Enhancing 4D Systems with PSO Algorithm for Optimal Parameters

Karam Adel Abed, Omar Saber Qasim, Saad Fawzi Al-Azzawi
0Department of Operations Research and Intelligence Techniques, College of Computer Sciences and Mathematics, University of Mosul, Iraq,
Department of Mathematics, College of Computer Science and Mathematics, University of Mosul, Mosul, Iraq
karamadel@uomosul.edu.iq

Abstract

This investigation focuses on the pivotal contribution of Particle Swarm Optimization (PSO) in designing and optimizing a novel hyperchaotic system. We develop a four-dimensional hyperchaotic system with seven nonlinear terms through topological modification of the three-dimensional Liu system (six-term basis). The PSO algorithm is strategically employed to maximize two positive Lyapunov exponents, thereby ensuring robust hyperchaotic characteristics - a significant advancement over traditional bifurcation-based parameter selection methods. Comprehensive dynamical analysis includes: (i) equilibrium point stability via Jacobian linearization, (ii) Lyapunov spectrum quantification, (iii) identification of multistability phenomena, and (iv) computation of the fractal (Kaplan-Yorke) dimension. Experimental validation through NI Multisim 14.3 circuit simulations confirms the practical realizability of the optimized system. This research demonstrates PSO's transformative potential in hyperchaotic system design, offering new possibilities for secure communications and nonlinear control applications.

Keywords: Hyperchaotic, Electronic circuit, Multisim software, PSO Algorithm.

تحسين الأنظمة رباعية الأبعاد باستخدام خوارزمية تحسين سرب الجسيمات (PSO) لتحقيق معاملات مثالية

كرم عادل عبد، عمر صابر قاسم، سعد فوزي العزاوي
قسم بحوث العمليات وتقنيات الذكائية، كلية علوم الحاسوب والرياضيات، جامعة الموصل، العراق
قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة الموصل، الموصل، العراق
karamadel@uomosul.edu.iq

الملخص

يركز هذا البحث على المساهمة المحورية لخوارزمية تحسين سرب الجسيمات (PSO) في تصميم وتحسين نظام فوضوي فائق جديد. تم تطوير نظام فوضوي فائق رباعي الأبعاد بسبعة حدود غير خطية من خلال التعديل الطوبولوجي لنظام ليو ثلاثي الأبعاد (على أساس ستة حدود). تُستخدم خوارزمية تحسين سرب الجسيمات (PSO) بشكل استراتيجي لتعظيم أُسّي ليابونوف موجبين، مما يضمن خصائص فوضوية فائقة - وهو تقدم كبير مقارنةً بأساليب اختيار المعاملات التقليدية القائمة على التشعب. يتضمن التحليل الديناميكي الشامل: (أ) استقرار نقطة التوازن عبر خطية جاكوبي، (ب) تحديد كمية طيف ليابونوف، (ج) تحديد ظواهر تعدد الاستقرار، و(د) حساب البعد الكسري (كابلان-يورك). يؤكد التحقق التجريبي من خلال محاكاة الدوائر

NI Multisim 14.3 إمكانية تحقيق النظام الأمثل عمليًا. يوضح هذا البحث الإمكانات التحويلية لـ PSO في تصميم الأنظمة فائقة الفوضى، مما يوفر إمكانيات جديدة للاتصالات الآمنة وتطبيقات التحكم غير الخطية.
الكلمات المفتاحية: مفرط الفوضى، الدائرة الإلكترونية، برنامجMultisim ، خوارزمية.PSO

1. Introduction

Continuous dynamic systems form the cornerstone of mathematical and physical studies, offering profound insights into complex behavior. The first three-dimensional chaotic system was introduced by Lorenz in 1963 [1], which paved the way for the development of numerous other 3D systems featuring increasing complexity such as Rossler 3D system 1976[2], Sprott system 1994[3], jerk systems[4-6]. Advances im computing tools have allowed researchers to ingvestigate higher-dimensional systems, extending the bounds chaos and broading its apllicability, whereas early studies mostly concentrated on understanding and paths of low systems. Notably amore thorough framework for comprehending chaotic behavior has been made available by the shift from studying three-dimensional systems to high dimensional systems four-dimensional systems [7-9], five-dimensional systems [10-12], six-dimensional systems [13- 15]. These systems are characterized by their chaotic nature and unpredictability, making them widely applicable in various practical fields such as chaos control [16-18], chaos synchronization [19] anti-synchronization [14, 20], hybrid function projective synchronisation [11], electronic circuits [21, 22], encryption[23-25].

Recent research has highlighted the close connection between algorithm and dynamic systems chaotic [26,27], demonstrating how optimization techniques may identify optimal parameter values that enhance chaotic behavior. Algorithm such as particle swarm optimization, Genetic algorithms, a have shown encouraging potential in this area. Though repeated optimization of parameter values to enhance chaos, these methods generate systems with distinct dynamic properties that are appropriate for specific application needs. By examining the complex parameter spaces of higher-dimensional dynamic systems, the PSO technique, has shown itself to be extremely effective at optimizing Lyapunov exponents (LE_s). It is therefore a useful instrument for increasing the required dynamic attributes [28-33]. The objective of this research is to bridge the gap between the theoretical study of dynamical systems and the practical application of optimization techniques. Specifically, the work focuses on systematically expanding a three-dimensional system and enhancing its properties using Particle Swarm Optimization (PSO) algorithms, with the objective of generating a hyperchaotic system characterized by more than one positive Lyapunov exponent. By holistically integrating historical perspectives, theoretical, advancements, and cutting-edge algorithmic techniques, this proposal aims to drastically transform the domains of dynamic systems chaos theory. Both more recent advancements in dynamical systems optimization techniques and classical works. Study makes the, methodology's creative protentional possible by providing the groundwork for the incorporation of algorithms like PSO with extensions to dynamical systems [34-37]. A key step in a successful hyperchaotic process, parameter estimation has generated a lot of attention in the field of chaotic control research. In recent years, a novel approach to optimization algorithms known as the PSO has emerged. It was developed by Mir Jalili and his team and strikes a compromise between exploration and exploitation. It might be a powerful tool for solving difficult issues in engineering, machine learning, and other fields where optimization is essential. This is done to turn the dynamic system into a hyperchaotic state and obtain the most accurate and optimal results [38].

The Particle Swarm Optimization (PSO) algorithm will be employed to estimate the optimal parameters of a multidimensional chaotic dynamical system. This technique will help improve model performance by intelligently tuning the influencing parameters and computationally efficiencies over traditional methods. This improved estimate generated more complex dynamics, resulting in a hyperchaotic system with more than one positive Lyapunov exponent. Table 1 shows a comparison between the current work and other systems available in the literature.

Table 1. Comparison of some 4D mathematical models with the proposed model by algorithmic method.

Ref.	Equations	Method	Parameters	Total of terms	LE_S
2022, [39]	$\begin{cases} \dot{x}_1 = a(x_2 + 0.2(x_1 - \varepsilon\, s \\ \dot{x}_2 = bx_1 - x_2 + x_3 + x_4 \\ \dot{x}_3 = -c\, x_2 + x_4 \\ \dot{x}_4 = -dx_1 \end{cases}$	Bifurcation diagram	$\begin{cases} a = 8.15 \\ b = 0.8 \\ c = 12.5 \\ d = 0.5 \\ \varepsilon = 0.5 \end{cases}$	10-term	(+, +,0, -
2020, [40]	$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = -x_1x_3 + bx_2 - 5x_4 \\ \dot{x}_3 = x_1x_2 - cx_3 \\ \dot{x}_4 = dx_2 \end{cases}$	Bifurcation diagram	$\begin{cases} a = 30 \\ b = 20 \\ c = 3 \\ d = 0.1 \end{cases}$	9-term	(+, +,0, -
2017,[41]	$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = -bx_2 + nx_1x_3 + cx \\ \dot{x}_3 = d - e^{x_1x_2} \\ \dot{x}_4 = -mx_2 \end{cases}$	Bifurcation diagram	$\begin{cases} a = 1 \\ b = 0.5 \\ c = 0.2 \\ d = 2.5 \\ n = 1 \\ m = 0.5 \end{cases}$	8-term	(+, +,0, -

2022,[42]	$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + kx_1x_3 \\ \dot{x}_2 = -cx_2 - x_1x_3 \\ \dot{x}_3 = -b + x_1x_2 \\ \dot{x}_4 = -m\,x_2 \end{cases}$	Bifurcation diagram	$\begin{cases} a = 10 \\ b = 100 \\ c = 2.7 \\ k = -0. \\ m = 1 \end{cases}$	9-term	(+, +,0, -
2024,[43]	$\begin{cases} \dot{x}_1 = -24\,x_1 + 8x_2 \\ \dot{x}_2 = ax_1 + x_2 - 2x_1x_3 \\ \dot{x}_3 = bx_1x_2 - 4x_3 + x_4 \\ \dot{x}_4 = -x_1x_2 - 2x_3 - x_4 \end{cases}$	Bifurcation diagram	$\begin{cases} a = 20 \\ b = 1.1 \end{cases}$	11-term	(+, +,0, -
This work	$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = -c + x_1x_3 \\ \dot{x}_3 = b - x_2^2 \\ \dot{x}_4 = -dx_1x_4 \end{cases}$	**Particle Swarm Algorithm**	$\begin{cases} a = 4.07 \\ b = 4.77 \\ c = 0.04 \\ d = 2.47 \end{cases}$	**7-term**	(+, +,0, -

Comparing this work with several previous studies that relied on finding parameters randomly in order to achieve the principles of extreme chaos in a 4D system consisting of 7-terms. In contrast, this research proposed the (PSO) algorithm, which was able to accurately find optimal parameters, where the proposed system, which presents a significant challenge in dynamic systems. This algorithm demonstrated great efficiency in achieving extreme chaos in this system, reflecting the progress made in parameter determination compared to previous works.

The main contribution of this work is summarized as follows:

1. 4D System: A minimal seven-term hyperchaotic system with dual positive Lyapunov exponents is introduced.

2. PSO Optimization: Parameters are optimized via PSO, ensuring hyperchaotic behavior without bifurcation analysis and traditional methods.

3. Theoretical Minimum: The system achieves the lowest term count for hyperchaotic, enhancing implement ability.

4. Circuit Validation: NI Multisim 14.3 simulations confirm physical realizability.

The article is structured as: Section 2 introduces the proposed algorithm, followed by Section 3, which outlines the suggested method. Section 4 provides a detailed description of the constructed 4D hyperchaotic system, while Section 5 explains the dynamics of the system. Section 6 demonstrates the application of the proposed system through the design of an electronic circuit, implemented using Multisim 14.3. Section 7 presents the results, and finally, Section 8 concludes the paper with a summary.

2. Particle Swarm Algorithm

With the development of computational technologies, various swarm intelligence algorithms have been developed together as a consequence of this development. Among these alternatives is the Particle Swarm Optimization (PSO) algorithm that was introduced in 1995. Observing the movement of bird flocking and fish schooling, the PSO algorithm forms a group of individuals in the search space called particles which attempt to update their current positions considering personal velocities and social velocities. Although the PSO algorithm operates on the following simple principles, the algorithm was found to be quite successful on many occasions. The attraction that particles create among themselves, with the effects of the knowledge, is deducted from other particles either in the personal memory or in the neighborhood memory. That's why, through a simple mechanism, the particles quickly populate the conforming range. Over time, even though there is an attraction to a common area formed on the graph, the possibility of producing more volatile results is considered within the adversative vector of the particles. Otherwise, it will be possible to eliminate the global expectation on the operation of the PSO algorithm. It outperforms other metaheuristic algorithms especially in the solution of optimization problems with complex design spaces. Since its introduction, the PSO algorithm has been modified at many stages and has given rise to a wide range of studies. One of the important steps in using the PSO algorithm is the selection of parameter values. If meaningful parameter values are set, the PSO algorithm can successfully be applied to many optimization problems. However, since the PSO algorithm includes many parameters, finding the most suitable parameters is a complex and advanced problem. The PSO algorithm has been used for many purposes since its introduction. However, one of the novelties in the literature is a modification of the classic PSO algorithm. In this sense, various studies are conducted on this subject, and different versions of the PSO algorithm are put forward. For instance, a total of 45 different modifications made to the classical PSO algorithm have been found [44]. On the other hand, researchers from many fields, particularly engineers and scientists, query the superior feature-priced and easy-to-implement optimization problems. In this sense, the PSO algorithm is considered as an important option. However, the fact that the PSO algorithm is a dependent optimization algorithm brings along some problems. A recent survey of PSO applications revealed that sequential initialization of particle velocities and positions, premature convergence in high-

Print ISSN 2710-0952 Electronic ISSN 2790-1254

dimensional search spaces, and positional convergence to suboptimal solitons are considered the main disadvantages of the PSO algorithm.

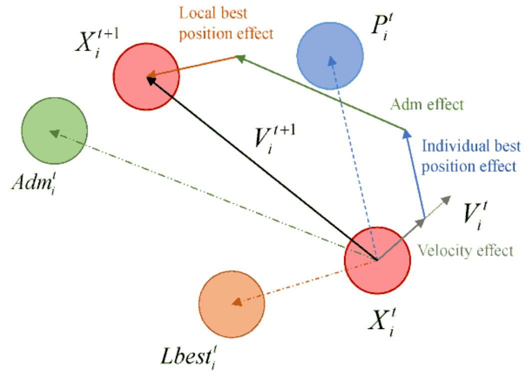


Figure 1. Geometric illustration of particle's movement in the particle swarm optimization Process [44].

2.1. Mathematical Formulation

The mathematical form of the n-dimension search space is given in with the learning experience of each other particles, particle swarm optimization has a strong global search ability. The PSO algorithm is a population-based optimization algorithm, which uses concepts inspired by the behavior of birds hunting for food. Birds examine their own food sources and cooperate with others to find the food source with the best quality. This is the basis of the PSO algorithm [45]. This is a swarm search algorithm in which particles cooperate with each other. If a particle can find the optimal solution, the entire particle swarm can quickly find the optimal solution. The PSO algorithm has been widely recognized. It has the advantages of fast convergence, powerful robustness, and user-friendly concepts. At the same time, the PSO algorithm is easy to implement. Over the years, the significant effect of the particle swarm optimization algorithm has attracted many scholars from the field to study the related issues continuously. Some researchers have proposed methods to improve the performance of PSO. There is much research applying the PSO algorithm to many scientific fields. Many optimization problems are resolved by the PSO algorithm. By using them in different applications, superior results are achieved. In recent years, PSO has successfully applied to image processing, neural network, function optimization, feature selection, data grouping, and mixed-variable optimization problems. With the increasing application of the PSO algorithm, PSO also faces some difficulties in optimization. Since the PSO algorithm is controlled and decided by some control parameters, proper parameters are essential for the performance of the PSO algorithm [46].

Therefore, any system that depends on this algorithm will initially be formed from a set of random solutions, and the search will be conducted within these random solutions for the best solution (optimum solutions) by updating the generations of solutions. Also, each particle in the PSO algorithm represents a position vector and is described as follows [47].

$$S_i = s_{i1}, s_{i2}, \dots, s_{id} \quad (1)$$

where d represents the dimension of the problem. The velocity of each particle is described as follows:

$$V_i = v_{i1}, v_{i2}, \dots, v_{id} \quad (2)$$

Since the swarm is formed from the sum of these particles, the best personal position obtained by the particle within the swarm, which gives the best fitness pbest, is described as follows:

$$P_i = p_{i1}, p_{i2}, \dots, p_{id} \quad (3)$$

The best location that the particle gets from among all the particles that make up the swarm is gbest, which is described as follows:

$$G_i = g_{i1}, g_{i2}, \dots, g_{id} \quad (4)$$

The position is also updated, where the position of each particle is added to the velocity of that particle, and from its calculation the new position of the particle is updated according to the following relationship:

$$S_i^{n+1} = S_i^m + V_i^{m+1} \quad (5)$$

Where m represents the generation frequency, and the velocity update can be done according to the following relationship:

$$V_i^{m+1} = V_i^m + c_1 r_1 (pbest_i^m - S_i^m) + c_2 r_2 (gbest^m - S_i^m) \quad (6)$$

Where m refers to the repetition of the generation, I refers to the number of iterations, V_i^m to the velocity of the current particle within the swarm, S_i^m refers to the position of the current particle within the swarm, $pbest_i^m$ refers the personal best position of the particle can be obtained by modifying the position of the particle itself, $gbest^m$ refers to the best global particle site within the entire

Print ISSN 2710-0952   Electronic ISSN 2790-1254

swarm, $c_1 c_2$ they are positive numerical constants used to adjust the equation and $r_1 r_2$ they are random variables between (0,1).

Pseudocode: A standard PSO

Begin PSO
Input:
N – Swarm size.
P – particle
V – velocity
LB – Lower bound of the search space.
UB – Upper bound of the search space.
Output:
Step A. Initialization
For each particle $i = 1, \dots, Np$, do
1. Put the (P) position with uniformly distribution as $Pi\ (0)\ \sim U\ (LB, UB)$.
2. Put *pbest* to its initial position: *pbest* $(I, 0)\ = Pi\ (0)$.
3. Put gbest to the minimal value of the swarm: *gbest* (0) =argmin $f\,[P_i\ (0)]$.
4. Put velocity: $V_i \sim U\ (-|\mathrm{UB} - \mathrm{LB}|, |\mathrm{UB} - \mathrm{LB}|)$.
Step B. Continue until a termination criterion is true.
For each (P) $i = 1, \dots, Np$, do
1. Put random numbers: r_1, r_2 between U (0,1).
2. Update P-V. (2.6)
3. Update (P) position. (2.5)
4. $if\ \ f[P_i(t)] < f[pbest(i,t)], do$
a. Update the best position of (P) i: $pbest(i,t) = P_i\ (t)$.
$if\ f[P_i(t)] < f[gbest(t)]$ update the swarm best known position *gbest*(t)= $P_i\ (t)$.
5. $t \leftarrow (t + 1)$.
Step C. Output $g\ b\ (t\ )$ which is the best solution obtained.
End PSO

3. Proposed Methodology

The study of chaotic systems has gained significant attention due to their unpredictable. Traditional approaches to generating hyperchaotic systems typically involve increasing the complexity of the system, such as adding additional variables, nonlinear terms, or coupling mechanisms. However, achieving hyperchaotic through these traditional approaches can be computationally intensive and less than optimal. To solve this challenge, biologically inspires algorithms are crucial for accurately and effectively transforming chaotic systems into hyperchaotic ones. These algorithms focus on adjusting the parameters of chaotic systems to get the optimal values that integrate hyperchaotic behavior. By analyzing the systems dynamics and continuously adjusting the parameter, they aim to improve specific features, such as the Lyapunov exponent. This gauges the level of chaos. In this study, particle swarm optimization (PSO) algorithms will be employed. This method employs a feed-forward techniques that consists of three stages: exploration, migration, and exploitation. Its primary objective is to navigate the large parameter space and identify values that enhance hyperchaotic properties, such as raising the complexity of phase dynamics and chaos dimensionality.

These algorithm's performance will also be evaluated on a range of chaotic systems to ascertain how different factors affect the algorithm's dynamic behavior. The ultimate objective is to determine the optimal parameter combinations that effectively transform conventional chaotic systems into hyperchaotic systems, increasing their complexity and boarding their applications in domains such as cryptography, secure communications, and nonlinear dynamics.

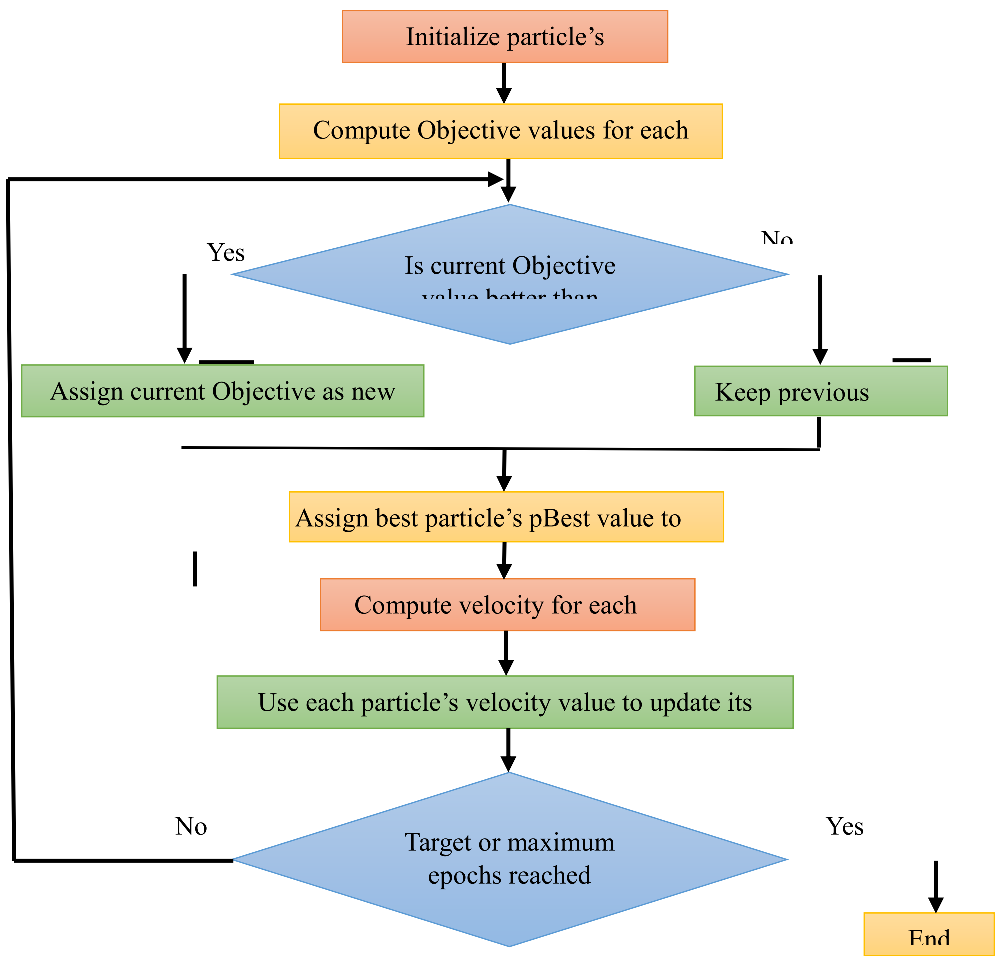


Figure 2. Flowchart of particle swarm optimization (PSO) algorithm.

Print ISSN 2710-0952 Electronic ISSN 2790-1254

4. A new 4D hyperchaotic system

In 2013, Liu et al. presented a unique 3D system consisting of six terms [48,49], as shown below:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = -c + x_1 x_3 \\ \dot{x}_3 = b - x_2^2 \end{cases}$$

(7)

where x_1, x_2, x_3 represent the state variables, a, b, c are the system parameters. This system exhibits chaotic attractor when $(a, b, c) = (1.5, 1.7, 0.05)$, whereas the corresponding LE_s as $(0.1928,\ 0.0002, -0.6443)$ and Lyapunov dimensions $(D_L = 2.2992)$ [48]. By applying a coupling control strategy, a new 4D hyperchaotic system is constructed by combining the original system (7) with Eq. ($\dot{x}_4 = -d x_1 x_4$) as shown below:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) \\ \dot{x}_2 = -c + x_1 x_3 \\ \dot{x}_3 = b - x_2^2 \\ \dot{x}_4 = -d x_1 x_4 \end{cases}$$

(8)

The variables x_1, x_2, x_3 and x_4 represent the state variables, with coupling parameter d ($d \neq 0$). Under the typical parameters and (IC) given in Eq. (9), Eq. (10), this system produces hyperchaotic attractors, the related (LE_s) and Lyapunov dimensions are described in Eq. (11) and Eq. (12), respectively. Fig. 3 shows the phase portraits of the suggest system.

$$(a, b, c, d)^{PSO} = \underbrace{(4.0728,\ 4.7726,\ 0.0487,\ 2.4745\ )}_{\text{Typical parameters}} \quad (9)$$

$$(x_o, y_o, z_o, w_o) = \underbrace{(0.1, 0.2, -0.25, 0.1)}_{IC} \quad (10)$$

$$\begin{cases} LE_1 = 0.8577 \\ LE_2 = 0.0014 \\ LE_3 = \mathbf{0.0008} \\ LE_4 = -3.1677 \end{cases}, \quad \sum_{i=1}^{4} LE_i = -2.3075$$

(11)

$$D_{KY}^{PSO} = J + \frac{1}{LE_{i+1}} \sum_{i=1}^{J} LE_i \Rightarrow D_{KY}^{PSO} = 3 + \frac{0.9454478}{|-2.011434|} = 3.3717 \quad (12)$$

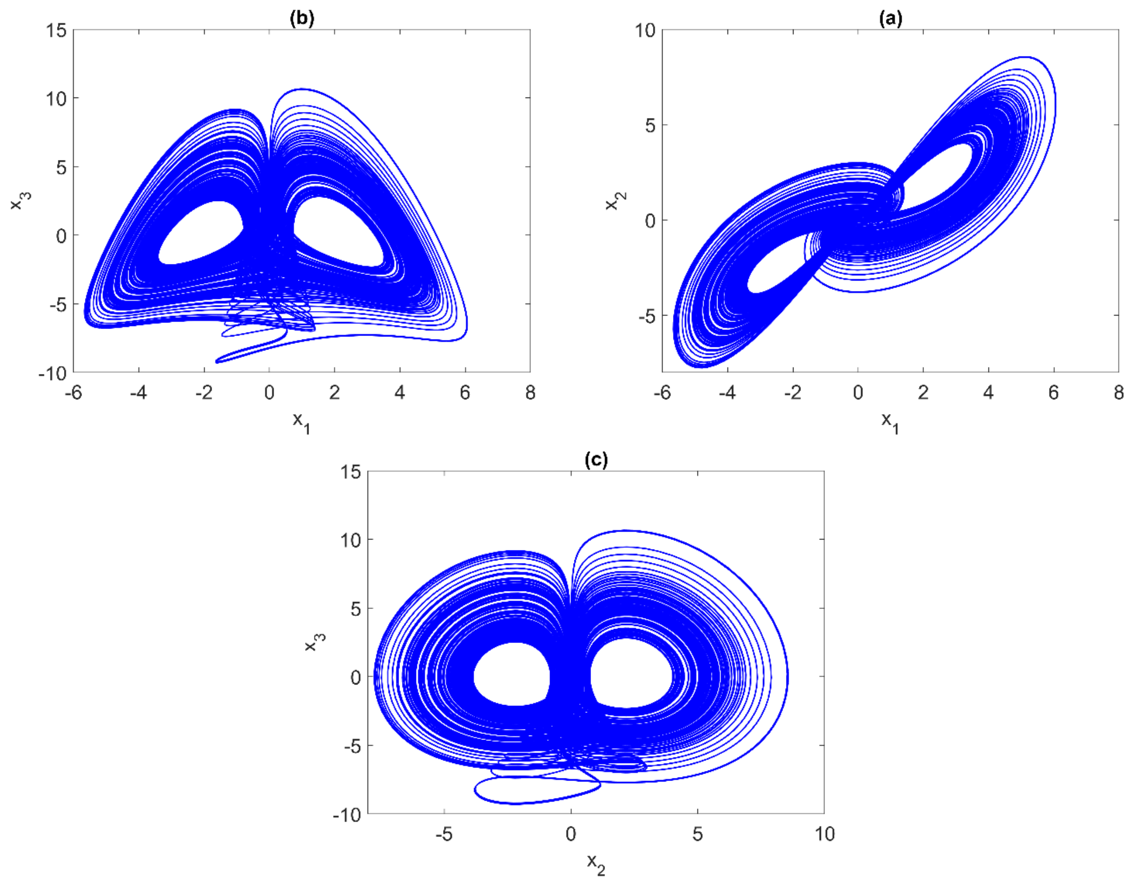


Figure 3. Phase portraits of the system (8) in planes, (a) $x_1 - x_2$, (b) $x_1 - x_3$, (c) $x_2 - x_3$.

5. Dynamic Analysis

The Lyapunov exponent, equilibrium, dissipation, and bifurcation diagram are among the common dynamical phenomena of this system that are examined.

The divergence (trace of a Jacobian matrix) of the system (8) is computed as:

$$Tr\,(J) = \sum_{i=1}^{4} \frac{\partial \dot{x}_i}{\partial x_i} = -a - dx_1 \quad (13)$$

5.1. Equilibrium with PSO

Setting $\forall\, \dot{x}_i = 0$ in system (8), and solved it yields two points:

Print ISSN 2710-0952 Electronic ISSN 2790-1254

$$E_{1,2}^{pso} = \left(\pm\sqrt{b}, \pm\sqrt{b}, \pm\frac{c}{\sqrt{b}}, 0\right) \tag{14}$$

The Jacobian matrix of the new system at the points $E_{1,2}$ can be expressed as follows:

$$J^{PSO} = \begin{bmatrix} -a & a & 0 & 0 \\ x_3 & 0 & x_1 & 0 \\ 0 & -2x_2 & 0 & 0 \\ -dx_4 & 0 & 0 & -dx_1 \end{bmatrix} \Longrightarrow J(E_{1,2}^{pso}) = \begin{bmatrix} -a & a & 0 & 0 \\ \pm\frac{c}{\sqrt{b}} & 0 & \pm\sqrt{b} & 0 \\ 0 & \pm 2\sqrt{b} & 0 & 0 \\ 0 & 0 & 0 & \mp d\sqrt{b} \end{bmatrix} \tag{15}$$

the characteristic equation and associated eigenvalues at parameters (9), are given in Eq. (16) and Eq. (17), respectively.

$$\lambda^4 + \underbrace{(a \pm d\sqrt{b})}_{p_1}\lambda^3 + \underbrace{(2b \pm ad\sqrt{b} \mp \tfrac{ac}{\sqrt{b}})}_{p_2}\lambda^2 + \underbrace{\left(2ab \pm 2bd\sqrt{b} - acd\right)}_{P_3}\lambda \pm \underbrace{2abd\sqrt{b}}_{P_4} = 0 \tag{16}$$

$$\underbrace{\begin{cases} \lambda_1 = -5.4060 \\ \lambda_2 = -4.0869 \\ \lambda_{3,4} = 0.0070 \pm 3.0841\, i \end{cases}}_{E_1^{pso}}, \quad \underbrace{\begin{cases} \lambda_1 = 5.4060 \\ \lambda_2 = -4.0586 \\ \lambda_{3,4} = -0.0070 \pm 3.0949\, i \end{cases}}_{E_2^{pso}} \tag{17}$$

Therefore, both equilibrium $E_{1,2}^{PSO} = \left(\pm\sqrt{b}, \pm\sqrt{b}, \pm\frac{c}{\sqrt{b}}, 0\right)$ are unstable saddle foci.

5.2. Lyapunov exponent with PSO

Lyapunov exponents are an effective tool for distinguishing between chaotic and hyperchaotic attractors. Several algorithms are available for computing Lyapunov exponents, with the Wolf algorithm being one of the most commonly used. Using the parameters $a = 4.0728,\ b = 4.7726,\ c = 0.0487,\ d = 2.4745$), a step size (sample time) of 0.25, an observation period of 1000, and an initial condition (10), the Lyapunov exponents of the proposed system are numerically calculated, as shown in Fig. 4.

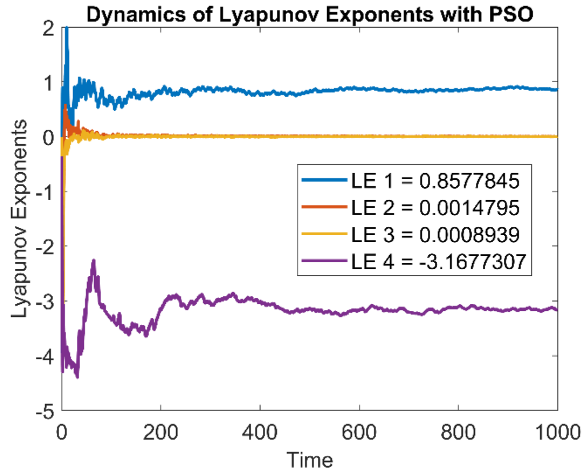


Figure 4. Lyapunov exponents of the system (8) for $(a, b, c) = (4.0728, 4.4426, 0.0487)$ and $d = 2.4745$.

6. Circuit Implementation

Hyperchaotic systems can be used to create many possible applications. As explained in this article, a readily modifiable analog op-amp circuit is used to implement the system's state variable. The circuit equations for integral operation, inverse operation, and nonlinear product are carried out by electronic parts such as multipliers, capacitor, resistors, and operational amplifiers. A compliant analog multiplier with an amplification factor of V is the operational amplifier supply, and the parameters are set at $a = 4.0728,\ b = 4.7726,\ c = 0.0487,\ d = 2.4745$).

$$\begin{cases} \dot{x}_1 = -4{,}072.8(x_3) - 4{,}072.8(-x_2) \\ \dot{x}_2 = -48.7 - 1000(-x_1)(x_3) \\ \dot{x}_3 = -4{,}772.6(-v_0) - 1000(-x_2)(x_2) \\ \dot{x}_4 = -2{,}474.5(x_1)(x_4) \end{cases}$$

(18)

By utilizing Kirchoff's law on the aforementioned system, we obtain

Print ISSN 2710-0952 Electronic ISSN 2790-1254

$$\begin{cases} \dot{x}_1 = -\frac{1}{R_1C_1}(x_1) - \frac{1}{R_2C_1}(x_2) \\ \dot{x}_2 = -\frac{1}{R_3C_2}(v_0) - \frac{1}{R_4C_2}(-x_1)(x_3) \\ \dot{x}_3 = -\frac{1}{R_5C_3}(-v_0) - \frac{1}{10R_6C_3}(-x_2)(x_2) \\ \dot{x}_4 = -\frac{1}{R_7C_4}(x_1)(x_4) \end{cases}$$

(19)

Select all the capacitors $(\forall C_i = 10\,nF,\quad i = 1,2,3,4)$ and $V_0\ is\ 1V$. By comparing them to Eqs. (18) and (19), the related resistors are found in Eq. (20), Fig. (5), Fig. (6) and Fig. (7) which show a screenshot from Multisim 14.3.

$$\begin{cases} \frac{1}{R_1C_1} = 0.000040728 \Rightarrow R_1 = 24.5531329\ \ K\Omega \\ \frac{1}{R_2C_1} = 0.000040728 \Rightarrow R_1 = 24.5531329\ \ K\Omega \\ \frac{1}{R_3C_2} = 0.0000004873 \Rightarrow R_3 = 2.052\ M\Omega \\ \frac{1}{10R_4C_2} = 0.0001 \Rightarrow R_4 = 10\ K\Omega \\ \frac{1}{R_5C_3} = 0.000047726 \Rightarrow R_5 = 20.952939\ K\Omega \\ \frac{1}{10R_6C_3} = 0.0001 \Rightarrow R_6 = 10\ K\Omega \\ \frac{1}{10R_7C_4} = 0.00024745 \Rightarrow R_7 = 4.04122\ K\Omega \end{cases}$$

(20)

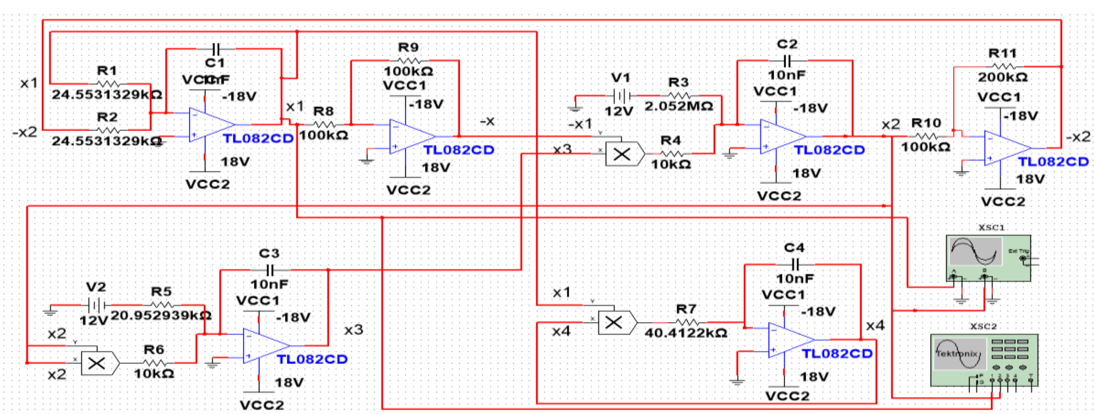
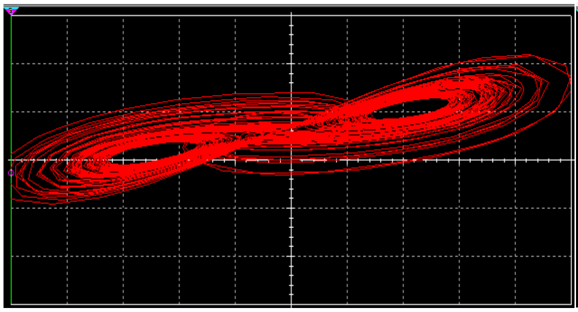
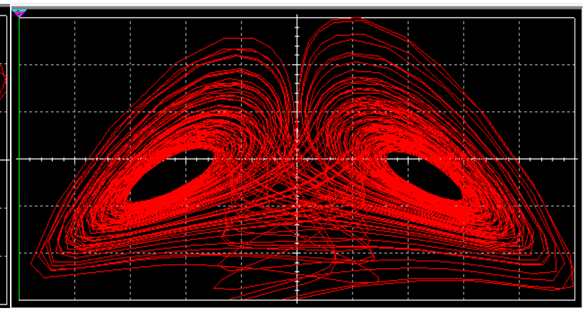


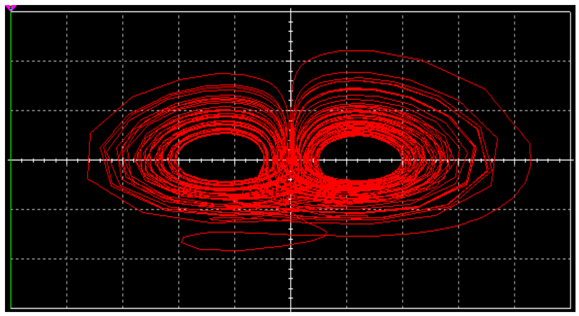
Figure 5. Diagram illustrating the circuit implementation of the proposed system.



(a)

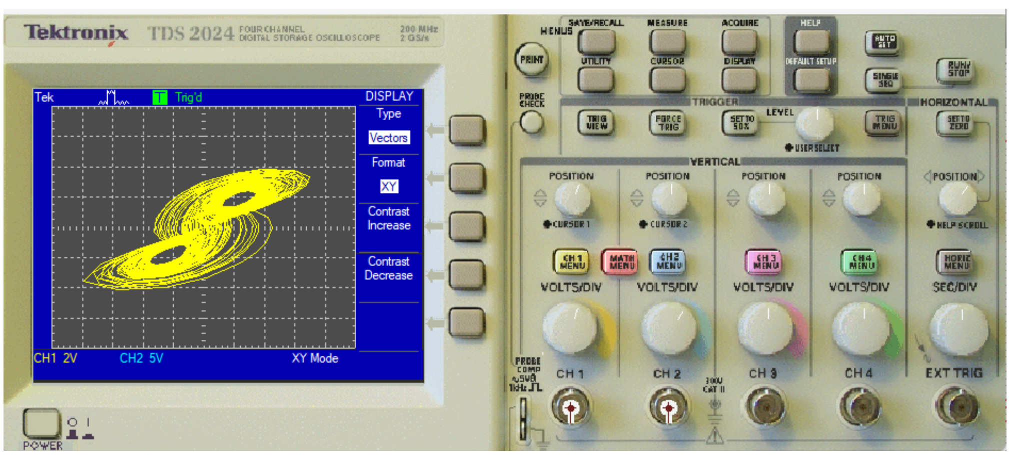


(b)



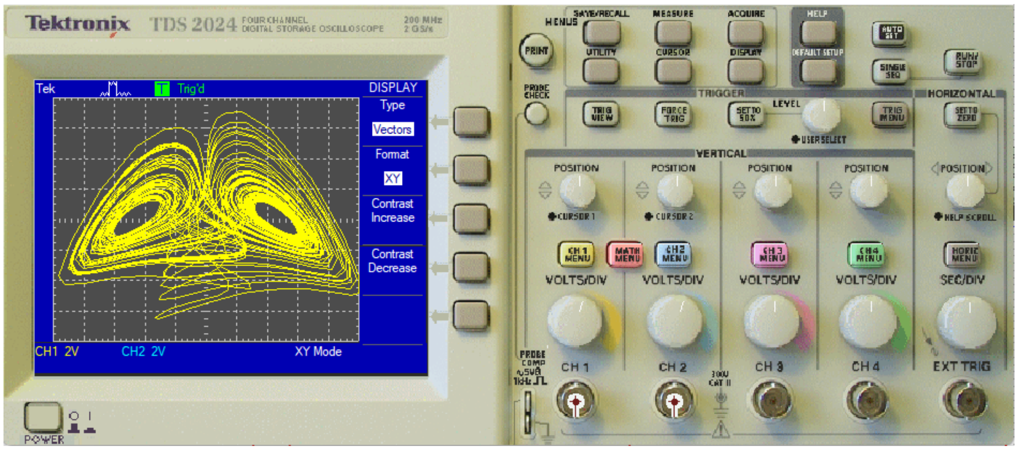
(c)

Figure 6. Simulation results in Multisim via oscilloscope in planes (a) $x_1 - x_2$, (b) $x_1 - x_3$, (c) $x_2 - x_3$.

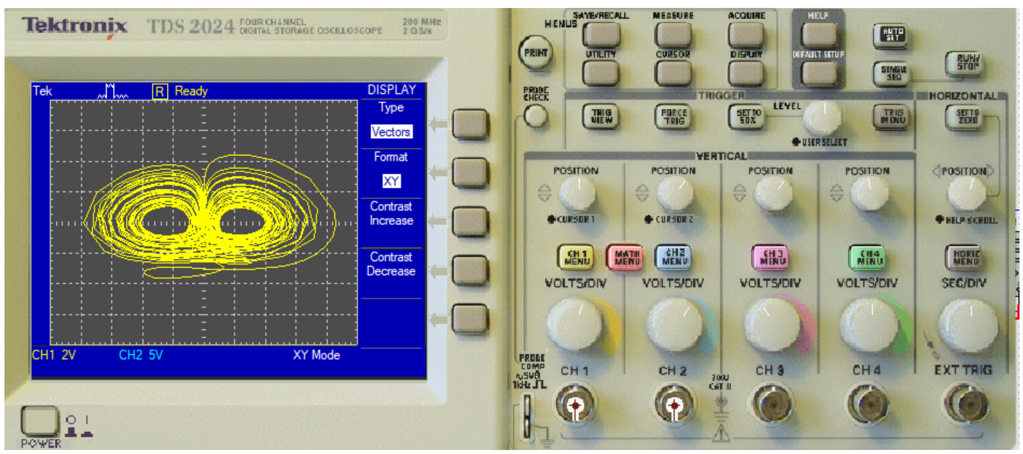


(a)

Print ISSN 2710-0952 Electronic ISSN 2790-1254



(b)



(c)

Figure 7. Screenshot from Multisim via Tektronix oscilloscope for simulation results of system (8), in planes (a) $x_1 - x_2$, (b) $x_1 - x_3$, (c) $x_2 - x_3$.

7. Discussion of Results

Following initial random parameter estimation of the chaotic system, the Particle Swarm Optimization (PSO) algorithm exhibited exceptional performance by increasing the maximal Lyapunov exponent (LE_s) from 0.45 to 0.8544745. This intelligent parameter optimization approach precisely identified critical parameter regions that maximize chaotic behavior, demonstrating both algorithmic precision and reliability. The quantitative results validate PSO effectiveness as a robust optimization tool for hyperchaotic systems, offering significant improvements over conventional parameter estimation methods. These findings advance the theoretical understanding of hyperchaotic system dynamics while enabling more sophisticated engineering applications in nonlinear dynamics research.

8. Conclusions

المجلة العراقية للبحوث الإنسانية والإجتماعية والعلمية
Iraqi Journal of Humanitarian, Social and Scientific Research
Print ISSN 2710-0952 Electronic ISSN 2790-1254

This study presents a novel seven-term four-dimensional hyperchaotic system derived through extension of the three-dimensional Liu system. The proposed system exhibits hyperchaotic characteristics, as evidenced by two positive Lyapunov exponents, and contains two unstable saddle-focus equilibrium points. Departing from conventional bifurcation analysis, we employ a Particle Swarm Optimization (PSO) algorithm to identify optimal parameter configurations that maximize the system's largest Lyapunov exponent. As a state-of-the-art metaheuristic optimization technique, PSO demonstrates exceptional efficacy in parameter estimation for nonlinear dynamical systems.

Through comprehensive theoretical and numerical analyses—including investigation of multistability phenomena, equilibrium point stability, Lyapunov spectrum computation, and Kaplan-Yorke dimension estimation—we elucidate the synergistic relationship between optimization algorithms and hyperchaotic system dynamics. The theoretical framework is experimentally validated via electronic circuit implementation in NI Multisim 14.3, successfully bridging theoretical modeling and practical realization. These findings collectively underscore the transformative role of computational optimization methods in advancing the analysis and control of complex nonlinear systems.

References:

[1] E. N. Lorenz: Deterministic nonperiodic flow. Journal of atmospheric sciences, 20(2), (1963), 130-141. DOI: 10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2.
[2] O. E. Rössler: An equation for continuous chaos. *Physics Letters A*, **57**(5), (1976), 397-398. DOI: 10.1016/0375-9601(76)90101-8
[3] J. C. Sprott: Some simple chaotic flows. *Physical review E*, **50**(2), (1994), R647. DOI: 1063-651X/94/50(2)/647(4)/$06. 00
[4] X. Hu, B. Sang and N. Wang: The chaotic mechanisms in some jerk systems. *AIMS Mathematics*, **7**(9), (2022), 15714-15740, DOI: 10.3934/math.2022861
[5] B. Li, B. Sang, M. Liu, X. Hu, X. Zhang and N. Wang: Some jerk systems with hidden chaotic dynamics. International *Journal of Bifurcation and Chaos*, **33**(06), (2023), 2350069, DOI: 10.1142/S0218127423500694
[6] S. F. Al-Azzawi: Dynamical behavior of a new jerk system inspired from chaotic memory oscillators. Archives of Control Sciences, **34**(1) (2024), 149-170. DOI: 10.24425/acs.2024.149656.
[7] K. E. Chlouverakis and J. Sprott: Chaotic hyperjerk systems. *Chaos, Solitons & Fractals*, **28**(3), (2006), 739-746, DOI: 10.1016/j.chaos.2005.08.019.

[8] S. F. Al-Azzawi and M. A. Al-Hayali: Coexisting of self-excited and hidden attractors in a new 4D hyperchaotic Sprott-S system with a single equilibrium point. *Archives of Control Sciences*, **32** (1), (2022), 37–56, DOI: 10.24425/acs.2022.140863.

[9] O. S. Ojoniyi and A. N. Njah: A 5D hyperchaotic Sprott B system with coexisting hidden attractors. *Chaos, Solitons & Fractals*, **87**, (2016), 172-181, DOI: 10.1016/j.chaos.2016.04.004.

[10] N. Cui and J. Li: A new 4D hyperchaotic system and its control. *AIMS Mathematics*, **8**(1), (2023), 905-923, DOI: 10.3934/math.2023044.

[11] D. Khattar, N. Agrawal and M. Sirohi: Dynamical analysis of a 5D novel system based on Lorenz system and its hybrid function projective synchronisation using adaptive control. *Pramana*, **97**(2), (2023), 76, DOI: 10.1007/s12043-023-02544-x.

[12] M.Yan and S. Li: Construction of a novel five-dimensional Hamiltonian conservative hyperchaotic system and its application in image encryption. *Soft Computing*, **29**, (2025), 53–67, DOI: 10.1007/s00500-025-10443-z.

[13] J. Wang, W.Yu, J. Wang, Y. Zhao, J. Zhang and D. Jiang: A new six-dimensional hyperchaotic system and its secure communication circuit implementation. *International Journal of Circuit Theory and Applications*, **47**(5), (2019), 702-717, DOI: 10.1002/cta.2617.

[14] S. F. Al-Azzawi and A. S. Al-Obeidi: Dynamical analysis and anti-synchronization of a new 6D model with self-excited attractors. Applied Mathematics-A Journal of Chinese Universities, **38**(1), (2023), 27-43, DOI: 10.1007/s11766-023-3960-0.

[15] J. Lv, Q. Xie and J. Zhang: Design, implementation of six-dimensional chaotic system and its application in color image encryption. *Physica Scripta*.**100**(2), (2025), 025251, DOI: 10.1088/1402-4896/adab99.

[16] J. Ma and Y. Yang: Hyperchaos numerical simulation and control in a 4D hyperchaotic system. *Discrete Dynamics in Nature and Society*, **2013**(1), (2013), 980578 , DOI: 10.1155/2013/980578.

[17] S.F. Al-Azzawi, M.A. Mohamed, H. Rubiani, M. Mamat, J. Titaley, and Y.A.R. Langi: Chaotic Lorenz system and it’s suppressed. *Journal of Advanced Research in Dynamical and Control Systems*, **12**(2), (2020), 548–555, DOI: 10.5373/JARDCS/V12I2/S20201076.

[18] W. Li, P. Li and M. Jia: Chaos control and chaos synchronization of a multi-wing chaotic system and its application in multi-frequency weak signal detection. *Aip Advances*, **11**(9), (2021), 095003, DOI: 10.1063/5.0062634.

[19] J. H. Park: Chaos synchronization of a chaotic system via nonlinear control. *Chaos, Solitons and Fractals*, **25** (2005), 579–584, DOI: 10.1016/j.chaos.2004.11.038.

[20] S. Y. Al-hayali and S. F. AL-Azzawi: An optimal nonlinear control for anti-synchronization of Rabinovich hyperchaotic system. *Indonesian Journal of Electrical Engineering and Computer Science*, **19**(1), (2020), 379-386, DOI: 10.11591/ijeecs.v19.i1.pp380-387.

[21] N.Yujun, W. Xingyuan, W. Mingjun and Z. Huaguang: A new hyperchaotic system and its circuit implementation. *Communications in Nonlinear Science and Numerical Simulation*, **15**(11), (2010), 3518-3524. DOI: 10.1016/j.cnsns.2009.12.005.

[22] Q. Lai, B. Bao, C. Chen, J. Kengne and A. Akgul: Circuit application of chaotic systems: modeling, dynamical analysis and control. *The European Physical Journal Special Topics*, **230**, (2021), 1691-1694. DOI: 10.1140/epjs/s11734-021-00202-0.

[23] Z. N. Al-Kateeb and M. Jader: Multi level of encryption and steganography depending on Rabinovich Hyperchaotic System & DNA. *Multimedia Tools and Applications*, **84**, (2025), 1211–1237. DOI: 10.1007/s11042-024-19057-3.

[24] N. M. Mohammed and Z. N. Al-Kateeb: A new secure encryption algorithm based on RC4 cipher and 4D hyperchaotic Sprott-S system. In *2022 Fifth College of Science International Conference of Recent Trends in Information Technology (CSCTIT)* (2022), (pp. 131-136). IEEE, DOI: 10.1109/CSCTIT56299.2022.10145711.

[25] S. J. Mohammed and Z. N. Al-Kateeb: Chao_SIFT based encryption approach to secure audio files in cloud computing . *Multimedia Tools and Applications*, **84**, (2025), 12683–12697, DOI: 10.1007/s11042-024-19424-0.

[26] K. A. Abed: A better approximation to Sawada-Kotera via differential transform method with hybrid grey Wolf and Cuckoo algorithm. *Journal of Interdisciplinary Mathematics*, **27**(6), (2024), 1231–1241, DOI: 10.47974/JIM-1513

[27] A. F. Qasim, K. A. Abed and O. S. Qasim: Optimal parameters for nonlinear Hirota-Satsuma coupled KdV system by using hybrid firefly algorithm with modified Adomian decomposition. *Journal of Mathematical and Fundamental Sciences*, *52*(3), (2020), 339-352, DOI: 10.5614/j.math.fund.sci.2020.52.3.7.

[28] S. P. S. Kumar and A. C. Kumar: Computer Aided Diagnosis of Brain Tumour Using Walrus Optimization Algorithm. *Journal of Electrical Systems*, **20**(3s), (2024), 2111-2126, DOI: 10.52783/jes.1811.

[29] E. H. Houssein, N. A. Samee, M. Alabdulhafith and M. Said: Extraction of PEM fuel cell parameters using Walrus Optimizer. *AIMS Mathematics*, **9**(5), (2024), 12726-12750, DOI: 10.3934/math.2024622.

[30] P. Trojovský and M. Dehghani: Walrus optimization algorithm: a new bio-inspired metaheuristic algorithm. *Scientific Reports* **13**, (2023), 8775 , DOI: 10.1038/s41598-023-35863-5.

[31] M. A. Shaheen, H. M. Hasanien, S. F. Mekhamer and H. E. Talaat: Walrus optimizer-based optimal fractional order PID control for performance enhancement of offshore wind farms. *Scientific Reports*, **14**(1), (2024), 17636, DOI:10.1038/s41598-024-67581-x.

[32] A. H. Alqahtani, H. M. Hasanien, M. Alharbi and S. Chuanyu: Parameters estimation of proton exchange membrane fuel cell model based on an improved Walrus optimization algorithm. *IEEE Access*. **12**, (2024), 74979 - 74992, DOI: 10.1109/ACCESS.2024.3404641.

[33] P. D. Kusuma and A. L. Prasasti: Migrating Walrus Algorithm: A New Metaheuristic that Hybridizes Migration Algorithm and Walrus Optimization Algorithm. *International Journal of Intelligent Engineering & Systems*, **16**(6), (2023), 673-683, DOI: 10.22266/ijies2023.1231.56.

[34] P. Trojovský and M. Dehghani: A new bio-inspired metaheuristic algorithm for solving optimization problems based on walruses behavior. *Scientific Reports*, **13**(1), (2023), 8775, DOI: 10.1038/s41598-023-35863-5.

[35] Z. Jalali Khalil Abadi, N. Mansouri, M. M. Javidi and B. Mohammad Hasani Zade: Load-balanced Task Scheduling Algorithm using Walrus Optimizer for Cloud Computing. *Journal of Algorithms and Computation*, **56**(1), (2024), 146-171.

[36] S. Vujošević, M. Ćalasan and M. Micev: Hybrid walrus optimization algorithm techniques for optimized parameter estimation in single, double, and triple diode solar cell models. *AIP Advances*, **14**(8), (2024), 085116, DOI: 10.1063/5.0223492.

[37] H. Kote and S. P. Siddique Ibrahim: Smart Organic Crop Rotation Methodology via Walrus-Optimized Morphable Schema Convolution Network with Precision Agriculture Data. *Journal of Computational Analysis & Applications*, **33**(5), (2024).

[38] J. Wang, B. Zhou and S. Zhou: An improved cuckoo search optimization algorithm for the problem of chaotic systems parameter estimation. *Computational intelligence and neuroscience*, **2016**(1), (2016), 2959370. DOI:10.1155/2016/2959370.

[39] T. Nestor, A. Belazi, B. Abd-El-Atty, M. N. Aslam, C. Volos, N. J. De Dieu and A. A. Abd El-Latif: A new 4D hyperchaotic system with dynamics analysis, synchronization, and application to image encryption. *Symmetry*, **14**(2), (2022), 424, DOI: 10.3390/sym14020424.

[40] P. Prakash, K. Rajagopal, I. Koyuncu, J. P. Singh, M. Alcin, B. K. Roy and M.Tuna: A novel simple 4-D hyperchaotic system with a saddle-point index-2 equilibrium point and multistability: design and FPGA-based applications. *Circuits, Systems, and Signal Processing*, **39**(9), (2020), 4259-4280, DOI: 10.1007/s00034-020-01367-0.

[41] D. Vo Hoang, S. Takougang Kingni and V. T. Pham: A No-Equilibrium Hyperchaotic System and Its Fractional-Order Form. *Mathematical Problems in Engineering*, **2017**(1), (2017), 3927184,
DOI: 10.1155/2017/3927184.
[42] C. Dong and J. Wang: Hidden and coexisting attractors in a novel 4D hyperchaotic system with no equilibrium point. *Fractal and Fractional*, **6**(6), (2022), 306, DOI:10.3390/fractalfract6060306.
[43] E. Ozpolat, V. Celik and A.Gulten: A Novel Four-Dimensional Hyperchaotic System: Design, Dynamic Analysis, Synchronization and Image Encryption Implementation. *IEEE Access*. (2024). DOI: 10.1109/ACCESS.2024.3454820
[44] J. Kennedy and R. Eberhart: Particle swarm optimization, *Proceedings of ICNN'95 - International Conference on Neural Networkss* **4**,(1995), 1942-1948.
DOI: 10.1109/ICNN.1995.488968
[45] Y. Shi and R. Eberhart: A modified particle swarm optimizer. *1998 IEEE International Conference on Evolutionary Computation Proceedings. IEEE World Congress on Computational Intelligence (Cat. No.98TH8360),* (1998), DOI: 10.1109/ICEC.1998.699146
[46] D. Freitas, L. G. Lopes and F. Morgado-Dias: Particle Swarm Optimisation: A Historical Review Up to the Current Developments. *Entropy*, **22**(3), (2020), 362, DOI: 10.3390/e22030362
[47] Y. Prasad, K. K. Biswas and M. Hanmandlu: A recursive PSO scheme for gene selection in microarray data. *Applied Soft Computing*, **71**, (2018), 213-225. DOI:10.1016/j.asoc.2018.06.019
[48] Y. Liu, S. Pang and D. Chen: An unusual chaotic system and its control. *Mathematical and computer modelling*, **57**(9-10), (2013), 2473-2493, DOI: 10.1016/j.mcm.2012.12.006
[49] K. A. Abed, S. F. Al-Azzawi and O. S. Qasim: Electronic circuit and image encryption using a novel simple 4D hyperchaotic system. *Physica Scripta*, **100**(1), (2024), 015210, DOI: 10.1088/1402-4896/ad941d