Mustansiriyah Journal of Pure and Applied Sciences, Print ISSN: 2957-9910. Online ISSN: 2957-9929, Vol. 3, No. 3, 2025, Pages 1-9
DOI: 10.47831/mjpas.v3i3.253

MJPAS

MUSTANSIRIYAH JOURNAL OF PURE AND APPLIED SCIENCES

Journal homepage:
https://mjpas.uomustansiriyah.edu.iq/index.php/mjpas

RESEARCH ARTICLE - MATHEMATICS

Dominance Rules for Solving Scheduling on Uniform Parallel Machines with Multi Objective Function

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Article Info.

Article history:

Received
26 June 2024

Accepted
5 August 2024

Publishing
30 June 2025

Abstract

The problem of uniform Parallel Machine Scheduling (UPMS) in manufacturing system is considered with utilizing the aims of minimizing total completion time, total tardiness and total earliness. Multi objective functions have been developed to select optimal solution in uniform parallel machine scheduling. In the present study, our focus is on the NP-hard problem for scheduling n jobs on a uniform parallel machine. We develop the dominance properties are incorporated, is proposed and tested on a large set of randomly generated instances.

The official journal published by the College of Education at Mustansiriya University

Keywords: Uniform parallel machine, total completion time, total tardiness, total earliness, Dominance properties.

1. Introduction

The parallel machine scheduling problem (PMSP) Where it can be defined as the process of structure or decision-making used in commonly, encountered in the machinery, electronics, textiles, transportation, telecommunications, pharmaceuticals, chemicals and service industries[1]. Since the first relevant discuss by McNaughton [2], various PMSPs have attracted extend look after among researchers. A classical parallel machine system (PMS) can be classified as identical, uniform, unrelated basically depends on the features of parallel machines[3]. Jobs in this identical PMS might occur handle on machine with the same speed ingredient denoted as P_m while jobs in an uniform PMS, might occur handle on machine with different speed ingredient denoted Q_m, finely the last type of machine is an unrelated PMS denoted R_m, each job some can be handled on privately but not all machines with diverse speed ingredient, and this machine have not the same rate with each one of other machine.

Owing of its significance in factual production framework, the uniform parallel machine scheduling problem (UPMSP) has been extensively investigated in recent decades. With respect to the precise techniques, Bulfin and Parker [4] made an assortment of alteration to uniform and unrelated parallel machine scheduling problem (PMSP) with two processors to minimize the makespan. Liao and Lin [5] studied the objective is to minimize the makespan, two machine uniform parallel machine scheduling problem (UPMSP). They transformed the uniform parallel machine scheduling problem (UPMSP) into a unique identical PMSP and then provided a precise technique to solve it in the best possible way. A branch-and-bound technique was presented by Azizoglu and Kirca [6] the basic goal of reduction the overall weighted flow time in same PMSP. The methodology was then extended to the UPMSP. In order to reduce the maximum lateness and solve the UPMSP, Koulamas and Kyparisis [7] expanded the EDD rule, which may result in a close to ideal solution.

Dessouky et al.'s work [8] produced a number of effective algorithms for uniform parallel machine scheduling that have scheduling criteria like makespan, total completion time, maximum lateness, and total tardiness that are non-decreasing in the job completion times. A branch and bound algorithm are produced by using the Dessouky target [9] for uniform parallel machine scheduling with ready times in order to reduce the maximum lateness. The Balakrishnan et al. [10] work in seek to minimize the sum of earliness and tardiness costs, thoughtful uniform parallel machine scheduling problems with sequence dependent setup times. They introduce a melding integer construction that has substantially small 0_1 variable for slim sized problems. Two heuristic techniques for uniform parallel machine scheduling were conclude by Lee et al. [11] to determine the best way to assign the machine operators with learning effects in order to reduce the makespan. The information which Elvikis et al. [12] did look at scheduling problem for uniform parallel machine with two jobs that consist of multiple operations. From this, Pareto optimal with makespan and cost functions. By extending a minimal complete Pareto set enumeration algorithm, Elvikis and T'kindt [13] Work on that task using several objectives relating to job completion times. To reduce the makespan, applied uniform parallel computers with arbitrary job sizes Zhou et al. [14] the work by strengthen a batch processing problem. They created an authentic differential expansion based hybrid algorithm a mixed integer programming frame .A hybrid approach that combines genetic algorithms and particle swarm optimization for uniform parallel machine scheduling with batch transportation was created mission for Jiang et al.[15].In their study, Zeng et al. [16] looked at electricity prices under time dependent or time of application electricity tariffs with a view resolve a bi-objective scheduling problem for uniform parallel machines. Mallek and Boudhar [17] addressed scheduling on uniform parallel machines with conflict graphs. The authors proposed a mixed-integer line program (MILP) formulation, along with lower and upper bounds, to minimize the makespan. Song et al. [18] studied the uniform parallel machine scheduling for a green manufacturing system to minimize the makespan and maximize the maximum lateness while incorporating an upper bound constraint on the total energy cost and authors proposed a polynomial-time algorithm for a preemptive uniform parallel machine scheduling and a two-approximation algorithm for a non-preemptive uniform parallel machine scheduling. Finally, as everyone can see, artificial intelligence and artificial neural networks are the future of most businesses and fields, as they can provide great benefit in the future[19], [20] present examples of the most famous of these uses.

This paper is structured as follows: The uniform parallel machine problem is described in Section 2. The formulation of the integer programming paradigm is described in Section 3. We provide dominance properties related to the problem in Section 4. Finally, Section 5 discusses the presents our conclusions and some suggestions additional research.

2. Problem Description

3. Page style

In this section, we give some notations which use characterization to solve for $Q_m // \sum_{i=1}^{m} \sum_{j=1}^{n} (C_{ij} + T_{ij} + E_{ij})$ problem.

The notations as follows:

J	Set of jobs.
i	Set of machines.
n	Set of jobs(n = |J|).
m	Set of the machines(m = |M|).
p_{ij}	Processing time of job j on machine i.
d_j	Due date of job j.
v_i	Relative processing speed of machine i
σ_i	Partial schedule sequence of the jobs assigned to machine i.
$c_{ij}\ (\sigma)$	Completion time of job j in partial schedule σ_i, $c_{ij}= \sum_{j=1}^{n} p_{ij}\ /v_i$.
$T_{ij}\ (\sigma)$	Tardiness of job j in partial schedule σ_i, $T_{ij}=$ max $\{\ c_{ij} - d_j, 0\}$.
$E_{ij}\ (\sigma)$	Earliness of job j in partial schedule σ_i, E_{ij} = max $\{d_j - \ c_{ij}, 0\}$.

3. Integer Programming

The $Q_m\ //\sum_{i=1}^{m}\sum_{j=1}^{n}(C_{ij} + T_{ij} + E_{ij})$ the problem can be described as a hypothetical integer programming problem (P).

$$Z_p = min\sum_{j=1}^{n}\sum_{k=1}^{m}\sum_{t=1}^{n}(C_{jk}^t X_{jk}^t + T_{jk}^t X_{jk}^t + E_{jk}^t X_{jk}^t) \quad (3.1)$$

Subject to

$$\left.\begin{array}{lll} \sum_{k=1}^{m}\sum_{t=1}^{n} X_{jk}^t = 1 & j = 1,2,\dots,n\} & (3.2) \\ \sum_{j=1}^{n} X_{jk}^t \leq 1 & \left.\begin{array}{l} t = 1,2,\dots,n \\ k = 1,2,\dots,m \end{array}\right\} & (3.3) \\ C_{jk}^t = \sum_{l=1}^{n}\sum_{s=1}^{t-1} p_{lk} X_{lk}^s & \left.\begin{array}{l} j,t = 1,2,\dots,n \\ k = 1,2,\dots,m \end{array}\right\} & (3.4) \\ T_{jk}^t = \max(0, C_{jk}^t - d_j) & \left.\begin{array}{l} j,t = 1,2,\dots,n \\ k = 1,2,\dots,m \end{array}\right\} & (3.5) \\ E_{jk}^t = \max(0, d_j - C_{jk}^t) & \left.\begin{array}{l} j,t = 1,2,\dots,n \\ k = 1,2,\dots,m \end{array}\right\} & (3.6) \\ X_{jk}^t = 0 \text{ or } 1 & \left.\begin{array}{l} j,t = 1,2,\dots,n \\ k = 1,2,\dots,m \end{array}\right\} & (3.7) \end{array}\right\} (P)$$

Where C_{jk}^t confirmed the completion time of job j is scheduling on machine k at t^{th} position and

$$X_{jk}^t = \begin{cases} 1 & if\ job\ j\ is\ scheduled\ in\ t^{th} position\ on\ machine\ k \\ 0 & o.w \end{cases}$$

With regard to Constraint (3.2) state that every job is displayed to minutely one position on machine, by contrast constraint (3.3) demands allocation of a maximum of one job to each position on each machine. The completion time of job j when it is scheduled at the t^{th} position on machine k appears by constraint (3.4). Constraint (3.5), (3.6) give the tardiness and earliness of job j, when it is completion time C_{jk}^tin the th position on machine k. Finally, constraint (3.7) is a variable that defines to show job j scheduled on machine k in position t, then $X_{jk}^t = 1$ otherwise 0.

4. Dominance Rules

We introduce some dominance rules for $Q_m//\sum_{i=1}^{m}\sum_{j=1}^{n}(C_{ij}+T_{ij}+E_{ij})$ problem has been shown in theorem (4.1).

Let $S_{1k}=(\beta_1\ jk\ \beta_2)$ and $S_{2k}=(\beta_1 kj\ \beta_2)$ think about two sequences :β_1 and β_2, that disjoint subsequences of the remaining *n-2* operations ,the two jobs (*j*) and (*k*) are adjacent jobs using same machine with $p_{ij}\leq p_{ik}$.

Let Φ be completion time of β_1, Let δ_{jki} be functions value where $\delta_{ikj}=\sum_{i=1}^{m}\sum_{j=1}^{n}(C_{ij}+T_{ij}+E_{ij})$ for the two jobs, subsequences (*j*), and (*k*) when (*j*) precedes (*k*) and δ_{jki} is functions value for the subsequences of the two jobs (*j*) and (*k*) when (*k*) precedes (*j*).

Currently, while we analyze the adjustments in $\Delta_{jki}=\delta_{jki}-\delta_{ikj}$ and, with the subsequent instances $\Delta_{jki}\leq 0$ it displays that

- If $\Delta_{jki}<0$ then, at time Φ, job *j* should come before job *k*.
- If $\Delta_{jki}>0$ then, at time Φ, job *k* should come before job *j*.
- If $\Delta_{jki}=0$ then, there is no difference in scheduling (*j*) or (*k*) first.

Theorem(4.1): For the $Q_m//\sum_{i=1}^{m}\sum_{j=1}^{n}(C_{ij}+T_{ij}+E_{ij})$ problem if $p_{ji}\leq p_{ki}$ for all *i*=1,2,…,m, , and $d_j\leq d_k$ where the jobs (*j*) and (*k*) are two adjacent jobs on the same machine ,then the job (*j*) precedes the job (*k*) in at least on optimal sequences.

S_{1k}	β_1	*j*	*K*	β_2
S_{2K}	β_1	*k*	*j*	β_2

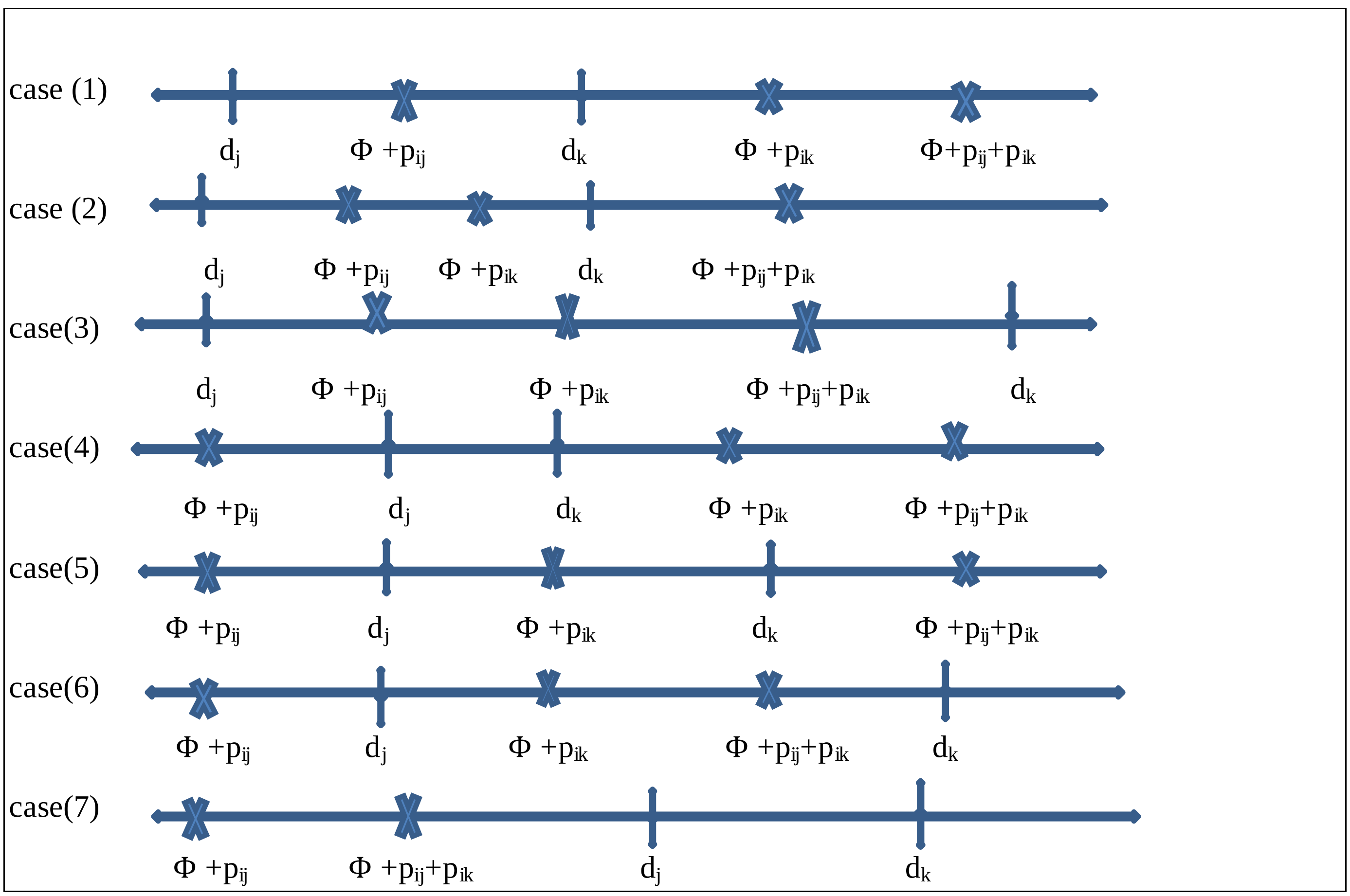

Figure (4.1): illustrates that there are seven cases.

Case 1: If $d_j \leq \Phi + p_{ij}, d_k \leq \Phi + p_{ik}$, jobs (j) and (k) remain tardy thus for any two adjacent jobs (j, k) on machine $i, i = 1{,}2$ then E_{ij}= $E_{ik} =\ 0$ (Refer to Figure 4.1, Case 1).

Proof: Δ_{jki}=$\delta_{jki} - \delta_{kji}$

$$=[(\Phi + p_{ij}) + (\Phi + p_{ij} - d_j) + 0 + (\Phi + p_{ij} + p_{ik}) + (\Phi + p_{ij} + p_{ik}) - d_k) + 0) - [(\Phi + p_{ik}) + (t + p_{ik}) - d_k) + 0 + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ij}) - d_j) + 0)]$$

$$= [4\Phi + 4p_{ij} + 2p_{ik} - d_j - d_k] - [4\Phi + 4p_{ik} + 2p_{ij} - d_j - d_k] = -2p_{ik} + 2p_{ij} \leq 0$$

Since $p_{ij} < p_{ik}$, then $j \to k$

Case 2: If $d_j \leq \Phi + p_{ij}$, $\Phi + p_{ij} \leq d_k \leq \Phi + p_{ij} + p_{ik}$, (j) be constantly tardy and (k) is tardy if not scheduled first ,then $E_{ij} = 0$, $T_{ik} = 0$ (Refer to Figure 4.1, Case 2)

Proof:

$$\Delta_{jki}(t) = [(\Phi + p_{ij}) + (\Phi + p_{ij} - d_j) + (\Phi + p_{ij} + p_{ik}) + (\Phi + p_{ij} + p_{ik}) - d_k) + o)] - [(\Phi + p_{ik}) + (d_k - \Phi - p_{ik}) + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ij} - d_j)]$$

$$= [4\Phi + 2p_{ik} + 4p_{ij} - d_j - d_k] - [2\Phi + 2p_{ij} + 2p_{ik} - d_j + d_k] = 2\Phi + 2p_{ij} - 2d_k$$

$$= 2(\Phi + p_{ij} - d_k) \leq 0 \; since \, \Phi + p_{ij} \leq d_k$$, then $j \to k$

Case 3: If $d_j \leq \Phi + p_{ij}$, $\Phi + p_{ji} + p_{ik} \leq d_k$, then(j) be constantly tardy and the (k) be constantly early. (Refer to Figure 4.1, Case 3).

Proof: Since (j) be constantly tardy and (k) be constantly early then $E_{ji} = 0 = T_{ki}$

$$\Delta_{jki}(t) = [(\Phi + p_{ij}) + (\Phi + p_{ij} - d_j) + 0 + (\Phi + p_{ij} + p_{ik}) + 0 + (d_k - \Phi - p_{ij} - p_{ik})] - [((\Phi + p_{ik}) + 0 + (d_k - \Phi - p_{ik}) + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ij} - d_j) + 0)]$$

$$= [2\Phi + 2p_{ij} - d_j + d_k] - [2\Phi + 2p_{ij} + 2p_{ik} - d_j + d_k] = -2p_{ik} \leq 0,$$

Since $p_{ik} > 0$, then $j \to k$

Case 4: If $\Phi + p_{ij} \leq d_j \leq d_k \leq \Phi + p_{ik}$, (i.e. the job (j) be constantly tardy and (k) is tardy if not scheduled firstly.(Refer to Figure 4.1, Case 4)

Proof:

$$\Delta_{jki}(t) = [(\Phi + p_{ij}) + 0 + (d_j - \Phi - p_{ij}) + (\Phi + p_{ij} + p_{ik}) + (\Phi + p_{ij} + p_{ik} - d_k) + 0] - [\Phi + p_{ik}) + 0 + (\Phi + p_{ik} - d_k) + 0 + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ij} - d_j) + 0)]$$

$$= \left[2\Phi + 2p_{ik} + 2p_{ij} + d_j - d_k\right] - \left[4\Phi + 2p_{ij} + 4p_{ik} - d_j - d_k\right]$$

$$= (-2\Phi - 2p_{ik} - 2d_j)$$

$$= -2(\Phi + p_{ik} - d_j) \leq 0 \quad since\, \Phi + p_{ik} \geq d_j \;, then \quad j \to k$$

Case 5: If $\Phi + p_{ij} \leq d_j$, $\Phi + p_{ij} \leq d_k \leq \Phi + p_{ij} + p_{ik}$, (i.e. both of the two jobs (j) and (k) are tardy if they not scheduled first).(Refer to Figure 4.1, Case 5)

Proof:

$$\Delta_{jki}(t) = \ [(\Phi + p_{ij}) \ + \ 0 + \ (d_j - \Phi - p_{ij}) + (\ \Phi + p_{ij} + p_{ik}) + (\ \Phi + p_{ij} + p_{ik} - d_k) + 0] \ - [(\ (\Phi + p_{ik}) \ + 0 + (d_k - \ \Phi - p_{ik}) \ + \ (\Phi + p_{ik} + p_{ij}) \ + \ (\Phi \ + p_{ik} + p_{ij} - d_j) + 0)]$$
$$= [2\Phi + 2p_{ij} + \ 2p_{ik} \ + d_j - d_k] - [\ 2\Phi + 2p_{ij} + 2p_{ik} - d_j + d_k] = 2d_j - 2d_k \leq \ 0$$

Since $d_{\ k} \geq d_j$, then $j \ \to k$

Case 6: If $\Phi + \ p_{ij} \leq \ d_j \ \leq \Phi + p_{ij} + p_{ik} \ \leq \ d_k$ (i.e. job (j) is tardy if not scheduled first, the job (k) be constantly early) (Refer to Figure 4.1, Case 6)

Proof:

$$\Delta_{ikj}(t) = \ [(\Phi + p_{ij}) \ + \ 0 + \ (d_j - \Phi - p_{ij}) + (\ \Phi + p_{ij} + p_{ik}) + \ 0 + (d_k - \Phi - \ p_{ij} \ - p_{ik}] - [(\ (\Phi + p_{ik}) \ + 0 + (d_k - \ \Phi - p_{ik}) \ + \ (\Phi + p_{ik} + p_{ij}) \ + \ (\Phi + p_{ik} + p_{ij} - d_j) + 0)]$$

$$= [d_j + d_k] - [\ 2\Phi + 2p_{ij} + 2p_{ik} - d_j + d_k] \ = \ -2\Phi - 2p_{ij} - 2p_{ik} + 2d_j \leq \ 0$$

Since $\Phi + p_{ij} + p_{ik} \geq d_j$, then $j \ \to k$

Case 7: If $\Phi + p_{ij} + p_{ik} \ \leq \ d_j \leq \ d_k$,(i.e. both (j) and (k) are early) (Refer to Figure 4.1, Case 7)

Proof:

$$\Delta_{jki}(t) = \ [(\Phi + p_{ij}) \ + \ 0 + \ (d_j - \Phi - p_{ij}) + (\ \Phi + p_{ij} + p_{ik}) + \ 0 + (d_k - \Phi - \ p_{ij} - p_{ik})] \ - \ [(\ (\Phi + p_{ik}) \ + 0 + (d_k - \ \Phi - p_{ik}) \ + \ (\Phi + p_{ik} + p_{ij}) \ + \ 0 + \ (d_j - \Phi - p_{ij} - p_{ik}$$
$$= [d_j + d_k] \ - \ [d_j + d_k] \ = 0$$

Theorem (4.2): For the $Q_m/d_j, k = d \ / \sum_{j=1}^{m} \sum_{i=1}^{n} (C_{ij} \ + T_{ij} + E_{ij})$ problem if $p_{ij} \leq \ p_{ik}$, assuming the jobs(j), and (k) are adjacent jobs on the same machine as i, then job (j) should precedes job (k) for at least one sequences with the optimum value (SPT rule).

Proof: Let $S_{1k} = (\beta_1 \ jk \ \beta_2)$ and $S_{2k} = (\ \beta_1 kj \ \beta_2)$ where β_1, and β_2 are disjoint subsequences. Let (Φ) be completion time of β_1, we will investigate the value of modifications Δ_{jki}=$\delta_{jki} - \delta_{kji}$

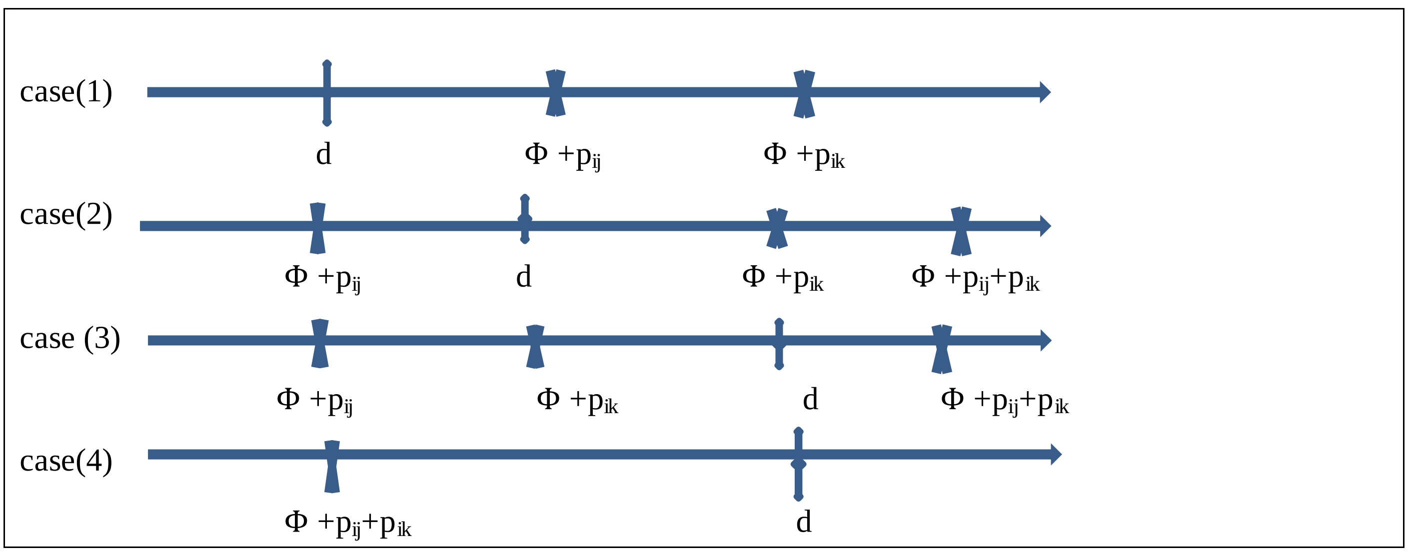

figure (4.2): illustrations for the theorem (4.2)

Case 1: If $d \leq \ \Phi + p_{ij} \leq \ \Phi + p_{ik}$ **(**i.e. both of the jobs (j), and (k) are always tardy) (Refer to Figure 4.2, Case 1)

Proof: Since the jobs (j), and (k) are both tardy then $E_{ji} = E_{ki} = 0$

$$\Delta_{jki}(t) = [(\Phi + p_{ij}) + (\Phi + p_{ij} - d) + 0 + (\Phi + p_{ij} + p_{ik}) + (\Phi + p_{ij} + p_{ik} - d) + 0] - [(\Phi + p_{ik}) + (\Phi + p_{ik} - d) + 0 + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ik} - d) + 0)]$$

$$= [4\Phi + 4p_{ij} + 2p_{ik} - 2d] - [\Phi + 4p_{ik} + 2p_{ij} - 2d)] = 2p_{ij} - 2p_{ik}$$

$$= 2(p_{ij} - p_{ik}) \le 0$$

Since $p_{ij} \le p_{ik}$, then $\Delta_{jki}(t) \le 0$, and so $j \rightarrow k$

Case 2: If $+p_{ij} \le d \le \Phi + p_{ik}$ **,** the job (j) is tardy if not scheduled first and the job (k) is tardy always) (Refer to Figure 4.2, Case 2)

Proof: As the job (k) is tardy always, then $E_{ki} = 0$, and the job (j) is tardy if not scheduled first, then

$$\Delta_{ikj}(t) = [(\Phi + p_{ij}) + 0 + (d - \Phi - p_{ij}) + (\Phi + p_{ij} + p_{ik}) + (\Phi + p_{ij} + p_{ik} - d) + 0] - [\Phi + p_{ik}) + (\Phi + p_{ik} - d) + 0 + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ij} - d) + 0]$$

$$= [2\Phi + 2p_{ij} + 2p_{ik}] - [4\Phi + 4p_{ik} + 2p_{ij} - 2d] = -2\Phi - 2p_{ik} - 2d$$

$$= -2(\Phi + p_{ik} + d) \le 0$$

As $+p_{ik} \ge d\, , then\ \ \Delta_{jki}(t) \le 0$, so $j \rightarrow k$

Case 3: If $\Phi + p_{ik} \le d \le \Phi + p_{ij} + p_{ik}$ (both jobs (j) and (k) are tardy if not scheduled firstly)(Refer to Figure 4.2, Case 3)

Proof:

$$\Delta_{jki}(t) = [(\Phi + p_{ij}) + 0 + (d - \Phi - p_{ij}) + (\Phi + p_{ij} + p_{ik}) + (\Phi + p_{ij} + p_{ik} - d) + 0] - [((\Phi + p_{ik}) + 0 + (d - \Phi - p_{ik}) + (\Phi + p_{ik} + p_{ij}) + (\Phi + p_{ik} + p_{ij} - d) + 0)]$$

$$= [2\Phi + 2p_{ij} + p_{ik}] - [2\Phi + 2p_{ik} + p_{ij}] = 0$$, then $j \rightarrow k$

Case 4: If $\Phi + p_{ji} + p_{ki} \le d$**,**(both jobs (j),and (k) are tardy) (Refer to Figure 4.2, Case4)

Proof:

$$\Delta_{jki}(t) = [(\Phi + p_{ji}) + 0 + (d - \Phi - p_{ji}) + (\Phi + p_{ji} + p_{ki}) + 0 + (d - \Phi - p_{ji} - p_{ki})] - [((\Phi + p_{ki}) + 0 + (d - \Phi - p_{ki}) + (\Phi + p_{ki} + p_{ji}) + 0 + (d - \Phi - p_{ki} - p_{ji})]$$

$$= [2d] - [2d] = 0\, , then\ \ j \rightarrow k$$

5. Conclusions

In this research, the questions of its schedule of regular and unconditional parallel machines have been addressed on business. The aim is to find the best solution to the hunger of the goal function. Because this is a difficult issue, a multi-border algorithm cannot be found and a time is right to find the best solution, and in this search a set of dominance rules has been derived, which are considered to be an important focus, which is used in optimal and unsurpassable resolution methods because they reduce the time in the search for a contract that leads to the best solution. These cases were taken with conditions for business.

The proposed future work is to use its branching and restriction method with the creation of the upper limits and the derivative of the lower boundary. And so you can take this issue with conditions on business.

Acknowledgments

The authors would like to thank the University of Mustansiriyah, College of Science, Department of Math, for helping us write this article .

Conflict of interest

The authors declare that they have no conflict of interest.

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