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***RESEARCH ARTICLE - PHYSICS***

# Investigation of nuclear structure of even isotones $^{134}$Ce and $^{136}$Nd with IBM and GBM

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**Abstract**

In this research the nuclear structure each of isotopes even-even $^{134}$Ce and $^{136}$Nd within a frame Interaction Boson Model (IBM:1), Model (IBM – code, IBMT – cod, and IBMP – code) was studied. Based on the calculation of energy ratios $(E^{+}{}_{4_1}/E^{+}{}_{2_1})$ $(E^{\dagger}{}_{6_1}/E^{+}{}_{2_1})$ and, $(E^{+}{}_{8_1}/E^{+}{}_{2_1})$, The probability of an electric transition $B(E2)$ is reduced, the electric quadrupole moments $Q^{+}$for first positive parity level $(2^{+})$, and potentiel energy surfaces $P.E.S.$The results of the current study were confirmed by comparison with the practical results of the latest dissolution schemes that can be obtained, every one of $^{134}$Ce and $^{136}$Nd they are located within the transitional region between the limit a Gamma-unstable $O(6)$and SU(3) .This has a small effect $a_0$ parameter and cancel effect $\varepsilon$ parameter,according to the contour shapes of the nuclei using Interaction Boson Geometric (GBM) ,The nuclei are clearly deformed.


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***Keywords**: IBM1, IBMT, GBM, IBMP, Contour diagrams*

## 1. Introduction

As a result of the intense interactions between protons and neutrons with each other, which are called nucleons, and on the basis of studying nuclear structure, several models were developed including the liquid drop model, the shell model and the collective model [1]. To latter was unable explain that nucleons outside closed shells behave in the form of pairs that interact with each other to form bosons, and they can operate the ground state –s when angular momentum L=0 and it's called (s-Boson) or to take the place of the levels of irritated states when L=1 it's called ( d- Boson) [2], [3] After these models, a more comprehensive model emerged that is based on both the shell model and the collective model. and to study the nuclear structure of isotopes even-even $^{134}$Ce and $^{136}$Nd within a frame Interaction Boson Model (IBM:1). There are many studies on these isotopes and with different ideas about their nuclear structure, which can be observed through the following studies, as an example: In the year 2000 ,the researcher Z.Jinfu.et studied isotopes with mass numbers Ce (128-150) and concluded that both light and heavy isotopes are going through a transitional phase from the vibrational region to the rotational[4], researchers study in the year 2007 Nuclear structure of $^{128\text{-}140}$Nd in IBM it was explained that they are transition nuclei O(6)- U(5) [6],and in the year 2019 researchers work K.A.Hussein ,et study the nuclear structure for $^{134}$Ce use IBM:1she indicated that it is located in the area O(6) [7].

## 2. Interacting Boson Model:1 IBM:1

The Interacting Boson Model I B M: 1is one of the most successful nuclear models that provided a description of the nuclear structure of medium and heavy nuclei [8]. And this model was developed by Iachello and Arima [9]. The I B M: 1does not distinguish between proton$(s_\pi, d_\pi)$ and neutron bosons $(s_v, d_v)$ it is described as pairs of particles(N= Nπ+Nν) located outside the closed shell, from the closest closed shell to the middle of the next shell[10]. In order to study any even- even nuclei, influences are required, including the Hamiltonian effect of the system energy, The boson-boson interacting energy can be written as :[11],[12].

$$H = \varepsilon_s(s^\dagger s) + \varepsilon_d \sum_m d_m^\dagger d_m + \sum_{L=0,2,4} \frac{1}{2}(2L+1)^{\frac{1}{2}} C_L\left[(d^\dagger d^\dagger)^{(L)}.\ (dd)^{(L)}\right]^{(0)} + \frac{1}{\sqrt{2}}\upsilon_2\left[(d^\dagger d^\dagger)^{(2)}.(ds)^{(2)} + (d^\dagger s^\dagger)^{(2)}.(dd)^{(2)}\right]^{(0)} + \frac{1}{2}\upsilon_0\left[(d^\dagger d^\dagger)^{(0)}.(ss)^{(0)} + (s^\dagger s^\dagger)^{(0)}.(dd)^{(0)}\right]^{(0)} + u_2\left[(d^\dagger s^\dagger)^{(2)}.(ds)^{(2)}\right]^{(0)} + \frac{1}{2}u_0\left[(s^\dagger s^\dagger)^{(0)}.(ss)^{(0)}\right]^{(0)} \qquad (1)$$

Where $C_L, \upsilon_L, u_L$ describe the boson interaction.
The synonym equation of Hamiltonian is [9,11]:-

$$H = \varepsilon n_d + a_0\, P^\dagger P + a_1\, L.L + a_2\, Q.Q + a_3 T_3 T_3 + a_4 T_4 T_4 \qquad (2)$$

where $= \varepsilon_d - \varepsilon_s$ is the boson enargy, $\varepsilon_s$ is set equal to zero only $\varepsilon = \varepsilon_d$ appears, $a_0, a_1, a_2, a_3$, and $a_4$ represents the strength of different interactions between identical bosons respectively. The $d-$ five components and the single component of $s-$ It extends across a six-dimensional space [13], [14]. For a specified number of bosons $N_{Tot.}$ the collection construction of the tricky is $U(6)$. Having seen the diverse concessions of $U(6)$, three dynamical proportions appear, namely $U(5)$*;* $SU(3)$; and $O(6)$; These symmetries are related to the geometric idea of the ball vibrator, the deformed blade and the symmetrically deformed (γ-smooth) rotor, separately, separately [10], [15]. One significant approach to exploring nuclear structure involves observing how nuclear reactions interact with an external electromagnetic field. These reactions originate from the distribution and movement of nucleons within the nucleus, where the influence of the electromagnetic field on nucleon motion is relatively minor compared to other forces. The decay of excited nuclear states results in the emission of electromagnetic radiation, including gamma rays and quadrupole electric transitions.
To compute transition rates, the simplest of I B M: 1utilizes the one-body transition operator, which is expressed as follows [16], [17]:

$$T_m^l = \alpha_2 \delta_{l2}[d^\dagger s + s^\dagger d]_m^{(2)} + \beta_l [d^\dagger d]_m^{(l)} + \gamma_0 \delta_{l0}\delta_{m0}[s^\dagger s]_0^{(0)} \qquad (3)$$

Where $\alpha_2, \beta_l, \gamma_{0;}$ they are the various coefficients associated with the three symmetries, which can be written in the form corresponding to the hexagonal space according to the theory of groups and subgroups as follows. [16], [17]:-

$$U(6): \begin{array}{l} SU(5): O(5): O(3): O(2) \\ SU\,(3): O(3): O(2) \\ O(6): O(5): O(3): O(2) \end{array} \qquad (4)$$

The quadrupole moments $Q^{+}_{2_1}$, in $U(5)$, $SU(3)$ and $O(6)$ limits defined as

$$Q^{+}_{2_1} = \beta_2 \frac{4\sqrt{\pi}}{\sqrt{5}}\sqrt{\frac{2}{7}}, Q^{+}_{2_1} = -\alpha_2 \frac{4\sqrt{\pi}}{\sqrt{5}} 2/7(4N+3) \text{ And } Q^{+}_{2_1} = 0 \quad (5)$$

### 3. Geometrical Boson Model GBM

The $\mathrm{P-E-S}$ as a function of geometrical variables $\beta$ and $\gamma$ expression as

$$V(\beta,\gamma) = \frac{N(\varepsilon_s+\varepsilon_d\beta^2)}{1+\beta^2} + \frac{N(N+1)}{(1+\beta^2)^2}(\mathrm{F}_1\beta^4 + \mathrm{F}_2\beta^3 Cos3\gamma + \mathrm{F}_3\beta^2 + \mathrm{F}_4) \quad (6)$$

Where $\mathrm{F}_1, \mathrm{F}_2,\ \mathrm{F}_3$ and $\mathrm{F}_4$ related to parameters in Eq. (1)

The nuclear shapes using polar angles have been put as in Figure (1) [18]

a) γ values of 0◦,120◦and 240◦ yield prolate spheriods with the 3,1,2 axes as symmetry axes;
b) γ =180◦,300◦and 60◦ give oblate shape;
c) with γ not multiple of 60◦, triaxial shape resulet;
d) the intervial ,( γ=0◦≤γ≤60◦) is sufficient to describe all posible quadrupole deformation shapes;

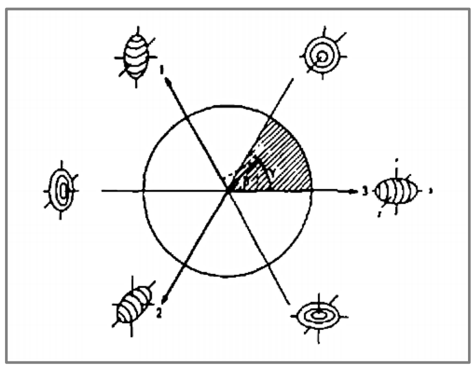


Figure1**:** Varios nuclearshapes in the (β-γ) plane.

The full complexity of possible V (β-γ) surfaces become clear in figure (2 ) where some typical V (β-γ) plots are depicted corresponding to avibtator β $^2$ variation ,a prolate equilibrium shape ,a γ-soft vibrator nucleus and trixaial rotor system respectively.

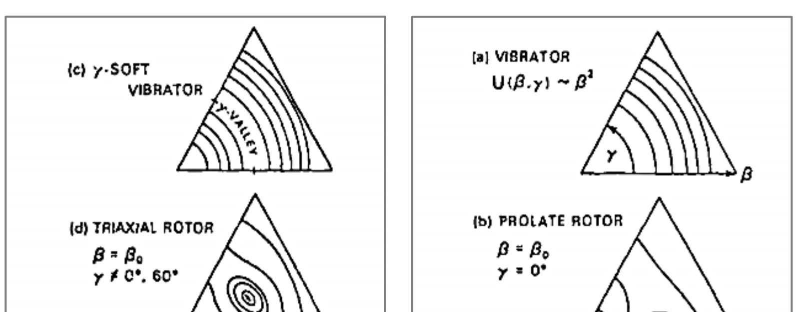


Figure2: Different potential energy shapes V (β-γ) in the β,( γ=0→ γ=60) sector corresponding to a spherical vibrator ,aprolate rotor, a , γ-soft vibrator and trixaial rotor respectively.

## 4. Calculations and Results:

The Ce: 134and Nd136, isotones, have $N = 76$ which equivalent (three - hole bosons) and ($Z = 58\ and\ 60$) respectively, which equivalent (4 and 5) particle proton boson number. By calculating a number of variables specified in the formulas for the Hamiltonian component equations (1)&(2), the low-lying energy levels, for Ce: 134and Nd136, isotones have been calculated using the software packages $IBM - code$ for IBM: 1. The paramaters estimated for the low-lying calculations of the excited energy levels for Ce: 134 and Nd136, isotones are given in Table (1).

Figure (3-5) show the energy ratios of ($E4_1^+/E2_1^+$), ($E6_1^+/E2_1^+$) and ($E8_1^+/E2_1^+$) respectively as a function of mass numbers for Ce: 134 and Nd136, isotones, The levels of the present work energy compared with the empirical data Ce: 134and Nd136 isotones have $B(E2)$ transition probabilities been shown in figure (6).

To calculate $B(E2)$ transition probabilities usethe effective boson charges estimated from equations (3) have been used to calculate and branching ratios Eqs. (5) for $SU(3)$, $U(5)$, and $O(6)$ respectively, $E2SD$ and $E2DD$, $IBMT$ paramaters where $\alpha_2$ and $\beta_2$ are the boson effective charge for $IBM1$ $(E2SD = \alpha_2), \left(E2DD = \sqrt{5}\beta_2\right)$ where $\beta_2 = \frac{-0.7}{\sqrt{5}}, \alpha_2, \frac{-\sqrt{7}}{2}\alpha_2\ \ and\ (\beta_2 = 0)$ in $U(5), SU(3)$, and $O(6)$ respectively [19].

**Table 1**. Values of parameters used in a program IBM: 1, GBM and E. transition probabiolity $B(E2)$ for even Ce: 134and Nd: 136 isotons(in Mev) with $N_n = 3$.

| **Isotopes** / **Param.′ s** | **Ce134** | **Nd136** | **Isotopes** / **Param.′ s** | **Ce134** | **Nd136** |
|---|---|---|---|---|---|
| $N_p$ | 4 | 5 | E2SD | 0.118 | 0.124 |
| $N_{Tot.}$ | 7 | 8 | E2DD | -0.173 | -0.197 |
| $\varepsilon$ | 0 | 0 | $\varepsilon_s$ | -0.225 | -0.19 |
| $a_0$ | 0.016 | 0.014 | $\varepsilon_d$ | 0.153 | 0.16 |
| $a_1$ | 0.023 | 0.023 | $\alpha_1$ | 0.004 | 0.003 |
| $a_2$ | -0.043 | -0.038 | $\alpha_2$ | -0.008 | 0.003 |
| $a_3$ | 0.043 | 0.043 | $\alpha_3$ | -0.18 | -0.152 |

| | | | | | |
|---|---|---|---|---|---|
| $\alpha_4$ | 0 | 0 | $\alpha_4$ | 0 | 0 |
| $\chi$ | -0.08 | -0.04 | Binding Energy **(B.E)** | -4.44314 | -3.50093 |

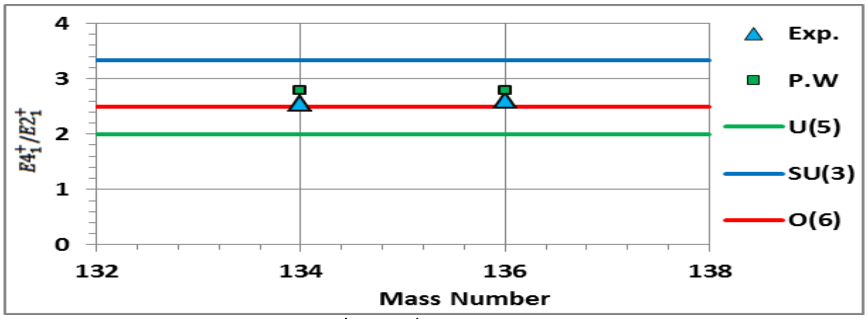



Figure3: Ratio energy The experiment($E4_1^+/E2_1^+$) , current work and standard [19]for even N- even Z 134Ce and 136Nd , isotones respectively

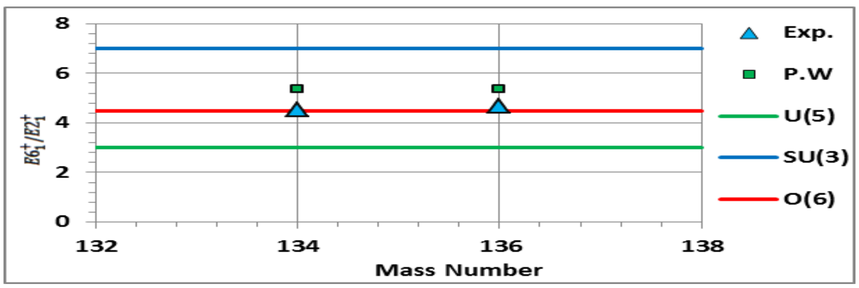



Figure4**:** Ratio energy The experiment($E6_1^+/E2_1^+$) , current work and standard [19] for even N- even Z 134Ce and 136Nd , isotones respectively.

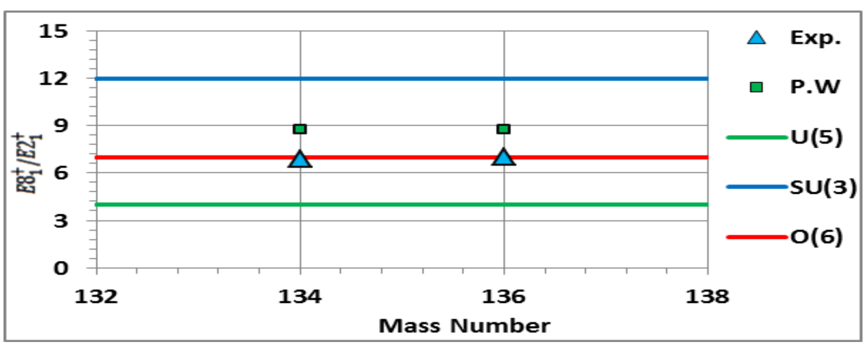
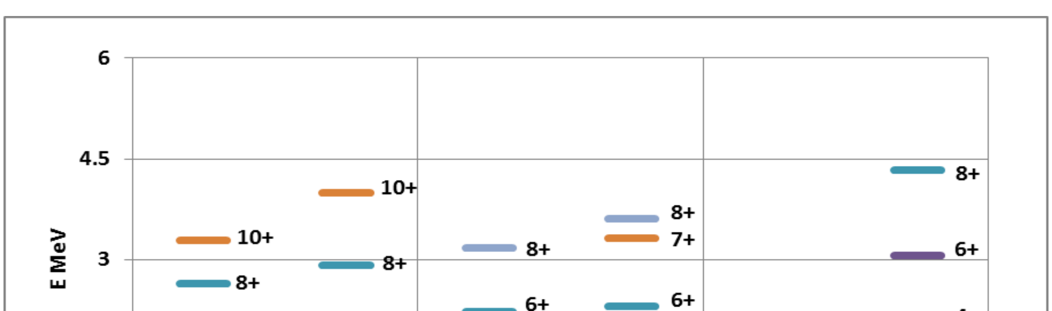



Figure5: Ratio energy the experiment ($E8_1^+/E2_1^+$) ,current work and standard [19] for even N- even Z 134Ce and 136Nd , isotones respectively.



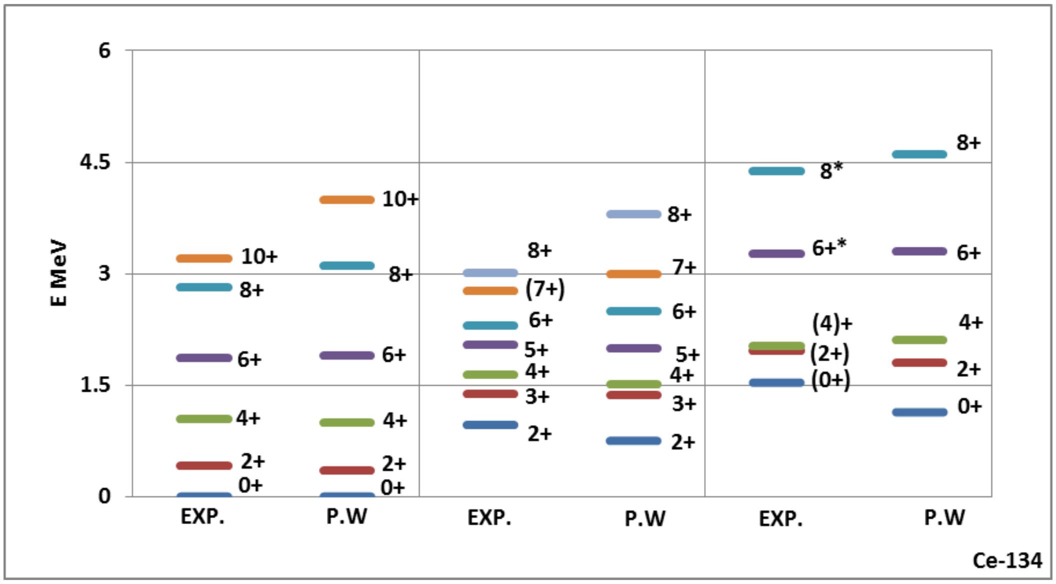



Figure 6: The levels of the present work energy it was compared with empirical for even N- even Z Ce:134 and Nd:136 isotones respectively.

Table2. Represent the empirical values [20] with present work to calculate reduced electric quadruple transitions probablitiy B(E2) and electric quadruple moment of $2_1^+$state for even N- even Ce:134 and Nd:136 , isotones.

| Isotopes | ${}^{134}_{58}Ce$ | | ${}^{136}_{60}Nd$ | |
|---|---|---|---|---|
| $J_i^+ \rightarrow J_f^+$ | $IBM1$ | $Expe.$ | $IBM1$ | $Expe.$ |
| $2_1 \rightarrow 0_1$ | 0.2178 | 0.21 | 0.3222 | 0.332 |
| $4_1 \rightarrow 2_1$ | 0.291 | 0.158 | 0.3874 | 0.0872 |
| $6_1 \rightarrow 4_1$ | 0.295 | 0.057 | 0.462 | 0.0149 |
| $8_1 \rightarrow 6_1$ | 0.2675 | --- | 0.4408 | --- |
| $2_1 \rightarrow 0_2$ | 0.0132 | --- | 0.0203 | --- |
| $2_2 \rightarrow 0_2$ | 0.0512 | --- | 0.0762 | --- |
| $2_2 \rightarrow 2_1$ | 0.1872 | --- | 0.3263 | --- |
| $4_2 \rightarrow 2_2$ | 0.1173 | --- | 0.198 | --- |
| $3_1 \rightarrow 2_2$ | 0.2065 | --- | 0.3268 | --- |
| $3_1 \rightarrow 4_3$ | 0.1742 | --- | 0.2715 | --- |

| | | | | |
|---|---|---|---|---|
| $7_1 \rightarrow 5_1$ | 0.1424 | --- | 0.2578 | --- |
| $Q_{2_1^+}(eb)$ | -0.90532 | --- | -1.0152 | --- |

The P. E. S as a function to $\beta$ with contour diagrams for even N- even Z 134Ce and 136Nd isotones calculated from equation (6) with $IBMP$ computer code represented in figure (7).

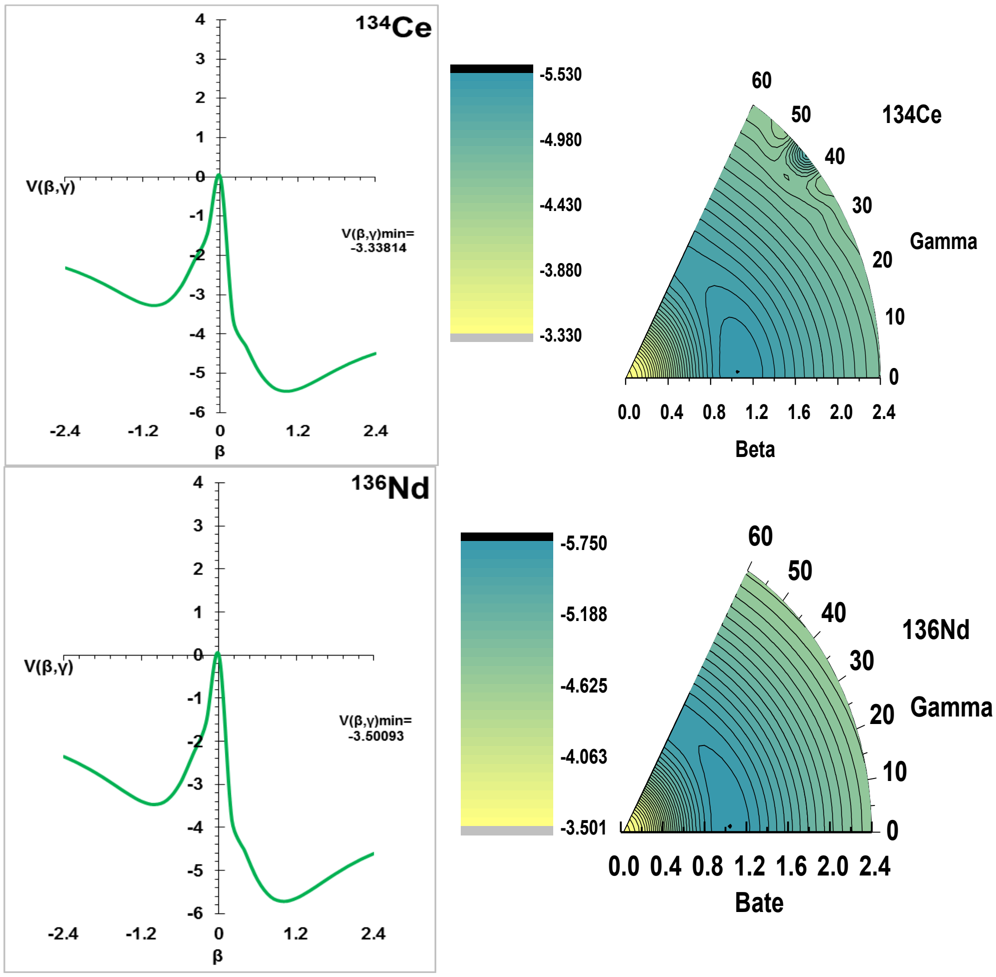



Figure 7: P.E.S as a function to $\beta$ *with contour* diagrams for even N- even Z 134Ce and 136Nd , isotones.

## 5. Discussion and Conclusions

In the current study, one of the effects of theoretical models was used to describe the low collective energy levels of atomic nuclei because to the ability of this model to study vibrational and rotational nuclei by applying the same Hamiltonian effect with a set of variables that change smoothly. The frequent updating of decay diagrams and empirical data pertaining to the 134Ce and 136Nd , isotones have been instrumental in the study of these isotopes and their rearrangement based on dynamic symmetry principles. Where we find that the minimum valye of the potential for the two134Ce and 136Nd , isotones is equal to(-3.33814,-3.50093)MeVat its value $\beta = 0$, as for the contour shapes they appear to be concentric . In this study , i concluded that each of 134Ce and 136Nd , isotones acts them as transitional nuclei between the Gamma –unstable O(6) and distortion limit SU(3), as shown fig (7) in the p. energy surface with contour diagrams .in IBM:1,a1 is the dominant, table (1). It has been shown that energy levels obtained Furthermore, the energy levels obtained in this study were found to be in acceptable agreement with empirical data. Numerous energy levels were predicted and confirmed and then compared with the practical energies of the three isotones, which were previously unknown.

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