

Investigated energy loss and energy straggling of alpha particles in Aluminum, Germanium and copper

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Abstract

In the study, the stopping power, range, and energy loss straggling were calculated using the Bethe theory for protons in Aluminum germanium and copper materials over an energy range of [0.01-1000] MeV. The mathematical equations were programmed and implemented using MATLAB to obtain the results for the relative mass stopping power, range. The computed results from the Bethe equation were compared with the experimental results from Astar. The results showed a difference between the experimental and theoretical results at the beginning of the range used in the Bethe equation . Hence, a correction term was proposed for the Bethe equation to correct the results at low energies, which produced negative values. The amount of energy loss due to scattering during collisions was also calculated using The Bohr equation. The range of the incident particle increased with increasing incident particle energy in the mentioned materials

Key words: energy loss, Bethe, Astar, energy straggling

1. Introduction:

One of the fundamental topics in modern physics is the investigation of the movement of charged particles through materials. Additionally, the development of numerous detectors was made possible by our understanding of the interactions that occur as charged particles move through space[1] The energy lost by a particle per unit of its journey length in a given medium is known as stopping power. The formula for this is (dE/dx) where (-dE)stands for energy loss and (dx)for path length increase. The spatial distribution of energy deposition in a particle track is described by the linear energy transfer (LET), or the amount of energy actually deposited per unit length along the path[2] It is important to know both the fluency and the energy of the charged particles in order to compute the stopping power at any given site in order to calculate the dosage at that location due to charged particle irradiations. Radiation therapy is interested in heavy charged particles because to their unique physical characteristics. The rate of stopping power or specific ionization of these charged particles increases with decreasing particle velocity as they move through a material. Proton and electron radiations are frequently employed in diagnostic and therapeutic processes in medicine[3]] Energy straggling is a phenomenon that occurs when charged particles pass through a material and lose energy due to collisions with atoms in the material. This loss of energy is not uniform, but rather fluctuates due to statistical variations in the number and energy of the collisions. As a result, the energy distribution of the particles broadens as they penetrate deeper into the material, leading to a phenomenon known as energy straggling. This effect is particularly important in the design and optimization of radiation detectors, as it can significantly impact the energy resolution and detection efficiency of the device. In this context, accurate modeling and characterization of energy straggling is essential for the development of high-performance radiation detectors[4]

2. Stopping power:-

Bethe was the first person to use quantum mechanical studies on stopping power. The Bethe theory of stopping power is valid when the projectile's velocity surpasses the Bohr velocity. In Bethe's theory, the goal is assumed to be charged particle[5] To reduce the effect of absorption density on stopping power, the penetration depth is often measured in units of (g/cm^2) , and the energy loss per unit length of the path is defined as the mass stopping power in units of $(MeV.cm^2/g)[6]$

$$-\frac{1}{\rho}\frac{dE}{dx} = \frac{4\pi Z_1^2 Z_2 e^4 N_A}{m_e c^2 \beta^2 A} L_{Bethe}$$
(1)
$$\beta = \frac{v}{c}$$
(2)
$$-\frac{1}{\rho}\frac{dE}{dx} = 0.30707 \frac{Z_1^2 Z_2}{\beta^2 A} L_{Bethe}$$
(2)
Where $\frac{4\pi e^4 N_A}{m_e c^2} = 0.30707$

 L_{Bethe} It is a number of Beth stopping, where its equation is derived using quantum mechanics and it is true when the velocity of the falling particle is higher than the Bohr velocity. The stopping power of high-energy particles is well described[7]

$$L_{Bethe} = ln \frac{2m_e v^2}{I}$$
(3)

Taking into account the relativistic corrections, the number of Beth stops becomes[8]

$$L_{Bethe} = \ln \frac{2m_e c^2 \beta^2}{1 - \beta^2} + \beta^2 - \ln I$$
(4)

The stopping power of Beth can be written as follows

$$-\frac{1}{\rho}\frac{dE}{dx} = 0.30707 \frac{Z_1^2 Z_2}{\beta^2 A} \left[\ln \frac{2m_e c^2 \beta^2}{1 - \beta^2} + \beta^2 - \ln I \right]$$
(5)

the average distance a particle beam will travel (also called range) in a medium by integrating the stopping power over the full energy spectrum of the incident particles, such as[9]

$$R = \int_0^E \frac{dE}{-\left(\frac{dE}{dx}\right)} \tag{6}$$

3. Energy Loss Straggling

Straggling can be described as the variation of energy loss or range of ions in matter. In stopping power, the straggling is usually divided into electronic and nuclear straggling. Both of these kinds of straggling have significant effects. Straggling is made quite complex by variation of the charge-state of the ion as it penetrates matter. In addition, straggling refers to the distribution of energy loss around a mean value. The study of particle straggling begins with deriving Bohr straggling[10].

If a beam of the mono-energetic charged particle, with initial energy, passes through an absorbing of a thickness, as a result of statistical fluctuations in energy loss, the variance can be defined as follows[11]

$$\Omega_B^2 = 4\pi e^4 Z_1 Z_2 N R \tag{7}$$

where R is the penetrating thickness through the target , Z_1 and Z_2 are the atomic masses of projectile and target respectively[12]

where

$$N = \frac{Z_2 N_A \rho}{A}$$

where ρ is density of the material in $(\frac{g}{cm^3})$, N_A = 6.022 × 10²³ mol⁻¹ is the Avogadro number and(A) is the atomic mass of the material $(\frac{g}{mol})$.

4. Results and discussion

Figures(1, 2,3) The calculations of energy loss per unit area, known as stopping power, for alpha particles in solid elements aluminum, copper, and germanium were explained

using the Bethe formula (5). The results were compared with those of the global program Astar [13]. It was observed that at low energies, the Bethe equation gives negative values, which are meaningless physically and are therefore disregarded. In this research, a semiempirical equation was derived and added to the Bethe equation for each selected element in the study to obtain approximate values comparable to those of the program Astar, as shown in Table (2). The equation was modified as follows

$$-\frac{1}{\rho}\frac{dE}{dx} = 0.30707 \frac{Z_1^2 Z_2}{\beta^2 A} \left[\ln \frac{2m_e c^2 \beta^2}{1 - \beta^2} + \beta^2 - \ln I \right] + f(E)$$
(8)

and it was noted from the figures that the stopping power after correction increases at low energies. This means that a charged particle with low energy loses more energy via ionization compared to a fast particle with high energy because the slower particle spends more time in the atom, increasing the likelihood of electronic transition within the atom.

Figures(4, 5, 6) illustrate calculating the range of alpha particles in aluminum, copper, and germanium at energy levels ranging from 0.01 to 1000. Equation 6 was used after substituting Equation 8, and the results were compared with the Astar program. It is observed that at lower energies, the speed of charged particles is low, allowing them to remain longer within the atom, resulting in a shorter range. Conversely, at higher energies, the charged particles move faster, leading to a shorter residency time in the target, thus resulting in a longer range.

Figures(7, 8, 9) The calculation of energy loss due to scattering, known as the energy straggling, can introduce errors in measuring energy loss using Equation(7). This energy loss was calculated for each shell of the selected elements in the study, and it is noted that it is dependent on the atomic number of the material, affecting the statistical fluctuations in energy loss. The highest scattering value is observed in aluminum in the second shell, while copper and germanium exhibit higher values in the third shell, with the lowest values in the first shell. Additionally, it depends on energy, as the ionization capability increases with higher energy levels The values are shown in Table (2)

Table (1): Mass stopping power for alpha particle in aluminum, copper, Germanium

E(MeV)	aluminum			copper			Germanium		
	Astar	Beathe(z)	p.w	Astar	Beathe(z)	p.w	Astar	Beathe(z)	p.w
0.01	0.2351	-382.05	0.3060	0.1051	-421.04	0.1051	0.1394	-414.79	0.1489

0.04	0.5504	-57.2132	0.5320	0.2032	-68.959	0.2032	0.2648	-68.6516	0.2075
0.06	0.6937	-30.6743	0.7906	0.2505	-38.89	0.2505	0.3184	-38.9342	0.3184
0.08	0.8059	-19.0319	0.8055	0.2907	-25.40	0.2907	0.3618	-25.5644	0.3379
0.1	0.8945	-12.7597	0.8319	0.3256	-17.9865	0.3256	0.3980	-18.1951	0.3520
0.15	1.0450	-5.5194	0.9834	0.3984	-9.1598	0.3984	0.4699	-9.3968	0.3815
0.2	1.1360	-2.5500	1.1471	0.4568	-5.3633	0.4568	0.5217	-5.5931	0.4444
0.4	1.2800	0.6400	1.4606	0.6070	-0.8667	0.6070	0.6274	-1.0443	0.6111
0.6	1.2970	1.1735	1.4845	0.6766	0.1300	0.6766	0.6628	-0.0129	0.7007
0.8	1.2690	1.2776	1.4283	0.7024	0.4742	0.7024	0.6723	0.3540	0.7166
1	1.2250	1.2688	1.3532	0.7049	0.6131	0.7049	0.6694	0.5089	0.7343
2	0.9849	1.0177	1.0306	0.6276	0.6698	0.6276	0.5994	0.6051	0.6742
4	0.6986	0.7009	0.7027	0.4834	0.5168	0.4834	0.4229	0.4781	0.4927
6	0.5357	0.5423	0.5429	0.3880	0.4156	0.3880	0.3615	0.3874	0.3929
8	0.4398	0.4468	0.4471	0.3257	0.3496	0.3257	0.3031	0.3271	0.3299
10	0.3760	0.3824	0.3825	0.2823	0.3033	0.2823	0.2638	0.2845	0.2860
20	0.2271	0.2304	0.2304	0.1773	0.1886	0.1773	0.1664	0.1779	0.1782
40	0.1344	0.1353	0.1353	0.1081	0.1133	0.1081	0.1020	0.1073	0.1073
60	0.0981	0.0984	0.0984	0.0801	0.0833	0.0801	0.0758	0.0790	0.0790
80	0.0783	0.0784	0.0784	0.0645	0.0667	0.0645	0.0611	0.0633	0.0633
100	0.0657	0.0657	0.0657	0.0545	0.0561	0.0545	0.0517	0.0533	0.0533
150	0.0478	0.0477	0.0477	0.0400	0.0410	0.0400	0.0380	0.0390	0.0390
200	0.0382	0.0381	0.0381	0.0322	0.0329	0.0322	0.0306	0.0313	0.0313
250	0.0321	0.0320	0.0320	0.0272	0.0278	0.0272	0.0260	0.0264	0.0264
300	0.0298	0.0279	0.0279	0.0238	0.0242	0.0238	0.0227	0.0231	0.0231
350	0.0249	0.0248	0.0248	0.0213	0.0216	0.0213	0.0202	0.0206	0.0206
400	0.0225	0.0225	0.0225	0.0193	0.0196	0.0193	0.0184	0.0187	0.0187
450	0.0208	0.0207	0.0207	0.0178	0.0181	0.0178	0.0169	0.0172	0.0172
500	0.0193	0.0192	0.0192	0.0165	0.0168	0.0165	0.0157	0.0160	0.0160
600	0.0170	0.0169	0.0169	0.0146	0.0148	0.0146	0.0139	0.0141	0.0141
700	0.0153	0.0153	0.0153	0.0132	0.0134	0.0132	0.0126	0.0128	0.0128
800	0.0140	0.0140	0.0140	0.0121	0.0123	0.0121	0.0116	0.0117	0.0117
900	0.0131	0.0130	0.0130	0.0113	0.0114	0.0113	0.0108	0.0109	0.0109
1000	0.0123	0.0122	0.0122	0.0106	0.0107	0.0106	0.0101	0.0103	0.0103

Table (2): Equations to correct the Bethe equation for the alpha particle

Elements and compounds	Correction equation	Constants
Aluminium(Al)	$f(E) = p_1/(E^3 + q_1 E^2 + q_2 E + q_3)$	$p_1 = 0.1289q_1 = 0.491q_2 = 0.0365q_3 = -7.818 \times 10^{-5}$
Copper(Cu)	$f(E) = p_1/(E^2 + q_1 E + q_2)$	$p_1 = 0.3334q_1 = 0.08428q_2 = -0.0001513$
Germanium(Ge)	$f(E) = p_1/(E^3 + q_1 E^2 + q_2 E + q_3)$	$p_1 = 2.735q_1 = 7.533q_2 = 0.7255q_3 = -0.001418$

E(MeV)	Aluminum		Copper			Germanium			
	Astar	p.w	straggling	Astar	Astar	straggling	Astar	p.w	straggling
0.01	0.000009	0.000007	1.1352E-55	0.000004	0.000004	1.4441E-55	0.0000055	0.0000045	1.2820E-55
0.04	0.000029	0.000031	2.1272E-55	0.000014	0.000014	2.9308E-55	0.0000108	0.0000100	2.5129E-55
0.06	0.000039	0.000036	3.7386E-55	0.000021	0.000026	5.7937E-55	0.0000207	0.0000283	4.8337E-55
0.08	0.000048	0.000041	5.0533E-55	0.000027	0.000030	8.4486E-55	0.0000298	0.0000320	6.9689E-55
0.10	0.000056	0.000046	6.1937E-55	0.000033	0.000036	1.0904E-54	0.0000383	0.0000400	8.9513E-55
0.15	0.000074	0.000060	7.2292E-55	0.000046	0.000040	1.3202E-54	0.0000463	0.0000612	1.0820E-54
0.2	0.000090	0.000086	9.5387E-55	0.000057	0.000046	1.8403E-54	0.0000647	0.0000706	1.5112E-54
0.4	0.000164	0.000131	1.1633E-54	0.000096	0.000085	2.3039E-54	0.0000815	0.0000916	1.9042E-54
0.6	0.000206	0.000240	2.1077E-54	0.000129	0.000112	3.8597E-54	0.0001414	0.0000995	3.3024E-54
0.8	0.000263	0.000369	2.6531E-54	0.000160	0.000127	5.2012E-54	0.0001966	0.0001644	4.5935E-54
1.0	0.000322	0.000416	3.3884E-54	0.000191	0.000177	6.4676E-54	0.0002510	0.0002531	5.8642E-54
2.0	0.000658	0.000459	4.1476E-54	0.000356	0.000301	7.7204E-54	0.0003054	0.0003537	7.1348E-54
4.0	0.001558	0.001127	8.4794E-54	0.000762	0.000614	1.4373E-53	0.0005942	0.0006005	1.3881E-53
6.0	0.002777	0.003582	2.0084E-53	0.001280	0.001598	3.0759E-53	0.0013099	0.0018297	3.0600E-53
8.0	0.004308	0.004167	3.5807E-53	0.001910	0.001551	5.1650E-53	0.0022336	0.0021985	5.2179E-53
10	0.006129	0.006314	5.5543E-53	0.002649	0.003591	7.7068E-53	0.0033645	0.0080036	7.8596E-53
20	0.019252	0.036143	7.9020E-53	0.007806	0.005581	1.0687E-52	0.0046822	0.0111854	1.0938E-52
40	0.063249	0.090512	2.4823E-52	0.024540	0.034727	3.1492E-52	0.0138561	0.0316372	3.2369E-52
60	0.128534	0.139765	8.1549E-52	0.048890	0.062169	9.9004E-52	0.0434766	0.0894834	1.0156E-51
80	0.213472	0.369143	1.6572E-51	0.080213	0.126508	1.9724E-51	0.0864299	0.1643915	2.0191E-51
100	0.316987	0.415893	2.7523E-51	0.118049	0.176800	3.2361E-51	0.1079252	0.2530972	2.5212E-51
150	0.651740	0.747755	4.0870E-51	0.239574	0.324802	4.7625E-51	0.2082243	0.3537141	4.8642E-51
200	1.088083	1.459165	8.4030E-51	0.396525	0.420066	9.6652E-51	0.4214953	0.6498143	9.8464E-51
250	1.618431	2.039245	1.4029E-50	0.586211	0.698863	1.5997E-50	0.6968224	1.0004545	1.6278E-50
300	2.236862	2.680657	2.0867E-50	0.806614	0.918679	2.3650E-50	1.0293458	1.3981777	2.4046E-50
350	2.938564	2.878014	2.8840E-50	1.055942	1.157668	3.2541E-50	1.4155140	1.8379523	3.3067E-50
400	3.719467	3.527141	3.7888E-50	1.332960	1.414399	4.2600E-50	1.8517757	2.3160847	4.3259E-50
450	4.574389	4.624681	4.7956E-50	1.634529	1.687721	5.3776E-50	2.3364486	2.8297128	5.4581E-50
500	5.499630	5.467856	5.8979E-50	1.961883	1.976683	6.5942E-50	2.8654206	3.3765341	6.6938E-50
600	7.553664	7.582043	7.0908E-50	2.684978	2.698417	7.9149E-50	3.4373832	3.9546438	8.0299E-50
700	9.851962	9.554466	9.7391E-50	3.493274	3.274380	1.0832E-49	4.7009346	5.1985143	1.0982E-49
800	12.379719	11.673319	1.2702E-49	4.381166	4.000525	1.4093E-49	6.1140187	6.5508767	1.4283E-49
900	15.114730	13.929102	1.5961E-49	5.339686	4.773597	1.7675E-49	7.6616822	7.0036363	1.7898E-49
1000	18.038490	16.313959	1.9488E-49	6.364350	5.590904	2.1542E-49	9.3364486	9.4502805	2.1810E-49

Table(3) :alpha range in Aluminumand Copper



Figure(2) stopping power of alpha particle in copper



Figure(3) stopping power of alpha particle in Germanium



Figure(4) alpha particle range in Aluminium



Figure(6) alpha particle range in Germanium



Figure(7) the energy straggling of the alpha particle in Aluminum



Figure(8) the energy straggling of the alpha particle in Copper



Figure(9) the energy straggling of the alpha particle in Germanium

5. Conclusion

- 1. The Bethe formula is a good formula for calculating the stopping power of heavy charged particles in the studied targets using the atomic number, except at some low energies.
- 2. The semi-empirical equations derived and added to the Bethe formula improve the results significantly, giving more accurate results at low energies.
- 3. Energy loss straggling increases with the increase in atomic number

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