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A NEW EXPONETIAL TYPE KERNEL INTEGRAL TRANSFORM OF THE CLASSICAL DERIVATIVE TO NON-INTEGER ORDERS

Anmar Saleh Mahdi Alsaloomee

anmar.salih@iku.edu.iq

Abstract

This paper aims to use a new integral transform, specifically the innovative kernel integral transform, as a means to deal with differential equations involving fractional derivatives using both RiemannLiouville and computing fractional derivatives. The paper includes a discussion several outcomes and findings were observed the proof of the existence of this new integral transform. Additionally, the paper discusses the harmony between the Shehu transform and the innovative kernel integral transform. Numerical illustrations are included to validate the feasibility and accuracy the proposed technique for resolving differential equations that encompass fractional derivatives.

Key words: A fractional differential equation involves, the Riemann fractional derivative, requires the computation.

1. INTRODUCTION

His field of fractional differential extends beyond traditional calculus operations, and he practices masturbation. Although this idea dates back to the contributions of "Euler, Laplace, and Fourier " during the eighteenth and ninteenth centuries, it has obtained considerable interest in the past few years due to its practical uses in various differential equations are commonly solved in scientific and engineering disciplines involving fractional derivatives solving problems may be easier compared to dealing conventional differential equations, so numerical and analytical methods[1-5] were designed for this particular aim. Integral transformations various fields benefit from the utilization of algorithms to streamline and resolve intricate issues. The Laplace transform is [6,7] one of the most famous integral transforms, it is extensively utilized in the fields of the theory of control and the processing of signals. The Fourier transform is another common integral transform used in fields such as analysis of signals and processing of images. These transforms are used to solve differential equations, analyze signals, and study Integral transforms are a valuable tool in simplifying the solution of differential equations by converting them into algebraic equations. This conversion allows for a more straightforward approach in solving these equations[1]. Additionally, integral transforms aid in examination of the frequency constituents of functions is carried out through the analysis of integral transforms. solve partial differential equations in engineering, such as solving heat transfer equations. Integral transforms are used in image processing for image analysis, enhancement, feature extraction, and compression. By applying new integral transform methods of the Askey type, illustrative solutions to differential equations involving fractional derivatives can be obtained using RiemannLiouville integration and fractional differentiation in calculus. The new integral kernel transform method [8-12] can be adapted to obtain Atangana-Baleanu derivatives and others, and there is a relationship between the new integral kernel transform and all these transforms. Applying new integral kernel transform methods to differentia of non-linea equations involving fractional derivatives can face challenges such as the complexity of analytical solutions and convergence problems. The new integral kernel transform methods rely on the concept of the integral kernel, which is a function that expresses the effect of integration on neighboring points. These methods require the use of digital or analytical approximations of integration and fractional differentiation and are an active field of research and development. Fractional calculus can be used in applications such as signal processing, control theory, medical imaging, electrical engineering, mechanical engineering, computer science, mthematical physics, quantum symmetry, biology, environmental science, economics, and other fields. It should be noted that fractional calculus is still an active field of research, and there are many challenges and open issues in this field. If you need more details about transformation methods and practical applications, it may be helpful to refer to recent articles and research in this field or to communicate with experts in fractional calculus[11-27].

1. Definition and properties

Definition 1: The operator $I_{0\beta}$, which is the fractional integral of order β with respect to the Riemann-Liouville, applied to a function .

$f:(0,\infty) \rightarrow R \; for \; all \; \beta \in R+$ is defined as,

$$I^{0}{}_{\beta} f(u) = \left(\frac{1}{\Gamma(\beta)}\right) \int^{0}{}^{u} (u-v)^{\beta-1} f(v) dv, \qquad (1)$$

The function Γ(.) refers to the well-known pseudo gamma function, which is defined as:.

$$\Gamma p = \int_{x}^{0} u^{p-1} e^{-u} du, \qquad p \in \mathbb{C}. \qquad (2)$$

Definition 2: The operator RiemannLiouville derivative of a fraction.$R^{D}{}_{\beta}$ of order $_{\beta}$ acting on a function .$f: (0, \infty) \rightarrow R \; for \; all \; _{\beta} \in R+$ is defined as,

$$R^{D}{}_{\beta} f(u) = D_{u}{}^{n} I^{n-}{}_{\beta} f(u) = \frac{d^{n}}{du^{n}} \int_{t}^{0} (u-v)^{n-{}_{\beta}-1} f(v) dv, \qquad u > 0. \qquad (3)$$

where $n - 1 < {}_{\beta} \leq n, and \; n \in \mathbb{N}$.

Definition 3: The derivative of a fraction of a function in computing. $f: (0, \infty) \rightarrow R \; for \; all \; _{\beta} \in R+$ is defined as,

$$c^{D}{}_{\beta} f(u) = I^{n-}{}_{\beta} D_{u}{}^{n} f(u) = \left(\frac{1}{\Gamma(n-{}_{\beta})}\right) \int_{u}^{0} (u-v)^{n-{}_{\beta}-1} f^{n}(v) dv, \quad u > 0. \quad (4)$$

where $n - 1 < {}_{\beta} \leq n, and \; n \in \mathbb{N}$.

Definition 4: The Mittag-Leffler function serves as a generalization of the exponential function $E_{\beta}(k)$ and is formally defined as such[21,23].

$$E\delta(k) = \sum_{r} =^{0}{}_{i}{}^{f} k^{r} \, \Gamma(\delta r + 1), \qquad (5)$$

$$E\delta, \gamma(k) = \sum_{r} =^{0}{}_{i}{}^{f} k^{r} \, \Gamma(\delta r + \gamma), \qquad (6)$$

where $\delta, \gamma \in R+$ and $k \in \mathbb{C}$. Prabhaker introduces a generalization of the Mittag-Leffler function in the following manner:

$$E\varepsilon\delta, \gamma(k) = \sum_{r} =^{0}{}_{i}{}^{f} (\varepsilon)^{r} \frac{\Gamma(\delta n + \gamma) k^{r}}{r!}, \qquad (7)$$

where $\delta, \gamma, \varepsilon \in R+ \; and \; k \in \mathbb{C}$.

2. An innovative integral transform with an exponential-type kernel.

A new exponential-type kernel integral transform has been introduced for a given function[17,18]. $f(u) \in [0, \infty] \rightarrow R$ of exponential order $\beta > 0$ as follows:

$$\xi = \{ f(u): \exists \, K, \beta > 0, |f(u)| < K \exp(\beta u), for \; all \; u \in [0, \infty] \},$$

utilizing the specified integral:

$$Kh[f(u)] = k(s,\lambda,\eta) = s\int^{0}\infty \exp(-su)\, f(\lambda\eta u)du, \qquad (8)$$

It can also be defined as.

$$Kh[f(u)] = k(s,\lambda,\eta) = s\lambda\eta\int^{0}\infty \exp(-su\lambda\eta)\, f(u)du,$$

or

$$= limx \to \infty\; s\lambda\eta\; \int^{0}{}_{\varphi} \exp(-su\lambda\eta)\, f(u)du, \qquad (10)$$

The variables s, λ, η > 0 represent parameters for the revolutionary exponential kernel integral transform. The real number β is involved, and the integral is computed with the limit u = ϕ. Equations (1–10) provide fundamental information about fractional operators and the innovative exponential type kernel integral transform. Given the recent discovery of the revolutionary exponential kernel integral transform, there questions may emerge concerning its efficacy and trustworthiness. However, it is reassuring to know that this transform can be trusted, as it exhibits strong associations with other well-established transforms. Several considerations need to be taken into account when extending the revolutionary exponential kernel integral transform approach for managing sets of differential equations involving fractional derivatives(FDEs) in order to enhance efficiency and accuracy.

2. Integral Transforms

In this section, we will explore two effective techniques for solving differential equations: the Fourier transform and the Laplace transform. Not only do these methods have practical applications, but the Fourier transform also plays a fundamental role in quantum mechanics. It creates a link between the position and momentum representations, aiding in the simplification of Heisenberg's uncertainty principles. The integral transform proves to be advantageous by converting complex problems into simpler ones. The transforms covered in this part of the course are generally useful for solving differential equations and, to a lesser extent, integral equations. The underlying concept behind the transform is remarkably straightforward. To illustrate, let's consider a differential equation involving an unknown function f. We begin by applying the transform to the equation, which transforms it into a more manageable equation typically involving the transform F of f. Next, we solve one of these equations to determine F, and finally, we apply the inverse transform to obtain f. This entire process can be visually represented as a circle (or square) of ideas.

1.1 . The Fourier series of a periodic function

Consider a function f(x) that is complex-valued and exhibits repetition every L units. This repetition is denoted by the property f(x + L) = f(x) for all x. As a result of this periodicity, the behavior of the function within a single period, such as [0, L], determines its behavior across the entire real line. In simpler terms, if we have knowledge of f(x) within one period, we can extend that knowledge to encompass the entirety of the real line**.** This concept can be extended to any period [x0, x0 + L], as long as f(x0) = f(x0 + L). This flexibility is particularly valuable when dealing with functions that are discontinuous. The Fourier expansion, which is capable of seamlessly handling continuous functions, can also accommodate discontinuous functions as long as the number of discontinuities within a period remains finite1. As previously mentioned, the series converges not pointwise, but rather in the sense of ,indicating that it converges pointwise almost everywhere.

The functions $en(x) \equiv \exp\left(\frac{i2\pi nx}{L}\right)$ are periodic with period L, since $en(x\ +\ L) = en(x)\exp(i2\pi n) =\ en(x)$.

Hence, we can attempt to extend the function $f(x) = \sum_{-\infty}^{\infty} conx\ =\ en(x)$, where {cn} represents complex coefficients. This particular series is referred to as a trigonometric or Fourier series of the periodic function f, and the {cn} coefficients are known as the Fourier coefficients. When subjected to complex conjugation, the exponentials en(x) fulfill the condition $en(x) * =\ e^{-n(x)}$, and they also exhibit the following orthogonality property.

$$\int [0, \mathrm{L}]\mathrm{em}(\mathrm{x}) *\ \mathrm{en}(\mathrm{x})\mathrm{dx}\ =\ \mathrm{L}, \mathrm{if}\ \mathrm{n}\ =\ \mathrm{m}, \mathrm{and}\ \ 0, \mathrm{otherwise}.$$

Hence, by multiplying em(x)* to both sides of equation and performing integration, we obtain the subsequent expression for the Fourier coefficients:

$$cn\ =\ \left(\frac{1}{L}\right)\int [0, L]em(x) *\ f(x)dx.$$

It is crucial to understand that the exponential functions en(x) fulfill the orthogonality relation for any given period, which may not necessarily be the same.

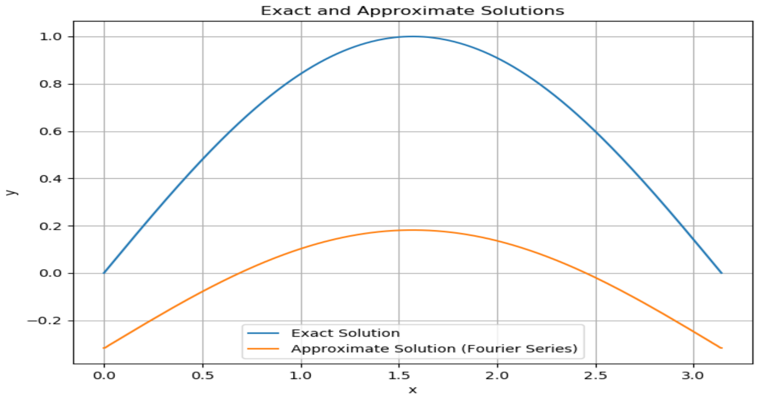
$$[0, L]: \int one\ period\ em(x) *\ en(x)dx\ =\ L, if\ n\ =\ m, and\ \ 0, otherwise.$$

The Fourier coefficients can be acquired by integrating over any single period.

$$cm\ =\ \left(\frac{1}{L}\right)\int one\ period\ em(x) *\ f(x)dx.$$

Once more, it can be asserted without providing evidence of the series' pointwise convergence almost everywhere within a specified timeframe., in the following manner:

$$\lim N \to \infty \int one\ period\ |f(x) -\ \Sigma N\ n = -N\ cn\ en(x)|^2 dx\ =\ 0,$$



whenever the {cn} are given by. **some example** Let us proceed to calculate several instances of Fourier series for the function f(x) = |sin x|. The graph of this function demonstrates its periodicity with a period of π. **As a second example**, 1. Let's examine the function f(x) which is defined in the interval [−π, π] as follows: $f(x) = (-1 - 2\pi x, if\ -\pi \leq x \leq 0$, and −1 + 2 π x ,if 0 ≤ x ≤ π. This function is extended periodically to the entire real line. The plot of this function for x ∈ [−2π, 2π] is illustrated in Figure 2. It is evident from the graph that f(x) exhibits periodicity of 2π, hence we anticipate a Fourier series representation in the form $f(x) = \sum_{n}^{\infty} cn\, e^{inx}$.

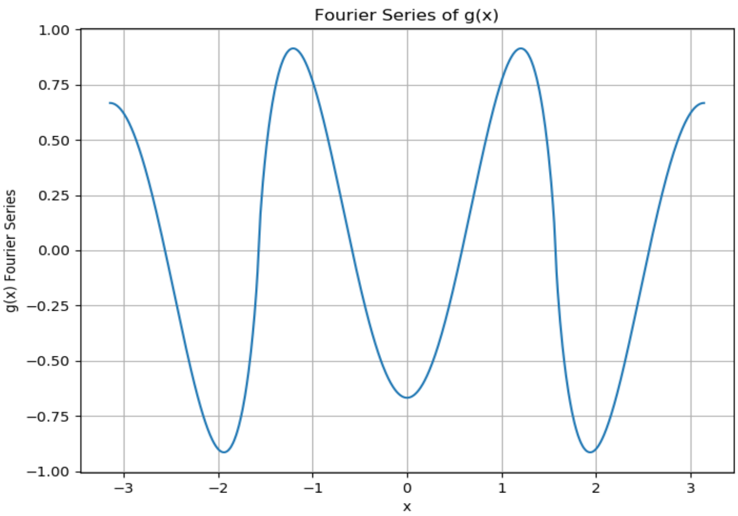
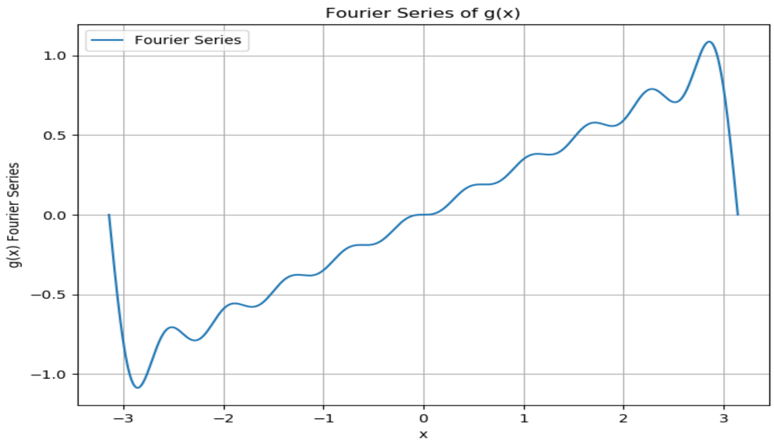


Fig1

The Fourier integral can be understood as follows: Let's consider a function f(x) defined on the entire real line. If f(x) were periodic with a period of L, we could represent it using a Fourier series that approximates f(x) within each period. This series takes the form f(x) = $\sum_{-\infty}^{\infty} cn\, e^{\frac{i2\pi nx}{L}}$, where the coefficients {cn} are calculated using $cn = \left(\frac{1}{L}\right)\int \left[\frac{L}{2},-\frac{L}{2}\right] f(x)e^{-\frac{i2\pi nx}{L}}dx$. For convenience, we choose the period as [-L/2, L/2]**.** Even if f(x) is not periodic, we can define a function $fL(x) = \Sigma\, cn\, e^{\frac{i2\pi nx}{L}}$ using the same coefficients {cn} as before. This function fL(x) is constructed to be periodic with period L and approximately coincides with f(x) for almost all x ∈ [-L/2, L/2]. As we increase the value of L, the agreement between fL(x) and f(x) extends to a larger portion of the real line. Naturally, as L tends to infinity, we expect fL(x) to converge to f(x) in some manner. Our goal now is to find suitable expressions for the limit as L approaches infinity for both fL(x) and the coefficients. Third example: Let



$f(x) = \frac{1}{4+x^2}$. This function is square-integrable, and its Fourier transform is given by:

$$f(k) = \left(\frac{1}{2\pi}\right)\int_{-\infty}^{\infty} \frac{e^{-ikx}}{4\,+\,x^2}dx.$$

By using the residue theorem, we can compute this integral and find:

$$f(k) = \left(\frac{1}{4}\right)e^{-2|k|}.$$

The inversion formula can also be verified, showing that $f(x) = \frac{1}{4+x^2}$.

Fourth example : Consider the pulse function:

$$f(x) = 1, for\ |x| < \pi,$$

$$0, otherwise.$$

This function is square-integrable, and its Fourier transform is given by:

$$f(k) = \left(\frac{1}{2\pi}\right) \int_{-\infty}^{\infty} f(x) e^{-ikx} dx = \frac{\sin(\pi k)}{\pi k}.$$

The inversion formula can be used to find f(x) for values other than x = ±π.

Now : Let's examine the Fourier transform of a finite wave train:

$$f(x) = sin(x), for\ |x| \leq 6\pi,$$

$$0, otherwise.$$

This function is square-integrable, and its Fourier transform is given by:

$$f(k) = \left(\frac{1}{2\pi}\right) \int \infty - \infty\, f(x) e^{-ikx} dx = \frac{\sin(6\pi k)}{\pi(1 - k^2)}.$$

The inversion formula can be applied for the continuous function f(x) in this case.

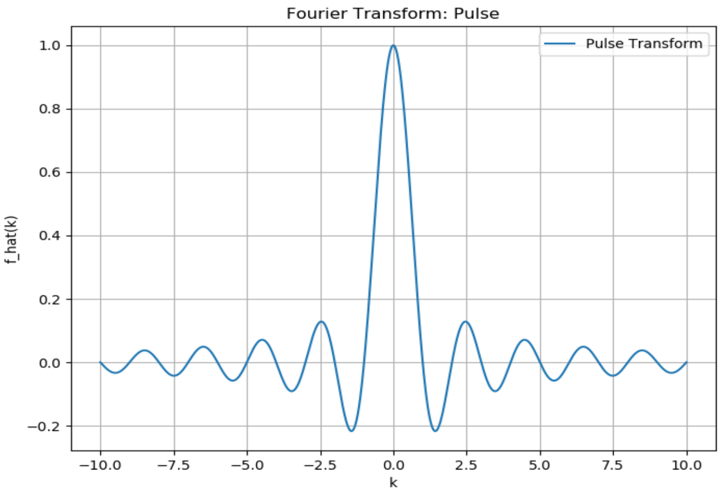
Table 1: pulse_transform

itreation	k	f_hat(k)
1	-0.10	0.3183098861837907
2	-0.99	0.3183098861837907
3	-0.98	0.3183098861837907
4	0.00	0.1
5	0.10	0.3183098861837907
6	0.99	0.3183098861837907
7	0.98	0.3183098861837907

Table 2: wave_transform

itreation	k	f_hat(k)
1	-0.10	0.045377994533445
2	-0.99	0.045377994533445
3	-0.98	0.045377994533445
4	0.00	0.0
5	0.10	0.045377994533445
6	0.99	0.045377994533445

7	0.98	0.045377994533445



4 Applications

In this section, we will explore specific applications of a new exponential-type kernel integral transform in the solution of fractional differential equations. By employing fractional order, we focus on a linear ordinary differential equation [46]..

$$c\,D\,u\ \ \beta\, y(u) = \sum j = 1\, n\, b\, j\, y\,(j)\,(u) + g(u) \quad (26) \quad cDu\,\beta y(u) = \sum j = 1nbjy(j)(u) + g(u) \quad (26)$$

with initial conditions:

$$y\,(j)\,(0) = a\,j\,, j = 0, \dots n-1, a\,j\,, b\,j\ \in R,\quad g(u) \in A \quad (27)\ \ y(j)(0) = aj, j = 0, \dots n-1, aj, bj \in R,\quad g(u) \in A \quad (27)$$

We apply the revolutionary exponential kernel integral transform to Equation (26), ensuring its stability.

$$Kh[\ c\,D\,u\ \ \beta\, y(u)] = Kh[\sum j = 1\, n\, b\, j\, y\,(j)\,(u) + g(u)]\ Kh[\ cDu\,\beta y(u)] = Kh[\sum j = 1nbjy(j)(u) + g(u)]$$

Utilizing the linearity property of a recently developed exponential-type kernel integral transform, we get.

$$Kh[\ c\,D\,u\ \ \beta\, y(u)] = \sum j = 0\, n\, b\, j\, Kh\big(y\,(j)\,(u)\big) + Kh\big(g(u)\big)Kh[\ c\,D\,u\ \ \beta\, y(u)] = b\,0\,y(u) + \sum j = 1\,n\,b\,j\,Kh\big(y\,(j)\,(u)\big) + Kh\big(g(u)\big)\ \ Kh[\ cDu\,\beta y(u)] = \sum j = 0nbjKh\big(y(j)(u)\big) + Kh\big(g(u)\big)Kh[\ cDu\,\beta y(u)] = b0y(u) + \sum j = 1nbjKh\big(y(j)(u)\big) + Kh\big(g(u)\big)$$

By applying theorem 3 and property (b), we derive

$$(\lambda\eta s) - \beta\, Y(s,\lambda,\eta) - \sum k = 0\, n - 1\, (\lambda\eta s) - \beta + k\, y\, k\,(0) = b\,0\, Y(s,\lambda,\eta) + \sum j$$
$$= 1\, n\, b\, j + [s\, j\, \lambda\, j\, \eta\, j\ Y(s,\lambda,\eta) - \sum k$$
$$= 0\, j - 1\, (s\lambda\eta)\, j - k\, y\, k\,(0)] + Kh\big(g(u)\big) + (\lambda\eta s) - \beta\, Y(s,\lambda,\eta)$$
$$- \sum j = 0\, n\, b\, j\, s\, j\, \lambda\, j\, \eta\, j\ Y(s,\lambda,\eta) = \sum k$$
$$= 0\, n - 1\, a\, k\, (\lambda\eta s) - \beta + k - \sum j = 1\, n\, b\, j \sum k$$
$$= 0\, j - 1\, a\, k\, (s\lambda\eta)\, j - k + Kh\big(g(u)\big)\ (\lambda\eta s) - \beta Y(s,\lambda,\eta) - \sum k$$
$$= 0n - 1(\lambda\eta s) - \beta + kyk(0) = b0Y(s,\lambda,\eta) + \sum j$$
$$= 1nbj + [sj\lambda j\eta jY(s,\lambda,\eta) - \sum k = 0j - 1(s\lambda\eta)j - kyk(0)]$$
$$+ Kh\big(g(u)\big) + (\lambda\eta s) - \beta Y(s,\lambda,\eta) - \sum j = 0nbjsj\lambda j\eta jY(s,\lambda,\eta)$$
$$= \sum k = 0n - 1ak(\lambda\eta s) - \beta + k - \sum j = 1nbj\sum k$$
$$= 0j - 1ak(s\lambda\eta)j - k + Kh\big(g(u)\big)$$

Alternatively, by employing Equation (27), we obtain

$$Y(s,\lambda,\eta) = \big((\lambda\eta s) - \beta - \sum j = 0\, n\, b\, j\, s\, j\, \lambda\, j\, \eta\, j\ \big) - 1\ \times \sum k$$
$$= 0\, n - 1\, a\, k\, (\lambda\eta s) - \beta + k - \sum j = 1\, n\, b\, j \sum k$$
$$= 0\, j - 1\, a\, k\, (s\lambda\eta)\, j - k + Kh\big(g(u)\big) \quad (28)$$

By employing the inverse of an exponential type kernel integral transform on Equation (28), we obtain the outcome of Equation (26) as a secure result.

$$y(u) = Kh - 1\Big[\Big[\big((\lambda\eta s) - \beta - \sum j = 0\, n\, b\, j\, s\, j\, \lambda\, j\, \eta\, j\ \big) - 1 \times \sum k$$
$$= 0\, n - 1\, a\, k\, (\lambda\eta s) - \beta + k - \sum j = 1\, n\, b\, j \sum k$$
$$= 0\, j - 1\, a\, k\, (s\lambda\eta)\, j - k + Kh\big(g(u)\big)\ \Big]\Big] \quad (29)$$

Example 1

Let us consider the homogeneous heat $equation$

$$u(x,t) = \cos(\pi^2 * t) * \sin(\pi * x)$$

$$4\partial u(x,t)\partial t = \partial\, 2\, u(x,t)\partial x\, 2$$

$$4\partial u(x,t)\partial t = \partial 2u(x,t)\partial x2$$

with initial condition

$$u(x,0) = sin\pi 2\, x, x, t > 0 \quad (37)\ u(x,0) = sin\pi 2x, x, t > 0$$

I added x_data and t_data as variables to represent the spatial and temporal grids, respectively. You can replace these variables with your actual data if you have specific values you want to use.

the solution u(x, t) at the point (x, t) = $(x\_values[i], t\_values[j])$.

Table 2: wave equation

itreation	*X*	*Y*	*T*	U(X,Y,T)

1	0.0	3.172	6.34	(6.342, 3.172, 1.2246)
2	0.0	3.1570	6.3109	(6.310, 3.157, 1.218)
3	0.0	3.1570	6.310	(6.3109, 3.1570, 1.2185)
4	0.0	3.1099	6.2167	(6.2167, 3.1099, 1.200)
5	0,0	-3.077	-6.152	(6.15 , 3.077, -1.1879)
6	0.0	-3.077	-6.152	(-6.1523, -3.077, 1.1879)
7	0.0	-3.07770	-6.15231	(-6.152, -3.0777, -1.187944)
8	0.0	-3.077705	-6.15231	(-6.1523, -3.0777, -1.187944)
9	0.0	-2.985	-5.96838	(-5.96838, 3.0777, -1.1879)
10	0.0	-3.077	-6.152	(6.15, -2.864, -1.10547)

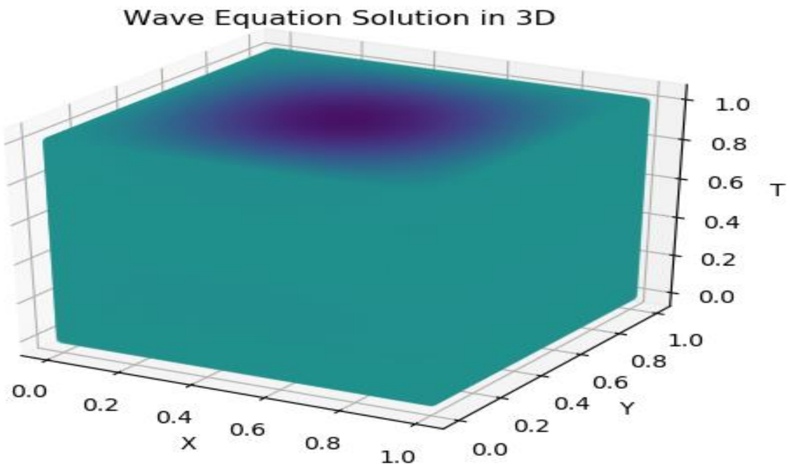


Fig4

Parameter	**Value**
Shape of U	**(100, 100, 100)**
Minimum value of U	**-0.9999999999999999**
Maximum value of U	**0.9999999999999999**

5 Conclusion

The new method for transforming the fractional integral kernel-based approach emerges as a promising and sophisticated method for handling RiemannLiouville fractional derivatives, integrals, and computing fractional derivatives. The method has demonstrated its effectiveness and efficiency in solving various examples, showcasing its capability to provide accurate solutions for differential equations involving fractional derivatives that require derivative and partial derivative computations. This notable success suggests that the kernel-based approach has the potential to make significant contributions to the advancement of solving complex mathematical problems in the field of fractional calculus in the future.

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