



Developing a novel heuristic method to solve the unbalanced allocation problem



Faten F. Al-Obaidi^{a*} , Alla Eldin H. Kassam^b, Sawsan S. Al-Zubaidi^b

^a Control & System Engineering Dept., University of Technology-Iraq, Alsina'a street, 10066 Baghdad, Iraq.

^b Production and Metallurgy Engineering Dept., University of Technology-Iraq, Alsina'a street, 10066 Baghdad, Iraq.

*Corresponding author Email: faten.f.abdulrazzaq@uotechnology.edu.iq

HIGHLIGHTS

- The research offers a new heuristic method for solving the unbalanced assignment problem.
- The proposed method relies on load balancing between machines to ensure that all jobs are performed effectively.
- A comparison with the modified Hungarian method showed the superiority of the proposed method.
- The new method reduces the total assignment time by 7% and idle time by 71%.
- An increase in machine utilization by 26% indicates the efficiency of the proposed method.

Keywords:

Assignment problem, Efficient assignment, Heuristic approach, Modified Hungarian method, Unbalanced assignment problem.

ABSTRACT

Globalization and technological advancements have emphasized the importance of efficient resource allocation for production, efficiency, cost reduction, and optimal use. The assignment problem, a long-term issue, has gained researchers' attention due to its significant impact on institutional success. It prioritizes resource allocation to minimize time and cost while ensuring the feasible execution of activities. The assignment issue can be balanced (the number of activities matches the number of resources) or unbalanced (the number of activities does not match the number of resources). This can lead to resources being left without assignments or activities without implementation when resources exceed activities. A novel heuristic method was proposed to allocate multiple activities to resources efficiently. The proposed method was unrelated to the Hungarian method and did not involve adding dummy tasks or machines. It was founded on lost opportunity cost, with all occupations implemented. The method was implemented numerically and showed a 7% reduction in total assignment time compared to the modified Hungarian method. It also reduced total idle time by 71% and increased machine utilization by 26%. The method's efficiency was further enhanced to evaluate the effectiveness of the proposed method by comparing results with the modified Hungarian method, demonstrating its practical relevance.

1. Introduction

Because of the competition and rapid development that organizations are facing today, organizations are continually looking for ways to incorporate the allocation of resources into their major strategy because of its important relevance to the performance of the institution by concentrating on techniques that guarantee optimal usage of these resources since they affect productivity and efficiency [1]. The assignment problem is a real-world issue that has garnered researchers' attention, leading to the expansion of the literature on resource allocation methods. These methods originate from various applications, such as healthcare [2], education [3], transportation [4], and architecture [5]. The goal is to find the most optimal way to allocate resources for optimal outcomes for institutions.

The assignment problem is a crucial optimization problem involving the allocation of activities on resources [6]. When activities (m) and resources (n) are equal, the allocation issue should be balanced [7]. The allocation matrix's parameters, such as cost and time, are represented as a square matrix [8]. However, the Matrix becomes rectangular when activities are less than resources, creating an imbalanced problem [9]. To solve this, the rectangular Matrix must be transformed into a square matrix by adding dummy activities or resources using the conventional Hungarian method [10].

An unbalanced assignment problem has two possible situations: $m < n$ or $m > n$. In the first situation, the number of tasks is smaller than the number of machines, indicating that not every machine is fully loaded to do every task. Dummy tasks can be introduced [11], to balance the Matrix. In the second case, $m > n$, the same strategy is used, which can result in skipped tasks. A more comprehensive approach is necessary to ensure all tasks are accomplished efficiently [12].

Due to the need to assign all tasks to machines, various researchers have looked at tackling the imbalanced assignment problem by changing the Hungarian technique or applying different approaches to overcome the restrictions of the conventional Hungarian method. One such method is to load the machine with many tasks, which allows each work to be distributed to several machines. These techniques aim to discover the optimum option that reduces the cost of allocating tasks while ensuring that all tasks are done in the best feasible way. These approaches may include a modified Hungarian algorithm, ant colony optimization algorithm, and genetic algorithms, among other approaches.

Kumar [13], offered an approach for tackling the unbalanced assignment problem by optimally assigning all jobs to machines. The approach separates the unbalanced Matrix into many balanced submatrices, solved using the classic Hungarian method. However, this strategy has a disadvantage because it frequently fails to provide a low overall cost. Iampang et al. [14], provided a new efficient solution for unbalanced assignment problems using linear space rather than polynomial complexity. According to an experiment employing 100,000 cost matrices, this method yields a lower optimum cost than Kumar [13]. To tackle the assignment problem, Kotwal and Dhope [15], presented a modified assignment model. A numerical example was given to show the model's effectiveness and comparison to the Hungarian technique. The suggested model outperforms the Hungarian technique in resolving an unbalanced assignment problem. Yadaiah and Haragopal [16], suggested an innovative solution for the unbalanced assignment problem. The problem was divided into two sub-problems to ensure all jobs were completed. A Lexi-search algorithm was used to solve the sub-problems. Unlike the Hungarian method, the Lexi-Search algorithm achieved the assignment without assigning jobs to dummy machines. Betts and Vasko [17], modified the numerical illustration in Yadaiah and Haragopal [16], by preserving the original assignment cost matrix and appending dummy rows to equalize the assignments. The solution was obtained using the Hungarian method. Majumdar and Bhunia [18], proposed an alternate methodology using a genetic algorithm to address the unbalanced assignment problem. The strategy incorporated new enhancements in initialization, crossover, and mutation. The algorithm showed great versatility and achieved fast calculation results, ensuring all jobs were assigned to agents. Wang et al. [19], employ twin cost matrices and an ant colony optimization algorithm based on graph structure. This approach can solve balanced or unbalanced assignment problems. The twin cost matrices are related to independent pheromones related to the assignment and traveling salesman problems. The mutation method prevents the ant colony from stumbling into the local optimal situation, making it easier to find the optimal solution. Based on experimental results, this method is superior to other existing methods.

Mondal et al. [20], employed the Hungarian method in cloud computing by proposing an innovative approach. The goal was to take advantage of distributed resources to perform tasks in a distributed manner with unequal assignments of tasks. This technique is generally known as load balancing for the unbalanced assignment problem. Rabbani et al. [21], represented a modified version of the Hungarian method. The algorithm solved the unbalanced assignment problem while ensuring that all tasks were completed without leaving any unexecuted by loading machines with more than one task. The authors compared their proposed method with other methods using the same example, and the results showed the superiority of the proposed method. Rabbani et al. [22], used a modified Hungarian method with included time parameters to manage job distribution to machines. A numerical example was used to illustrate the proposed method.

The previous literature shows a diversity of approaches to address the problem of unbalanced allocation, focusing on improving efficiency and reducing cost. However, most of these studies face challenges associated with the complexity of solutions, limited flexibility in dealing with multidimensional or large-scale problems, and reliance on techniques that increase execution time or reduce the effectiveness of results in some cases. The need remains to develop more holistic and efficient methodologies, combining the ability to address the complexity of the problem and achieve optimal results in different scenarios. Therefore, this study aims to solve the unbalanced assignment problem by loading machines with multiple jobs. Accordingly, a new heuristic approach was proposed, and the steps of the proposed method were applied to the same example for the researchers Rabbani et al. [22], and the results of the two methods were compared.

2. Mathematical formulation

This study focused on addressing the problem of unbalanced assignment in the case of jobs (m) in the row $>$ machines (n) in the column. The goal was to complete all jobs without introducing additional machinery or leaving any jobs undone. The study explored the possibility of assigning multiple jobs to a single machine to reduce the completion time.

Let $m \times n$ unbalanced assignment matrix where $m > n$, P_{ij} be the assignment parameters (such as cost, time ... etc.); Table 1 represents the parameter assignment matrix.

The mathematical formulation of the unbalanced assignment problem is shown in Equations below [21].

$$\text{Minimize} = \left\{ \sum_{i=1}^m \sum_{j=1}^n P_{ij} x_{ij} \right\} \quad (1)$$

Let x_{ij} denote the i^{th} job assigned for a j^{th} machine such that

$$x_{ij} = \begin{cases} 1 & \text{if the job is assigned to machine} \\ 0; & \text{otherwise} \end{cases}$$

P_{ij} = Parameter allocation from job i to machine j ; the parameter may be cost, time, ..., etc.

Subject to jobs and machine constraints:

$$\sum_{i=1}^m x_{ij} = 1 ; \text{For } j = 1, 2, \dots, n. \quad (2)$$

$$\sum_{j=1}^n x_{ij} \geq 1 ; \text{For } i = 1, 2, \dots, m. \quad (3)$$

$$x_{ij} = 0 \text{ or } 1 \quad (4)$$

Equation (1) is the objective function that minimizes the total of the assignment parameters. Equation (2) depicts the job constraint, in which each work should be given to just one machine. It is not permitted to quit a single job without being assigned. Equation (3) shows the machine constraint, which states that several jobs can be allocated to the same machine. Furthermore, Equation (4) represents the binary limitations.

Table 1: Parameter assignment matrix

	Machines				
	M ₁	M ₂	...	M _n	
Jobs	J ₁	P ₁₁	P ₁₂	...	P _{1n}
	J ₂	P ₂₁	P ₂₂	...	P _{2n}
	⋮	⋮	⋮	⋮	⋮
	J _m	P _{m1}	P _{m2}	...	P _{mn}

3. Proposed method

The suggested methodology depended on Fouad et al. [23], which presents a novel heuristic method for dealing with unbalanced assignment problems. The proposed method is unrelated to the Hungarian method and does not involve adding dummy tasks or machines. It is founded on lost opportunity cost, with all occupations implemented. To finish all jobs in the lowest amount of time or cost, the load must be distributed across the machines with the least variation, ensuring complete equilibrium in machine allocation. It can help accomplish work more successfully, reduce idle time (when the resource is not in use), and enhance resource utilization. The proposed method is divided into several steps, as seen in the illustration in Figure 1.

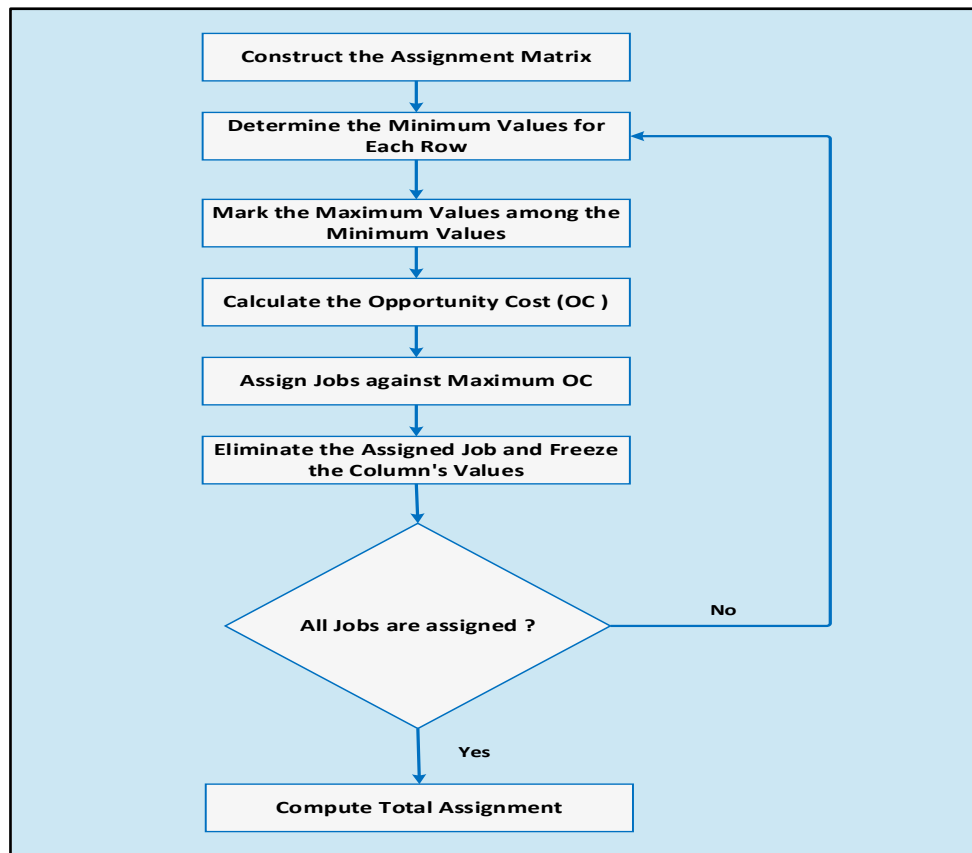


Figure 1: Flow diagram of the proposed heuristic method

The proposed heuristic method effectively integrates with the mathematical formulation by leveraging the concept of lost opportunity cost in job allocation. The method ensures that job constraints are met by allocating each job to only one machine, while machine constraints allow multiple jobs to be assigned to a single machine. At each step, the heuristic method calculates the lost opportunity cost and prioritizes the allocations that minimize the total cost. Constraints are dynamically verified and applied throughout the process, ensuring that solutions remain feasible and consistent with the mathematical objectives of the problem.

4. Validation of the proposed method

The proposed method is straightforward and applicable to an unbalanced assignment problem. To illustrate the steps of the proposed method, a numerical example of an unbalanced assignment problem was employed according to the research by Rabbani et al. [22], where the representation of the assignment matrix consists of eight jobs located in the rows and five machines located in the columns, and the cell parameters show the completion times (in minutes) of each job by each machine. The 8×5 unbalanced assignment matrix, as shown in Table 2, was created to represent the jobs, while the columns represent the machines. The cells of the matrix contain the time values associated with each job.

Table 2: Unbalanced assignment matrix

		Machines				
Jobs		1	2	3	4	5
	1	30	40	20	25	35
	2	25	30	40	20	35
	3	40	20	30	35	50
	4	50	20	40	30	40
	5	35	20	30	25	40
	6	25	45	50	30	60
	7	40	20	30	35	50
	8	25	25	40	30	40

As shown in Table 3, the time values listed in each row of the matrix are examined to determine the minimum time value of the jobs in each row. Since each row represents a specific job, the lowest time value is selected within the row, reflecting the least time required to complete the job based on the specified machine.

The selected minimum values of each row are checked, and the maximum value is selected among them. This value, representing the maximum between the minimum values, reflects the maximum time required to complete a job within the minimum times in each row. For example, the maximum value specified is (25) in the column of machine 1, as shown in Table 4.

Based on the column with the highest value specified in advance, the opportunity cost is accurately calculated to analyze the available options and make optimal allocation decisions. The opportunity cost is calculated by finding the absolute difference between the minimum value specified for each row and the minimum value following it within the same row. The results of the opportunity cost are recorded in a new column called (OC), as demonstrated in Table 5.

The values in the column (OC) are analyzed to determine the maximum value, which reflects the allocation priority. According to Table 6, job 6 is assigned to machine 1 as the most suitable, recording this assignment in the first assignment row to ensure clarity of the sequence of operations and support for the next steps.

Table 3: Determine minimum time values

		Machines				
		1	2	3	4	5
Jobs	1	30	40	20	25	35
	2	25	30	40	20	35
	3	40	20	30	35	50
	4	50	20	40	30	40
	5	35	20	30	25	40
	6	25	45	50	30	60
	7	40	20	30	35	50
	8	25	25	40	30	40

Table 4: Mark maximum time value

		Machines				
Jobs		1	2	3	4	5
	1	30	40	20	25	35
	2	25	30	40	20	35
	3	40	20	30	35	50
	4	50	20	40	30	40
	5	35	20	30	25	40
	6	25	45	50	30	60
	7	40	20	30	35	50
	8	25	25	40	30	40

Table 5: Opportunity cost calculation

		Machines					OC
		1	2	3	4	5	
Jobs	1	30	40	20	25	35	
	2	25	30	40	20	35	
	3	40	20	30	35	50	
	4	50	20	40	30	40	
	5	35	20	30	25	40	
	6	25	45	50	30	60	5
	7	40	20	30	35	50	0
	8	25	25	40	30	40	

Table 6: First assignment of a job to amachine

		Machines					OC
Jobs		1	2	3	4	5	
	1	30	40	20	25	35	
	2	25	30	40	20	35	
	3	40	20	30	35	50	
	4	50	20	40	30	40	
	5	35	20	30	25	40	
	6	25	45	50	30	60	
	7	40	20	30	35	50	
8	25	25	40	30	40	0	
First Assignment	25						

The previous steps were followed to assign the remaining jobs, resulting in the distribution of jobs (6, 3, 1, 2, and 4) for machines from 1 to 5. Table 7 shows the assignment's final results, showing the job distribution according to the methodology used to ensure efficiency and consistency.

Table 7: Final assignment of all jobs to machines

		Machines				
		1	2	3	4	5
Jobs	1	30	40	20	25	35
	2	25	30	40	20	35
	3	40	20	30	35	50
	4	50	20	40	30	40
	5	35	20	30	25	40
	6	25	45	50	30	60
	7	40	20	30	35	50
	8	25	25	40	30	40
First Assignment		25	20	20	20	40
Final Assignment		25	20	0	25	0

5. Results and discussion

After implementing the proposed approach using the numerical example provided by Rabbani et al. [22], the results indicated that machines (1, 2, and 4) were assigned more than one job. In comparison, machines (3 and 5) were given only one job. By analyzing the allocation results, the total time for the whole allocation is 195 minutes, and the total idle time is only 55 minutes through the utilization of each machine, with an average of 78%. The results of the proposed approach were compared with those obtained by the modified Hungarian method. In the modified Hungarian method, the total assignment time was 210 minutes. This resulted in the machines being idle for 190 minutes through machine utilization, averaging 52%. The assignment results of the proposed and modified Hungarian methods are summarized in Table 8.

Table 8: Summarized assignment results by the proposed and the modified Hungarian methods

Machines	Proposed method				Modified Hungarian Method			
	Jobs	Assignment values (minutes)	Idle Time (minutes)	Machine Utilization (%)	Jobs	Assignment values (minutes)	Idle Time (minutes)	Machine Utilization (%)
1	6, 8	25+25=50	0	100	6	25	55	31
2	3, 7	20+20=40	10	80	3, 7	20+20=40	40	50
3	1	20	30	40	1	20	60	25
4	2, 5	20+25=45	5	90	2, 5	20+25=45	35	56
5	4	40	10	80	4, 8	40+40=80	0	100
Total		195	55	78	Total	210	190	52

In the proposed method, Machine 1 was fully utilized 100% with a time allocation of 50 minutes and without idle time. This suggests that the proposed method distributed the tasks accurately since the time available to the machine was entirely used to perform jobs 6 and 8. In the modified Hungarian method, the utilization of Machine 1 was only 31%; 25 minutes were allocated, and 55 minutes were left idle. This significant difference is because the Hungarian method may not effectively distribute jobs to machines that can fully accommodate them, resulting in less utilization and waste of a substantial part of the available resources.

The proposed method achieved 80 % utilization of Machine 2, where 40 minutes were allocated, meaning the idle time was only 10 minutes. This shows that the proposed method was able to allocate jobs 3 and 7 to the machine in such a way as to minimize idle time. In contrast, the Hungarian method exploited Machine 2 by only 50%, with an allocation of 40 minutes and an idle time of 40 minutes. This difference shows that the Hungarian method lacks flexibility in job distribution, making it less efficient to utilize the machine when the available jobs are fully unbalanced. Machine 3 was allocated just job 1 to perform in 20 minutes, resulting in a 40% utilization rate and 30 minutes of idle time. The Hungarian technique allocated the same amount of time, 20 minutes, to accomplish the same job, and 60 minutes were attained as idle time, resulting in a machine utilization of just 25% for Machine 3. In addition, jobs 2 and 5 were allocated to machine 4, and the total assignment time for the proposed and Hungarian methods was 45 minutes. In the proposed method, the idle time was only 5 minutes, which means that the jobs were scheduled to be carried out in such a way as to minimize time gaps, and therefore, the utilization of the machine was 90%. As for the Hungarian method, the idle time amounted to 35 minutes, which means that the method could not effectively exploit the time gaps resulting from the distribution of jobs, which was reflected in a decrease in the utilization rate of the machine to 56%.

In the proposed method, only job 4 was allocated to machine 5 with a total allocation of 40 minutes, resulting in 10 minutes of idle time and 80% machine utilization. This suggests that the proposed method focused on a balanced distribution of jobs among all machines while accepting some waste to achieve flexibility in distribution. As for the Hungarian method, jobs 4 and 8 were allocated on the same machine with a total allocation of 80 minutes, which led to 100% full utilization without any idle time. This difference reflects the concept of the Hungarian method, which focuses on maximizing the use of each machine individually, regardless of the impact on the distribution of work on the rest of the machines.

Figure 2 compares the suggested and the Hungarian methods regarding the total time each machine took to complete all the jobs. The horizontal axis represents the machines, and the vertical axis represents the assignment time of the jobs on the machines. The blue bars show the assignment time of the proposed method, while the orange bars show the assignment time of the modified Hungarian method.

As noted, the first machine was loaded in the proposed method with two jobs, and the total time was 50 minutes, while the modified Hungarian method was loaded with one job, and the total time was 25 minutes. The second, third, and fourth machines were loaded with equal jobs for both methods and equal total time: 20 minutes for the second, 40 minutes for the third, and 45 minutes for the fourth. Figure 2 shows that the fifth machine was loaded with one job in the proposed method, taking 40 minutes, while in the modified Hungarian method, the machine was loaded with two jobs, taking 80 minutes.

Figure 3 shows the comparison of the idle time of each machine between the proposed and the modified Hungarian method, where the values for the proposed method were represented using a line and dots in blue. In contrast, the values for the

modified Hungarian method were represented using orange. Figure 3 clearly shows the performance of the two methods, which makes it possible to notice the differences in the idle time of each machine and how efficiently each method reduces idle time.

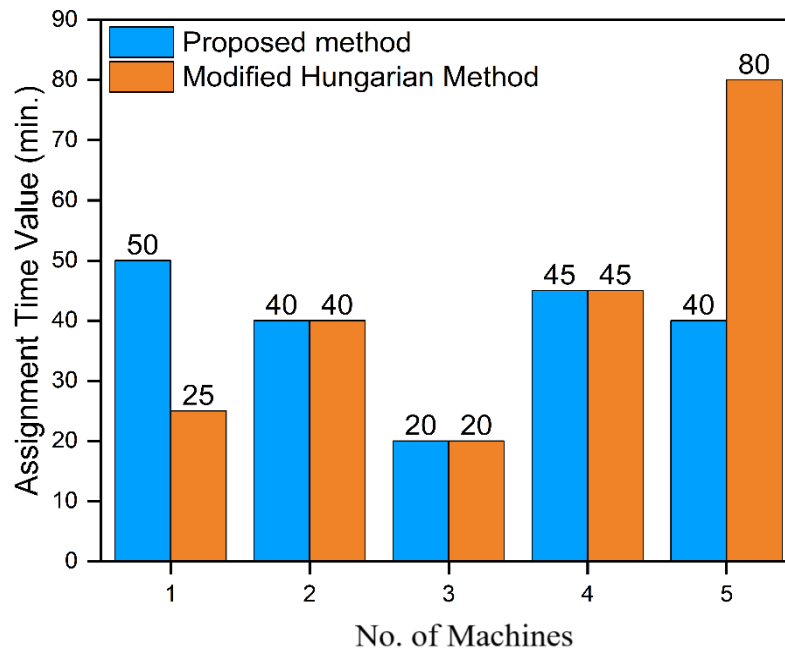


Figure 2: Total assignment results for each machine by the proposed and modified Hungarian methods

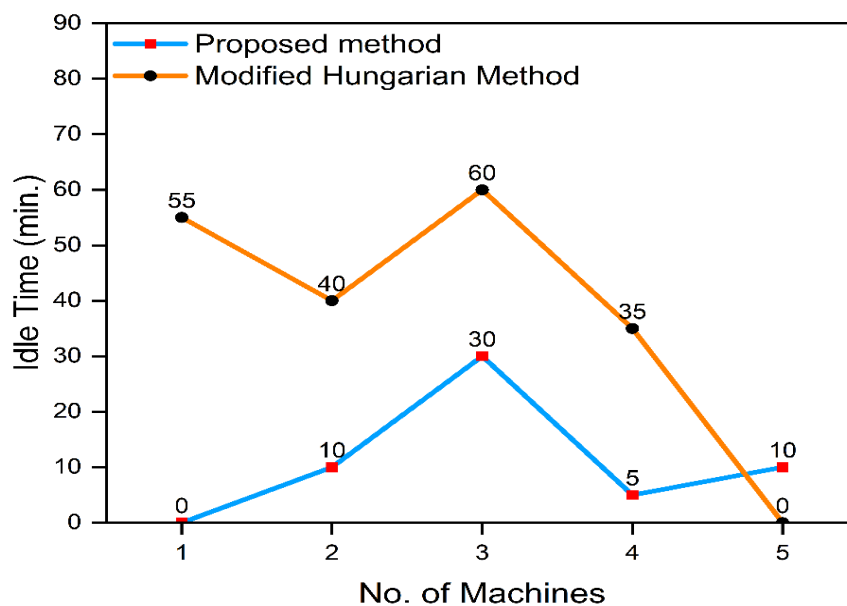


Figure 3: Idle time results for each machine by the proposed and modified Hungarian methods

For the first machine, the idle time of the proposed method is zero, while in the modified Hungarian method, it is 55 minutes. Therefore, the idle time has been completely reduced, meaning the reduction rate is 100%. As for the second machine, the idle time is 10 minutes in the proposed method, while in the modified Hungarian method, it is 40 minutes. Here, the idle time was reduced by 75% compared to the modified Hungarian method. In the third machine, the idle time of the proposed method was 30 minutes, while in the Hungarian method, it was 60 minutes. Thus, the idle time was reduced by 50%. For the fourth machine, the idle time in the proposed method is 5 minutes, while in the Hungarian method, it is 35 minutes. Here, the idle time was reduced by 86 %. As for the fifth machine, the idle time of the proposed method is represented by 10 minutes, while in the Hungarian method, there is no idle time (that is, zero).

Figure 4 compares the utilization ratio of each machine using the proposed method and the modified Hungarian method. The values for the proposed method are displayed in blue columns, while the values for the modified Hungarian method are displayed in orange columns. The scheme allows a clear view of the difference in utilization ratios between the two methods, which helps assess each method's efficiency in making the most of the machines.

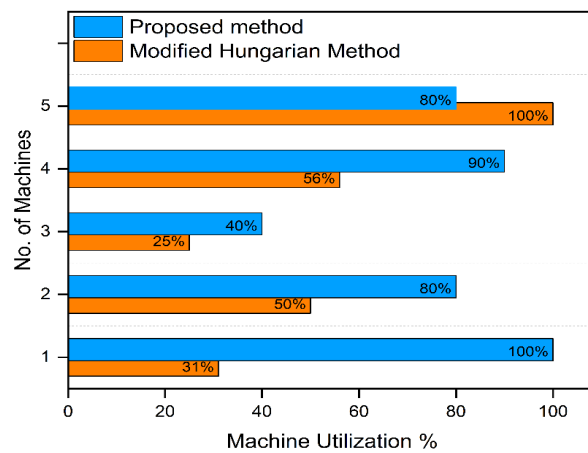


Figure 4: Machine utilization results for each machine by the proposed and modified Hungarian methods

For the first machine, the percentage of its utilization by the proposed method was much higher than that of the modified Hungarian method, where the difference was significant, which means that the percentage of its utilization increased significantly compared to the modified Hungarian method. As for the second machine, the increase in utilization was about 60% with the proposed method compared to the Hungarian, which indicates a significant improvement in the utilization of the machine. The third machine also showed a 60% increase in utilization with the proposed method, reflecting a similar improvement to the second machine. For the fourth machine, the increase in its utilization was almost 60%, the same trend as in the previous machines. The fifth machine was the only one that experienced a decrease in its utilization, as it decreased by 20% compared to the modified Hungarian method. Thus, the result shows that most of the machines experienced an improvement in their exploitation using the proposed method, except the fifth machine, which recorded a reduction.

Finally, the results showed that the proposed method is characterized by superior performance compared to the modified Hungarian method in terms of several operational aspects. It has reduced the total assignment time by 7%, reflecting a direct improvement in the efficiency of the assignment processes. Moreover, the method significantly contributed to reducing idle time, as it decreased by a significant 71 %, which indicates a clear improvement in the continuity of operations and a reduction in time losses. The method also showed a positive impact on the use of the machines, as this led to an increase in the utilization rate of machines by 26%, reflecting higher efficiency in the distribution of jobs. Together, these improvements confirm the effectiveness of the proposed method in improving the overall performance and reducing the waste of time and resources.

6. Conclusion

The problem of unbalanced assignment arises when the number of jobs is not equal to the number of machines, and one of the solutions is to load the machines with more than one job while ensuring that each machine is loaded evenly and sufficiently to accomplish all the jobs. To solve this problem, a new heuristic approach, which was easy to implement, was proposed based on calculating the cost while ensuring that all jobs were carried out by balancing the load on the machines. The effectiveness of the proposed approach was proven by applying it to a numerical example and comparing the results obtained with the results of the modified Hungarian method. The results revealed that the proposed method outperformed the modified Hungarian method, reducing total assignment time by 7%. It also decreased idle time by 71% while increasing machine usage by 26%.

For future work, the proposed method can be applied to address the unbalanced assignment problem when the goal is to maximize the assignment problem. Further research could also explore extending the evaluation of the effectiveness of the method by comparing it with other heuristic or optimization techniques.

Acknowledgment

Acknowledgments may be directed to individuals or institutions that have contributed to the research or a government agency.

Author contributions

Conceptualization, F. Al-Obaidi and S. Al-Zubaidi; data curation, F. Al-Obaidi; formal analysis, F. Al-Obaidi and S. Al-Zubaidi; investigation, F. Al-Obaidi; methodology, F. Al-Obaidi; project administration, F. Al-Obaidi, A. Kassam, and S. Al-Zubaidi; resources, F. Al-Obaidi; software, F. Al-Obaidi; supervision, A. Kassam and S. Al-Zubaidi; validation, F. Al-Obaidi, A. Kassam, and S. Al-Zubaidi; visualization, F. Al-Obaidi; writing—original draft preparation, F. Al-Obaidi; writing—review and editing, F. Al-Obaidi, A. Kassam, and S. Al-Zubaidi. All authors have read and agreed to the published version of the manuscript.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Data availability statement

The data that support the findings of this study are available on request from the corresponding author.

Conflicts of interest

The authors declare that there is no conflict of interest.

References

- [1] S. Bouajaja, N. Dridi, A Survey on Human Resource Allocation Problem and Its Applications, *Oper. Res.*, 17 (2017) 339–369. <https://doi.org/10.1007/s12351-016-0247-8>
- [2] E. Lanzarone, A. Matta, The Nurse-To-Patient Assignment Problem in Home Care Services, *Oper. Res. Manag. Sci.*, 173 (2012) 121–139. https://doi.org/10.1007/978-88-470-2321-5_8
- [3] M. Tounsi, A Heuristic-Based Technique for University Resource Allocation Problems, *IEEE GCC Conference (GCC)*, (2006) 1–6. <https://doi.org/10.1109/IEEGCC.2006.5686243>
- [4] G. Jordan, S. Martello, M. Monaci, The Assignment and Loading Transportation Problem, *Eur. J. Oper. Res.*, 289 (2021) 999–1007. <https://doi.org/10.1016/j.ejor.2019.07.039>
- [5] M. Lone, S. Mir, M. Wani, An Application of Assignment Problem In Agriculture Using R, *J. Sci. Res. Rep.*, 13 (2017) 1–5. <https://doi.org/10.9734/JSRR/2017/31902>
- [6] F. S. Hiller, S., G. J. Lieberman, *Introduction-Operations Research*, McGraw-Hill Higher Education, (2014).
- [7] N. Rai, A. J. Khan, A Brief Review on Classic Assignment Problem and Its Applications, *IOSR Journal of Engineering (IOSRJEN)*, 9 (2019) 73–81. www.iosrjen.org
- [8] Y. O. Aderinto, M. O. Raji, A. Rauf, On One-to-One Correspondence Mapping and Its Application to Linear Assignment Problem, *Am. J. Sci.*, 14 (2018) 62–68. <http://www.jofamericanscience.orgonline>
- [9] S. Dhoubib, Novel Optimization Method for Unbalanced Assignment Problems with Multiple Jobs: The Dhoubib-Matrix-AP2, *Intell. Syst. Appl.*, 17 (2023) 1–8. <https://doi.org/10.1016/j.iswa.2023.200179>
- [10] R. Abdur, An Alternative Approach for Solving Unbalanced Assignment Problems, *Jahangirnagar Univ. J. Sci.*, 40 (2017) 45–56. <https://www.juniv.edu/journal/6688/file>
- [11] E. R. Wulan, A. Pratiwi, Q. Y. Zaqiah, Mahmud, The Analysis of Unbalanced Assignment Problems Using the Kotwal-Dhope Method to Develop A Massive Open Online Course, *International Conference on Wireless and Telematics (ICWT)*, IEEE, (2020) 1–5. <https://doi.org/10.1109/ICWT50448.2020.9243644>
- [12] G. Ramesh, G. Sudha, K. Ganesan, Method of Finding an Optimal Solution for Interval Balanced and Unbalanced Assignment Problem, *IOP Conference Series Material Science Engineering*, 912 (2020) 062031. <https://doi.org/10.1088/1757-899X/912/6/062031>
- [13] A. Kumar, A Modified Method for Solving the Unbalanced Assignment Problems, *Appl. Math. Comput.*, 176 (2006) 76–82. <https://doi.org/10.1016/j.amc.2005.09.056>
- [14] A. Iampang, V. Boonjing, P. Chanvarasuth, A Cost and Space Efficient Method for Unbalanced Assignment Problems, *IEEE Int. Con. Industrial Eng. & Eng. Man.*, IEEE, (2010) 985–988. <https://doi.org/10.1109/IEEM.2010.5674228>
- [15] J. G. Kotwal, T. S. Dhope, Unbalanced Assignment Problem by Using Modified Approach, *Int. J. Advanced Research in Computer Science and Software Engineering*, 5 (2015) 451–456. www.ijarcse.com
- [16] V. Yadaiah, V. V. Haragopal, A New Approach of Solving Single Objective Unbalanced Assignment Problem, *Am. J. Oper. Res.*, 6 (2016) 81–89. <https://doi.org/10.4236/ajor.2016.61011>
- [17] N. Betts, F. J. Vasko, Solving the Unbalanced Assignment Problem: Simpler Is Better, *Am. J. Oper. Res.*, 6 (2016) 296–299. <https://doi.org/10.4236/ajor.2016.64028>
- [18] J. Majumdar, A. K. Bhunia, An Alternative Approach for Unbalanced Assignment Problem Via Genetic Algorithm, *Appl. Math. Comput.*, 218 (2012) 6934–6941. <https://doi.org/10.1016/j.amc.2011.12.070>
- [19] L. Wang, Z. He, C. Liu, Q. Chen, Graph Based Twin Cost Matrices for Unbalanced Assignment Problem with Improved Ant Colony Algorithm, *Results Appl. Math.*, 12 (2021) 100207. <https://doi.org/10.1016/j.rinam.2021.100207>
- [20] R. K. Mondal, P. Ray, E. Nandi, M. K. Sanyal, D. Sarddar, Load Balancing of Unbalanced Assignment Problem with Hungarian Method, *Int. J. Ambient Comput. Intell.*, 10 (2019) 46–60. <https://doi.org/10.4018/IJACI.2019010103>
- [21] Q. Rabbani, A. Khan, A. Quddoos, Modified Hungarian Method for Unbalanced Assignment Problem with Multiple Jobs, *Appl. Math. Comput.*, 361 (2019) 493–498. <https://doi.org/10.1016/j.amc.2019.05.041>
- [22] Q. Rabbani, A. Khan, A. Quddoos, Assignment of Multiple Jobs Scheduling to A Single Machine, *Advances in Math. Sci. J.*, 10 (2020) 1003–1011. <https://doi.org/10.37418/amsj.10.2.29>
- [23] Faten Fouad, Alla Eldin H. Kassam, Sawsan S. Al-Zubaidi, A New Heuristic Method for Solving Unbalanced Multi-Objective Assignment Problem, *Eng. Res. Express*, IOP Publishing Ltd., 6 (2024) 045429. <https://doi.org/10.1088/2631-8695/ad9888>