14}$	0.188861	0.191858	0.257119	0.255423	0.268764	0.261152	0.296339	0.27268

$\hat{m}_{15}$	0.188861	0.229263	0.257119	0.278434	0.268764	0.282684	0.296339	0.292669
$\hat{m}_{16}$	0.188861	0.243941	0.257119	0.278525	0.268764	0.280327	0.296339	0.293478
$\hat{m}_{17}$	0.188861	0.720107	0.257119	0.722016	0.268764	0.786384	0.296339	0.938624
$\hat{m}_{18}$	0.188861	0.611027	0.257119	0.651126	0.268764	0.708193	0.296339	0.888694
$\hat{m}_{19}$	0.188861	0.631602	0.257119	0.693356	0.268764	0.739714	0.296339	0.890664
$\hat{m}_{20}$	0.188861	0.245993	0.257119	0.280098	0.268764	0.281467	0.296339	0.295182
$\hat{m}_{21}$	0.188861	0.244208	0.257119	0.278588	0.268764	0.280359	0.296339	0.293493
$\hat{m}_{22}$	0.188861	0.823689	0.257119	0.806608	0.268764	0.821587	0.296339	0.787857
$\hat{m}_{23}$	0.188861	0.466608	0.257119	0.306471	0.268764	0.308641	0.296339	0.388753
$\hat{m}_{24}$	0.188861	0.331901	0.257119	0.395659	0.268764	0.406314	0.296339	0.418933
$\hat{m}_{25}$	0.188861	0.33879	0.257119	0.309506	0.268764	0.302042	0.296339	0.313861
$\hat{m}_{26}$	0.188861	0.312079	0.257119	0.473546	0.268764	0.540102	0.296339	0.524037
$\hat{m}_{27}$	0.188861	0.382249	0.257119	0.298097	0.268764	0.283038	0.296339	0.322205
$\hat{m}_{28}$	0.188861	0.194337	0.257119	0.258457	0.268764	0.263562	0.296339	0.271072
$\hat{m}_{29}$	0.188861	0.191432	0.257119	0.255482	0.268764	0.264624	0.296339	0.27206
$\hat{m}_{30}$	0.188861	0.548579	0.257119	0.578576	0.268764	0.578321	0.296339	0.594005
$\hat{m}_{31}$	0.188861	0.981973	0.257119	0.886852	0.268764	0.904358	0.296339	0.990127
$\hat{m}_{32}$	0.188861	0.954964	0.257119	0.842107	0.268764	0.784787	0.296339	0.977603
$\hat{m}_{33}$	0.188861	0.191651	0.257119	0.256119	0.268764	0.266499	0.296339	0.28792
$\hat{m}_{34}$	0.188861	0.825052	0.257119	0.807815	0.268764	0.822447	0.296339	0.789272
$\hat{m}_{35}$	0.188861	0.261815	0.257119	0.289249	0.268764	0.292211	0.296339	0.321322
$\hat{m}_{36}$	0.188861	0.373518	0.257119	0.405557	0.268764	0.413166	0.296339	0.427372
$\hat{m}_{37}$	0.188861	0.301711	0.257119	0.325714	0.268764	0.323071	0.296339	0.333957
$\hat{m}_{38}$	0.188861	0.464208	0.257119	0.518312	0.268764	0.564399	0.296339	0.558008
$\hat{m}_{39}$	0.188861	0.211216	0.257119	0.265712	0.268764	0.287078	0.296339	0.278918
$\hat{m}_{40}$	0.188861	0.189768	0.257119	0.256282	0.268764	0.267829	0.296339	0.274758
$\hat{m}_{41}$	0.188861	0.292699	0.257119	0.317401	0.268764	0.319558	0.296339	0.343543
$\hat{m}_{42}$	0.188861	0.352964	0.257119	0.381466	0.268764	0.385228	0.296339	0.397942
$\hat{m}_{43}$	0.188861	0.270488	0.257119	0.302444	0.268764	0.30041	0.296339	0.319925
$\hat{m}_{44}$	0.188861	0.392343	0.257119	0.428092	0.268764	0.44683	0.296339	0.44765
$\hat{m}_{45}$	0.188861	0.190194	0.257119	0.256412	0.268764	0.267222	0.296339	0.282913
$\hat{m}_{46}$	0.188861	0.189057	0.257119	0.25705	0.268764	0.268621	0.296339	0.294852
$\hat{m}_{47}$	0.188861	0.189067	0.257119	0.256989	0.268764	0.26847	0.296339	0.292973
$\hat{m}_{48}$	0.188861	0.192289	0.257119	0.25512	0.268764	0.262987	0.296339	0.269718
$\hat{m}_{49}$	0.188861	0.189958	0.257119	0.256139	0.268764	0.265964	0.296339	0.270983
$\hat{m}_{50}$	0.188861	0.194811	0.257119	0.255007	0.268764	0.261657	0.296339	0.269726
$\hat{m}_{f1}$	0.188861	0.067727	0.257119	0.190354	0.268764	0.222525	0.296339	0.229775
$\hat{m}_{f2}$	0.188861	0.065394	0.257119	0.187334	0.268764	0.218436	0.296339	0.225733
$\hat{m}_{f3}$	0.188861	0.065394	0.257119	0.187238	0.268764	0.219436	0.296339	0.227531
$\hat{m}_{f4}$	0.188861	0.139448	0.257119	0.256843	0.268764	0.258164	0.296339	0.240289
$\hat{m}_{f5}$	0.188861	0.131979	0.257119	0.217055	0.268764	0.252331	0.296339	0.238389
$\hat{m}_{f6}$	0.188861	0.101285	0.257119	0.127374	0.268764	0.232442	0.296339	0.252984
$\hat{m}_{f7}$	0.188861	0.066555	0.257119	0.189007	0.268764	0.220989	0.296339	0.228754
$\hat{m}_{f8}$	0.188861	0.065394	0.257119	0.187738	0.268764	0.219536	0.296339	0.227733
$\hat{m}_{f9}$	0.188861	0.169773	0.257119	0.236698	0.268764	0.267841	0.296339	0.247597
$\hat{m}_{f10}$	0.188861	0.065394	0.257119	0.125738	0.268764	0.216536	0.296339	0.227733

Table 4: Estimated MSEs of m when n = 100 and $\sigma = 1$

φ	0.75		0.85		0.95		0.99	
	OLS	ORR	OLS	ORR	OLS	ORR	OLS	ORR
$\hat{m}_1$	0.246976	0.251611	0.298358	0.302419	0.340027	0.332903	0.346606	0.320681
$\hat{m}_2$	0.246976	0.250066	0.298358	0.301013	0.340027	0.331466	0.346606	0.319678
$\hat{m}_3$	0.246976	0.253195	0.298358	0.304156	0.340027	0.326164	0.346606	0.317121
$\hat{m}_4$	0.246976	0.285948	0.298358	0.324339	0.340027	0.327754	0.346606	0.320525
$\hat{m}_5$	0.246976	0.249109	0.298358	0.300187	0.340027	0.333826	0.346606	0.323828
$\hat{m}_6$	0.246976	0.24853	0.298358	0.299683	0.340027	0.334348	0.346606	0.323976
$\hat{m}_7$	0.246976	0.613883	0.298358	0.665008	0.340027	0.63777	0.346606	0.648463
$\hat{m}_8$	0.246976	0.399191	0.298358	0.421239	0.340027	0.33901	0.346606	0.354161
$\hat{m}_9$	0.246976	0.48947	0.298358	0.358503	0.340027	0.319936	0.346606	0.322838
$\hat{m}_{10}$	0.246976	0.687441	0.298358	0.415068	0.340027	0.321363	0.346606	0.32752
$\hat{m}_{11}$	0.246976	0.934089	0.298358	0.531291	0.340027	0.323735	0.346606	0.337545
$\hat{m}_{12}$	0.246976	0.248408	0.298358	0.299532	0.340027	0.334607	0.346606	0.324644
$\hat{m}_{13}$	0.246976	0.378597	0.298358	0.409302	0.340027	0.401019	0.346606	0.410083
$\hat{m}_{14}$	0.246976	0.248011	0.298358	0.300559	0.340027	0.323034	0.346606	0.33196
$\hat{m}_{15}$	0.246976	0.283351	0.298358	0.325837	0.340027	0.329991	0.346606	0.337155

$\hat{m}_{16}$	0.246976	0.301486	0.298358	0.341904	0.340027	0.33247	0.346606	0.340532
$\hat{m}_{17}$	0.246976	0.981482	0.298358	0.796375	0.340027	0.796095	0.346606	0.949314
$\hat{m}_{18}$	0.246976	0.937187	0.298358	0.660913	0.340027	0.703581	0.346606	0.922423
$\hat{m}_{19}$	0.246976	0.760218	0.298358	0.630166	0.340027	0.719841	0.346606	0.934154
$\hat{m}_{20}$	0.246976	0.305977	0.298358	0.347976	0.340027	0.334913	0.346606	0.343645
$\hat{m}_{21}$	0.246976	0.302048	0.298358	0.342639	0.340027	0.332567	0.346606	0.34055
$\hat{m}_{22}$	0.246976	0.782598	0.298358	0.758572	0.340027	0.763611	0.346606	0.752574
$\hat{m}_{23}$	0.246976	0.816136	0.298358	0.562957	0.340027	0.373122	0.346606	0.397468
$\hat{m}_{24}$	0.246976	0.369875	0.298358	0.402456	0.340027	0.426395	0.346606	0.445254
$\hat{m}_{25}$	0.246976	0.431503	0.298358	0.473182	0.340027	0.377233	0.346606	0.373942
$\hat{m}_{26}$	0.246976	0.345377	0.298358	0.457434	0.340027	0.557488	0.346606	0.541887
$\hat{m}_{27}$	0.246976	0.498956	0.298358	0.411315	0.340027	0.341625	0.346606	0.353882
$\hat{m}_{28}$	0.246976	0.24795	0.298358	0.300813	0.340027	0.321933	0.346606	0.322676
$\hat{m}_{29}$	0.246976	0.252804	0.298358	0.303785	0.340027	0.326701	0.346606	0.317161
$\hat{m}_{30}$	0.246976	0.629784	0.298358	0.675449	0.340027	0.645181	0.346606	0.656794
$\hat{m}_{31}$	0.246976	0.999718	0.298358	0.994914	0.340027	0.974518	0.346606	0.994734
$\hat{m}_{32}$	0.246976	0.99178	0.298358	0.958282	0.340027	0.900948	0.346606	0.985311
$\hat{m}_{33}$	0.246976	0.248928	0.298358	0.29946	0.340027	0.336834	0.346606	0.341178
$\hat{m}_{34}$	0.246976	0.789145	0.298358	0.768901	0.340027	0.769255	0.346606	0.758269
$\hat{m}_{35}$	0.246976	0.299844	0.298358	0.340637	0.340027	0.337114	0.346606	0.350772
$\hat{m}_{36}$	0.246976	0.438763	0.298358	0.470107	0.340027	0.451027	0.346606	0.46069
$\hat{m}_{37}$	0.246976	0.348766	0.298358	0.386337	0.340027	0.375024	0.346606	0.383443
$\hat{m}_{38}$	0.246976	0.564326	0.298358	0.600874	0.340027	0.609904	0.346606	0.59429
$\hat{m}_{39}$	0.246976	0.277667	0.298358	0.333768	0.340027	0.319898	0.346606	0.323752
$\hat{m}_{40}$	0.246976	0.248997	0.298358	0.299783	0.340027	0.325413	0.346606	0.317119
$\hat{m}_{41}$	0.246976	0.334858	0.298358	0.372941	0.340027	0.363653	0.346606	0.376801
$\hat{m}_{42}$	0.246976	0.412188	0.298358	0.445381	0.340027	0.428072	0.346606	0.437115
$\hat{m}_{43}$	0.246976	0.322357	0.298358	0.36197	0.340027	0.351387	0.346606	0.364198
$\hat{m}_{44}$	0.246976	0.466189	0.298358	0.501572	0.340027	0.493233	0.346606	0.491659
$\hat{m}_{45}$	0.246976	0.249873	0.298358	0.300721	0.340027	0.334369	0.346606	0.325397
$\hat{m}_{46}$	0.246976	0.247241	0.298358	0.298514	0.340027	0.339536	0.346606	0.344363
$\hat{m}_{47}$	0.246976	0.247264	0.298358	0.298537	0.340027	0.339076	0.346606	0.34152
$\hat{m}_{48}$	0.246976	0.254612	0.298358	0.305619	0.340027	0.322904	0.346606	0.319438
$\hat{m}_{49}$	0.246976	0.248496	0.298358	0.299326	0.340027	0.331889	0.346606	0.317234
$\hat{m}_{50}$	0.246976	0.254868	0.298358	0.304003	0.340027	0.323566	0.346606	0.317264
$\hat{m}_{f1}$	0.246976	0.044296	0.298358	0.109287	0.340027	0.274685	0.346606	0.262701
$\hat{m}_{f2}$	0.246976	0.039031	0.298358	0.098657	0.340027	0.263754	0.346606	0.258535
$\hat{m}_{f3}$	0.246976	0.039031	0.298358	0.098657	0.340027	0.263754	0.346606	0.258535
$\hat{m}_{f4}$	0.246976	0.24781	0.298358	0.258878	0.340027	0.248022	0.346606	0.237705
$\hat{m}_{f5}$	0.246976	0.094943	0.298358	0.163916	0.340027	0.252703	0.346606	0.27432
$\hat{m}_{f6}$	0.246976	0.227288	0.298358	0.149285	0.340027	0.238757	0.346606	0.292609
$\hat{m}_{f7}$	0.246976	0.041643	0.298358	0.103947	0.340027	0.268862	0.346606	0.260622
$\hat{m}_{f8}$	0.246976	0.039031	0.298358	0.098657	0.340027	0.263754	0.346606	0.258535
$\hat{m}_{f9}$	0.246976	0.22827	0.298358	0.269177	0.340027	0.236975	0.346606	0.234104
$\hat{m}_{f10}$	0.246976	0.039031	0.298358	0.098657	0.340027	0.213754	0.346606	0.218535

Table 5: Estimated MSEs of m when n = 100 and $\sigma = 5$

φ	0.75		0.85		0.95		0.99	
	OLS	ORR	OLS	ORR	OLS	ORR	OLS	ORR
$\hat{m}_1$	0.947036	0.921495	0.732089	0.711801	0.741935	0.691785	1.010597	0.73859
$\hat{m}_2$	0.947036	0.917251	0.732089	0.71185	0.741935	0.690315	1.010597	0.743166
$\hat{m}_3$	0.947036	0.915852	0.732089	0.719715	0.741935	0.700092	1.010597	0.728555
$\hat{m}_4$	0.947036	0.938798	0.732089	0.719962	0.741935	1.563713	1.010597	0.721173
$\hat{m}_5$	0.947036	0.922078	0.732089	0.715502	0.741935	0.69584	1.010597	0.734771
$\hat{m}_6$	0.947036	0.917784	0.732089	0.712029	0.741935	0.694551	1.010597	0.733148
$\hat{m}_7$	0.947036	0.984955	0.732089	0.958458	0.741935	0.948008	1.010597	0.860763
$\hat{m}_8$	0.947036	0.931621	0.732089	0.770549	0.741935	0.73647	1.010597	0.729935
$\hat{m}_9$	0.947036	0.928297	0.732089	0.730122	0.741935	0.761379	1.010597	0.727688
$\hat{m}_{10}$	0.947036	0.935752	0.732089	0.737412	0.741935	0.698556	1.010597	0.730055
$\hat{m}_{11}$	0.947036	0.945158	0.732089	0.746038	0.741935	0.980642	1.010597	0.731631
$\hat{m}_{12}$	0.947036	0.93372	0.732089	0.716306	0.741935	0.690142	1.010597	0.768926
$\hat{m}_{13}$	0.947036	0.931767	0.732089	0.714068	0.741935	0.699477	1.010597	0.731506
$\hat{m}_{14}$	0.947036	0.941159	0.732089	0.724488	0.741935	0.725748	1.010597	0.829743
$\hat{m}_{15}$	0.947036	0.939545	0.732089	0.71515	0.741935	0.689811	1.010597	0.733661

$\hat{m}_{16}$	0.947036	0.915539	0.732089	0.726243	0.741935	0.706368	1.010597	0.731758
$\hat{m}_{17}$	0.947036	0.975588	0.732089	0.862563	0.741935	0.995127	1.010597	0.976708
$\hat{m}_{18}$	0.947036	0.958256	0.732089	0.840325	0.741935	0.983413	1.010597	0.964661
$\hat{m}_{19}$	0.947036	0.947482	0.732089	0.858314	0.741935	0.93852	1.010597	0.969832
$\hat{m}_{20}$	0.947036	0.920252	0.732089	0.738123	0.741935	0.71575	1.010597	0.735335
$\hat{m}_{21}$	0.947036	0.927519	0.732089	0.733079	0.741935	0.711558	1.010597	0.7478
$\hat{m}_{22}$	0.947036	0.938357	0.732089	0.803448	0.741935	0.786557	1.010597	0.864464
$\hat{m}_{23}$	0.947036	0.942958	0.732089	0.770606	0.741935	0.933154	1.010597	0.75618
$\hat{m}_{24}$	0.947036	0.918026	0.732089	0.762745	0.741935	0.74622	1.010597	0.783121
$\hat{m}_{25}$	0.947036	0.928047	0.732089	0.734797	0.741935	0.71927	1.010597	0.741867
$\hat{m}_{26}$	0.947036	0.91698	0.732089	0.771499	0.741935	0.769124	1.010597	0.807664
$\hat{m}_{27}$	0.947036	0.926825	0.732089	0.741848	0.741935	0.713029	1.010597	0.74179
$\hat{m}_{28}$	0.947036	0.94302	0.732089	0.723235	0.741935	0.732907	1.010597	0.821418
$\hat{m}_{29}$	0.947036	0.915785	0.732089	0.719258	0.741935	0.699568	1.010597	0.729366
$\hat{m}_{30}$	0.947036	0.98976	0.732089	0.960673	0.741935	0.959389	1.010597	0.968833
$\hat{m}_{31}$	0.947036	0.999395	0.732089	0.990818	0.741935	0.999982	1.010597	0.997746
$\hat{m}_{32}$	0.947036	0.993525	0.732089	0.983307	0.741935	0.989692	1.010597	0.988843
$\hat{m}_{33}$	0.947036	0.941121	0.732089	0.726217	0.741935	0.72872	1.010597	0.946773
$\hat{m}_{34}$	0.947036	0.951834	0.732089	0.843329	0.741935	0.81947	1.010597	0.873116
$\hat{m}_{35}$	0.947036	0.916735	0.732089	0.71583	0.741935	0.701868	1.010597	0.734509
$\hat{m}_{36}$	0.947036	0.927525	0.732089	0.776999	0.741935	0.758494	1.010597	0.788777
$\hat{m}_{37}$	0.947036	0.916477	0.732089	0.733363	0.741935	0.718647	1.010597	0.748936
$\hat{m}_{38}$	0.947036	0.946399	0.732089	0.834134	0.741935	0.812173	1.010597	0.837797
$\hat{m}_{39}$	0.947036	0.915367	0.732089	0.726413	0.741935	0.75577	1.010597	0.744943
$\hat{m}_{40}$	0.947036	0.940867	0.732089	0.722272	0.741935	0.699268	1.010597	0.727353
$\hat{m}_{41}$	0.947036	0.915621	0.732089	0.728688	0.741935	0.715796	1.010597	0.746845
$\hat{m}_{42}$	0.947036	0.923737	0.732089	0.763861	0.741935	0.746679	1.010597	0.77667
$\hat{m}_{43}$	0.947036	0.915527	0.732089	0.72724	0.741935	0.712918	1.010597	0.743368
$\hat{m}_{44}$	0.947036	0.931652	0.732089	0.788981	0.741935	0.769775	1.010597	0.797539
$\hat{m}_{45}$	0.947036	0.916658	0.732089	0.711702	0.741935	0.689417	1.010597	0.776369
$\hat{m}_{46}$	0.947036	0.946505	0.732089	0.731467	0.741935	0.739407	1.010597	0.947171
$\hat{m}_{47}$	0.947036	0.946562	0.732089	0.731328	0.741935	0.736576	1.010597	0.888009
$\hat{m}_{48}$	0.947036	0.916909	0.732089	0.725807	0.741935	0.713758	1.010597	0.728473
$\hat{m}_{49}$	0.947036	0.944886	0.732089	0.727943	0.741935	0.703029	1.010597	0.727247
$\hat{m}_{50}$	0.947036	0.929567	0.732089	0.714589	0.741935	0.68961	1.010597	0.730589
$\hat{m}_{f1}$	0.947036	0.825369	0.732089	0.5599	0.741935	0.614568	1.010597	0.715804
$\hat{m}_{f2}$	0.947036	0.706317	0.732089	0.472457	0.741935	0.575989	1.010597	0.700728
$\hat{m}_{f3}$	0.947036	0.706317	0.732089	0.472457	0.741935	0.575989	1.010597	0.700728
$\hat{m}_{f4}$	0.947036	0.945498	0.732089	0.729954	0.741935	0.731334	1.010597	0.83445
$\hat{m}_{f5}$	0.947036	0.831026	0.732089	0.574765	0.741935	0.617159	1.010597	0.724339
$\hat{m}_{f6}$	0.947036	2.036744	0.732089	1.708542	0.741935	0.650267	1.010597	0.871941
$\hat{m}_{f7}$	0.947036	0.773405	0.732089	0.518447	0.741935	0.596105	1.010597	0.706566
$\hat{m}_{f8}$	0.947036	0.706317	0.732089	0.472457	0.741935	0.575989	1.010597	0.700728
$\hat{m}_{f9}$	0.947036	0.94467	0.732089	0.728845	0.741935	0.726485	1.010597	0.795981
$\hat{m}_{f1(}$	0.947036	0.706317	0.732089	0.472457	0.741935	0.575989	1.010597	0.700728

Table 6: Estimated MSEs of m when n = 150 and $\sigma = 0.01$

φ	0.75		0.85		0.95		0.99	
	OLS	ORR	OLS	ORR	OLS	ORR	OLS	ORR
$\hat{m}_1$	0.163065	0.163066	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_2$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_3$	0.163065	0.163066	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_4$	0.163065	0.221515	0.187262	0.236112	0.234814	0.268813	0.246175	0.277365
$\hat{m}_5$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_6$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_7$	0.163065	0.492519	0.187262	0.513024	0.234814	0.550879	0.246175	0.559618
$\hat{m}_8$	0.163065	0.164324	0.187262	0.188486	0.234814	0.235795	0.246175	0.247598
$\hat{m}_9$	0.163065	0.163257	0.187262	0.187349	0.234814	0.234892	0.246175	0.246603
$\hat{m}_{10}$	0.163065	0.163665	0.187262	0.187595	0.234814	0.235326	0.246175	0.24957
$\hat{m}_{11}$	0.163065	0.177626	0.187262	0.188746	0.234814	0.235622	0.246175	0.257135
$\hat{m}_{12}$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{13}$	0.163065	0.313697	0.187262	0.325765	0.234814	0.351284	0.246175	0.357798
$\hat{m}_{14}$	0.163065	0.284414	0.187262	0.302052	0.234814	0.28693	0.246175	0.248488
$\hat{m}_{15}$	0.163065	0.265531	0.187262	0.287419	0.234814	0.294966	0.246175	0.278062
$\hat{m}_{16}$	0.163065	0.221489	0.187262	0.236084	0.234814	0.268795	0.246175	0.277365

$\hat{m}_{17}$	0.163065	0.518835	0.187262	0.576388	0.234814	0.790018	0.246175	0.944388
$\hat{m}_{18}$	0.163065	0.437333	0.187262	0.483623	0.234814	0.730761	0.246175	0.909043
$\hat{m}_{19}$	0.163065	0.469014	0.187262	0.512266	0.234814	0.772076	0.246175	0.91896
$\hat{m}_{20}$	0.163065	0.22149	0.187262	0.236085	0.234814	0.268795	0.246175	0.277366
$\hat{m}_{21}$	0.163065	0.221489	0.187262	0.236084	0.234814	0.268795	0.246175	0.277365
$\hat{m}_{22}$	0.163065	0.997351	0.187262	0.99713	0.234814	0.996688	0.246175	0.99683
$\hat{m}_{23}$	0.163065	0.255803	0.187262	0.215517	0.234814	0.251741	0.246175	0.310553
$\hat{m}_{24}$	0.163065	0.470681	0.187262	0.429851	0.234814	0.448229	0.246175	0.428142
$\hat{m}_{25}$	0.163065	0.189307	0.187262	0.225053	0.234814	0.26375	0.246175	0.283733
$\hat{m}_{26}$	0.163065	0.859711	0.187262	0.8654	0.234814	0.833288	0.246175	0.645342
$\hat{m}_{27}$	0.163065	0.172574	0.187262	0.194413	0.234814	0.242621	0.246175	0.2665
$\hat{m}_{28}$	0.163065	0.222549	0.187262	0.270637	0.234814	0.278836	0.246175	0.249806
$\hat{m}_{29}$	0.163065	0.163066	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{30}$	0.163065	0.51772	0.187262	0.53324	0.234814	0.564011	0.246175	0.571447
$\hat{m}_{31}$	0.163065	0.91322	0.187262	0.65952	0.234814	0.733376	0.246175	0.991134
$\hat{m}_{32}$	0.163065	0.349918	0.187262	0.353367	0.234814	0.503347	0.246175	0.939064
$\hat{m}_{33}$	0.163065	0.163135	0.187262	0.187311	0.234814	0.234842	0.246175	0.246187
$\hat{m}_{34}$	0.163065	0.997351	0.187262	0.99713	0.234814	0.996688	0.246175	0.99683
$\hat{m}_{35}$	0.163065	0.204245	0.187262	0.211161	0.234814	0.250475	0.246175	0.280273
$\hat{m}_{36}$	0.163065	0.482212	0.187262	0.43764	0.234814	0.451684	0.246175	0.435203
$\hat{m}_{37}$	0.163065	0.224316	0.187262	0.25802	0.234814	0.292778	0.246175	0.308831
$\hat{m}_{38}$	0.163065	0.861843	0.187262	0.866796	0.234814	0.835377	0.246175	0.666941
$\hat{m}_{39}$	0.163065	0.163068	0.187262	0.187264	0.234814	0.234818	0.246175	0.246177
$\hat{m}_{40}$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{41}$	0.163065	0.243358	0.187262	0.24635	0.234814	0.278803	0.246175	0.308766
$\hat{m}_{42}$	0.163065	0.388089	0.187262	0.37592	0.234814	0.395749	0.246175	0.392607
$\hat{m}_{43}$	0.163065	0.190557	0.187262	0.210433	0.234814	0.254787	0.246175	0.272831
$\hat{m}_{44}$	0.163065	0.592623	0.187262	0.60073	0.234814	0.575931	0.246175	0.486568
$\hat{m}_{45}$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{46}$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234814	0.246175	0.246175
$\hat{m}_{47}$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{48}$	0.163065	0.163066	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{49}$	0.163065	0.163065	0.187262	0.187262	0.234814	0.234815	0.246175	0.246175
$\hat{m}_{50}$	0.163065	0.163146	0.187262	0.187334	0.234814	0.234872	0.246175	0.246225
$\hat{m}_{f1}$	0.163065	0.151806	0.187262	0.186036	0.234814	0.223841	0.246175	0.243846
$\hat{m}_{f2}$	0.163065	0.141806	0.187262	0.186036	0.234814	0.213854	0.246175	0.242846
$\hat{m}_{f3}$	0.163065	0.146806	0.187262	0.186026	0.234814	0.223584	0.246175	0.242846
$\hat{m}_{f4}$	0.163065	0.123065	0.187262	0.187262	0.234814	0.218015	0.246175	0.246175
$\hat{m}_{f5}$	0.163065	0.152141	0.187262	0.186361	0.234814	0.212097	0.246175	0.245146
$\hat{m}_{f6}$	0.163065	0.152603	0.187262	0.186811	0.234814	0.234455	0.246175	0.235652
$\hat{m}_{f7}$	0.163065	0.161806	0.187262	0.186266	0.234814	0.233842	0.246175	0.234846
$\hat{m}_{f8}$	0.163065	0.131806	0.187262	0.186136	0.234814	0.213384	0.246175	0.224846
$\hat{m}_{f9}$	0.163065	0.151065	0.187262	0.187262	0.234814	0.234815	0.246175	0.236175
$\hat{m}_{f10}$	0.163065	0.112806	0.187262	0.176036	0.234814	0.213384	0.246175	0.144846

Table 7: List of Abbreviations.

EMSE	**Estimated Mean Squared Error**
MSE	**Scalar Mean Squared Error**
OLS	**Ordinary Least Square Estimator**
ORR	**Ordinary Ridge Regression**
m	**Shrinkage Estimator**
LSE	**Least Square Estimator**

Conclusions

This paper examined the properties of ridge regression estimators under varying sample sizes (n), error term variance (σ^2), and levels of predictor correlation (φ). The analysis is conducted through a simulation study, in which the proposed estimators are evaluated and compared with existing ridge estimators. Each simulation scenario is repeated 10,000 times, and the MSEs are computed to assess estimator performance. The results indicate that the proposed estimators perform well across a wide range of multicollinearity levels among predictors. Based on the simulation results and the numerical example, the proposed estimators ($\hat{m}_{f1}$,$\hat{m}_{f2}$,$\hat{m}_{f3}$,$\hat{m}_{f4}$,$\hat{m}_{f5}$,$\hat{m}_{f6}$,$\hat{m}_{f7}$,$\hat{m}_{f8}$,$\hat{m}_{f9}$ and $\hat{m}_{f10}$) performed better than the rest in terms of small MSE and may thus be recommended to practitioners.

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We are dedicating this article to those who lost their lives in Gaza and Palestine.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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أداء بعض نماذج انحدار معلمات الحرف المقدرة الجديدة

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الخلاصة:

في ظل وجود علاقة خطية متعددة بين المتغيرات المستقلة في نموذج الانحدار الخطي والتي تعرف بمشكلة التداخل الخطي تؤدي إلى ان مقدرات المربعات الصغرى الاعتيادية تنتج تباينات كبيرة في العينة. وللتغلب على هذه المشكلة تمت التوصية بالعديد من المقدرات، من احدى المقدرات التي تم اقتراحها للتقليل من تأثير مشكلة التداخل الخطي هو مقدر انحدار الحرف الذي يؤدي الى معاملات تقديرية متحيزة ولكنها تمتلك تباين اصغر من مقدرات المربعات الصغرى الاعتيادية مما قد يكون لها متوسط مربعات خطأ اصغر (MSE). في هذا البحث ، نستعرض 50 نوعا من معلمات الحرف (m) التي تمت ادراجها لتقدير مقدر انحدار الحرف العادي (ORR) بالإضافة إلى ذلك ، تم اقتراح 10 مقدرات جديدة للمعلمة m بناء على مناهج مختلفة. باستخدام اسلوب المحاكاة ، قمنا بمقارنة هذه المقدرات ل m باستخدام تقدير متوسط الخطأ التربيعي (EMSE) لدراسة أدائها. تظهر نتائج المحاكاة أنه يمكن استخدام المقدرات المقترحة الجديدة ل m بدلا من تدقيق الآخرين