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Jordan Higher ⋆-Left (Accordingly Right)-Centralizers on Prime Rings With Involution

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Abstract:

In this paper we introduce the concepts higher ⋆-left (accordingly right) centralizer, Jordan higher ⋆-left (accordingly right) centralizer. We prove Any J H⋆L(AR) C on prime ring R has characteristic different from 2 with involution is H⋆L(AR) C on R.

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1. Introduction

Assume R be a ring with involution, (In short ⋆-ring) the concept of prime ring was presented in [5] the concept of 2-torsion free was presented in [2].The concept of higher left centralizer on R was presented in [6]. For more information see [3,4],[7-9]. Let $\star: k \mapsto k^\star$ be an additive map of R satisfying: i)$(kq)^\star = q^\star k^\star$ and ii)$(k^\star)^\star = k$, for all $k,q \in R$. then R is called ring with involution [1]. The objective of this article specify the relation between two concepts: higher ⋆-left (accordingly right) centralizer and Jordan higher ⋆-left (accordingly right) centralizer within certain conditions.

2. Higher ⋆-Left (Accordingly Right) Centralizers on Ring with Involution

Definition 2.1: A family of additive mappings $g = (g_i)_{i\in\mathbb{N}}$ is called higher ⋆ - left (accordingly right) centralizers (abbreviation H⋆L(AR) C) of R if ∀ η, ϑ ∈ R and n∈ ℕ then

$g_n(\vartheta\eta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)$ (accordingly $g_n(\vartheta\eta) = \sum_{i=1}^{n} g_{i-1}(\vartheta^\star)\, g_i(\eta)$)

f is said to be jordan higher ⋆-left (accordingly right) centralizers (J H⋆L(AR) C) of R if

$g_n(\vartheta^2) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\vartheta^\star)$ (resp. $g_n(\vartheta^2) = \sum_{i=1}^{n} g_{i-1}(\vartheta^\star) g_i(\vartheta)$)

g is said to be Jordan higher triple ⋆-left (accordingly right) centralizers (J HT⋆L(AR) C) of R if

$g_n\vartheta\eta\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star)$

(resp. $g(\vartheta\eta\vartheta) = \sum_{i=1}^{n} g_{i-1}(\vartheta^\star)\, g_{i-1}(\eta^\star)\, g_i(\vartheta)$).

Example 2.2: Let ℤ be the ring of integers, and $R = \left\{\begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix} : u \in \mathbb{Z}\right\}$ be a ring , $g = (g_i)_{i\in\mathbb{N}}$

defined on R by

$g_n\begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3nu \\ 0 & 0 \end{pmatrix}$, for all u∈ℤ, n∈ℕ.

$\star: R \to R$ defined by $\begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix}^\star = \begin{pmatrix} 0 & 7u \\ 0 & 0 \end{pmatrix}$, for all $\begin{pmatrix} 0 & u \\ 0 & 0 \end{pmatrix} \in R$. Then g is H⋆L(AR) C on R.

Lemma 1: If $g = (g_i)_{i\in\mathbb{N}}$ is J H⋆L(AR) C on prime rings with involution R then ∀ ϑ, η ∈ R

i) $g_n(\vartheta\eta + \eta\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_i(\eta)\, g_{i-1}(\vartheta^\star)$

ii) $g_n(\vartheta\eta w + w\eta\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(w^\star) + g_i(w)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star)$

iii) If R is 2-torsion free and ϑ η=ηϑ, ∀ ϑ, η∈ R, then

$g_n(\vartheta\eta w) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(w^\star)$

Proof:i) $g_n((\vartheta+\eta)(\vartheta+\eta)) = \sum_{i=1}^{n} g_i((\vartheta+\eta)(\vartheta+\eta))$

$= \sum_{i=1}^{n} g_i(\vartheta+\eta)\, g_{i-1}(\vartheta+\eta)$

$= \sum_{i=1}^{n} g_i(\vartheta) g_{i-1}(\eta^\star) + g_i(\vartheta) g_{i-1}(\eta^\star) + g_i(\eta)\, g_{i-1}(\vartheta^\star) + g_i(\eta)\, g_{i-1}(\eta^\star)$ ... (1)

Again

$g_n((\vartheta+\eta)(\vartheta+\eta)) = g_n(\vartheta^2 + \vartheta\eta + \eta\vartheta + \eta^2)$

$= \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\vartheta^\star) + g_i(\eta)\, g_{i-1}(\eta^\star) + g_n(\vartheta\eta + \eta\vartheta)$ ...(2)

So, by (1) and (2) we have

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$g_n(\vartheta\eta+\eta\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_i(\eta)\, g_{i-1}(\vartheta^\star)$

ii) Replacing ϑ+w for ϑ in definition2.1

$g_n((\vartheta+w)\eta(\vartheta+w)) = \sum_{i=1}^{n} g_i(\vartheta+w)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star+w^\star)$
$= \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(\eta^\star) + g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(w^\star) + g_i(w)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star) + g_i(w)\, g_{i-1}(\eta^\star)\, g_{i-1}(w^\star)$ ...(3)

Again

$g_n((\vartheta+w)\eta(\vartheta+w)) = g_n(\vartheta\eta\vartheta + \vartheta\eta w + w\eta\vartheta + w\eta w)$
$= \sum_{i=1}^{n} g_i(\vartheta) g_{i-1}(\eta^\star) g_{i-1}(\vartheta^\star) + g_i(w) g_{i-1}(\eta^\star) g_{i-1}(w^\star) + g_n(\vartheta\eta w + w\eta\vartheta)$ ...(4)

By (3) and (4) we get

$g_n(\vartheta\eta w + w\eta\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(w^\star) + g_i(w)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star)$

iii) By (ii) and R is commutative 2-tortion free.

Remark 2.3: If $g = (g_i)_{i\in\mathbb{N}}$ is J H⋆L(AR) C on R with involution we define $\varphi_n(\vartheta,\eta)$ by

$\varphi_n(\vartheta,\eta) = g_n(\vartheta\eta) - \sum_{i=1}^{n} g_i(\vartheta) g_{i-1}(\eta^\star)$, $\forall\, \vartheta,\eta \in R$.

Lemma 2 : If $g = (g_i)_{i\in\mathbb{N}}$ is J H⋆L(AR) C on R with involution then $\forall\, \vartheta,\eta,w \in R$.

1) $\varphi_n(\vartheta,\eta) = -\varphi_n(\eta,\vartheta)$

2) $\varphi_n(\vartheta+w,\eta) = \varphi_n(\vartheta,\eta) + \varphi_n(w,\eta)$

3) $\varphi_n(\vartheta,\eta+w) = \varphi_n(\vartheta,\eta) + \varphi_n(\vartheta,w)$

Proof: 1) By lemma 1(i)

$g(\vartheta\eta+\eta\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_i(\eta) g_{i-1}(\vartheta^\star)$

$g(\vartheta\eta) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) = -g(\eta\vartheta) - \sum_{i=1}^{n} g_i(\eta)\, g_{i-1}(\vartheta^\star)$

$\varphi_n(\vartheta,\eta) = -\varphi_n(\eta,\vartheta)$

2) $\varphi_n(\vartheta+w,\eta) = g_n((\vartheta+w)\eta) - \sum_{i=1}^{n} g_i(\vartheta+w)\, g_{i-1}(\eta^\star)$
$= g_n(\vartheta\eta+w\eta) - \sum_{i=1}^{n}(g_i(\vartheta)+g_i(w)) g_{i-1}(\eta^\star)$
$= g_n(\vartheta\eta) + g(w\eta) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_i(w) g_{i-1}(\eta^\star)$
$= g_n(\vartheta\eta) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_n(w\eta) - \sum_{i=1}^{n} g_i(w)\, g_{i-1}(\eta^\star)$
$= \varphi_n(\vartheta,\eta) + \varphi_n(w,\eta)$

3) $\varphi_n(\vartheta,\eta+w) = g_n(\vartheta(\eta+w)) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta+w)^\star$
$= g_n(\vartheta\eta+\vartheta w) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_i(\vartheta)\, g_{i-1}(w^\star)$
$= g_n(\vartheta\eta) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star) + g_n(\vartheta w) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(w^\star)$
$= \varphi_n(\vartheta,\eta) + \varphi_n(\vartheta,w)$

Remark 2.4: Let R be a ring with involution then $g = (g_i)_{i\in\mathbb{N}}$ is H⋆L(AR) C on R iff $\varphi_n(\vartheta,\eta)=0$, $\forall\, \vartheta,\eta\in R$ and $n\in\mathbb{N}$.

3. The Main Results

Lemma 3: If $g = (g_n)_{i\in\mathbb{N}}$ is on prime ring with involution R then $\forall\, \vartheta,\eta,w\in R$ and $n\in\mathbb{N}$

$\varphi_n(\vartheta,\eta)\, g_{n-1}(w^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(\eta^\star)] = 0$

Proof: Suppose that the result satisfies $\forall p\in\mathbb{N}$, where p<n

$\varphi_p(\vartheta,\eta)\, g_{p-1}(\eta^\star)\, [g_{p-1}(\vartheta^\star), g_{p-1}(\eta^\star)] = 0$

then by using mathematical induction on n, we have.

Now if m= ϑηwηϑ + ηϑwϑη, then

$g_n(m) = g_n((\vartheta\eta w)(\eta\vartheta)) + g_n((\eta\vartheta w)(\vartheta\eta))$
$= g_n((\vartheta\eta w)(\eta\vartheta)) + g_n((\eta\vartheta w)(\vartheta\eta))$
$= \sum_{i=1}^{n} g_i(\vartheta\eta w) g_{i-1}(\eta\vartheta)^\star + g_i(\eta\vartheta w) g_{i-1}(\vartheta\eta)$
$= \sum_{i=1}^{n} g_i(\vartheta) g_{i-1}(\eta^\star) g_{i-1}(w^\star)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(\eta^\star) + g_i(\eta)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(w^\star)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star)$

$= \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{n-1}(w^\star)\, g_{n-1}(\vartheta^\star)\, g_{n-1}(\eta^\star) + \sum_{i=1}^{n-1} g_i(\vartheta) g_{i-1}(\eta^\star) g_{n-1}(w^\star) g_{n-1}(\vartheta^\star) g_{n-1}(\eta^\star)$
$+ \sum_{i=1}^{n} g_i(\eta)\, g_{i-1}(\vartheta^\star)\, g_{n-1}(w^\star)\, g_{n-1}(\eta^\star) g_{n-1}(\vartheta^\star)$
$+ \sum_{i=1}^{n-1} g_i(\eta)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(w^\star)\, g_{i-1}(\eta^\star)\, g_{n-1}(\vartheta^\star)$ ...(5)

Again

$g_n(m) = g_n((\vartheta\eta)w(\eta\vartheta) + g_n(\eta\vartheta)w(\vartheta\eta))$
$= \sum_{i=1}^{n} g_n(\vartheta\eta) g_{i-1}(w^\star) g_{i-1}(\eta\vartheta)^\star + g_i(\eta\vartheta)\, g_{i-1}(w^\star) g_{i-1}(\vartheta\eta)^\star$
$= g(\vartheta\eta)\, g_{n-1}(w^\star)\, g_{n-1}(\eta\vartheta)^\star + \sum_{i=1}^{n-1} g_i(\vartheta\eta) g_{i-1}(w^\star) g_{i-1}(\eta\vartheta)^\star$
$+ g(\eta\vartheta)\, g_{n-1}(w^\star)\, g_{n-1}(\vartheta\eta)^\star + \sum_{i=1}^{n-1} g_{i-1}(\eta\vartheta) g_{i-1}(w^\star) g_{i-1}(\vartheta\eta)^\star$ ...(6)

By comparing (5) , (6) and the assumption, we get

$0 = (g_n(\vartheta\eta) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star))\, g_{n-1}(w^\star) g_{n-1}(\vartheta^\star)\, g_{n-1}(\eta^\star) + (g(\eta\vartheta) - \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star))\, g_{n-1}(w^\star)\, g_{n-1}(\eta^\star) g_{n-1}(\vartheta^\star)$
$= \varphi_n(\vartheta,\eta) g_{n-1}(w^\star) g_{n-1}(\vartheta^\star) g_{n-1}(\eta^\star) + \varphi_n(\eta,\vartheta) g_{n-1}(w^\star)\, g_{n-1}(\eta^\star)\, g_{n-1}(\vartheta^\star)$
$= \varphi_n(\vartheta,\eta)\, g_{n-1}(w^\star) g_{n-1}(\vartheta^\star) g_{n-1}(\eta^\star) - \varphi_n(\vartheta,\eta) g_{n-1}(w^\star)\, g_{n-1}(\eta^\star)\, g_{n-1}(\vartheta^\star)$
$= \varphi_n(\vartheta,\eta)\, g_{n-1}(w^\star)\, [g_{n-1}(\vartheta^\star),\, g_{n-1}(\eta^\star)]$

Lemma 4 : Let $g = (g_i)_{i\in\mathbb{N}}$ be J H⋆L(AR) C on prime ring with involution then $\forall\, \vartheta,\eta,q_1,q,p\in R$ and $n\in\mathbb{N}$

$\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(p^\star)] = 0$

Proof: Replacing ϑ +q for ϑ in lemma 3

$\varphi_n(\vartheta+q,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star+q^\star), g_{n-1}(\eta^\star)] = 0$

$\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(\eta^\star)] + \varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(\eta^\star)] +$
$\varphi_n(q,\eta)\, g_{n-1}(q^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(\eta^\star)] + \varphi_n(q,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(\eta^\star)] = 0$

And by using lemma 3 we have

$\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(\eta^\star)] + \varphi_n(q,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(\eta^\star)] = 0$

Therefore

$0 = \varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(\eta^\star)]\, q_1\, \varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(p^\star)]$

$=-\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(\eta^\star)]\, q_1\, \varphi_n(q,\eta)\, \mathcal{T}\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(\eta^\star)]$

Since R is prime we have

$\varphi_n(\vartheta,)\, g_{n-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(\eta^\star)] = 0$

...(7)

On the other hand by replacing k+p with k in Lemma3 and using the same above argument, we get

$\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(q^\star)] = 0$ ... (8)

Now, $\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star + q^\star), g_{n-1}(\eta^\star + p^\star)] = 0$

$\varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{n-1}(\vartheta^\star), g_{n-1}(\eta^\star)] + \varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{m-1}(\vartheta^\star), g_{m-1}(p^\star)] +$
$\varphi(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{m-1}(q^\star), g_{m-1}(\eta^\star)] + \varphi_n(\vartheta,\eta)\, g_{n-1}(q_1^\star)\, [g_{m-1}(q^\star), g_{m-1}(p^\star)] = 0$

Therefore by using lemma 3, (7) and (8)

$\varphi_n(\vartheta,\eta)\, g_{m-1}(q_1^\star)\, [g_{n-1}(q^\star), g_{n-1}(p^\star)] = 0$

Theorem 5: Any J H⋆L(AR) C on prime ring R has characteristic different from 2 with involution is H⋆L(AR) C on R.

Proof: Assume that $g = (g_i)_{i\in\mathbb{N}}$ is J H⋆L(AR) C on prime ring R. So, by Lemma 4 we have $\varphi_n(\vartheta,\eta) = 0$ or $[g_{n-1}(q^\star), g_{n-1}(p^\star)] = 0$, $\forall\, \vartheta,\eta,q,p \in R$; $n \in \mathbb{N}$.

If $[g_{m-1}(q^\star), g_{n-1}(p^\star)] \neq 0$, $\forall\, q,p \in R$, n∈N, then $\varphi_n(\vartheta,\eta) = 0$, $\forall\, \vartheta,\eta \in R$; n ∈ N. So, by remark2.4 we have g is higher ⋆- left (accordingly right) centralizers on R.

If $[g_{n-1}(q^\star), g_{n-1}(p^\star)] = 0$, ∀q, p∈ R, then R is commutative ring therefore, by proposition 1(i)

$g_n(2\,\vartheta\,\eta) = 2\sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^*)$

Since R is of characteristic different from 2 therefor $\mathcal{T}$ satisfy the require result.

Proposition 6: Any J H⋆L(AR) C on characteristic different from 2 ring R with involution is JT H⋆L(AR) C on R.

Proof: Let $g = (g_i)_{i\in\mathbb{N}}$ is J H⋆L(AR) C on ring R

Linearizing by ϑη+η in proposition 1(i)

$g(q(\vartheta\eta + \eta\,\vartheta) + (\vartheta\,\eta + \eta\,\vartheta)\,\vartheta) = \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}((\vartheta\eta + \eta\,\vartheta)^\star) + g_i((\vartheta\eta + \eta\,\vartheta))\, g_{i-1}(\vartheta^\star)$

$= \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star) + g_i(\vartheta)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(\eta^\star) + g_i(\vartheta)\, g_{i-1}(\boldsymbol{\eta}^\star)\, g_{i-1}(\vartheta^\star) + g_i(\eta)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(\vartheta^\star)$

...(9)

On the other hand

$g(\vartheta(\vartheta\,\eta + \eta\,\vartheta) + (\vartheta\,\eta + \eta\vartheta)\,\vartheta) = g(\vartheta\vartheta\eta + \vartheta\eta\vartheta + \vartheta\eta\vartheta + \eta\vartheta\vartheta)$

$= \sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(\eta^*) + g_{i-1}(\eta)\, g_{i-1}(\vartheta^\star)\, g_{i-1}(\vartheta^\star) + g_n(\vartheta\eta\vartheta + \vartheta\eta\vartheta)$

...(10)

Therefore by comparing (9) and (10), we get

$g_n(2\vartheta\eta\vartheta) = 2\sum_{i=1}^{n} g_i(\vartheta)\, g_{i-1}(\eta^\star)\, g_{i-1}(\vartheta^\star)$

Since R is of characteristic different from 2 we get g is satisfying the required result.

Corollary 7: Every J H⋆L(AR) C of R with involution on characteristic different from 2 is J HT⋆L(AR) C on R.

Proof: If $g = (g_i)_{i\in\mathbb{N}}$ is H⋆L(AR) C on R so g is J H⋆L(AR) C and by Proposition 6 we have g is Jordan higher triple ⋆ - left (acco. right) centralizer on R.

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