Tikrit Journal of Pure Science Vol. 30 (3) 2025

DOI: https://doi.org/10.25130/tjps.v30i3.1707

IRAQI Academic Scientific Journals

Tikrit Journal of Pure Science

ISSN: 1813 – 1662 (Print) --- E-ISSN: 2415 – 1726 (Online)

Journal Homepage: https://tjpsj.org/

Soft M-Algebras and Soft M-Ideals

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Received: 5 Aug. 2024 **Received in Revised Form:** 3 Nov. 2024 **Accepted :** 19 Nov. 2024

Final Proof Reading : 17 Apr. 2025 **Available Online :** 25 Jun. 2025

ABSTRACT

In this paper, I start by introducing and going over the new M-algebra classes. Additionally, I present new classes of soft algebras, which I refer to as soft M-algebras. I next introduce and explore new ideas in soft M-algebras, like soft M-subalgbera and soft M-ideals, using our new connotations. I also presented the theorem of soft M-ideals and soft M-algebras.

Keywords: M-algebra, Soft ideal, Soft M-ideal, Soft M-algebra, Soft M-Subalgebra, Soft set.

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الجبر الناعم من الخط **M** والمثاليات الناعمة من الخط **M**

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قسم الكيمياء، كلية التربية، جامعة كرميان، العرا ق

الملخص

في هذه الدراسة، قمنا في البداية بتقديم وتحليل فكرة جديدة تمامًا عن جبر **M**. كما قدمنا فئات جديدة من الجبر الناعم، والتي نشير إليها باسم جبر الناعم **M**. ثم قمنا بتقديم واستكشاف أفكار جديدة في جبر الناعم **M** ، مثل المثاليات الناعمة لجبر **M** والجبر الفرعي الناعم **M**، باستخدام الدلالات التي اكتسبناها حديثًا. بالإضافة إلى ذلك، قدمنا نظرية جبر الناعم **M** والمثاليات الناعمة**M**.

INTRODUCTION

The idea of a soft set was first presented by [1], and it is considered a newly developed mathematical instrument to manage uncertainty. You can use it to resolve challenging issus in economics, environment and the engineering. Moreover, a pair (G, B) a mapping $G: B \rightarrow P(U)$, is referred to as a soft set over U. Additionally [2], have provided several soft set theory's new operations. In 1966, Imai and Isek proposed two classes of abstract algebras [3, 4], called BCK-algebras and BCI-algebras. "They also discussed the notion of soft groups [5] introduced and investigated the notion of soft BCK/BCI-algbras[6] discussed the applications of soft sets in the ideal theory BCK/BCI-algbras". They introduced the notions of soft BCH-subalgebra and soft BCH-ideals[7, 8]. Many operations on soft sets were specified by [9, 10]. In 2009, the auothors [11] introduced let (α, A) is a soft set over X. Then $\alpha(x)$ is said to be a soft d-algebra over X if $(\alpha(e), *, 0)$ is said to be a d-algebra for all $x \in A$ [12, 8] provided new ideas definition of soft sets in BH-algebra and soft BCH-algebra. In [8] for any a soft set (γ, C), the set $Supp(\gamma, D) = \{x \in D : \gamma(x) \neq \emptyset\}$ is said to be the support of the soft set (γ, D), the soft set (γ, D) is said to be a non-null if $Supp(\gamma, D) \neq \emptyset$. The authors [8] introduced a soft d-subalgebra, let (α, A) and (β, B) are soft d-algebras over X. Then (β, B) is said to be a d-subalgebra of (α, A) represented by $(\beta, B) \lesssim_s (\alpha, A)$ if it meets the requirements listed below:

i. $B \subseteq A$,

ii. $\beta(x)$ is a d-subalgebra of $\alpha(x)$ for all $x \in Supp(\beta, B)$. This is recalled [8, 13]. Let (F, A) is a soft BCI-algebra over X. A soft set (G, I) over X is said to be a soft p-ideal of (F, A), represented by $(G, I) \ \widetilde{\lhd}_p \ (F, A)$,if it satisfies:

i. $I \subseteq A$,

ii. $(\forall x \in I)(G(x) \lhd_p F(x))$.

In this piece of work, the notion of M-algebra is now presented an algebra $(W, \#, 0)$ of types $(2,0)$ is said to be a M-algebra if it satisfies the following conditions:

$(M_1)(w\#w = 0)$, $(M_2)(w\#0 = w)$,
$(M_3)(0\#w = w)$, $(M_4)(w\#(x\#y) = (w\#x)\#y)$,
$\forall\, w, x, y \in W$.

After presenting the notion of soft M-algebra suppose that (γ, C) is a soft set over W. Then (γ, C) is said to be a soft M-algebra over W. If $(\gamma(w), \#, 0)$ is a M-algebra and $\forall\, w \in C$. I extended the concepts of soft M-algebra to the context of soft M-subalgebra. Assume that (γ, C) and (λ, D) are soft M-algebras over W. Then (λ, D) is said to be a soft M-subalgebra of (γ, C), represented by $(\lambda, D) \lesssim (\gamma, C)$, if it satisfies the following conditions:

i. $D \subseteq C$,

ii. $\lambda(w)$ be a M-subalgebra of $\gamma(w), \forall\, w \in Supp(\lambda, D)$.

Moreover, I extend the notion of soft M-ideal, suppose that (G, C) is a soft M-ideal over W. A soft set (λ, D) over W is said to be a soft M-ideal of (G, C), represented by $(\lambda, D) \ \widetilde{\lhd}_M \ (G, C)$, if it satiefies:

i. $D \subseteq C$,

ii. $(\forall\, w \in D)\ (\lambda(w) \lhd_M G(w))$.

The structure of this paper is as follows: Sections two and three present definition of M-algebras and soft M-algebras, and then gave several theorem, proposition, lemma and examples. In the final section, I introduce the concept of (soft M-subalgebras and soft M-ideals) and survey several properties.

PRELIMINARIES

Firstly, I give the definition, examples and a proposition about M-algebras.

Definition 1

An algebra $(W, \#, 0)$ of types $(2,0)$ is said to be a M-algebra if it satisfies the following conditions:

$(M_1)(w\#w = 0)$,
$(M_2)(w\#0 = w)$, $(M_3)(0\#w = w)$,
$(M_4)(w\#(x\#y) = (w\#x)\#y)$,

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$\forall\, w, x, y \in W$.

Example 1

i. Assume that $W = \{0,1,2\}$ with the cayle table as follows:

#	**0**	**1**	**2**
0	0	1	2
1	1	0	2
2	2	1	0

Then $W = (\{0,1,2\}, \#, 0)$ is a M-algebra.

ii. Assume that $W = \{0,1,2\}$ with the cayle table as follows:

#	**0**	**1**	**2**
0	0	0	0
1	1	0	1
2	2	1	0

$(W = (\{0,1,2\}, \#, 0)$ is not a M-algebra, since (M_3) does not hold $(0\#1 = 0)$.

Proposition 1

If $(W, \#, 0)$ is a M-algebra, then for any $w, x, y \in W$

1. $w\,\#w = 0$.
2. $0\,\#w = w$.
3. $w\,\#(w\#x) = x$.
4. $(w\,\#(w\#x))\#x = 0$.

Proof: (1 and 2) they are clearly.

3.Suppose $w\,\#(w\#x) = (w\#w)\#x$.

Then $(0\#x) = x$ (by Definition $1(M_3)$).

4.Suppose $(w\,\#(w\#x))\#x = 0$.

Then $(w\#w)\#(x\#x) = 0\#0 = 0$ (by Definition $1(M_1)$).

Lemma 1

Let $(W, \#, 0)$ is a M-algebra. Then for all $w, y \in W$

1. $0\,\#(0\#w) = w$,
2. If $0\,\#w = 0\#y$, then $x = y$,
3. $w\#(0\#w)\#w = w$.

Proof: 1. we have that $0\,\#(0\#w) = w$, since by Definition $1(M_3)$ implies $0\,\#w = w$.

2. If $0\,\#w = 0\#y$, since by Definition $1(M_3)$ implies $w = y$.

3. Let $w\,\#(0\#w)\#w = w$, since by Definition 1 (M_1, M_3). Then $0\#w = w$.

SOFT M-ALGEBRAS

In this section, I cover seveeral basic a soft M-algebra topics, as well as some additional ideas that are relevant to our study.

Definition 2

Suppose that (γ, C) is a soft set over W. Then (γ, C) is said to be a soft M-algebra over W. If $\gamma(w), \#, 0$ is a M-algebra and $\forall\, w \in C$.

Example 2

Assume that $(W, \#, 0)$ is a M-algebra in Example 2 (i). Assume that (γ, C) be a soft over $W$and $C = W$ defined by:

$\gamma(w) = \{\, x \in W\colon x\,\#\,(x\,\#\,w) \in \{0,1\}\,\}$, be a set valued function and $\gamma\colon C \to P(W), \forall\, w \in C$. Then, (γ, C) is a soft M-algebra over W, since $\gamma(0) = \gamma(1) = \gamma(2) = \{0,1\}$ are M-algebras.

Example 3

Assume that $(W, \#, 0)$is a M-algebra in Example 2 (ii). Assume that $(L, C)$is a soft over $W$and $(C = \{\,0,1,2\} = W)$, with $L : C \to P(W)$being the set-valued function and $\forall\, w \in C$, $L(x) = \{\, x \in W\colon x\,\#\,(x\#\,w) \in \{0,1\}$.

Since $L(0) = L(1) = W$isn't a M-algebra, $L(0) = \{0,1\}$ and $L(0) = W$ aren't soft M-algebras over W.

Definition 3

Suppose that W be a M-algebra and suppose that S be a subset of WIf $w\,\#\,x \in W$, whenever $\forall\, w, x \in S$, then S is a subalgebra of W.

Proposition 2

Let $\{(H_j, D_j)\ ;\, j \in \gamma\} \neq \emptyset$ be a soft M-algebras over W. Then the bi-intersection $\tilde{\sqcap}_{j\,\in\,\gamma}\,(H_j, D_j)$ is a soft M-algebra over W, if it isn't null.

Proof: Let $\{(H_j, D_j)\ ;\, j \in \gamma\} \neq \emptyset$ be a soft M-algebras over W. By (See Definition 2 (ii)) [8], we may compose $(G, C) = \tilde{\sqcap}_{j\,\in\,\gamma}\,(H_j, D_j)$,where:

$C = \cap_{j\in\gamma}\, D_j$, and $G(w) = \cap_{j\in\gamma}\, H(D_j)$for all $w \in C$.

Let us assume $w \in Supp\,(G, C)$. Consequently, we get $H_j(w) \neq \emptyset$, $\forall j \in \gamma$, where:

$\cap_{j\in\gamma}\, H_j(w) \neq \emptyset$. Given that:

$\{(H_j, D_j) \ ; \ j \ \in \ \gamma\} \neq \emptyset$; be a soft M-algebras over W, $H_j(w)$ is a M-Subalgebra of W for all $j \in \gamma$, and its intersection is likewise a M-subalgebra of W,in other words:

$G(w) = \cap_{j\in\gamma} H_j(w)$; is a soft M-algebra over W, because $\tilde{\sqcap}_{j\in\gamma} (H_j, D_j)$ is a M-subalgebra of W and $\forall\ w \in Supp\ (G, C)$.

Theorem 1

Assume that $\{ (L_j, D_j) \ ; \ j \ \in \ \gamma\} \neq \emptyset$; be a soft M-algebras over W. Then the extended intersection $\tilde{\sqcap}_{j\in\gamma} \ (L_j, D_j \ )$ is a soft M-algebra over W.

Proof: Let $\{ (L_j, D_j) \ ; \ j \ \in \ \gamma\} \neq \emptyset$ be a soft M-algebras over W. By (See Definition 1) [8], we may compose $\tilde{\sqcap}_{j\in\gamma} \ (L_j, D_j \ ) \ = (H, E)$, where $E = \tilde{\cup}_{j\in\gamma} \ D_j$,

$H(w) = \cap_{j\in\gamma} L_j(w)$and $\forall\ w \in \ E$.

Let $w\epsilon Supp(H, E)$, then $F_j(w) \neq \emptyset$ for every $w \ \in \ \gamma$ since $\cap_{j\in\gamma} L_j(w) \neq \emptyset$. Because (L_j, D_j) is a soft M-algebras over W for all $w \ \in \ \gamma$, thus, we conclude that $L_j(w)$ is a M-algebras of W, for all $j \in \gamma$,. It follows that $H(w) = \cap_{j\in\gamma(w)} \ L_j(w)$ is a M-subalgebra of W. For any $w \ \in \ Supp(H, E)$. As a result, the extended intersection $\tilde{\sqcap}_{j\in\gamma} \ (L_j, D_j \ )$ is a soft M-algebra over W.

Theorem 2

Let $\{ (G_j, C_j \ ; \ j \ \in \ \gamma\} \neq \emptyset$ be a soft M-algebras over W_j.

Then the cartesian product $\tilde{\prod}_{j\in\gamma}(G_j, C_j)$ is a soft M-algebra over $\prod_{j\in\gamma}(w_j)$.

Proof: By (See Definition 2) [8], we may compose $\tilde{\prod}_{j\in\gamma}(G_j, C_j) = \ (H, B)$, where:

$B = \prod_{j\in\gamma} C_j$, $H(w) = \ \prod_{j\in\gamma} G_j(w)$, and:

$\forall w = \left(w_j\right)_{j\in\gamma} \in B$.

Let $w = Supp(H, B) = \left(w_j\right)_{j\in\gamma}$

Then $H(w) = \ \prod_{j\in\gamma} G_j(w_j) \ \neq \emptyset$

Consequentaly, we have $G_j\left(w_j\right) \neq \emptyset$ for every $j \in \gamma$. Because $\left\{ \left(G_j, C_j\right); \ j \ \in \ \gamma\right\}$is a soft M-algebras over w_j ,$\forall\ j \in \gamma$. Consequently, $G_j\left(w_j\right)$ is a M-subalgebra of w_j, and as such, $\prod_{j\in\gamma} G_j(w_j)$ is a M-subalgebra of $\prod_{j\in\gamma} w_j$ for all $w = \left(w_j\right)_{j\in\gamma} \in Supp(H, B)$. As a result, the cartesian product $\tilde{\prod}_{j\in\gamma}(G_j, C_j)$ is a soft M-algebra over $\prod_{j\in\gamma}(w_j)$.

Definition 4

Suppose that a function g from set Y to set Z is mapping of M-algebras and that Y, Z are two M-algebras. If (H, C) and (L, D) are soft sets over Y and Z respectivily, then $(f(H), C)$ is a soft set over Z, whereas $f(H): C \ \rightarrow P(Z)$ is characterized with:

$f(H)(y) = \ f\big(H(y)\big), \ \forall y \in C$

and $(f^{-1}(L), D)$ is a soft set over Y, whereas $f^{-1}(L): D \ \rightarrow P(Y)$ characterized with $f^{-1}(L)(y) = \ f^{-1}\big(L(y)\big), \ \forall\ y \ \in D.$

Theorem 3

Suppose that $g : W \rightarrow Y$ is an onto homomorphism of M-algebras.

1. If (H, D) is a soft M-algebra over W, then $(g(H), D)$ be a soft M-algebra over Y.
2. If (L, C) be a soft M-algebra over Y, $then$ $(g^{-1}(L), C)$ be a soft M-algebra over W if it is non-null.

Proof: 1. Assume that (H, D) is a soft M-algebra over W, It's obvious that $(g(H), D)$ is a non-null soft set over Y.

Let$\forall\ w \ \in Supp\ (g(H), D)$. We have $g(H)(w) = g\big(H(w)\big) \neq \ \emptyset$. Because $H(w)$ is a M-subalgebra of Y, it's onto homomorphic image $g(H(w))$ is a M-subalgebra of W. As a result, $g(H(w))$ is a M-subalgebra of $Y \ \ \forall\ w \in Supp(g(H), D)$, that is, $(g(H), D)$ is a soft M-algebra over Y.

3. It is easily to see that:

$Supp\ (g^{-1}(L), C) \subseteq Supp\ (G, C)$. Let $y \in Supp(g^{-1}(L), C)$. Then $L(y) \ \neq \emptyset$. Since the nonempty set $L(y)$ is a soft M-algebra of W, it homomorphic inverse image of $g^{-1}(L(y))$ is also a M-subalgebra of W as it is a M-subalgebra of Y. Hence $g^{-1}(L(y))$ is a M-subalgebra of Y, $\forall\ y \ \in Supp\ (g^{-1}(L), C)$. That is, $(g^{-1}(L), C)$ is a soft M-algebra over W.

Remark 3.10- Let a function g from set Y to set Z be an algebraic homomorphism of M-algebras. If

(γ, D) be a soft M-algebra over Y, then $(g(\gamma), D)$ be a soft M-algebra over Z.

Proof: Straightforward.

SOFT M-IDEALS

Now, I define some special soft M-subalgebras, soft M-ideals and present some results on them.

Definition 4

Assume that (γ, C) and (λ, D) are soft M-algebras over W. Then (λ, D) is said to be a soft M-subalgebra of (γ, C), represented by $(\lambda, D) \lesssim (\gamma, C)$), if it satisfies the following conditions:

i.$D \subseteq C$

ii.$\lambda(w)$ be a M-subalgebra of $\gamma(w), \forall\, w \in Supp(\lambda, D)$.

Definition 5

Assume that S be a M-subalgebra of W. A non-empty subset I of W is said to be ideal of W related to W (brifly, S-ideal of W), represented by $I \lhd S$, if it is satisfies:

i.$0 \in S$

ii.$(\forall\, w \in S)(\forall y \in I)(w\#y \in I \Rightarrow w \in I)$

Example 4

Consider $(W, \#, 0)$is a M-algebra given in Example 3. We create a set-valued function $\gamma : C \to P(W)$ for $C = \{0,1,2\} \subseteq W$. for every $w \in C$ by $\gamma(w) = \{y \in W : y\#(y\#w) \in \{0,1\}\}$. Then (γ, C) is a soft M-subalgebra over X. now let (λ, D)be a soft set over W. $D = \{0,1\} \subseteq C$ and $\lambda: D \to P(W)$ is a set-valued function as specified by:

$\lambda(w) = \{y \in W : y\#(y\#w) = 0\}$

for all $w \in D$. It is clear to understand that (λ, D) is a soft M-subalgebra over W. Then $\lambda(0) = \{0\}$, $\gamma(0) = \{0,1\}$, $\lambda(1) = \{0\}$, $\gamma(1) = \{0,1\}$.

Therefore $(\lambda, D)$is a soft M-subalgebra of (γ, C).

Proposition 3

Let (F, C) is a soft M-algebra over W, and $\{(K_j, C_j); j \in \gamma\} \neq \emptyset$ be a soft M-algebras over (F, C). Then the bi-intersection $\tilde{\sqcap}_{j\in\gamma}(K_j, C_j)$ be a soft M-subalgebra (F, C) if it is non-null.

Proof: Simliar to Theorem 1 proof .

Theorem 4

Let (G, C) be a soft M-algebras of W and $\{(L_j, C_j); j \in \gamma\} \neq \emptyset$ be a soft M-subalgebras over (G, C). Then the extend intersection $\tilde{\sqcap}_{j\in\gamma}$ (L_j, C_j) is a soft M-subalgebra over (G, C).

Proof : Simliar to Theorem 2 proof.

Theorem 5

Let (γ, C) and (λ, D) are two soft M-algebra over W and $(\lambda, D) \tilde{\subseteq} (\gamma, C)$. Then $(\lambda, D) \lesssim_s (\gamma, C)$.

Proof: Straightforward.

Theorem 6

Assume that (L, C) is a soft M-algebra over W and $\{(G_j, C_j); j \in \gamma\} \neq \emptyset$ be a soft M-subalgebra of (L, C) . Then the Cartesian product of $\tilde{\prod}_{j\in\gamma}(G_j, C_j)$ be a soft M-subalgebra $\prod_{j\in\gamma}(L, C)$

Proof: By (See Definition 2) [8], may compose $\tilde{\prod}_{j\in\gamma}(G_j, C_j) = (G, B)$, where $B = \prod_{j\in\gamma} C_j$ and G(w)= $\prod_{j\in\gamma} G_j \in B\ \forall\, w = (w_j)_{j\in\gamma} \in B$.

Suppose that $w = (w_j)_{j\in\gamma} \in Supp(G, B)$. Then $G(w) = \prod_{j\in\gamma} G_j(w) \neq \emptyset$, and so we've got $G_j(w_j) \neq \emptyset\ \forall j \in \gamma$

Because of this $\{(G_j, C_j); j \in \gamma\} \neq \emptyset$ is a soft M-subalgebras over (L, C), we've that $G_j(x_j)$ is a M-subalgebra of $G(w_j)$, from which we obtain that $\prod_{j\in\gamma} G_j(w_j)$ be a M-subalgebra of $\prod_{j\in\gamma} L_j(w_j), \forall\, w = (w_j)_{j\in\gamma} \in Supp(G, B)$.

Therefore $\tilde{\prod}_{j\in\gamma}(G_j, C_j)$ be a soft M-subalgebra $\prod_{j\in\gamma}(L, C)$.

Proposition 4

Let $g : W \to Y$ is a homomorphism of M-algebras and (γ, C), (λ, D) are soft M-algebras over W. If $(\lambda, D) \lesssim_s (\gamma, C)$, then $(g(\lambda), D) \lesssim_s (g(\gamma), C)$.

Proof: Let $(\lambda, D) \lesssim_s (\gamma, C)$. Let us assume $w \in Supp(\lambda, D)$, then $w \in (\gamma, C)$. By Definition 4 [8], C is a subset of D and $\lambda(w)$ is a M-subalgebra of $\gamma(w)$, $\forall\, w \in Supp(\lambda, D)$. since g is homomorphism and $g(\lambda)(w) = g(\lambda(w))$ is the M-subalgebra of $g\gamma(w)) = g(\gamma)(w)$. Hence $(g(\lambda), D) \lesssim_s (g(\gamma), C)$.

Proposition 5

Let $g : W \rightarrow Y$ is an onto homomorphism of M-algebras and (γ, C), (λ, D) are soft M-algebras over W. if $(\lambda, D) \precsim_s (\gamma, C)$, implies $(g^{-1}(\lambda), D) \precsim_s (g^{-1}(\lambda), C)$,

Proof: Let $(\lambda, D) \precsim_s (\gamma, C)$

Let $y \in Supp(g^{-1}(\lambda), D)$. By Definition 4 [8] D is a subset of C, and for every $y \in$ D, $\lambda(y)$ is a M-subalgebra of $\gamma(y)$, Because of this, g is homomorphism with $g^{-1}(\lambda(y) = g^{-1}(\gamma(y))$ is the M-subalgebra of $g^{-1}(\gamma(y)) = g^{-1}(\gamma)(y)$ for any $y \in Supp(g^{-1}(\lambda), D)$.

Hence $(g^{-1}(\lambda), D) \precsim_s (g^{-1}(\gamma), C)$.

Definition 6

Assume that S be a M-subalgebra of W. A subset $I \neq \emptyset$ of W is said to be M-ideal of W related to W (brifly, S-M-ideal of W), represented by $I \lhd_M S$, if it is satisfies:

i. $0 \in S$,

ii. $(\forall\, w, x \in S)(\forall y \in I)(w\#y)\#(x\#y) \in I \Rightarrow w \in I)$.

Definition 7

Suppose that (G, C) is a soft M-ideal over W. A soft set (λ, D) over W is said to be a soft M-ideal of (G, C), represented by $(\lambda, D) \trianglelefteq_M (G, C)$, if it satiefies:

iii. $D \subseteq C$,

iv. $(\forall\, w \in D)\ (\lambda(w) \lhd_M G(w))$.

Proposition 6

Let (L, D) be a soft M-algebra. There exists some soft sets (λ_1, E_1) and (λ_2, E_2) over W, where $E_1 \cap E_2 \neq \emptyset$, we have $(\lambda_1, E_1) \trianglelefteq_M (L, D)$, $(\lambda_2, E_2) \trianglelefteq_M (L, D)$ implies $(\lambda_1, E_1) \cap (\lambda_2, E_2) \trianglelefteq_M (L, D)$.

Proof: By (see Definition 2) [8], we may write $(\lambda_1, E_1) \tilde{\cap} (\lambda_2, E_2) = (\beta, C)$, where $C = E_1 \cap E_2$ and $\beta(w) = \lambda_1(x)$ or $\lambda_2(x)$ or $\forall\, w \in C$, obviously $C \subseteq D$ and $\beta : C \rightarrow P(W)$ is mapping. Thus (β, C) is a soft set over W. Because $(\lambda_1, E_1) \trianglelefteq_M (L, D)$ and $(\lambda_2, E_2) \trianglelefteq_M (L, D)$, we know that $\beta(w) = \lambda_1(w) \lhd_M L(w)$ or $\beta(w) = \lambda_2(w) \lhd_M L(w)\ \forall w \in C$

Hence $(\lambda_1, E_1) \tilde{\cap} (\lambda_2, E_2) \trianglelefteq_M (L, D)$.

Theorem 7

Assume that the soft M-algebra over W is (F, A). If (λ_1, E_1) and (λ_2, E_2) are any soft sets and we get $(\lambda_1, E_1) \trianglelefteq_M (F, A)$, $(\lambda_2, E_2) \trianglelefteq_M (F, A)$ over W with w in which E_1 and E_2 are disjoint, then $(\lambda_1, E_1) \tilde{\cup} (\lambda_2, E_2) \trianglelefteq_M (F, A)$.

Proof: Let us assume that $(\lambda_1, E_1) \trianglelefteq_M (F, A)$, and $(\lambda_2, E_2) \trianglelefteq_M (F, A)$. By (See Definition 3) [11], we may write $(\lambda_1, E_1) \cap (\lambda_2, E_2)$=(G,D), where $D = E_1 \cap E_2$ and for every $w \in D$,

$$\begin{cases} \lambda_1(w) & \text{if } w \in E_1 \backslash E_2 \\ \lambda_2(w) & \text{if } w \in E_2 \backslash E_1 \\ \lambda_1(w) \cup \lambda_2(w) & \text{if } w \in E_1 \cap E_2 \end{cases}$$

Because $E_1 \cap E_2 = \emptyset$, either $w \in E_1 \setminus E_2$ or $w \in E_2 \setminus E_1$. If $w \in E_1 \backslash E_2$, then $G(w) = \lambda_1(w) \lhd F(w)$ since $(\lambda_1, E_1) \trianglelefteq_M (F, A)$. if $w \in E_2 \setminus E_1$, then $G(w) = \lambda_2(w) \lhd F(w)$. because $(\lambda_2, E_2) \trianglelefteq_M (F, A)$. For every x in D, $G(w) \lhd F(w)$, and so $(\lambda_1, E_1) \tilde{\cup} (\lambda_2, E_2) = (G, D) \trianglelefteq_M (F, A)$.

CONCLUSIONS

A new mathematical technique for overcoming uncertainly is the use of soft sets. I studied an algebraic structure known as M-algberas using the soft sets theory. I presented the notions of soft M-algebras,soft M-subalgebras and soft M-ideal. Many related properties were surveyed.

Conflict of interests: The author declared no conflicting interests.

Sources of funding: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Author contribution: The author independently conducted all components of the research, including problem formulation, theoretical development, proofs, and preparation of the manuscript.

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