



Reliability Assessment Methods in Geotechnical Design A Review

Zuhair Abd Hacheem^{1*}, Hayder A. Hasan^{1*}, Karim Rashid Gubashi^{1*}

¹ Department of Water Resources Engineering, College of Engineering,
Mustansiriyah University, Palestine St., PO BOX 14022, Baghdad, Iraq.

* zuhairabd@uomustansiriyah.edu.iq

Abstract

The stochastic finite element technique is one of the best tools for computational stochastic mechanics (SFEM). Static and dynamic stochastic mechanical issues can be addressed using the traditional deterministic finite element (FE) approach., geometric, and/or loading aspects using SFEM, a stochastic framework extension. The substantial care that SFEM has got over the past 10 years is partly due to the dramatic growth in computer power that has enabled efficient treatment of large-scale challenges possible. This study aims to present a the reviews of historical and recent improvements in the SFEM area, along with defining future directions and certain open concerns that the computational mechanics community will need to solve in the future. Using random finite elements (RFEM), by combining the finite element approach with spatially variable random field ground models, a reliability-based geotechnical methodology that considers weakest path failure reasons as well as spatial variability is created. The main benefit of RFEM is the capacity to accurately represent spatial heterogeneity in ground parameters, as well as the advantage to follow the weakest in the soil and mass, and allow site information to be used throughout the design process.

Keywords: Stochastic, SFEM, Finite element, RFEM, Reliability.

1-Introduction

Geotechnical engineering is one of the most important fields of civil engineering in general. It is the primary science of soil studies. Where the interrelationship between soil and foundations is studied. Soil properties are studied and classified, and important laboratory experiments that measure ideal criteria between soil and foundations are determined. The constant pursuit of researchers in order to determine the performance of structures and provide all the data necessary to make designs that simulate reality makes studies and research and a continuous escalation in order to reach the best. There are many studies and research on the reliability of evaluating the methods of geotechnical design. In order to study the problems facing designers and engineers and to be able to reach the best analysis between design and RBD tools.

The design based on reliability is the most important and most accurate engineering approach today. Where the possibilities expected to occur during the design or the tangible physical condition of the materials for which it is designed are taken into account. It is also more compatible with the code and conforms to the required limits. Reliability studies help specialists and scholars to perform accurate design calculations. The reliability-based design



aims to study the behavior of complex materials and analyze the principles of uncertainty resulting from many different laboratory experiments.

2-Literature Review

The researchers prepared a Reliability-Based Design (RBD) diagram with the aim of separating the geotechnical design part from the analysis part and thus making it easier to perform a reliability-based design by the laboratories. In this paper, many problems that occur in reliability-based design have been studied. Uncertainty is one of the major problems facing design. With this study, researchers were able to move to a higher degree in the geotechnical engineering zone.

The study proved that the necessary statistical information must be available to study all the previously mentioned parts. These factors helped to obtain accurate real values, accuracy in the values of the equations, and the correctness of their calculations. The R language was used to make calculations and statistical analyzes. Which helped in accuracy and save a lot of time and effort. [1]

The researchers studied the role of reliability analysis in decision-making. Whereas uncertainty and its clear impact on reliability were studied. Many studies have been conducted to find out the appropriate approach to studying the role of uncertainty. It has been approved to give examples of marine geotechnical design in a comprehensive way for improving the design and work to become familiar with everything related to the subject. What is clear is that there are many sources of uncertainty.

But reducing it helps improve reliability. The analyses showed that the problems facing the reliability analysis are as follows:

- field measurement software
- Site exploration plans
- Update site characteristics and performance estimates with new data
- Provide guidelines for creating reliability-based design codes
- Comparing the reliability of different structures or design alternatives

More research has helped to highlight the benefits that accrue from the use of reliability methods. [2]

Geotechnical engineering is affected by two important factors, risk, and uncertainty. Reliability-based optimization is one of the design methods, taking into account the above two things in order to obtain an economical design at a smaller cost. This was evident through this study, which seeks to use Reliability-based optimization in order to reliably control the cost. This integration was done using MATLAB, fmincon in order to get the requirements of the geotechnical design and the results proved to be accurate. The results also showed that the success of the method of combining reliability assessment and cost minimization. [3]

As mentioned earlier in the geotechnical design, the principle of uncertainty and risks must be taken into account. Therefore, it is necessary to resort to different methods for a comprehensive study of the matter. One of these techniques is the Finite Element Model

(FEM) in order to study the variable factors and their relationship to reliability. The combination of PLAXIS 2D and Probabilistic tool-kit (PTK) was also studied.

All this for the purpose of analysing reliability and indicators indicating it in an easy way and using different technical programs to study the matter from different sides. The results proved that these studies and techniques helped to design the geotechnical structures well. These techniques help to combine more than one failure mechanism into the same model. The use of PTK helped to collect all the reliability methods with different possibilities. This technique is simple and smooth for everyone who deals with it. [4]

The problems and obstacles facing the geological design are still a pure study and analysis that researchers seek to solve and make them accurately reflect the reality. Structure's reliability analysis is the password or the black box that contains all the details with which the improvement takes place. A two-pronged study was conducted. The first axis is a study of the challenges facing the improvement of the geotechnical designs of the old traditional structures. Where uncertainty has been carefully studied and its impact on geotechnical designs, with a study of its statistics. The second axis is a study of the challenges facing structural designs in geotechnical engineering in general. Where it was sought to develop the design by re-evaluating it for reliability through additional notes that are added upon review. All for the purpose of obtaining more flexible designs while developing the concept of performance design.

This study proved that with the availability of a large database, important geological information can be extracted to solve many problems, and this has been tested. A step has also been taken to switch from static reliability analysis to dynamic reliability analysis based on the amount of data available. Further studies of the PBD are recommended for the importance of facilities management with the importance of knowledge of societal requirements and continuous evaluation of the performance of structures with great care for good design. [5]

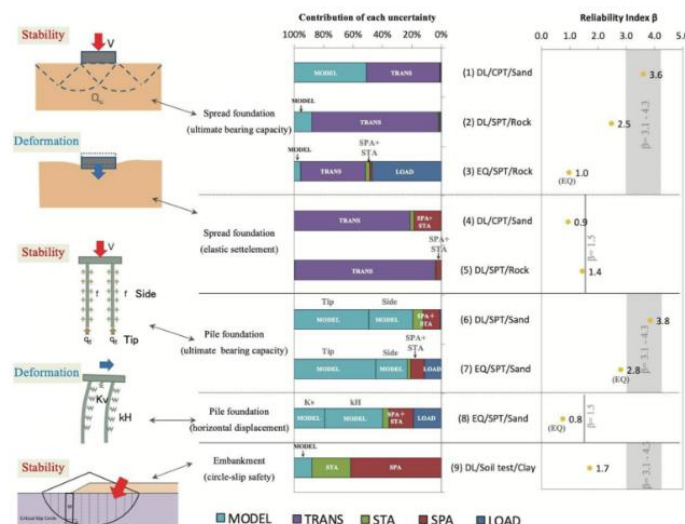


Figure 1. Brief of the contributions for the uncertainty sources to geotechnical structures [5]

3-Methods

3.1-The finite element method

The finite element technique makes it simple to simulate issues with spatially varying attributes, which is one of its many advantages. For instance, a particular soil deposit may be composed of layers with various levels of permeability, in which rows of elements may be given different features. By analysing issues where each factor in the mesh has a special feature depending on some underlying statistical distribution, the RFEM pushes this capacity to its maximum limit. Smith and Griffiths provide a comprehensive description of the finite element method used in the RFEM[6].

To correctly portray the ground, random fields are employed, allowing the properties of the ground to fluctuate spatially as they do in nature. The normal distribution is used in the easiest random field models. This is due to how straightforward it is to simulate and use the multi-variate normal distribution. A normal random field has a mean μ , a variance σ^2 , a correlation structure, and a variance of 2. When a trend has been discovered at the location being modelled, it is appropriate to allow the mean to fluctuate spatially, $\mu(x)$.

the variance could also change spatially, $\sigma^2(x)$, albeit this is rarely used because it would take a very thorough site examination to even fairly estimate the variance trend.

Typically, this variance is regarded as stationary, or steady across time. The correlation formation of random field models is the most challenging to comprehend and measure. Points near to one another will have similar traits, whereas points far apart could have rather different qualities. This is the purpose of a correlation structure: to give the random field some "persistence." Since real soils often also have a tendency to be in similar features at neighbouring locations and less comparable properties farther away, this characteristic of random fields is what makes it a actual soil form. The most difficult model of random field models to be understood and measured is their correlation formation. The purpose of a correlation formation is to give the random field some "persistence"; points that are close to one another will have similar features, whereas those that are far apart may have quite distinct characteristics. This feature of random criteria is what makes it a realistic soil mode because real soils typically have similar properties at local areas and less comparable qualities at increasing separations. Even with a big data set, it is unfortunately exceedingly challenging to predict the correlation structure of a soil. Because of this, correlation structures utilised in practise are frequently relatively straightforward and nearly never require more than one parameter. The Markov correlation function is one of the most basic and popular correlation structures,

$$\rho(\tau) = e^{\frac{-2|\tau|}{\theta}} \quad (1)$$

This displays the correlation coefficient for two sites that are geographically apart. The only parameter is the length of correlation which often directs to the scale of volatility. Beyond these two locations in the field, it is about the distance between them, and they are mostly uncorrelated (which distributes normally, also means large independence). By collecting data at a sequence of n evenly spaced locations along a line and fitting Eq. 1 to the observed correlation function, it is possible to effectively determine the (radial) correlation length.

$$\hat{p} = \frac{1}{(n-j-1)\sigma^2} \sum_{i=1}^{n-j} (X_i - \widehat{\mu_x})(X_{i+j} - \widehat{\mu_x}) \quad (2)$$

Once the theoretical nature of the random field has been established, the subsequent stage in RFEM is the modelling of versions of the arbitrary field. The Local Average Sub (LAS) approach is used to construct constructions of the local averages of the random field. [7], with each local average being obtained across an area that is the identical size as the finite elements it is subsequently mapped to. The authors have chosen to use the finite element approach in combination with their method despite the fact that there are several possible simulation methodologies accessible (see, for example, Fenton, 1994). Combining finite element analysis with local averages of ground characteristics has a number of important benefits.

1. Bounded factors are essentially continuous approximations of the material they show, with different assumptions being made to simplify the internal strain field. As an illustration, if loads or displacements are applied at the nodes and the material properties inside the element remain consistent, the form functions of a 4-node quadrilateral element will be correct. The arbitrary field's average value over the element domain should be provided as a constant attribute.
2. According to statistical theory, the statistics of local averages vary as the size of the averaging domain does (mean and variance). Adopting a local average random field is similarly compatible with the finite element technique since both the FE and LAS models' representations progress smoothly toward the point-wise changing random field as the element (averaging domain) grows smaller.
3. In any case, the bulk of ground attributes are measured using regional average. For particular, the flow through a particular volume of permeable material is usually always measured in order to assess hydraulic conductivity. This surely represents an average of some kind. At the atomic level, hydraulic conductivity is not measurable. These physical measures show a decrease in variation with sample volume, just like the local averages of a random field. As a result, there is agreement between regional averages and actual measurements of ground characteristics. The fundamental concept of Local Average Subdivision is depicted in Figure 2 (for more details). [8]. The method randomly creates a local average for the all field before going on to the next iteration. Z01 with the must of statistics for an average of that dimension. Then, the field is spirited into equal pieces, and the local averages and Z11 are repeated in a way that guarantees they have the proper average statistics, are correctly connected to one another, and average to the value



Z_0^1 Repeating the process will cause the field's size to steadily shrink until the required resolution is attained.

Stage0	z_1^0							
Stage1	z_1^1				z_2^1			
Stage2	z_1^2		z_2^2		z_3^2		z_4^2	
Stage3	z_1^3	z_2^3	z_3^3	z_4^3	z_5^3	z_6^3	z_7^3	z_8^3
Stage4								

Figure 2. Top-down LAS construction of a local average random process [8]

To assess probabilities and statistics of the responding for Monte Carlo representation generally requires creating a realisation of the spatially varying ground parameters at random, using finite elements to analyse the geotechnical system's response, and then repeatedly doing so. How many realisations must be made before probability may be estimated, such the probability of failure p_f , to an acceptable level of precision, is a crucial concern that emerges. By realising that each realisation is a Bernoulli random variable that either succeeds or fails, this question may be answered quite simply. The likelihood estimate's standard deviation,

$$\sigma_f \cong \sqrt{\frac{p_f q_f}{n}} \quad (3)$$

Generally speaking, if the confidence max level allowable error on p_f is

$$e = 1 - \alpha \quad (4)$$

The number which is required for realizations to reach the accurate value is ,

$$n = p_f q_f \left(\frac{Z_\alpha}{2/e} \right)^2 \quad (5)$$

3.2-Ground-water modelling

Laplace's equation is used to model continuous groundwater flow in this instance,

$$\nabla [K \nabla \phi] = 0 \quad (6)$$

The use of the Method of Fragments [9] is growing. Conventional approaches can account for anisotropic characteristics and stratification, but they are deterministic in that they assume uniform (everywhere the same) soil permeability.

A more sensible way to modelling clay is to assume that the permeability of the clay under a pattern, as shown in Figure 3, is random—that is, that the soil is a domain that is "random," with an actually imply, standard deviation, and some sort of correlation formation Although higher joint moments are theoretically feasible, only the first two joint moments (average and co structure) are routinely computed efficiently. [10]

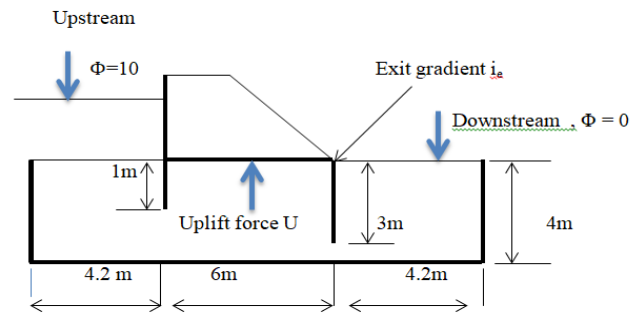


Figure 3: Problem with the boundary value of confined seepage. The hashed boundaries and two vertical walls are taken to be impenetrable [10]

Analytical solutions to the stochastic flow problem shown in Figure 3 are not even remotely feasible (or would be useless due to the necessary simplifying assumption). The RFEM outlined above makes it simple to determine the probabilities connected to flow, uplift, and exit gradients. The simulated field of permeabilities is mapped using the finite element mesh., and the boundary restrictions for the potential and stream functions are specified. In a finite element setting, the nodal potential values across the mesh are determined completing a series of linear "equilibrium" formulas derived from the governing elliptic equation for steady flow (Laplace).

4-Stochastic Flow Models

4.1-Uncertainty Model

4.1.1-Gaussian stochastic process and field simulation techniques

Because it is straightforward and there are few relevant experimental data, the gaussian considerations is widely used in engineering systems even if the huge of the unknown variables have non-Gaussian characteristics (– for example, substances, dimensional properties, wind, and seismic stresses). The central boundary theorem naturally causes gaussian random fields to appear in applications because when information on the second-order moments is all that is provided, they represent the model of maximum entropy [11].

The two most frequently used techniques for simulating stochastic treatment and domains are the spectral representation approach and the (KL) expansion [12]. A thorough approach to developing techniques for simulating Gaussian random fields, that includes simulating of the spectral and also K-L expansion, were proposed [13].

4.1.2-Method of spectral representation

Trigonometric functions with nonlinear phase angles and amplitudes are used to depict the stochastic field spectrally in the general case. Since it produces sample functions with ergodic mean values and autocorrelations, the version with just random phase angles is used in the majority of cases [14]. As a result, the amplitudes are deterministic and totally reliant on the spectra of the stochastic field: The amplitudes are then predictable and solely depend on the stochastic field's predetermined power spectrum.:

$$\hat{f} = \sum_{n=0}^{N-1} A_n \cos (k_n x + \phi_n^{(i)}) \quad (7)$$



Where :

$$A_n \sqrt{2S_{ff}(k_n)\Delta k}, \Delta k = nk \text{ and } n = 0, 1, 2, \dots, N-1$$

$$A_0 = 0 \text{ or } S_{ff}(k = 0) = 0$$

K_u is the active region

$$\Delta k \rightarrow 0 \text{ as } N \rightarrow \infty$$

$$\phi_n \in [0, 2\pi]$$

The characteristics of each sample function provided by Eq. (7) are as follows [15]:

1. A Gauss stochastic domain is t. asymptotically as $N \rightarrow \infty$
2. t has the same mean value and autocorrelation as the associated objectives $N \rightarrow \infty$
3. In the case of $A_0 = 0$ or $S_{ff}(k = 0) = 0$, it can be presented that f^i is periodic with period $T_0 = 2\pi/\Delta k$.

The idea of the evolutionary power spectrum works as the foundation for the representing for non-stochastic area [16]. By Using the Fourier transform, the computing costing of creating homogeneous Gaussian model functions digitally can be significantly decreased (FFT). The use of spectral representation is advantageous in the approximation of non-Gaussian domains. There are different methods that make use of the translation idea to present model functions of the underlying Gaussian domain [17]. Additionally, the stochastic finite element method has been successfully applied to real-world problems using spectral representation inside the context of Monte Carlo simulation (MCS). [18]

4.1.3-The K-L Method

Since the auto covariance formula serves as the kernel (covariance decomposition) in the K-L expansion is a particular case of the orthogonal series expansion and is a second order Fredholm integral formula. [19]

$$\hat{f}(x) = \bar{f}(x) + \sum_1^N \sqrt{\gamma_n} \zeta_n \phi_n \quad (8)$$

$$\int C_{ff}(x_1, x_2) \phi_n(x_1) dx = \gamma_n \phi_n(x_2) \quad (9)$$

Where:

$\bar{f}(x)$ is field mean and $= 0$ most of time .

$\int C_{ff}(x_1, x_2)$ are the auto covariance function's eigenvalues and Eigen functions of γ_n and ϕ_n .

N represents the K-L terms number .

The K-L expansions provides a coherent modelling framework for both homogeneous and heterogeneous stochastic domains, notwithstanding certain difficulties with the uniformity of the sample functions that were created [19]. According to the fact that only a small number of terms, which correspond to the N larger eigenvalues, may adequately reflect the majority of the field's random fluctuations, Strongly connected stochastic fields may be represented using it particularly well.. (shown in Figure 4) .

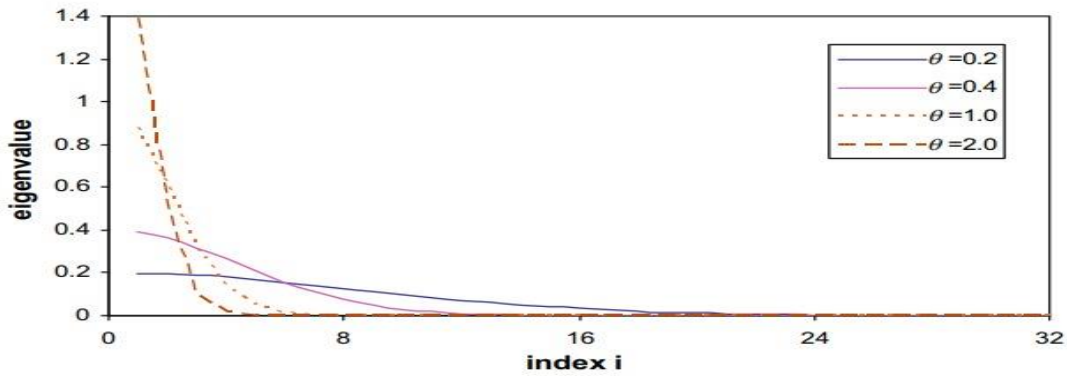


Figure 4. Entropy decay for the size of the change in the K-L expand for θ shown [20]

Although theoretically important, K-L expansion is typically not used in practise because of how challenging it is to solve the Fredholm integral formula . Since the only known analytic solutions to this integral equation are the auto covariance function and simple geometries, real-world problems involving complex domains necessitate a particular numerical strategy. The essential equations for the dense matrices generated by these numerical techniques [21] are often quite expensive to compute. By using just N (20) terms in Eqs. (7) and (8), it is demonstrated how the K-L growth and spectral representation differ from one another. This shows that the K-L growth may make it simpler to generate strongly linked random domains with a smooth auto covariance function (Figure 5). But when more words are retained, spectral approximation performs better. Because of the central limit theorem, the spectral representation technique typically requires a lowest amount of 128 terms for N to guarantee some degree of convergence to gaussianity.. Finally, the K-L series' specimen functions' uniformity and monotonicity are contested, and its processing efficiency is inferior to that of spectral approximation.

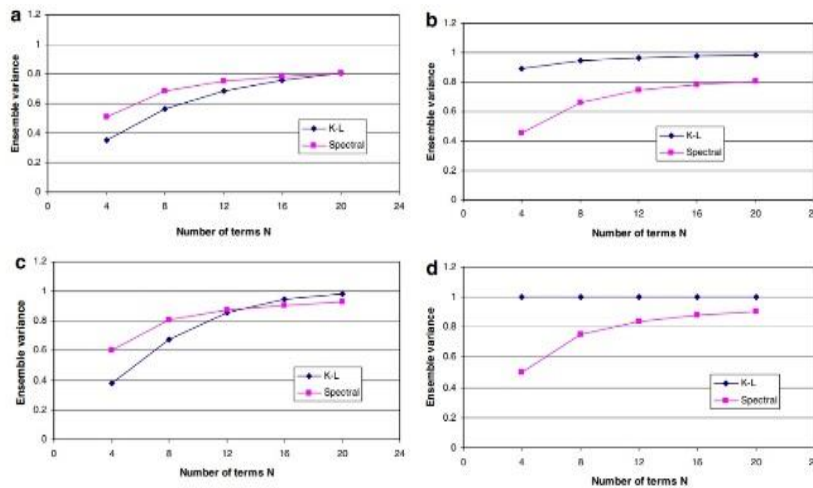


Figure 5. As a function of N retained terms, K-L growth to the target range and completion of the harmonic approach are shown in Equations (7) and (8).) [22].

4.1.4-Method of Stochastic finite element (SFEM)

It may be seen the difference between the K-L expansion and the spectral approximation by using the few terms N (20) in Eqs. (7) and (8). This shows that the K-L expansion may make it simpler to simulate efficiently correlated stochastic domains with a soft auto covariance (Figure 5). But when more terms are retained, spectral representation performs better. Regularly calls for the harmonic modeling technique. It requires a minimum of 128 elements for N in order to ensure some range of convergence to Gaussianity using the central limit theorem. The last thing can be mentioned here, the K-L series' sample functions' homogeneity and ergodicity are contested, and its computing efficiency is inferior to that of spectral representation [23].

The perturbation technique, which is based on a Taylor series growth of the response vector, and the sinusoidal stochastic finite element approach, in which each response quantity is represented using a series of random Hermite polynomials, are the two main implementations of SFEM in the literature. For these two versions, another choice is Modeling Tool [24]. A determinate issue is solved (many times) using the MCS method, and the solution heterogeneity is computed using straightforward statistical connections. MCS is frequently used in the literature as a reference technique to assess the correctness of other approaches due to its robustness and simplicity, and it is occasionally paired with the two SFEM variants. The generation of the stochastic matrices (first for each individual component, then for the entire system), the partial differential of the random domains that represent the ambiguous system properties, and ultimately the estimation of response variability (response statistics). These acts' computational components are detailed in the sections that follow:

Discretization Of Stochastic Field And Process: The stochastic process is discretized as the first fundamental step in SFEM. The methods/fields that represent the hazy mechanical and qualities of a geometric system. The process of discretizing a continuous stochastic field involves replacing it with a non-uniform vector generated of a finite number of non-uniform variables :

$$f(x) \rightarrow \hat{f}(x) = \{ f_i \} \quad (10)$$

Two broad categories of discretization techniques can be distinguished in the literature [48] :

1. The values of the stochastic domains at certain locations in the system are the sole final random variables when using point discretization techniques.
2. Finite difference techniques of the average kind, in which the probabilities are represented as (valued) integrals of the stochastic field over all numerical techniques. The interpolation, midpoint, projection, integration, and techniques make up the majority of the first group's contributors.

The Construction of the Random Finite Element Matrix: The random matrix, which has the following form in the case of random huge changes in Tensile properties caused by a zero mean, uniform stochastic ground $f(x, y, z)$ (e), is created using the variational random domains for each finite element:

$$K(e) = k_0^{(e)} + \Delta k^{(e)} \quad (11)$$

Where $\bar{k}_0^{(e)}$ and $\Delta k^{(e)}$ are the stochastic finite element matrix's mean (deterministic) and fluctuating elements.

5-Other Methods

5.1-The Turning Bands Method (TBM)

To imitate random areas in two or more dimensions, the TBM places a succession of 1D area along lines that span the area. Understanding of the 1D autocorrelation function $R_1(\zeta)$ is required for the formulation of TBM (n). If this function is known, one or more effective 1D algorithms can be used to generate the line fields [22]. The geometry of the multivariate association $R_n(\zeta)$ is reflected over the ensemble thanks to the selection of the autocorrelation function $R_1(\zeta)$ as shown in Figure (6).

By comparing how closely the target autocorrelation function specified by the user and the produced example features and the related example autocorrelation function coincide, such time-series production success is commonly evaluated..

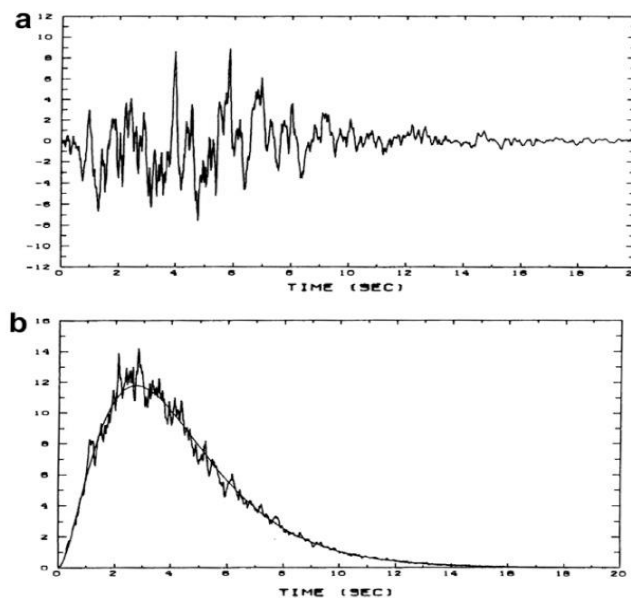


Figure 6. (a) An example of an ARMA model-generated non-stationary process function (b) The generated sample was used to determine the goal's (non-discert line) and sexample's (harmonic line) autocorrelation formulas [22]

5.2-The Autoregressive Moving average Method (ARMA)

The creation of specific coefficients linking a Gaussian white noise process with the process to be replicated is made possible by the use of recursive formulations in the ARMA-AR frames, allowing for the synthesis of fixed and quasi random variables. [25].

5.3-The Optimal linear Estimation Method (OLE)

The OLE approach, which was first presented in [26], is also known as the Kriging approximation. The stagnation on linearly functional approach is used in this situation. A linear function of dimensional values,

$f = f(x_1), \dots, f(x_n)$, defines the approximated field $f(x)$ in the inclusion of OLE as follows:

$$f(x) = \alpha(x) + \sum_{n=1}^N b(x)_n f_n = \alpha(x) + bT(x)f \quad (12)$$

where :

$\alpha(x)$ and $b(x)_n$ are defined by the minimum value of each point x with error $\text{Var} [f(x) - \Delta f(x)]$

5.4-The Expansion linearly Estimation Method (ELOE)

The defect for a specific number of maintained words is ascertained using the EOLE technique, an extend of OLE [7]. Furthermore, the K-L point-wise variation failure approximation is less than the EOLE failure within the normalisation domain but bigger at the frontiers for a given order of expansion. However, the K-L technique has the lowest mean defect throughout the whole domain..

6-Non-Stochastic Flow

Designed to simulate non-Gaussian system dynamics and domains has received a lot of interest recently in stochastic mechanics. This is due to the non-Gaussian probabilistic elements of a variety of parameters that are present in engineering problems that arise in the real world, including material, structural properties, soil characteristics, winds, waves, and earthquake stresses. For stochastic systems devoid of spectral distributions, upper limits on the responding difference can be estimated using non-Gaussian domains [27]. It is now widely accepted that simulating up skewing narrow-banding random processes and areas can be used to test the effectiveness of current simulation methods [28].

6.1-Simulation Methods of Non- Stochastic Flow

Simulation approximations for non-Gaussian random domains processes fall into two main types . There are two different kinds of sample functions: those that aim to produce specimen functions that are compatible with all currently known probabilistic information, as well as those that attempt to generate sample functions, for example, that match a target random field's required spectral density power function and lower-order statistics (differences, standard error, variance, and kurtosis) [29]. The first group of strategies is suited for modelling wind and wave loads and will yield accurate results for the random response by constructing non-Gaussian example functions in line with specified lower-order moments. [29].

However, example functions with only the specified smaller moments are insufficient for solving issues where correct distribution tail characterisation is crucial (for example, soil liquefaction [30]). This is because a non-Gaussian field that is exclusively defined by its smaller -order moments may not be uniquely realised according to the marginal probability distribution of those realisations. According to studies, the chance of soil liquefaction greatly depends on the tails of the cumulative random variable of the randomly selected field data used in the study.. According to [30], inverse relation estimations will produce noticeably different amounts of observed soil liquefaction, even when their lower order moments are comparable..

6.1.1-Techniques for deformation in similarity

Changing domains The techniques in the 2nd type are more difficult since they aim to provide sample functions that are compatible with all available probabilistic data, including the random field's and negative estimation . gerate sample functions that are compatible with all available probabilistic information, specifically the stochastic dynamical field's SDF and the marginal probability distribution [31].

All of these methods rely on a nonlinear memoryless transformation accordinf to the formula :

$$f(x) = F^{-1} \phi[g(x)] \quad (13)$$

6.1.2-Extensions to the Translation Field Methods

These Methods are used at the case with the following conditions [32]:

1-an iterative process where the underlying Gaussian stochastic field of g's SDF is repeatedly updated g(x)

2-random field f(x) with the specified F and $S(k)^T_{ff}$ is generated via an extended empirical uniform to non-Gaussian mapping.

The Equation is used to Extension as follow :

$$S_{gg}^{(j+1)}(k) = \frac{S_{gg}^{(j)}(k)}{S_{ff}^{(j)}(k)} S_{ff}^T(k) \quad (14)$$

This technique yields unexpectedly good results for fields with broad-banded SDFs that are slightly non-Gaussian. Deodatis and Micaletti point out that there is a bounded with regards to the modelling of steeply skewing random fields [33]. The non-gaussian example formulas that are produced in this scenario have the works as SDF, but their random probability density function (PDF) deviates highly from the needed one (Figure 7). This restriction results from the unique format of Eq's updating formula (13). The main issue is that, for reasons that are fully explained in, the underlying gaussian area is no longer uniform and homogenous after the first fame.

Another attempt with an algorithm with the same structure as Yamazaki and Shinozuka's was proposed by Deodatis and Micaletti [34], but with the following improvements:

$$S_{gg}^{(j+1)}(k) = \left[\frac{S_{gg}^{(j)}(k)}{S_{ff}^{(j)}(k)} \right]^\alpha S_{ff}^T(k) \quad (15)$$

The authors came to the conclusion that a result = 0.3 provides the best outcomes in terms of uniformity after conducting considerable numerical experiments.

1. random random extended empirical mapping,

$$f(x) = F^{-1} \cdot F^*[g(x)] \quad (16)$$

2. Frequency shifting is used to get around some convergence problems that appear around $k=0$.

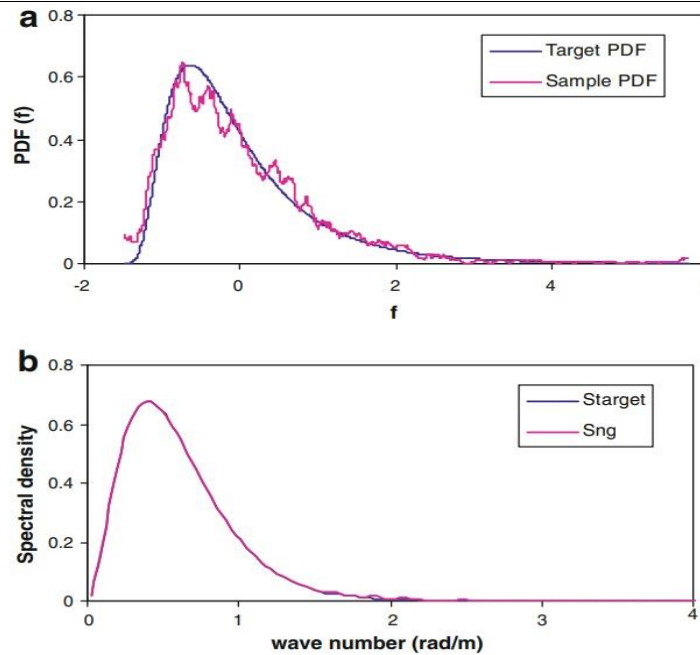


Figure 7. Using the Yamazaki- Shinozuka algorithm [34]

6.1.3-Random Media Simulation

An 1D binary simulating a 2-phase functionally graded composite matter was effectively implemented using a changing example for not a stationary, non-gaussian random treatment that was proposed in [35]. However, because the criteria of positively determinations is frequently not met, translation models are frequently insufficient to adequately characterise the micro-struct features of non uniform media [36].

Koutsourelakis and Deodatis [37] recently presented an alternate methodology for the simulating of double random areas according to their stipe auto correlation function. Essentially, it is composed of two elements. An algorithm for getting examples of a binary field from a nonlinear transforming with memory of a uniform field is introduced in the first section. The underlying uniform domain's probabilistic properties are determined using an iterative process in the second stage, resulting in a binary area with a predetermined auto correlation function. The approach can be used in a variety of contexts and has a low computing cost.

6.1.4-Polynomial Chaos Expansion Method

To create sample functions of quasi-non-stationary random variables in accordance with their suggested (quasi) insignificant PDF and correlation function, Sakamoto and Ghanem [38] provided an alternate methodology. This uses traditional polynomial complexity (PC) decomposition to overattribute the non-Gaussian process at certain places.

$$\mu(x) = \sum_{j=0}^p \mu_j(x) \psi_j \quad (17)$$

By utilising a more all-encompassing PC framework termed Askey chaos, Xiu and Karniadakis [39] proposed an ideal description of different distributing patterns. They specifically offered a new approach to Wiener's polynomial chaos and Galerkin projections for the solution of uniform differential framework. This framework uses the optimal as an

attempt to basis from the Askey family of orthogonal polynomials to represent stochastic processes , that causes the system's dimensions to be reduced and the error to exponentially converge:

$$\mu(x) = \sum_{j=0}^p \mu_j(x) \phi_j \quad (18)$$

Where , $\{\phi_j\}_{j=0}^p$ P is the value of the number of generalised PC expansion terms, and signifies a collection of (non-Gaussian) random variables.

7-Other Methods For Non-Stochastic Flow

A novel spectral depiction model has recently been created for the simulating of a class of non-Gaussian processes [40]. The 2nd moment characteristics and other second - order moments of any quasi process may be fit by the model, which is based on the spectral reproduction theorem for weakly stable processes. The model may be applied to simulations by Monte Carlo and analytical studies to determine how linear and nonlinear systems react to quasi noise. It consists of a superposition of harmonics with independent, randomly varying amplitudes [41].

Elishakoff L. [42] proposed a conditional simulating approach for quasi stochastic field. It was a conditional stochastic field expansion of Yamazaki's [43] unconditional simulation method. Their approach consists on doing sample simulations in the Gaussian stochastic field after converting random variables into uniform ones without taking into account correlation. Then, calculations are repeated until the correlation constants between samples that are converted to a specific random area converges to the desired value. Numerical simulations are the only technique utilised to validate this tactic. An improved simulating of the Elishakoff technique is offered by Hoshiya [44] for the simulation with limited translation stochastic domains . This technique has an efficient formulation , theoretically.

8-Conclusion

The article's goals were to offer an analysis of recent and ongoing breakthroughs in the SFEM, REEM, and other domains, to recommend future directions, and to address certain open issues that the engineering community should consider going forward::

1. A list of precise and effective simulation methods has been provided for Gaussian and some quasi nonlinearities and fields, including severely slanted quasi scalar processes. For the dependability evaluation of uncertain physical systems using MCS, the procedures based on the interpretation zone idea hold great promise since they combine accuracy and processing efficiency with a variety of traits (analytical calculation of crossing rates and extreme value distributions). It has also been noted how crucial it is to provide effective methods for modelling ou pas vector processes and fields.
2. The perturbation technique, MCS and its modifications, and SSFEM—the three most important alternative SFE analysis formulations—have all been thoroughly examined and summarised. In-depth discussion has also been presented to the topics of the producing of the uniform finite element matrices and the discretization of the uniform fields reflecting the uncertain system features. For significant time savings in computing



for large-scale applications, the ability to combine two different meshes with a broad method of the random field realisation onto the grid looks to be crucial.

3. Applying SFEM successfully to situations requiring temporal dependency, stochastic inverse issues, and nonlinear issues is still a challenge. MCS is the only method that can be used universally to solve SFE issues of this complexity without incurring prohibitive processing costs. SSFEM is a capable replacement in some circumstances with space for improvement.
4. To boost SFEM's acceptance by the scientific community, the theoretical underpinning must be strengthened by in-depth evidence of convergence properties and error estimation studies.
5. The previously mentioned non uniform Finite Element Method provides such a means. The combination of random area modelling and the finite technique decreases model defect by many correctly mimicking the ground's different sorts and by allowing defect to occur where it "wants to" naturally. There are several alternative probabilistic methods. Additionally, the tool is easily adapted to look into how site emplaning plans impact as generated dependability, which improves the potential to develop reliability-based geotechnical design standards.
6. Understanding the earth's spatial variability is one of the key challenges in assessing a geotechnical system's dependability. To precisely measure the characteristics of the spatial variability, site sampling is required over a somewhat big area (such as the correlation length). Fortunately, it appears that the worst-case scenario of spatial variability is commonly present and can be used to generate cautious designs. It is still unknown how much it would cost to construct geotechnical systems using this "worst case" correlation length, and the price might be higher or lower than the cost of the additional study required to estimate the correlation length. It will take a lot of time to decide this issue.
7. By creating dependable and efficient solution approaches appropriate for a parallel treatment surrounding properly make up to order the particular problem at hand, SFEM's potential will be even further boosted. The potential of SFEM will be further increased by the improvement of reliable and effective solution methodologies appropriate for a co-linear processing environment suitably configured to address the specific issue at hand.
8. It is also critical to develop user-friendly, specialised SFEM software that can cooperate with powerful external codes and resolve enormous stochastic problems in a reasonable amount of time..
9. The figures presented above still represent the "average location," making them quite generic. They are important because they educate the geotechnical community examines the fundamental probabilistic behaviour of geotechnical systems, in particular how spatial variability influences the likelihood of failure. Future studies should concentrate on site-specific behaviour: Sites frequently include numerous layers, are devoid of isotropic correlation patterns, and may not be properly modelled by a single spatially



varying random field. Conceptually, it is simple to model a specific site using the above-discussed methods, but not all issues have the required computer models constructed. These improvements still need to be made, but they are progressing.

9-References

1. Otake, Y. and Y. Honjo, Challenges in geotechnical design revealed by reliability assessment: Review and future perspectives. *Soils and Foundations*, 2022. **62**(3): p. 101129.
2. Hu, B., et al., Characterizing uncertainty in geotechnical design of energy piles based on Bayesian theorem. *Acta Geotechnica*, 2022. **17**(9): p. 4191-4206.
3. Mahmood, Z., Reliability-based optimization of geotechnical design using a constrained optimization technique. *SN Applied Sciences*, 2020. **2**(2): p. 168.
4. Sudret, B. and A. Der Kiureghian, Stochastic finite element methods and reliability: a state-of-the-art report. 2000, Berkeley, California: Department of Civil and Environmental Engineering, University of California.
5. Phoon, K.-K., Reliability-based design in geotechnical engineering: computations and applications. 2008, NY, USA: CRC Press.
6. Honjo, Y., Challenges in geotechnical reliability based design. *Geotechnical Safety and Risk. ISGSR 2011*, 2011: p. 11-28.
7. Fenton, G.A., Error evaluation of three random-field generators. *Journal of engineering mechanics*, 1994. **120**(12): p. 2478-2497.
8. Fenton, G.A. and E.H. Vanmarcke, Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics*, 1990. **116**(8): p. 1733-1749.
9. Law, A.M., W.D. Kelton, and W.D. Kelton, Simulation modeling and analysis. 5th ed. Vol. 3. 2007, New York: Mcgraw-hill.
10. Fenton, G.A. and D.V. Griffiths, Statistics of block conductivity through a simple bounded stochastic medium. *Water Resources Research*, 1993. **29**(6): p. 1825-1830.
11. Spanos, P.D. and B.A. Zeldin, Monte Carlo Treatment of Random Fields: A Broad Perspective. *Applied Mechanics Reviews*, 1998. **51**(3): p. 219-237.
12. Shinozuka, M. and G. Deodatis, Simulation of Stochastic Processes by Spectral Representation. *Applied Mechanics Reviews*, 1991. **44**(4): p. 191-204.
13. Poirion, F. and B. Puig. A unified approach for generating Gaussian random field simulation methods. in *Proceedings of Ninth International Conference on Structural Safety Reliability (ICOSSAR 2005)*, Rome, Italy. 2005.
14. Grigoriu, M., On the spectral representation method in simulation. *Probabilistic Engineering Mechanics*, 1993. **8**(2): p. 75-90.
15. Deodatis, G., Simulation of Ergodic Multivariate Stochastic Processes. *Journal of Engineering Mechanics*, 1996. **122**(8): p. 778-787.



16. Field, R.V. and M. Grigoriu, On the accuracy of the polynomial chaos approximation. Probabilistic Engineering Mechanics, 2004. **19**(1): p. 65-80.
17. Matthies, H.G. and C. Bucher, Finite elements for stochastic media problems. Computer Methods in Applied Mechanics and Engineering, 1999. **168**(1): p. 3-17.
18. Witteveen, J.A.S. and H. Bijl, An alternative unsteady adaptive stochastic finite elements formulation based on interpolation at constant phase. Computer Methods in Applied Mechanics and Engineering, 2008. **198**(3): p. 578-591.
19. Zhang, J. and B. Ellingwood, Orthogonal Series Expansions of Random Fields in Reliability Analysis. Journal of Engineering Mechanics, 1994. **120**(12): p. 2660-2677.
20. Stefanou, G. and M. Papadrakakis, Assessment of spectral representation and Karhunen–Loève expansion methods for the simulation of Gaussian stochastic fields. Computer Methods in Applied Mechanics and Engineering, 2007. **196**(21): p. 2465-2477.
21. Grigoriu, M., Evaluation of Karhunen–Loève, Spectral, and Sampling Representations for Stochastic Processes. Journal of Engineering Mechanics, 2006. **132**(2): p. 179-189.
22. Mantoglou, A. and J.L. Wilson, Simulation of random fields with the turning bands method. Rep. No. 264, Ralph M. Parsons Laboratory Hydrology and Water Resources Systems, Dept. of Civil Engineering, Massachusetts Institute of Technology, 1981.
23. Matthies, H.G., et al., Uncertainties in probabilistic numerical analysis of structures and solids-Stochastic finite elements. Structural Safety, 1997. **19**(3): p. 283-336.
24. Schenk, C.A. and G.I. Schuëller, Buckling analysis of cylindrical shells with random geometric imperfections. International Journal of Non-Linear Mechanics, 2003. **38**(7): p. 1119-1132.
25. Li, C.C. and A.D. Kiureghian, Optimal Discretization of Random Fields. Journal of Engineering Mechanics, 1993. **119**(6): p. 1136-1154.
26. Deodatis, G. and M. Shinozuka, Regressive Model for Nonstationary Stochastic Processes. Journal of Engineering Mechanics, 1988. **114**(11): p. 1995-2012.
27. Papadopoulos, V., G. Deodatis, and M. Papadrakakis, Flexibility-based upper bounds on the response variability of simple beams. Computer Methods in Applied Mechanics and Engineering, 2005. **194**(12): p. 1385-1404.
28. Bocchini, P. and G. Deodatis, Critical review and latest developments of a class of simulation algorithms for strongly non-Gaussian random fields. Probabilistic Engineering Mechanics, 2008. **23**(4): p. 393-407.
29. Lagaros, N.D., G. Stefanou, and M. Papadrakakis, An enhanced hybrid method for the simulation of highly skewed non-Gaussian stochastic fields. Computer Methods in Applied Mechanics and Engineering, 2005. **194**(45): p. 4824-4844.
30. Grigoriu, M., Crossings of Non-Gaussian Translation Processes. Journal of Engineering Mechanics, 1984. **110**(4): p. 610-620.



31. Grigoriu, M., Non-Gaussian models for stochastic mechanics. Probabilistic Engineering Mechanics, 2000. **15**(1): p. 15-23.
32. Popescu, R., G. Deodatis, and J.H. Prevost, Simulation of homogeneous nonGaussian stochastic vector fields. Probabilistic Engineering Mechanics, 1998. **13**(1): p. 1-13.
33. Yamazaki, F. and M. Shinozuka, Digital Generation of Non-Gaussian Stochastic Fields. Journal of Engineering Mechanics, 1988. **114**(7): p. 1183-1197.
34. Masters, F. and K.R. Gurley, Non-Gaussian Simulation: Cumulative Distribution Function Map-Based Spectral Correction. Journal of Engineering Mechanics, 2003. **129**(12): p. 1418-1428.
35. Phoon, K.K., S.P. Huang, and S.T. Quek, Simulation of second-order processes using Karhunen–Loeve expansion. Computers & Structures, 2002. **80**(12): p. 1049-1060.
36. Ferrante, F.J., S.R. Arwade, and L.L. Graham-Brady, A translation model for non-stationary, non-Gaussian random processes. Probabilistic Engineering Mechanics, 2005. **20**(3): p. 215-228.
37. Graham-Brady, L. and X.F. Xu, Stochastic Morphological Modeling of Random Multiphase Materials. Journal of Applied Mechanics, 2008. **75**(6): p. 061001-061011.
38. Koutsourelakis, P.-S. and G. Deodatis, Simulation of Binary Random Fields with Applications to Two-Phase Random Media. Journal of Engineering Mechanics, 2005. **131**(4): p. 397-412.
39. Sakamoto, S. and R. Ghanem, Polynomial Chaos Decomposition for the Simulation of Non-Gaussian Nonstationary Stochastic Processes. Journal of Engineering Mechanics, 2002. **128**(2): p. 190-201.
40. Grigoriu, M., Spectral Representation for a Class of Non-Gaussian Processes. Journal of Engineering Mechanics, 2004. **130**(5): p. 541-546.
41. Grigoriu, M. Simulation of non-Gaussian stochastic processes and fields with applications to structural engineering problems. in III European Conference on Computational Mechanics. 2006. Dordrecht: Springer Netherlands.
42. Elishakoff, I., Y.J. Ren, and M. Shinozuka, Conditional simulation of non-Gaussian random fields. Engineering Structures, 1994. **16**(7): p. 558-563.
43. Cai, G.Q. and Y.K. Lin, Generation of non-Gaussian stationary stochastic processes. Physical Review E, 1996. **54**(1): p. 299-303.
44. Arwade, S.R., Translation vectors with non-identically distributed components. Probabilistic Engineering Mechanics, 2005. **20**(2): p. 158-167.