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Quality Control Charts for EWMA with Wavelet Shrinkage: A Simulation Study

E. A. Haydier(1), D. M. Saleh(2), T. H. Ali (3)*, B. S. Sedeeq(4)

(1,2,,3*,4) Department of Statistics and Informatics, College of Administration and Economics, University of Salahaddin, Erbil, Kurdistan Region, Iraq

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Correspondence:
Taha Hussein Ali
taha.ali@su.edu.krd

Abstract

The study aims to utilise modern statistical techniques to analyze data often sensitive to small changes. Quality control charts are valuable tools for monitoring and analyzing data and identifying and extracting points that fall outside the control limits. On the other hand, wavelet shrinkage is an effective technique for removing noise from data and identifying deviations while preserving significant signals, thus enhancing the accuracy of the analysis. Alongside using actual data that represents the qualitative feature of children's weights- a crucial indicator for evaluating overall health and early detection of nutritional or health issues- this chart was simulated using MATLAB. However, weight measurements are frequently susceptible to noise and interference due to factors such as measurement errors or natural variations. The study concluded that the proposed chart (weighted moving average using wavelet shrinkage) is superior to the classical chart because it performs better in identifying the differences between the upper and lower limits of the two charts. We also observed that the difference between the two limits in the proposed chart is smaller

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1. Introduction

Many manufacturing and non-manufacturing processes employ control charts, the most crucial tool of statistical process control (SPC), to identify changes in the quality characteristic of interest.

When Roberts first presented the exponentially weighted moving average (EWMA) graphic (Technometrics 1959) [1], it was known as a geometric moving average chart. Since exponential smoothing is the foundation of EWMA charts, the name was modified to reflect this.

An exponentially weighted moving average chart, or EWMA chart, is a form of control chart used in statistical quality control to track variables or attribute-type data, utilising the full output history of the business or industrial process under observation. The EWMA chart tracks the exponentially weighted moving average of all previous sample averages, whereas other control charts handle logical subsets of data separately [2]. Samples are weighted by EWMA in a geometrically decreasing sequence, with the most recent samples contributing the most and the most distant samples contributing the least. The SPC literature has suggested several EWMA chart extensions and tweaks to enhance the traditional EWMA chart's detection capabilities for particular shift ranges.

Wavelet shrinkage is a method of reducing noise in signals. e-noising is a technique for lowering signal noise because of wavelet shrinking [3]. One method of signal denoising that relies on thresholding the wavelet coefficients is wavelet shrinkage. Donoho et al. (1995a) [4] first presented the wavelet shrinkage method for broad curve estimation issues. There are several good reasons why wavelet shrinkage can be used for estimation functions. Wavelet shrinkage is an effective tool for removing noise to filter data and detect deviations in a way that preserves important signals, which contributes to improving the accuracy of the analysis [5].

A quality control chart is a graphical representation of whether a firm's products or processes are meeting their intended specifications. If problems appear to arise, the quality control chart can be used to identify the degree to which they vary from those specifications and help in error correction. A quality control chart can also be univariate or multivariate, meaning that it can show whether a product or process deviates from one or more than one desired result [6].

The exponential moving average (EMA) is a technical chart indicator that tracks the price of an investment, such as a stock or a commodity, over time. An exponential moving average (EMA) is a type of moving average (MA) that places greater weight and significance on the most recent data points. The exponential moving average is also called the exponentially weighted moving average. An exponentially weighted moving average reacts more significantly to recent price changes than a simple moving average (SMA), which applies equal weight to all observations in the period. The exponentially weighted moving average (EWMA) is discussed [7].

The EWMA is often used to smooth irregular fluctuations (i.e., noise) in a time series, allowing data analysts to better reveal trend and cycle patterns over time. Additionally, the EWMA is frequently used to compute short-term forecasts of time series data (e.g., sales and stocks). This discussion includes common applications of the EWMA in management science and quality engineering. Furthermore, properties of the EWMA, such as the expected value, variance, and one-step ahead prediction variance, are also explored [8].

Donoho and Johnstone (1995) pioneered the concept of wavelet shrinkage for signal denoising, and its integration into control charting has gained increasing attention [9]. For example, Zhao et al. (2020) demonstrated that incorporating wavelet denoising into multivariate process monitoring can significantly enhance fault detection sensitivity [10]. Li and Wang (2021) proposed a wavelet-EWMA hybrid model that showed improved detection of small shifts in mean processes under high-noise conditions [11].

Applications of SPC in healthcare contexts have also become prominent. According to Kumar and Alvarado (2022), control charts have been effectively used to monitor pediatric weight and growth metrics, aiding early detection of nutritional imbalances [12]. The relevance of such applications is heightened in scenarios involving frequent measurement noise due to non-standardized weighing equipment and physiological variability in children.

Recent simulation-based studies have further validated the use of advanced SPC tools. For instance, Ahmed et al. (2023) compared classical Shewhart, CUSUM, and wavelet-enhanced EWMA charts under various noise scenarios, reporting the consistent superiority of wavelet-based methods in terms of Average Run Length (ARL) performance [13]. Meanwhile, Chen et al. (2023) emphasized the importance of tightening control limits, facilitated by denoising, to detect subtle shifts that would otherwise go unnoticed in traditional frameworks [14].

Yilmaz and Ozdemir (2024) conducted a comprehensive simulation study on hybrid control charts and found that charts integrating adaptive thresholds and noise reduction techniques, such as wavelets, performed best under small sample sizes and short monitoring periods [15].

Collectively, these studies support using wavelet-based enhancements for classical EWMA control charts, especially in applications involving health-related, noise-prone data, such as monitoring children's weight. The current study expands on this growing body of work by demonstrating, through MATLAB simulations, that wavelet shrinkage EWMA charts provide superior sensitivity and tighter control bounds than their classical counterparts, as well as processing data noise.

2. Quality Control Charts

It is a statistical measurement used to monitor production operations and ensure their compliance with the required standards. These plates aim to maintain the overall production operation, ensuring the required specifications of high quality and fewer units than the production factors within a limited time frame. However, rapid intervention is necessary when any issues affect the quality of the final product and its timely delivery as soon as possible [16].

Contents of Control Charts:

Control charts are used to monitor the stability of production processes and evaluate their performance. They rely on three main lines [17]:

1. **Middle Line:**
   - This line represents the average or central value of the analyzed data.
   - It serves as a reference point to determine the deviation of other values.
2. **Upper Control Limit (UCL):**
   - This is the maximum value allowed for the process before it is considered unstable.
   - Points above this line indicate a potential issue that needs to be addressed.
3. **Lower Control Limit (LCL):**
   - This is the minimum acceptable value for the process before it becomes unstable.
   - Values below this line indicate a problem that requires immediate action.

These lines facilitate the quick identification of any deviations, thereby ensuring process stability and maintaining the quality of the final product.

Exponential Moving Average Chart

An EWMA (Exponential Weighted Moving Average) chart is a control chart used to monitor a process's stability over time. It is beneficial for detecting small shifts in the process because it gives more weight to recent observations while still considering past data [8], [18].

Key Components of an EWMA Chart:

1- Data point (x_t): The individual observations or measurements over time.

2- EWMA statistic (z_t):

$$z_t = \lambda x_t + (1-\lambda) z_{t-1} \quad (1)$$

Where:

z_t is the EWMA value at time t.

λis the smoothing parameter (0 < λ < 1). Smaller values give more weight to past data.

$x_t$is the observation at time t.

$z_{t-1}$is the EWMA value at time t-1 (starting with the process mean or a specified initial value).

3- Center Line (CL): The target of the mean value of the process.

4- Control Limits:

$$UCL = CL + L.\sigma_z \quad (2)$$
$$LCL = CL - L.\sigma_z \quad (3)$$

Where:

L is the control limit multiplier (typically set to 3 for a 99.73% confidence interval).

$\sigma_z = \sigma\sqrt{\frac{\lambda}{2-\lambda}}$is the standard deviation of the EWMA statistic and σrepresent the standard deviation of the process.

How to Create an EWMA Chart [19]:

1. Collect Data: Gather the sequential process measurements.
2. Calculate EWMA: Apply the formula to compute z_t for each time point.
3. Determine Control Limits:
   - Use the target mean (CL) and known or estimated process standard deviation (σ).
   - Compute the upper and lower control limits.
4. Plot:
   - Plot the EWMA values (z_t) over time.
   - Add the center line and control limits.

4. Wavelet

Wavelets are mathematical functions that separate data into distinct frequency components and then analyze each one at a scale-appropriate precision. A general form of the wavelet is:

$$\psi(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right) \quad (4)$$

Where $\psi(t)$ is the mother wavelet, a is the scale parameter, and b is the translation parameter.

When studying physical scenarios where the signal comprises abrupt spikes and discontinuities, they are superior to conventional Fourier approaches [20]. In the fields of electrical engineering, seismic geology, quantum physics, and mathematics, wavelets were separately developed. Many novel wavelet applications, including image compression, turbulence, human vision, radar, and earthquake prediction, have emerged because of interactions between these domains over the past ten years [21].

- Daubechies Wavelet

Wavelet techniques, such as Daubechies, are used in signal and data processing to remove noise and improve quality. By applying small waves to EWMA data, we can improve the accuracy of measurements and uncover hidden patterns in the data [22].

- Coiflets Wavelet

At Ronald Coifman's request, Ingrid Daubechies created coiflets, which are discrete wavelets with scaling functions that have vanishing moments [23].

- Dmey Wavelet

The discrete variant of the Meyer wavelet function is called a Dmey wavelet. The two-scale equation can be solved using Mayer's wavelet, which is essentially a solvent approach [24]. Meyer used Fourier techniques to determine the DTFT of the two-scale education coefficients, giving a basis for the approximation space.

6. Discrete Wavelet Transformation

When dilation and translation parameters are discretized from a continuous representation, a discrete wavelet transform is produced, with the resultant set of wavelets forming a frame. According to [25], the translation parameter is usually discretized by integer multiples of a dilation-dependent step, and the dilation parameters are usually discretized by an exponential sampling with a fixed dilation step. Only a subset of scales and positions could be calculated, but it would take a substantial amount of effort and produce a lot of data to calculate wavelet coefficients at every scale. Surprisingly, the analysis will be equally as accurate and considerably more economical if we select scales and positions based on powers of two, or so-called dyadic scales and positions [26].

Discrete Wavelet Transform (DWT)provides this kind of analysis. Mallat (1999) [23] devised an effective filter-based implementation of this technique. As a two-channel sub-band coder, the Mallat method is a classical approach in the field of signal processing. This extremely useful filtering algorithm produces a quick wavelet transform or a box where a signal passes out of which wavelet coefficients quickly emerge. The original signal can be divided into numerous lower-resolution components by iterating the decomposition process, which breaks down consecutive approximations one after the other. The wavelet decomposition tree is the name given to this. Accordingly, the original signal goes through two complementary filters (high- and low-pass) in the filtering process at its most basic level, emerging as two signals known as approximations and details [27].

7. Proposed Charts

The proposed method relies on processing the data of the EWMA chart from noise before estimating its control limits using wavelet shrinking, which relies on the following algorithm:

1. Using one of the wavelets (Daubechies, Coiflets, and Dmey), quality characteristics data to obtain DWT coefficients.

$$\mathrm{DWT}(x_t) = (c_A, c_D) \qquad (5)$$

Where c_A is the approximation (low-frequency) coefficients and c_D is the detail (high-frequency) coefficients.

2. Estimate the thresholding level (γ) using the Universal thresholding from the first level for DWT coefficients. Compute the threshold γ using the universal threshold formula:

$$\gamma = \sigma\sqrt{2LnN} \qquad (6)$$

Where σ is the standard deviation of the detail coefficients$c_D$and N is the number of data points.

3. Use the soft threshold rule on the DWT coefficients at the estimated threshold level (γ) by keeping or killing (converting them to zero) to obtain modified coefficients with little noise. Modify the wavelet coefficients using the soft thresholding rule [28]:

$$\acute{c}_i = sign(c_i) \times max(|c_i| - \gamma, 0) \qquad (7)$$

for each wavelet coefficient $c_i \in c_A \cup c_D$ and$\acute{c}_i$ is the threshold coefficient, $sign(c_i)$represent the sign function of the coefficient c_i

4. Finding the inverse of the DWT coefficients for de-noise data while preserving 99.97% of the data energy. Reconstruct the denoised data by applying the IDWT on the threshold wavelet coefficients:

$$\breve{x}_t = \mathrm{IDWT}(\acute{c}_A, \acute{c}_D) \qquad (8)$$

Where $\acute{c}_A$ and $\acute{c}_D$ are the threshold approximation and detail coefficients, respectively, and $\breve{x}_t$ is the denoised data at time t.

5. Using de-noised data to create proposed EWMA charts.

8. Stimulation Study

To conduct a comparative study of the Discrete Wavelet Transform (DWT) using three wavelet families (Daubechies 20, Coiflet 5, and Dmey)- where the numbers (20, 5) refer to the number of coefficients used in the wavelet filter- with Shewhart Weighted Moving Average, 20 normally distributed random samples with a mean of (10) and a variance of (2) were generated. The original data, the data generated by the wavelet transform, and the weighted moving average curve were plotted in graphs to illustrate the differences between the four methods, as shown in Figure 1.

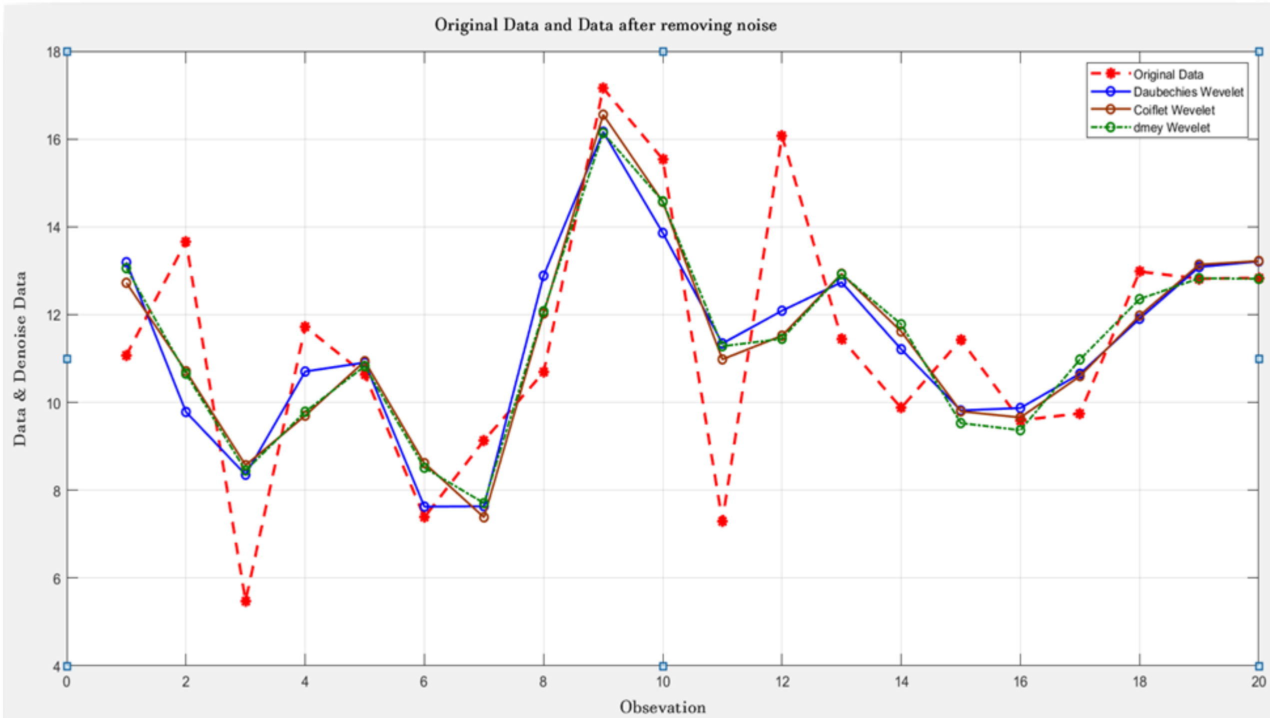

Figure 1. Original and Denoise Data (Simulation Data)

Table 1 shows the results of Simulation experiments that were conducted 1000 times to evaluate the performance of the classical chart (using formulas 1, 2, and 3) with the proposed chart (using formulas 5-8) for different wavelets using sample sizes (20, 40, 60, 80, 100), varying the sample size allows assessment of the control chart's sensitivity and stability under different data volumes. Smaller samples test robustness under data scarcity, while larger samples evaluate performance in detecting subtle shifts over time. Different arithmetic means (10, 100) and standard deviations (2, 10). The average, upper and lower limits were calculated for each case and summarized in Tables 1-3 for comparison.

Table 1. Results of the Simulation (Mean = 10, Sigma = 1)

Method	Sample Size	UCL	LCL	Target	Mean-D
Classical	20	10.7091	9.1206	9.9149	1.5885
db20		10.6037	9.2225	9.9131	1.3812
Coiflets		10.6049	9.2241	9.9145	1.3808
Dmey		10.5992	9.2255	9.9123	1.3737
Classical	40	11.1397	8.9428	10.0413	2.1969
db20		10.5566	9.5229	10.0398	1.0337
Coiflets		10.5708	9.5337	10.0522	1.0371
Dmey		10.5834	9.5185	10.0509	1.0649
Classical	60	11.3139	9.1743	10.2441	2.1396
db20		10.6704	9.8128	10.2416	0.8576
Coiflets		10.6813	9.8150	10.2482	0.8663
Dmey		10.6865	9.8148	10.2507	0.8717
Classical	80	11.1842	9.2678	10.2260	1.9164
db20		10.6153	9.8426	10.2289	0.7727
Coiflets		10.6035	9.8567	10.2301	0.7468
Dmey		10.6153	9.8426	10.2289	0.7727
Classical	100	11.2726	8.9944	10.1335	2.2782
db20		10.4682	9.8130	10.1406	0.6552
Coiflets		10.4478	9.8234	10.1356	0.6244
Dmey		10.6080	9.8500	10.2290	0.758

Table 2. Results of the Simulation (Mean = 10, Sigma = 2)

Method	Sample Size	UCL	LCL	Target	Mean-D
Classical	20	11.4182	8.2412	9.8297	3.177
db20		11.2075	8.4450	9.8262	2.7625
Coiflets		11.2098	8.4481	9.8290	2.7617
Dmey		11.1984	8.4509	9.8247	2.7475
Classical	40	12.2795	7.8856	10.0825	4.3939
db20		11.1132	9.0458	10.0795	2.0674
Coiflets		11.1415	9.0673	10.1044	2.0742
Dmey		11.1668	9.0370	10.1019	2.1298
Classical	60	12.6278	8.3486	10.4882	4.2792
db20		11.3407	9.6256	10.4832	1.7151
Coiflets		11.3626	9.6300	10.4963	1.7326
Dmey		11.3731	9.6296	10.5013	1.7435
Classical	80	12.3683	8.5355	10.4519	3.8328
db20		11.2305	9.6851	10.4578	1.5454
Coiflets		11.2071	9.7134	10.4602	1.4937
Dmey		11.2160	9.7000	10.4580	1.516
Classical	100	12.5453	7.9887	10.2670	4.5566
db20		10.9364	9.6261	10.2812	1.3103
Coiflets		10.8956	9.6468	10.2712	1.2488
Dmey		10.9211	9.6433	10.2822	1.2778

Table 3. Results of the Simulation (Mean = 100, Sigma = 10)

Method	Sample Size	UCL	LCL	Target	Mean-D
Classical	20	107.0908	91.2062	99.1485	15.8846
db20		106.0374	92.2249	99.1312	13.8125
Coiflets		106.0491	92.2405	99.1448	13.8086
Dmey		105.9923	92.2547	99.1235c	13.7376
Classical	40	111.3974	89.4278	100.4126	21.9696
db20		105.5661	95.2292	100.3976	10.3369
Coiflets		105.7077	95.3365	100.5221	10.3712
Dmey		105.8339	95.1848	100.5093	10.6491
Classical	60	113.1391	91.7429	102.4410	21.3962
db20		106.7035	98.1281	102.4158	8.5754
Coiflets		106.8128	98.1502	102.4815	8.6626
Dmey		106.8653	98.1482	102.5067	8.7171
Classical	80	111.8417	92.6777	102.2597	19.164
db20		106.1527	98.4256	102.2892	7.7271
Coiflets		106.0353	98.5670	102.3012	7.4683
Dmey		106.0799	98.4998	102.2898	7.5801
Classical	100	112.7265	89.9437	101.3351	22.7828
db20		104.6819	98.1303	101.4061	6.5516
Coiflets		104.4780	98.2341	101.3560	6.2439
Dmey		104.6054	98.2163	101.4108	6.3891

9. Discussion Simulation Study

From Tables 1-3, the results of the analysis show us the following:

The D values represent the width of the control limits (UCL - LCL) for each method. A smaller D indicates that the control limits are more tightly constrained, suggesting that the process is more stable and sensitive to small deviations. Conversely, a larger D means the control limits are wider, allowing for more variation in the data.

Table 1 shows the results for Simulations with Mean = 10, Sigma = 1

For a sample size of 20, the classical chart has a D value of 1.5885. The db20 chart has a D = 1.3812, indicating that the control limits are narrower, meaning better sensitivity to variations and dealing with the noise problem. The other wavelet methods (Coiflets and Dmey) also show smaller D values than the Classical method, with Coiflets (Mean-D = 1.3808) and Dmey (Mean-D = 1.3737) providing similarly narrow control limits.

For a sample size of 40, the classical chart has a D = 2.1969, showing wider control limits compared to the wavelet methods. The wavelet methods (db20, Coiflets, and Dmey) again show narrower D values, with db20 (D = 1.0337) and Coiflets (D = 1.0371) being particularly effective at narrowing the control limits and dealing with the noise problem. For a sample size of 60, the classical chart still has wider control limits (D = 2.1396) compared to the wavelet methods. Wavelet methods continue to perform better, with db20 (D = 0.8576), Coiflets (D = 0.8663), and Dmey (D = 0.8717) providing much narrower control limits.

For a sample size of 100, the classical chart has a D = 2.2782, indicating wider control limits. The wavelet methods continue to show narrower D values, with db20 (D = 0.6552), Coiflets (D = 0.6244), and Dmey (D = 0.758) being more effective at narrowing the control limits.

Table 2 shows the results for Simulations with Mean = 10, Sigma = 2

For a sample size of 20, the classical chart has a D = 3.177, indicating wider control limits compared to the wavelet methods. The wavelet methods (db20, Coiflets, and Dmey) have smaller D values, with db20 (D = 2.7625) providing the narrowest control limits. For a sample size of 40, the classical chart shows a D value of 4.3939, significantly wider than the wavelet methods. Wavelet charts (db20, Coiflets, and Dmey) provide much narrower control limits, with db20 (D = 2.0674) showing the best performance.

For a sample size of 60, the classical chart (D = 4.2792) has wider control limits than the wavelet charts. db20 (D = 1.7151) and Coiflets (D = 1.7326) have narrower control limits, which is more desirable for better sensitivity to small shifts in the data. For a sample size of 100, the classical chart (D = 4.5566) shows wide control limits. The wavelet charts again perform better, with db20 (D = 1.3103) providing the narrowest control limits, followed by Coiflets (D = 1.2488) and Dmey (D = 1.2778).

Table 3 shows the results for Simulations with Mean = 100, Sigma = 10

For a sample size of 20, the classical chart has a D = 15.8846, indicating very wide control limits. The wavelet charts have significantly narrower D values, with db20 (D = 13.8125) performing best in narrowing the control limits. For a sample size of 40, the classical chart (D = 21.9696) shows very wide control limits. The wavelet charts perform much better, with db20 (D = 10.3369) showing the best control limits, followed by Coiflets (D = 10.3712) and Dmey (D = 10.6491).

For a sample size of 60, the classical chart (D = 21.3962) still shows very wide control limits. The wavelet charts provide narrower control limits, with db20 (D = 8.5754) and Coiflets (D = 8.6626) performing well. For a sample size of 100, the classical chart shows a D = 22.7828, indicating very wide control limits. The wavelet charts show much narrower D values, with db20 (D = 6.5516) providing the best performance, followed by Coiflets (D = 6.2439) and Dmey (D = 6.3891).

The db20 wavelet chart consistently produces the narrowest control limits (smallest D) across all sample sizes and experiment conditions, indicating that it offers better performance in terms of sensitivity to small shifts in the data. The Classical chart, on the other hand, produces much wider control limits (larger D values), suggesting that it is less sensitive to smaller variations in the data. As the sample size and variability increase, the wavelet methods continue to perform better in providing narrower control limits, which is preferable in a quality control setting where detecting small shifts is critical.

10. Real Data

Children's weight data were collected from Lalla Private Hospital (Appendix) to design the proposed chart and the classical chart in the first stage. Where 60 observations were taken in the case of constructing the chart, and then 60 new observations were taken in the case of using these boards as an effective tool for monitoring and controlling the process in the second stage.

10.1. First Phase: Constructing and interpreting the proposed chart.

Real data of children's weights were taken from Lalla Private Hospital, and these real data and smoothing results were plotted using wavelet transforms (formulas 5-8) as shown in Figure 2:

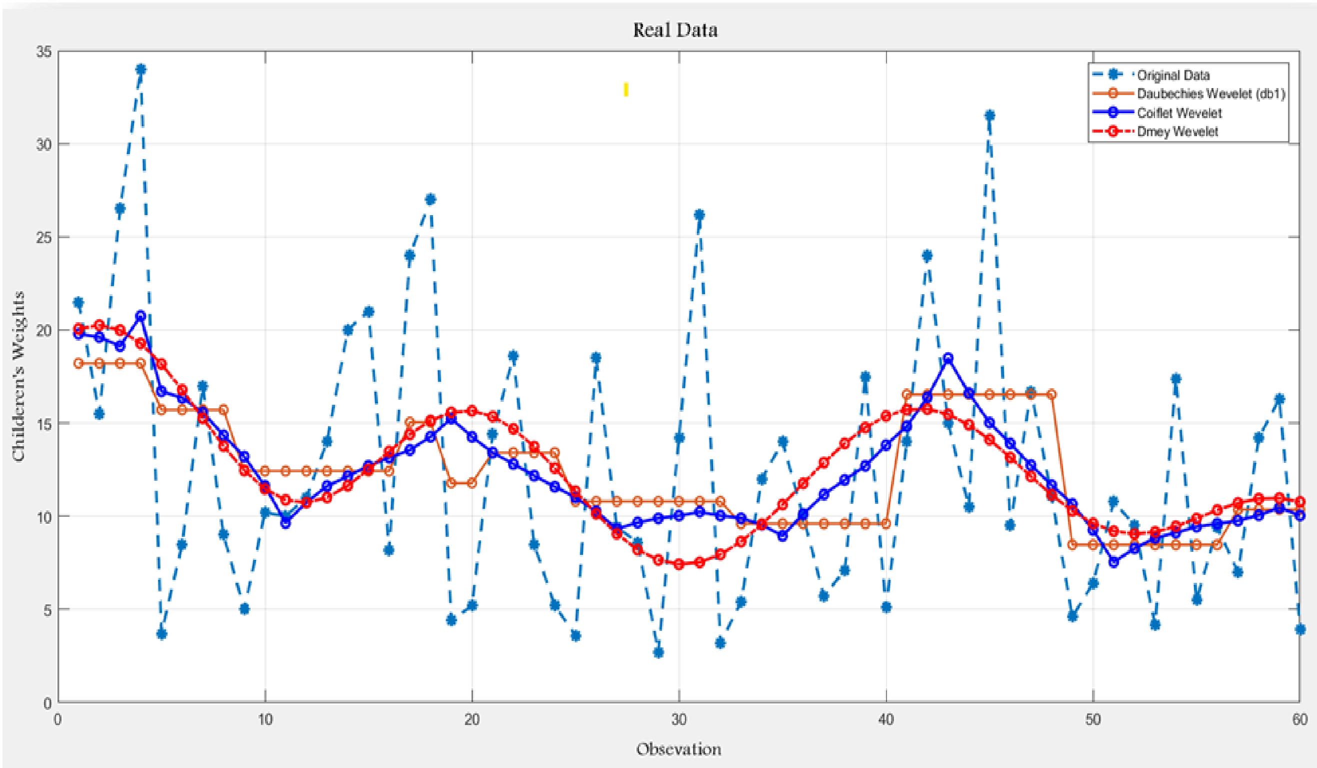

Figure 2. Original and Denoise for Real Data

To construct the proposed chart and compare it with the classical chart (EWMA chart) for data taken from children's weights (see Appendix A). Using the special programming of the MATLAB program, the classical chart analysis was performed using the proposed chart (EWMA wavelet chart), as shown in Figure 3:

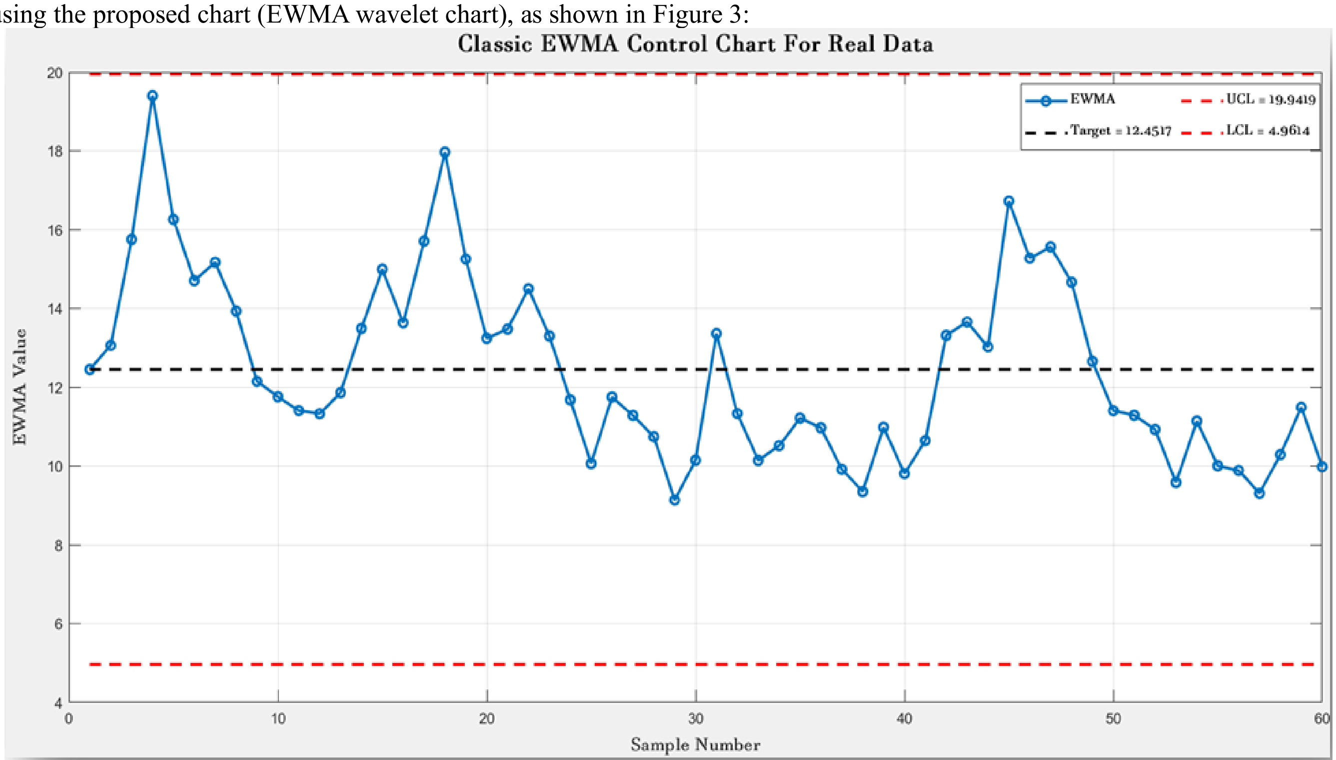

Figure 3. Classic EWMA Chart for Real Data

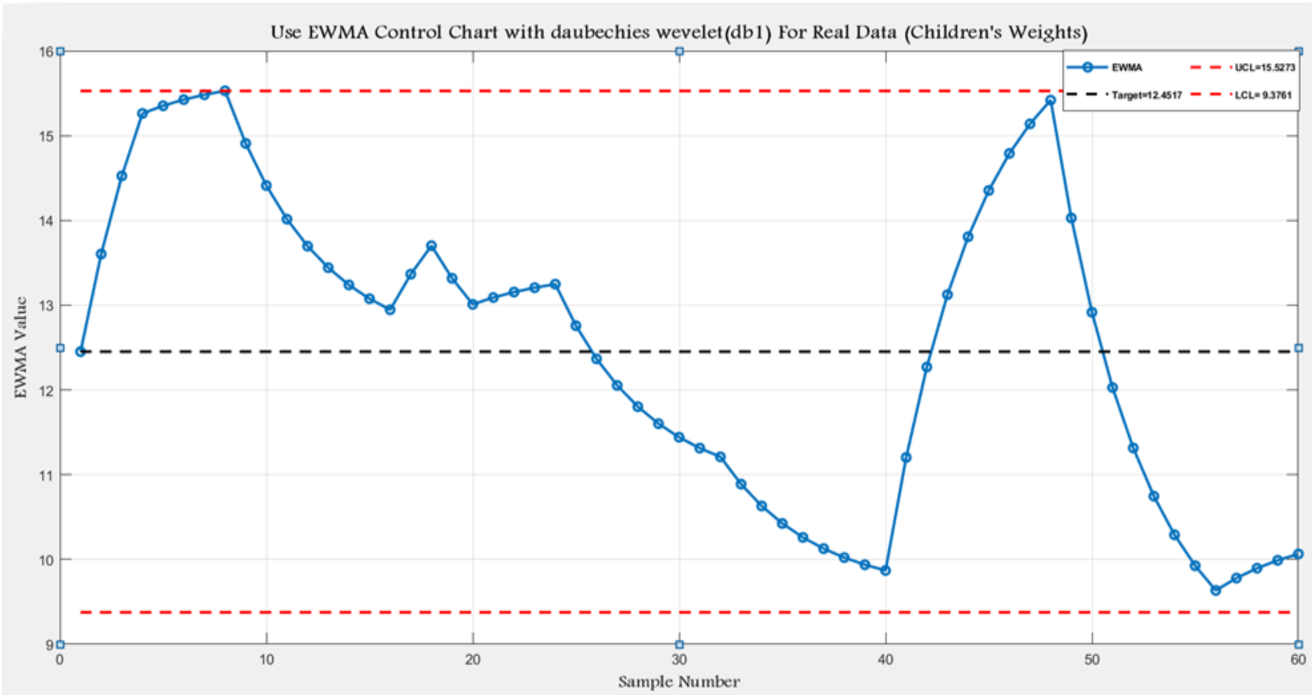

Figure 4. EWMA Control Chart with Daubechies Wavelet for Real Data

From Figures 3 and 4, notice the following:
Since Phase I charts are under control, it suggests that: The process (in this case, the data collection of children's weight) is stable and consistent. The control limits for both the classical and wavelet-smoothed charts have been properly set. Now, proceed to Phase II, where you apply these control charts to new data (the second set of 60 observations) and monitor if the process continues to be stable. The goal is to use these charts to detect any significant shifts or deviations from the expected range (i.e., out-of-control signals).

Table 4: Results between the classic chart and the proposed chart for real data

Method	UCL	LCL	Target	D
Classical	19.9419	4.9614	12.4517	14.9805
db20	15.5273	9.3761	12.4517	6.1512
Coiflets	15.5730	9.2830	12.4280	6.2900
Dmey	15.8390	9.3112	12.5751	6.5278

Table 4 Classical chart: The control limits for the classical chart have a wide range (UCL = 19.9419, LCL = 4.9614), which results in a large D value of 14.9805. This suggests that the classical chart has a broader tolerance for fluctuations, possibly due to its lack of noise-reduction techniques.

Proposed Charts (Wavelet Transforms): Wavelet shrinkage (using methods like db20, Coiflets, and Dmey) results in narrower control limits, with D values significantly smaller than the classical chart. db20: D = 6.1512, Coiflets: D = 6.2900, and Dmey: D = 6.5278. The reduction in D indicates that the wavelet-based methods have narrower control limits, which can lead to more sensitive detection of outliers or shifts in the process. Essentially, these charts are “tighter” and can detect small changes more effectively.

Target: The target value (12.4517 for the classical chart and around 12.4 for the wavelet methods) remains consistent across the methods. This indicates that the central tendency of the data is similar across both the classical and proposed charts.

Implications of Narrower Control Limits: Increased Sensitivity: The narrower control limits provided by the proposed charts (wavelet-based) are beneficial for the early detection of deviations from the target. The classical chart, with its wide limits, might miss small shifts or outliers that the proposed charts would detect.
Potential for Fewer False Alarms: Narrower control limits, especially when dealing with real data (which may have noise), can help reduce false alarms and more accurately reflect deviations in the process. The proposed charts (db20, Coiflets, and Dmey)

seem to provide better performance than the classical chart, as indicated by their smaller D values, meaning tighter control limits. The Classical chart might be less sensitive and could miss smaller deviations in the process, while the wavelet-based charts are more refined in detecting these shifts.

10.2. Second Phase: Use the classic and proposed chart.

These charts can be installed in Figures (3) and (4) and used in the future to control and monitor the exponentially weighted moving average (EWMA) of children's weights. On this basis, these charts were used to collect 60 new observations (see Appendix B) and summarized in the following table.

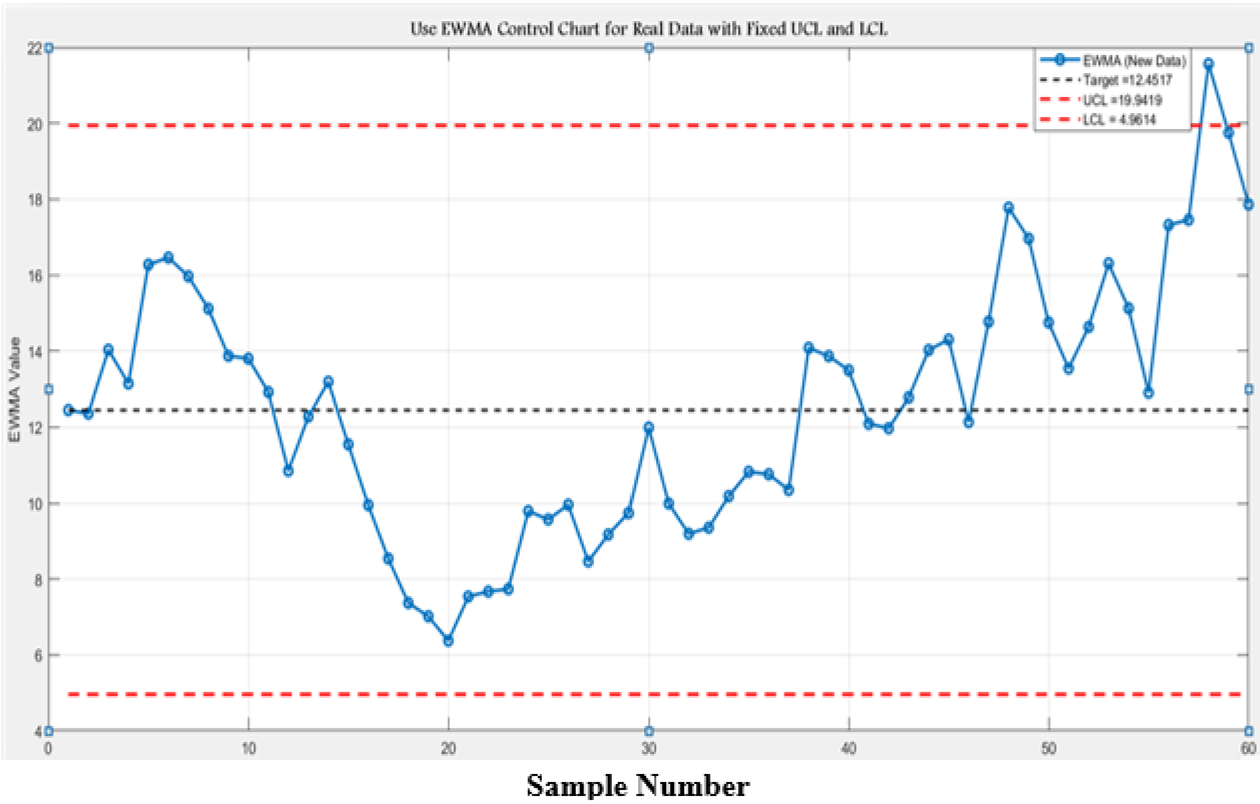

Figure 5. Use EWMA Control Chart for Real Data with Fixed UCL and LCL

After installing the classic EWMA Control Chart and using new data for these charts, notice the following:

1- One point occurs outside the control limits of the classical chart, and the process is out of control.

2- Several points occur outside the control limits of the proposed chart, and the process is out of control, which indicates the superiority of the proposed chart compared to the classic chart due to the removal of the noise present in the proposed chart and the narrowing of the control limits. This means that the proposed charts were more accurate and addressed part of the noise problem.

3- The process is out of control in two charts for Children's weight data from Lalla Private Hospital.

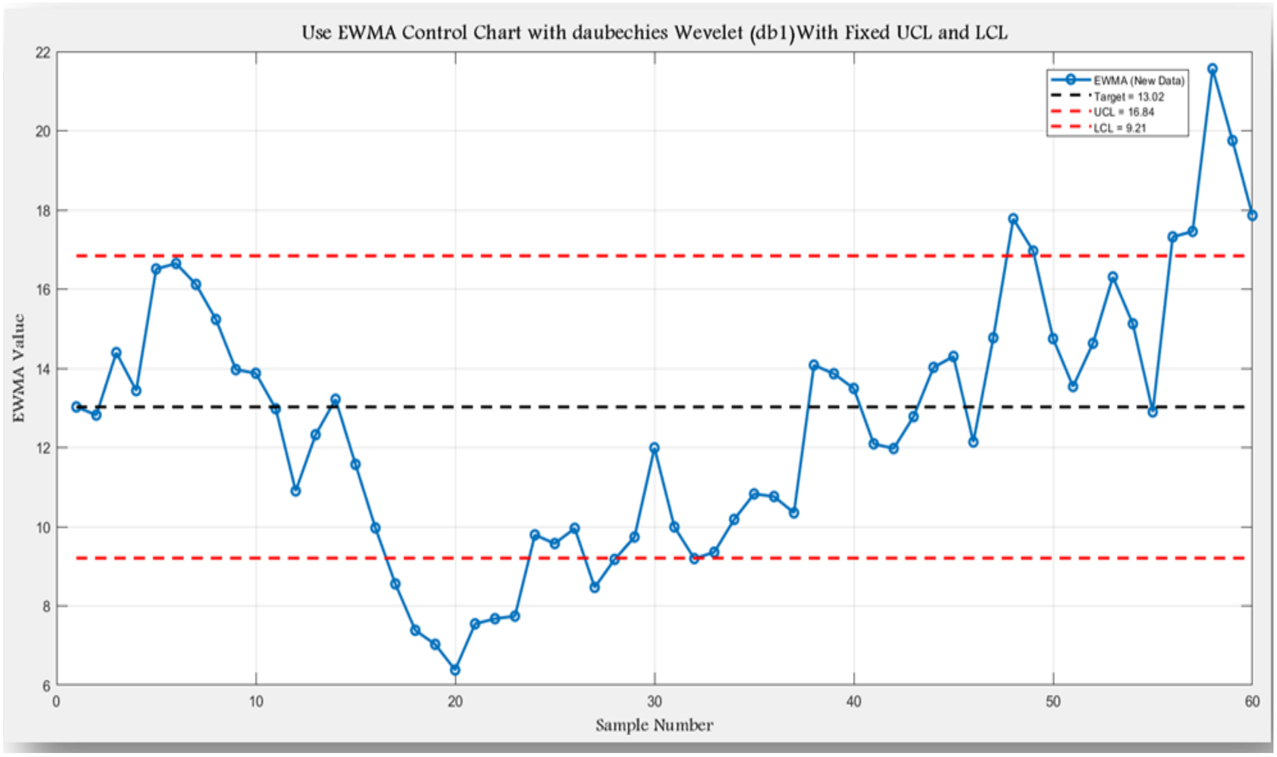

Figure 6. Use EWMA Control Chart with Daubechies(db1) Wavelet for Real Data

11. Conclusions

1. The proposed EWMA chart with wavelet shrinkage is superior to the classical EWMA chart in both simulation and real data analysis.
2. The proposed EWMA chart dealt with noise data, and the classical chart, with its broader control limits, may miss small shifts in the process, leading to less effective monitoring and control.
3. The proposed charts are handy for situations where data contains noise or outliers. They can be effectively applied in fields such as quality control, process monitoring, and health assessments, where accurate deviation detection is critical.
4. The proposed wavelet-based EWMA control chart is more effective than the classical chart for monitoring children's weights, especially in the presence of noise and natural fluctuations.

The classical chart, while useful, does not offer the same level of precision or ability to filter out noise, making the proposed chart a superior choice for real-time health data monitoring.

5. Further comparative studies are recommended to evaluate the performance of the proposed graph against other rapidly developing techniques, such as machine learning-based methods, to identify scenarios where the modified EWMA with wavelet shrinkage demonstrates superior performance.

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Appendix

Table A. Children's Weight Data in Lalla Private Hospital (Phase I)

1	21.5	21	14.4	41	14
2	15.5	22	18.6	42	24
3	26.5	23	8.5	43	15
4	34	24	5.2	44	10.5
5	3.7	25	3.6	45	31.5
6	8.5	26	18.5	46	9.5
7	17	27	9.4	47	16.7
8	9	28	8.6	48	11.1
9	5	29	2.7	49	4.6
10	10.2	30	14.2	50	6.4
11	10	31	26.2	51	10.8
12	11	32	3.2	52	9.5
13	14	33	5.4	53	4.2
14	20	34	12	54	17.36
15	21	35	14	55	5.49
16	8.2	36	10	56	9.4
17	24	37	5.7	57	7
18	27	38	7.1	58	14.2
19	4.4	39	17.5	59	16.3
20	5.2	40	5.1	60	3.95

Table B. Children's Weight Data in Lalla Private Hospital (Phase II)

1	17	21	12.2	41	6.5
2	12	22	8.2	42	11.5
3	20.7	23	8	43	16
4	9.6	24	18	44	19
5	28.8	25	8.7	45	15.4
6	17.2	26	11.5	46	3.5
7	14	27	2.5	47	25.3
8	11.7	28	12	48	29.8
9	8.9	29	12	49	13.7
10	13.5	30	21	50	5.900
11	9.4	31	2	51	8.7
12	2.6	32	6	52	19
13	18	33	10	53	23
14	16.8	34	13.5	54	10.4
15	5	35	13.4	55	4
16	3.55	36	10.5	56	35
17	2.9	37	8.7	57	18
18	2.7	38	29	58	38
19	5.6	39	13	59	12.5
20	3.8	40	12	60	10.300

لوحات السيطرة النوعية للمتوسط المتحرك الموزون الأسي مع التقليص المويجي: دراسة محاكاة

إسراء عوني حيدر [1]، دلشاد محمود صالح [2]، طه حسين علي [3]*، بيخال صمد صديق [4]

[1, 2, 3, 4] قسم الإحصاء والمعلوماتية، كلية الادارة والاقتصاد، جامعة صلاح الدين، أربيل، العراق

الخلاصة:
تهدف هذه الدراسة إلى توظيف تقنيات إحصائية حديثة لتحليل البيانات التي غالبًا ما تكون حساسة للتغيرات الطفيفة. تُعد لوحات السيطرة النوعية أدوات قيمة لمراقبة البيانات وتحليلها، وتحديد النقاط الخارجة عن حدود الرقابة واستخلاصها. ومن جهة أخرى، تُعد تقنية تقليص المويجات (Wavelet Shrinkage) من الأساليب الفعّالة في إزالة الضوضاء من البيانات وتحديد الانحرافات مع الحفاظ على الإشارات المهمة، مما يُحسّن من دقة التحليل. وبالإضافة إلى استخدام بيانات فعلية تمثل الخاصية النوعية لأوزان الأطفال — والتي تُعد مؤشراً مهماً لتقييم الصحة العامة والكشف المبكر عن المشكلات الصحية أو التغذوية — تمت محاكاة هذه اللوحات باستخدام برنامج MATLAB. ومع ذلك، فإن قياسات الوزن غالباً ما تكون عرضة للتشويش والتداخل بسبب عوامل مثل أخطاء القياس أو التغيرات الطبيعية. وقد خلصت الدراسة إلى أن اللوحات المقترحة (المتوسط المتحرك الموزون الأسي باستخدام تقليص المويجات) يتفوق على اللوحة التقليدية، نظراً لأدائه الأفضل في الكشف عن الفروق بين الحدين الأعلى والأدنى للوحتين. كما لوحظ أن الفرق بين الحدين في اللوحة المقترحة كان أقل.