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RESEARCH ARTICLE

A Two-Phase Service Access Beside Retrial Queues, Orbit Mechanism, Working Vacation Policies for Single Server Environments

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ABSTRACT

This paper analyzes the M/G (P1, P2)/1 queue using a single server that provides essential services in both the first phase (FPS) and the second phase (SPS). It was assumed that the server undertakes unreliability due to starting failure (SF) during the beginning of the first phase with probability (α) or service begins by the idle server successfully with probability $(1-\alpha)$ for the arriving or retrial customer. Then immediately took the failed server for repair and launched the customer into orbit. Also, the server operates at varying rates instead of fully ceasing service during its vacation. After returning from vacation, the server resumes normal operations for the customers. It also discussed the system, orbit size, and other significant metrics, utilizing the supplementary variable technique (SVT) and the probability generating function (PGF). Applications for this retrial queueing system incorporating the model and a general decomposition law were discussed. Numerical analysis was established to examine the impact of different factors on the system's efficiency. MATLAB software was used to establish several impacts of the system's behavioral measures.

Keywords: Dual phases, Performance measures, Retrial queues (RQ), Starting failure, Working vacation (WV)

Introduction

In our modern and congested society, all have experienced the frustration of waiting for service, even for basic needs. A queueing system is introduced to overcome those difficulties. The queueing model plays a crucial role in various real-life scenarios. Many beneficial applications of the queueing theory are traffic flaws (vehicles, aircraft, people, communications), organizing (patients in hospitals, jobs on machines, programs on computers), and facility design (banks, post offices, supermarkets).

Retrial queuing is given particular emphasis in the most recent study on queueing systems (QS). In queueing theory, "retrial queues" refer to a category of models where consumers encountering a crowded server upon arrival join a virtual queue known as the "orbit" instead of exiting the system. These clients sometimes request a "retrial" service after a random duration from the orbit. Due to many reasons in the queueing model, the server was unavailable for a specific interval because of unavoidable spectacles like maintenance of the server, breaks, engagement with secondary jobs, and so on.

We can conceptualize an orbit as a storage region where customers, initially unable to access a server due to its busyness, wait before attempting to access it again. Typically, a stochastic system controls the retry process, ensuring random intervals between trials that may follow different distributions, such as exponential or more intricate ones. An orbit serves as a fundamental concept in the modeling of systems, where blocked clients continue to seek service without stopping the process.

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Retrial queues play a major role in health centers, treasury, ticket reservations, electronic mail-transmission systems in computers, broadcasting networks, customer support centers, production and construction systems, etc. Yang Templeton,[1] and Falin[2] have conducted a fascinating investigation into retrial wait times.

Artalejo[3] investigated vacation lines and their significance. Kumar and Arivudainambi[4] delved into a retrial queueing model, incorporating variable retrial times.

After gaining access to the server, every consumer in a two-phase service system experiences two distinct phases of service. In most cases, the first stage is a preliminary or fundamental service that may include initial processing or validation. The second stage entails a more extensive service, which may in turn necessitate additional resources or time.

It's crucial to remember, though, that machinery with two-phase service (TPS) can be more advantageous. These advantages include better efficiency, fewer floor space requirements, faster changeover times between various manufacturing processes, and more production flexibility. Additionally, the price of multifunctional machinery may go down over time as economies of scale and technological advancements take effect. Since double-phase services have many real-world uses in locations including bank counters, operating ATMs, massive supermarkets, healthcare facilities, etc., numerous writers have studied RQ with two service phases when customers request both phases of service. Michael Mathavavisakan and Indhira[5] made an investigation on RQ with two-phase service under WV and with impatient customers. Bharathy and Saravanarajan[6] analyzed the Markovian queueing system with TPS and a single WV. Niranjan and Latha[7] investigated a model with two phases heterogeneous service, feedback customer, and breakdown in the system.

Researchers have extensively studied service during vacation (WV) in queuing systems, particularly in areas such as communication networks, assembly lines, convenience stores, cafeterias, health centers, and wireless sensor networks. Once the service is over for the customers, and if the orbit turns out to be vacant, the server goes on a working vacation. It was provided slow-rate service during the working vacation as an alternative to shutting down the system. In the final analysis, WV is essential for manufacturing requirements, production volume, and possible long-term gains in terms of output, adaptability, and operational efficiency. Two different queueing models for working during vacation were carried out by Sindhu et al.[8] GnanaSekar and Kandaiyan[9] have conducted an investigation into the single server retrial queueing structure and its relationship with WV. Li et al.[10] and Rajadurai[11] have deliberated on a model by combining the M/G/1 queue with WV together with interruption. Working under a Bernoulli schedule, Li and Liu,[12] along with Tian et al.,[13] focused on an M/G/1 queue with a single WV and interruption. Tian et al.[14] conducted an investigation using equilibrium strategies, which included balking, a single working vacation, and then an interruption in the vacation. Bouchentouf et al.[15] analyzed the Markovian queueing system at the time of working vacation containing single WV and impatient customers reneging.

Most existing literature on retrial queueing systems assumes a reliable (i.e., never fails) or always, permanently available server. However, this is unfeasible because server start failure has a significant impact in the real world at any time. Starting failures in the field of queueing systems pertain to instances where a service attempt may not commence correctly upon a customer's arrival or attempted retry. Starting failure adds an extra level of intricacy, as it mirrors real-life situations in which system components may malfunction or need many attempts before effectively engaging. In the context of retrial queues, starting failures refer to instances when a server, even while not in use, is unable to commence service to an incoming customer after a failure. A range of sources, such as mechanical problems, software malfunctions, or operational mistakes, can cause these failures.

Until the failed service facility is recovered, the waiting times for customers in the system increase, so it is critical to analyze retrial queues with an initial failure. Many writers have studied SF's queueing system. Rajadurai et al.[16] have framed a model by incorporating WV and SF. Yang and Wu[17] have integrated the concept of starting failure into the retrial model, which incorporates two phases of vital service, each subject to WV and starting failure. Liu, et al.[18] and Upadhyaya[19] made remarkable work on RQ by taking SF as an important part.

The concept of "repair" is crucial in the analysis of trial queues applied to systems with service mechanisms susceptible to failures. Repairing retry queues is the act of reinstating the service mechanism after its failure. Within some practical systems, such as telecommunications networks, manufacturing processes, or computer servers, service mechanisms are prone to experiencing failures. The repair procedure is critical to ensuring that the system is restored to its operational condition, allowing the service process to continue.

Supplementary variable technique (SVT)

The Supplementary Variable Technique (SVT) is a robust mathematical method used in queueing theory to analyze systems. These methodologies offer a useful instrument for measuring system efficiency and making well-informed judgments in diverse practical scenarios, including call centers, manufacturing technologies, and computer networks.

Probability generating function (PGF)

A probability-generating function (PGF) serves as a mathematical device employed to depict the probability distribution for a discrete random variable. Its purpose is to streamline system analysis by transforming the probability distribution of the number of customers in the system into a generating function. This approach is especially advantageous for handling the retrial queue, where the state space might acquire significant size. The process entails converting the probability distribution of the system's customer count into a generating function. Also, this simplifies the analysis by effectively converting convolutions of probability distributions into products.

Prominent contributions include the book by Falin and Templeton,[20] which offers an extensive examination of retry queues, encompassing the application of SVT and PGFs. The text delves into a diverse array of applications and examines distinct expansions of the fundamental M/G/1 retrial queue paradigm. In 2008, Artalejo and Gómez-Corral[21] looked into retrial queues with vacations using the supplementary variable technique (SVT) and probability-generating functions (PGFs) to test different system configurations.

The present study conducts an in-depth analysis of the SVT and PGF approaches within the framework of M/G/1 retrial queues. In our study of M/G/1 retrial queues with Poisson arrivals, starting failure, repair, two-phase service, and working vacation, the SVT method was employed to add more variables that make the complicated time-dependent state probabilities easier to understand. This simplifies the queueing system's data analysis.

This research focuses on retrying queues with working vacations and starting failures because of the practical applications of working vacations and service disruptions. In this queueing system, vacation models play an important role, escorted by the server moving to idle time for various purposes. The system provided service both during busy periods and during vacations. The system was designed to assume that failures only occur during normal busy periods. The service provided during a vacation is comparatively lower than during a normal busy period. We generally distributed the duration of service between normal busy and low-rate periods. To the great extent of our understanding, this may be the first instance of work that seamlessly combines two essential phases of service, with failure at the start besides working vacation. This model was optimized to predict the queue length and waiting time. Section 1 provided an introduction. Section 2 provides the model's visual and mathematical representations, as well as model descriptions. We have also deliberated the application of the suggested model. The number of consumers in the orbit/system and the steady state joint distribution of the server state are obtained in section 3. Section 4 discusses a few system performance metrics, including measures of reliability, the mean busy cycle, and the mean busy period. Section 5 deduces significant special cases. Section 6 presents a numerical analysis of the impact of different factors on the system's performance. Section 7 contains the paper's conclusion and summary.

Pictorial and mathematical illustration of the suggested design

Model description

In the M/G (P1, P2)/1 retrial queue-up sample, consideration was given to starting failure, dual segment essential facility, and working vacation. The schematic diagram of the framed model is given in Fig. 1. Also detailed procedure for the framework was given:

Arrival process

The Poisson stream of primary clients enters the system at a rate ($\Lambda > 0$). Also, it is assumed there will not be patiently waiting space. Customer arrivals follow a Poisson process, indicating that the intervals between arrivals are both exponentially distributed as well as independent. The Poisson process was chosen because of its simplicity and broad generality in representing random arrivals in a variety of real-world systems. Nevertheless, it is crucial to acknowledge that actual arrival patterns in the real world may display variations from Poisson behavior, such as bursty arrivals or linked arrivals.

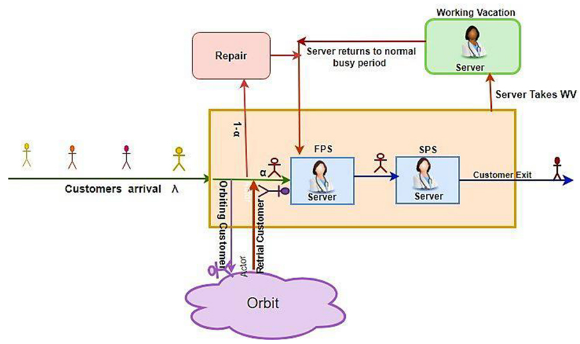

Fig. 1. Model's visual representations.

Procedure for retrial

The trial mechanism ensures that customers who arrive to discover the server busy or the system in a failure condition do not permanently exit the system. Instead, the system places them in a retry queue, where they endure a period of waiting and then attempt to regain access to the service after a random, predetermined time. This assumption captures the firmness of clients who are unsuccessful in obtaining service instantaneously. The time intervals between retrials are expected to conform to an exponential distribution with a rate (θ), which represents the stochastic character of the customer's choice to retry after a waiting period.

Service time distribution

Here, the expected service times follow a general distribution (G). This allows our model to incorporate a variety of service time behaviors, such as exponential, deterministic, or more intricate distributions, ensuring its adaptability to represent a broad spectrum of service scenarios. The service time distribution is selected based on the specific characteristics of the system under analysis. The concept incorporates a two-phase service mechanism, in which each consumer experiences two separate stages of service. But, on finding the busy or unavailable server, the arriving customer moves from the service area to orbit immediately according to FCFS discipline, and after some random period request for service is repeated. When a primary customer arrives, the consumer gets served right away if the server is available. A single server delivers essential service in two phases having the rate μ_{b} which was generally distributed for the first phase (FPS) P_1 besides μ_{sb} for the second phase (SPS) P_2. Every client receives from the server two successive stages of heterogeneous service, with the FPS coming first and the SPS second. Both phases are mandatory.

Working vacations or reduced service rates

When the orbit is depleted, the server takes a single WV. When the server is on vacation, it still serves customers, but at a lower rate, then this refers to a "working vacation policy." The vacation times happen at random, and while they do, the server's capacity is lower but not zero, so some services can still run. This assumption describes what could happen if the system requires maintenance or other work that reduces its service capacity without stopping all processes. This assumption was used to model scenarios where the server can perform other tasks during periods of low demand. During this vacation, the server attends the jobs at a lesser service rate μ_v accomplishment on the spot in the vacation period which follows an RV W_v.

Mechanisms for repairing starting failures

If a customer arrives and finds the server idle, they must turn it on. If they are successful in doing so, they will receive service right away (probability α); if not, they will experience a starting failure with probability $\bar{\alpha}$ resulting in service interruptions, after a client arrives but before the service commences. At that point, the server is immediately taken for repair, and the customer will have to join the orbit.

A starting failure prevents prompt attention to the customer and necessitates system repair before the service can commence. This assumption was frequently used to depict scenarios where system faults, resource

Table 1. Mathematical notations used for the model.

Process	Cumulative Distribution Function	Laplace Stieltijes Transform	Remaining Service Time	Conditional Completion Rate	First Two Moments
Retrial	$A(x)$	$A^*(\emptyset)$	$A^0(\tau)$	$\theta(x)$	$E(A)$, $E(A^2)$
FPS	$L_b(x)$	$L_b{}^*(\emptyset)$	$L_b{}^0(\tau)$	$\mu_b(x)$	$E(L_b)$, $E(L_b{}^2)$
SPS	$R_b(x)$	$R_b{}^*(\emptyset)$	$R_b{}^0(\tau)$	$\mu_{sb}(x)$	$E(R_b)$, $E(R_b{}^2)$
WV	$W_v(x)$	$W_v^*(\emptyset)$	$W_v^0(\tau)$	$\mu_v(x)$	$E(W_v)$, $E(W_v^2)$
Repair	$S_f(x)$	$S_f^*(\emptyset)$	$S_f^0(\tau)$	$\vartheta(x)$	$E(S_f)$, $E(S_f^2)$

shortages, or customer-generated issues could lead to unsuccessful service efforts. A prevalent strategy is to assume that the probability of initial failure remains constant for every service attempt. This implies that neither the system's operational condition nor any previous service attempts influence the probability of a failure. Actual data from systems characterized by frequent initial setup failures, such as industrial or telecommunications equipment supports this method.

For instance, within the telecommunications sector, network nodes frequently encounter early configuration problems that can be represented as starting failures.

Repair process

A broad distribution of repair times is associated with a starting failure. Efficient maintenance is vital for minimizing system downtime and ensuring overall dependability. An often-made assumption is that repair times adhere to an exponential distribution, which suggests a capacity for memoryless behavior. If failures are believed to be memoryless, repair times may have an exponential distribution. This approach streamlines the study and enables the derivation of straightforward mathematical answers.

The likelihood of initial failures and the duration required for repairs were approximated using empirical data from diverse systems such as call centers, industrial facilities, and computer networks. This empirical data can establish the validity of the assumptions underlying the model. Research in the manufacturing and service sectors frequently shows that exponential distributions, which accurately capture the varied characteristics of repair procedures, can represent repair durations. If failures are believed to be memoryless, repair times may have an exponential distribution. Service delivery will be halted and retried to the recently arrived client during the repair process. This follows an RV.

Table 1 provides the cumulative distribution function, Laplace Stieltijes transform, relapsed time, conditional completion rate, and required moments for each state.

The time intervals between the arrivals (P1, P2, WV, and repair) were mutually independent. Different stochastic processes were tangled with this model, which doesn't depend on one another.

Applications that embrace the model

Applications 1: Customer support ticketing system (CSTS)

A customer support ticketing system functions as a queueing system where support tickets are analogous to customers, and the support agent or automated system serves as the server. It can represent the system as an M/G/1 retrial queue. If a customer comes and the server is occupied or inaccessible, the customer is obstructed and may strive to establish contact (or make another attempt) at a later time. This phenomenon is prevalent in customer care systems, where clients may make several attempts at contact resolution if tickets that remain unresolved on the initial attempt (possibly due to insufficient information or intricate problems) are redirected to a retrial queue. The tickets will undergo a re-attempt for service following a delay, akin to clients making another attempt to retrieve service.

A Poisson process can represent the arrival of customer support tickets, where clients or tickets arrive randomly over time. This indicates that the likelihood of a client arriving within a certain period is consistent, irrespective of the preceding arrivals. In this context, the randomness observed is indicative of the inherent unpredictability and fluctuating frequency of consumer requests throughout the day.

Starting failure: In this concept of starting failure, the term "starting failure" refers to situations where the ticketing system cannot initiate the resolution process for a ticket. Several factors, such as system crashes, agent unavailability, technical issues, misrouting, or improper information, can cause this failure. Such circumstances can result in longer waiting periods and heightened customer dissatisfaction.

Repair: Before restoring service, it proceeds through a repair procedure following a server failure. In the event of a starting failure, the ticket necessitates a repair procedure, which may include redirecting it to the appropriate department, collecting further information, or fixing technical problems. The implementation of this measure guarantees the ticket's ultimate successful processing. The length of the repair procedure can have a direct impact on the system's performance and customer satisfaction.

Two-phase service: The process of resolving tickets generally consists of two distinct phases of service. The initial phase involves a thorough evaluation of the ticket to ascertain the extent of the problem and collect the essential information. Should the ticket necessitate more action, it proceeds to the second phase, during which a more comprehensive inquiry or settlement procedure occurs. This may encompass technical assistance, progression to a more advanced degree of technical knowledge, or direct interaction with the client.

Working vacation: The server may undertake a "working vacation," whereby it can manage low-priority tickets while giving priority to other duties. This refers to situations where support personnel or the system are not actively involved in resolving active tickets but are still vigilantly monitoring for pressing issues. In these situations, they may focus solely on addressing essential or high-priority tickets, leaving other less urgent complaints in a queue for further processing. This feature enables the system to effectively distribute workload and prevent exhaustion while still ensuring a fundamental level of service. This can facilitate workload management and improve the system's operational efficiency.

To improve the efficiency and dependability of customer support, strategies such as dynamic employee management, categorization, and routing, system reliability, working vacation effectiveness, trial management, and performance monitoring based on the M/G/1 replay queue model have to be implemented. Firms can enhance the efficiency of their customer support operations by integrating these approaches.

Customers can now reach you through phone calls, social media, or live chatbots integrated into their websites. Thus, "support ticket" refers to any communication that takes place between a customer and a customer service agent.

Applications 2: Live victim radar

Live victim radar serves as a crucial tool for emergency response, enabling real-time tracking of survivors from disasters such as earthquakes and building collapses. It can use the M/G/1 retrial queue model from queueing theory to better understand how it works and where it might get backed up by making comparisons between how the radar works and these queueing models. This model demonstrates how challenging it is to run a live victim radar in the real world. It takes into account things like random search requests, system breakdowns, phased search operations, and the fact that the system can work less well when it's down.

The Live Victim Radar can be thought of as an M/G/1 retrial waiting system. The radar is like a server, and the signals from potential real victims are like users entering the system. The radar may not pick up these signals on the first try, indicating a failed detection attempt. After a while, it was placed on a retry list and attempted to pick it up again.

Arrival of Poisson: The radar tries to find people using a Poisson arrival process, which means that signals from real people come at random times. This randomness highlights the uncertainty in locating a survivor, as it depends on factors such as the survivor's movement, the level of noise, and the signal's strength.

Starting to fail and correction: Starting failures happen when the radar doesn't try to find something, possibly because of a hardware problem or interference. Fixing these kinds of problems is necessary before the radar can resume its use. Recalibrating the radar, getting rid of interference, or fixing technical problems could all be part of the repair process. This is similar to the repair step in queueing systems. Repair time may alternatively be expressed as a stochastic variable.

Service in two phases: The radar operates on a two-phase service model. In the initial detecting phase, the radar searches for signals in the environment. The radar scans the area for possible signs of life. If the first phase successfully detects a signal, the radar proceeds to the second phase, where it locks onto the signal and conducts a more comprehensive search to confirm the presence of a victim and pinpoint their precise location.

The first step is a wide scan, and the second step is a tight detection. Each step has a distinct service time distribution, which the M/G/1 framework can display as any general distribution.

Work-based vacation: The radar system can go into a "working vacation" mode. In this mode, the radar can still find people and help them, but it is on low power or standby. Although not actively scanning, the radar is ready to respond to signals. This method allows the system to conserve power while maintaining basic surveillance. This working holiday ensures the system continues to function, albeit at a reduced efficiency.

A closer look at the retrial mechanism: The retrial process is a crucial component of the M/G/1 retrial queue. If a target gets there and the radar system is busy or down, they don't wait in line; instead, they leave the system for a short time and try to get in touch with the radar again later. In emergencies where time is of the essence, this trait is especially useful. You can also model the retry time, or the time between failed attempts, as a random variable.

System analysis in a steady state

The elapsed time of retrial, service in two phases, repair, and working vacation are treated as supplementary variables in this section while creating the steady-state difference differential equations for the retrial queueing system. Then determines the probability-generating function (PGF) for the server states, the PGF for several system customers, and the orbit.

Steady state equations

During steady state, it was assumed that

$$A(0) = 0,\ A(\infty) = 1,\ \ L_b(0) = 0,\ L_b(\infty) = 1,\ R_b(0) = 0,\ R_b(\infty) = 1,\ W_v(0) = 0,$$

$$W_v(\infty) = 1,\ \ S_f(0) = 0,\quad S_f(\infty) = 1.$$

Now steady state of differential-difference equations for the structure was discussed by considering the elapsed time of retrial, services, working vacation, and repair time by way of supplementary variables.

$$\theta(x) = \frac{dA(x)}{1 - A(x)};\quad \mu_b(x) = \frac{dL_b(x)}{1 - L_b(x)};\quad \mu_{sb}(x) = \frac{dR_b(x)}{1 - R_b(x)};\quad \mu_v(x) = \frac{dW_v(x)}{1 - W_v(x)};\quad \vartheta(x) = \frac{1 - S_f(x)}{S_f(x)}.$$

Farther, by suggesting the RV as

$$C(\tau) = \begin{cases} 0 & \text{The server is idle.} \\ 1 & \text{The server is occupied with P1.} \\ 2 & \text{The server is occupied with P2.} \\ 3 & \text{The server is on vacation from work.} \\ 4 & \text{The server is in repair.} \end{cases}$$

Let $\{\tau_n;\ n \in N\}$ be a series of periods corresponding to service finishing timings or vacation expiration periods. At τ, defining the model's state through the Markov progression $\{C(\tau),\ X(\tau);\ \tau \geq 0\}$, where $C(\tau)$ represent the state of server (0, 1, 2, 3, 4) server free, busy in P1, P2, WV, server on repair. $X(\tau)$ represents the customer's number in the orbit.

If $C(\tau) = 0$ with $X(\tau) > 0$, on that occasion $A^0(\tau)$ denotes the finished retrial time was τ.
If $C(\tau) = 1$ with $X(\tau) > 0$, on that occasion ${L_b}^0(\tau)$ denotes the finished FPS at the time τ.
If $C(\tau) = 2$ with $X(\tau) > 0$, on that occasion ${R_b}^0(\tau)$ denotes elapsed SPS at the time τ.
If $C(\tau) = 3$ with $X(\tau) > 0$, on that occasion $W_v^0(\tau)$ indicates the finished WV at τ.
If $C(\tau) = 4,\ X(\tau) > 0$, on that occasion $S_f^0(\tau)$ indicates the finished repair time at τ.
Then Markov chain is formed by a series of random vectors, $Z_n = \{C(\tau_n+), X(\tau_n+)\}$ is the embedded Markov chain used in our retrial waiting scheme. Its space of states has been designated by $S = \{0, 1, 2, 3 \text{ and } 4\} \times N$.

Theorem 1: *The embedded Markov chain $\{Z_n;\ n \in N\}$ is said to be ergodic iff* $\Lambda(E(L_b) + E(R_b)) + \bar{\alpha}(1 + \lambda E(S_f)) < A^*(\lambda)$.

Table 2. Probability of the system's various states.

$P_n(x,\tau)$	Prob of totally n customers stays in orbit during τ and expired retrial time to be x.
$M_{n,b}(x,\tau)$	Prob of totally n customers stays in orbit during normal FPS at τ besides expired FPS is x.
$N_{n,b}(x,\tau)$	Prob of totally n customers stays in orbit during normal SPS at τ besides expired SPS is x.
$G_{n,v}(x,\tau)$	Prob of totally n customers stays in orbit during WV at τ and the expired WV period is x.
$U_n(x,\tau)$	Prob of totally n customers stays in orbit during repair at τ and expired repair time is x.

Proof: Strongly saying that the chain $\{Z_n;\ n \in N\}$ was considered an irreducible, aperiodic Markov chain. Here Foster's criterion[22] was used to establish the sufficient requirement of the ergodicity circumstance, which states that an irreducible as well as aperiodic Markov chain has become ergodic if a non-negative function $f(j), j \in N$ and $\in\ > 0$ exist concerning mean drift $\chi_j = E[(f(z_{n+1}) + f(z_n)|z_n = j)]$ was limited for entirely $j \in N$ besides $\chi_j \le -\in$, along with $j \in N$, except for possibly a certain restricted number j. In this case, the function was deliberated as $f(j) = j$, and then,

$$\chi_j = \begin{cases} \alpha\Lambda\left(E\left(L_b\right) + E\left(R_b\right)\right) + \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right) - A^*\left(\lambda\right), & j = 1, 2, \ldots. \\ \alpha\Lambda\left(E\left(L_b\right) + E\left(R_b\right)\right) + \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right) - 1, & j = 0 \end{cases}$$

The inequality $\alpha\Lambda(E(L_b) + E(R_b)) + \bar{\alpha}(1 + \Lambda E(S_f)) < A^*(\Lambda)$ clearly defines the necessary and sufficient requirements for ergodicity. As Sennott et al.[23] shown, uncertainty in the Markov chain $\{Z_n;\ n \in N\}$ gratifying Kaplan's format, especially $\chi_j < \infty$ for every $j \ge 0$, then here arises $j_0 \in N$ concerning $\chi_j \ge 0$ for all $j \ge j_0$. Kaplan's criteria are fulfilled in the present instance because appear t such that $\varphi_{i,j} = 0$ every $j < i - t$ and $i > 0$, wherein $R = (\varphi_{i,j})$ is $\{Z_n;\ n \ge 1\}$'s one step transition matrix. The inequality $\alpha\Lambda(E(L_b) + E(R_b)) + \bar{\alpha}(1 + \Lambda E(S_f)) \ge A^*(\Lambda)$ then it has suggested that the Markov chain was non-ergodic.

The probability of various states during τ were denoted in Table 2.

Based on the Markov process $\{X(\tau);\ \tau \ge 0\}$, constructing the probability, $P_0(\tau) = \{C(\tau) = 0, X(\tau) = 0\}$ and its corresponding probability densities as

$$P_n(x,\tau)dx = \{C(\tau) = 0, X(\tau) = n, x \le A^0(\tau) < x + dx\};\quad n \ge 1, x \ge 0.$$

$$M_{n,b}(x,\tau)dx = \{C(\tau) = 1, X(\tau) = n, x \le {L_b}^0(\tau) < x + dx\};\quad n \ge 0, x \ge 0.$$

$$N_{n,b}(,\tau)dx = \{C(\tau) = 2, X(\tau) = n, x \le {R_b}^0(\tau) < x + dx\};\quad n \ge 0, x \ge 0.$$

$$G_{n,v}(x,\tau)dx = \{C(\tau) = 3, X(\tau) = n, x \le W_v^0(\tau) < x + dx\};\quad n \ge 0, x \ge 0.$$

$$U_n(x,\tau)dx = \{C(\tau) = 4, X(\tau) = n, x \le S_f^0(\tau) < x + dx\};\quad n \ge 1, x \ge 0.$$

Here, stability conditions seem to be satisfied, so, establishing $P_0 = \lim_{\tau\to\infty} P_0(\tau)$ for $x, \tau \ge 0$ and limiting densities

$$P_n(x) = \lim_{\tau\to\infty} P_n(x,\tau);\quad M_{n,b}(x) = \lim_{\tau\to\infty} M_{n,b}(x,\tau);\quad N_{n,b}(x) = \lim_{\tau\to\infty} N_{n,b}(x,\tau);$$

$$G_{n,v}(x) = \lim_{\tau\to\infty} G_{n,v}(x,\tau);\quad U_n(x) = \lim_{\tau\to\infty} U_n(x,\tau).$$

By using SVT the framework is regulated by the differential-difference equations listed below, which are based on the presumptions stated above.

$$\Lambda P_0 = \int_0^\infty G_{v,0}\left(x\right)\mu_v\left(x\right)\ dx \tag{1}$$

$$\frac{dP_n\left(x\right)}{dx} + \left(\theta\left(x\right) + \Lambda\right)P_n\left(x\right) = 0;\quad n \ge 1 \tag{2}$$

$$\frac{dM_{b,0}\left(x\right)}{dx} + \left(\Lambda + \mu_b\left(x\right)\right)M_{b,0}\left(x\right) = 0;\ \ n = 0 \tag{3}$$

$$\frac{dM_{b,n}\left(x\right)}{dx} + \left(\Lambda + \mu_b\left(x\right)\right)M_{b,n}\left(x\right) = \Lambda M_{b,n-1}\left(x\right);\quad n \ge 1 \tag{4}$$

$$\frac{dN_{b,0}(x)}{dx} + (\Lambda + \mu_{sb}(x))N_{b,0}(x) = 0;\ n = 0 \tag{5}$$

$$\frac{dN_{b,n}(x)}{dx} + (\Lambda + \mu_{sb}(x))N_{b,n}(x) = \Lambda N_{b,n-1}(x);\quad n \geq 1 \tag{6}$$

$$\frac{dG_{v,0}(x)}{dx} + (\Lambda + \mu_{v}(x))G_{v,0}(x) = 0;\ n = 0 \tag{7}$$

$$\frac{dG_{v,n}(x)}{dx} + (\Lambda + \mu_{v}(x))G_{v,0}(x) = \Lambda G_{v,n-1}(x);\ n \geq 1 \tag{8}$$

$$\frac{dU_{n}(x)}{dx} + (\Lambda + \vartheta(x))G_{v,0}(x) = \Lambda U_{n-1}(x);\ x > 0 \tag{9}$$

Every queueing model has solutions that involve constant probabilities, and the results are favorable if and only if $\alpha\Lambda(\mathrm{E(L_b)} + \mathrm{E(R_b)}) + \bar{\alpha}(1 + \Lambda \mathrm{E(S_f)}) < \mathrm{A}^*(\Lambda)$, i.e., only by satisfying the circumstance of ergodicity.

The following equations provide boundary conditions on $x = 0$.

$$P_n(0) = \int_0^\infty G_{v,n}(x)\mu_v(x)\ dx + \int_0^\infty N_{b,n}(x)\mu_{sb}(x)\ dx + \int_0^\infty U_n(x)\ \vartheta(x)\,dx;\ n \geq 1 \tag{10}$$

$$M_{b,0}(0) = \alpha\Lambda P_0 + \alpha\int_0^\infty P_1(x)\theta(x)\,dx \tag{11}$$

$$M_{b,n}(0) = \alpha\Lambda\int_0^\infty P_n(x)\ dx + \int_0^\infty \alpha P_{n+1}(x)\theta(x)\ dx;\quad n \geq 1 \tag{12}$$

$$N_{b,0}(0) = \int_0^\infty M_{b,0}(x)\mu_b(x)\ dx \tag{13}$$

$$N_{b,n}(0) = \int_0^\infty M_{b,n}(x)\mu_b(x)\ dx;\ n \geq 1 \tag{14}$$

$$G_{v,0}(0) = \int_0^\infty N_{b,0}(x)\mu_{sb}(x)\ dx \tag{15}$$

$$G_{v,n}(0) = 0;\ n \geq 1 \tag{16}$$

$$U_1(0) = \bar{\alpha}\Lambda P_0 + \bar{\alpha}\int_0^\infty P_1(x)\theta(x)\,dx \tag{17}$$

$$U_n(0) = \bar{\alpha}\Lambda\int_0^\infty P_{n-1}(x)\,dx + \bar{\alpha}\int_0^\infty P_n(x)\theta(x)\,dx;\ n \geq 2 \tag{18}$$

The required normalizing condition is

$$P_0 + \sum_{n=1}^{\infty}\int_0^\infty P_n(x)\,dx + \sum_{n=0}^{\infty}\int_0^\infty M_n(x)\,dx + \sum_{n=0}^{\infty}\int_0^\infty N_n(x)\,dx + \sum_{n=0}^{\infty}\int_0^\infty G_v(x)\,dx + \sum_{n=1}^{\infty}\int_0^\infty U_n(x)\,dx = 1 \tag{19}$$

The essential steady-state solution of our pattern is given by

Finding steady state results of the retrial queueing model, the PGF procedure is applied here. To solve the equations already mentioned, construct the generating function as follows: $|y| \leq 1$,

$$P(x, y) = \sum_{n=1}^{\infty} P_n(x) y^n \ ; \ P(0, y) = \sum_{n=1}^{\infty} P_n(0) y^n; \quad M_b(x, y) = \sum_{n=0}^{\infty} M_{b,n}(x) y^n;$$

$$M_b(0, y) = \sum_{n=0}^{\infty} M_{b,n}(0) y^n; \quad N_b(x, y) = \sum_{n=0}^{\infty} M_{b,n}(x) y^n; \quad N_b(0, y) = \sum_{n=0}^{\infty} M_{b,n}(0) y^n;$$

$$G_v(x, y) = \sum_{n=0}^{\infty} G_{v,n}(x) y^n; \quad G_v(0, y) = \sum_{n=0}^{\infty} G_{v,n}(0) y^n; \quad U(x, y) = \sum_{n=1}^{\infty} U_n(x) y^n;$$

$$U(0, y) = \sum_{n=1}^{\infty} U_n(0) y^n.$$

Theorem 2: *Once the server state is 0, 1, 2, 3, or 4, the generating functions $P(y)$, $M_b(y)$,$N_b(y)$, $G_v(y)$, $U(y)$ of the customer's quantity in orbit is 0, 1, 2, 3, and 4. Also, the server is idle, working towards regular first, second phase servicing, busy on WV, and repair during the ordinary busy time frame, as shown below.*

$$P(y) = \frac{y P_0 (1 - A^*(\Lambda))}{W_v^*(\Lambda)} \times \left\{ \frac{(1 - W_v^*(\Lambda(1-y))) + W_v^*(\Lambda)\left[1 - \alpha L_b{}^*(\Lambda(1-y)) R_b{}^*(\Lambda(1-y)) - \bar{\alpha} z S_f^*(\Lambda(1-y))\right]}{\left(\alpha L_b{}^*(\Lambda(1-y)) R_b{}^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y + (1-y) A^*(\Lambda)) - y} \right\} \tag{20}$$

$$M_b(y) = \frac{\alpha P_0 \left(1 - L_b{}^*(\Lambda(1-y))\right)}{W_v^*(\Lambda)(1-y)} \times \left\{ \frac{(1 - W_v^*(\Lambda(1-y)))(y + (1-y) A^*(\Lambda)) + (1-y) A^*(\Lambda) W_v^*(\Lambda)}{\left(\alpha L_b{}^*(\Lambda(1-y)) R_b{}^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y + (1-y) A^*(\Lambda)) - y} \right\} \tag{21}$$

$$N_b(y) = \frac{\alpha P_0 L_b{}^*(\Lambda(1-y)) \left(1 - R_b{}^*(\Lambda(1-y))\right)}{W_v^*(\lambda)(1-y)} \times \left\{ \frac{\{(1 - W_v^*(\Lambda(1-y)))(y + (1-y) A^*(\Lambda)) + (1-y) A^*(\Lambda) W_v^*(\Lambda)\}}{\left(\alpha L_b{}^*(\lambda(1-y)) R_b{}^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y + (1-y) A^*(\Lambda)) - y} \right\} \tag{22}$$

$$G_v(y) = \frac{P_0 \left(1 - W_v^*(\Lambda(1-y))\right)}{W_v^*(\Lambda)(1-y)} \tag{23}$$

$$U(y) = \frac{\bar{\alpha} y P_0 \left(1 - S^*_f(\Lambda(1-y))\right)}{W_v^*(\Lambda)(1-y)} \times \left\{ \frac{(1 - W_v^*(\Lambda(1-y)))(y + (1-y) A^*(\Lambda)) + (1-y) A^*(\Lambda) W_v^*(\Lambda)}{(\alpha L_b{}^*(\Lambda(1-y)) R_b{}^*(\Lambda(1-y)) + \bar{\alpha} y (\Lambda(1-y)))(y + (1-y) A^*(\Lambda)) - y} \right\} \tag{24}$$

The normalizing condition can be indicated using:

$$P_0 + P(1) + M_b(1) + N_b(1) + G_v(1) + U(1) = 1 \tag{25}$$

Proof: The following set of PDE is derived by multiplying Eqs. (2) to (7) by appropriate powers for y then summing over n.

$$\frac{\partial P(x, y)}{\partial x} + (\Lambda + \theta(x)) P(x, y) = 0 \tag{26}$$

$$\frac{\partial M_b(x, y)}{\partial x} + (\Lambda - \Lambda y + \mu_b(x))\ M_b(x, y) = 0 \tag{27}$$

$$\frac{\partial N_b(x, y)}{\partial x} + (\Lambda - \Lambda y + \mu_{sb}(x))\ N_b(x, y) = 0 \tag{28}$$

$$\frac{\partial (x, y)}{\partial x} + (\lambda - \Lambda y + \mu_v(x)) G_v(x, y) = 0 \tag{29}$$

$$\frac{\partial U(x)}{\partial x} + (\Lambda - \Lambda y + \vartheta(x)) U(x, y) = 0 \tag{30}$$

Resolving the partial differential Eqs. (26) to (30) above as

$$P(x, y) = P(0, y) e^{-\Lambda x}\ (1 - A(x)) \tag{31}$$

$$M_b(x, y) = M_b(0, y) e^{-\Lambda(1-y)x}\ (1 - L_b(x)) \tag{32}$$

$$N_b(x, y) = N_b(0, y) e^{-(1-y)x\,\Lambda}\ (1 - R_b(x)) \tag{33}$$

$$G_v(x, y) = G_v(0, y) e^{-(1-y)x\,\Lambda}\ (1 - W_v(x)) \tag{34}$$

$$U(x, y) = U(0, y) e^{-\Lambda(1-y)x}\ \left(1 - S_f(x)\right) \tag{35}$$

Also, obtaining another set of partial differential equations by multiplying Eqs. (10) to (18) by appropriate powers of y and summing over n.

$$P(0, y) = \int_0^\infty G_v(x, y) \mu_v(x) dx + \int_0^\infty N_b(x, y) \mu_{sb}(x)\ dx + \int_0^\infty U(x, y) \vartheta(x) dx - \Lambda P_0 - G_{v,0}(0) \tag{36}$$

$$M_b(0, y) = \frac{\Lambda}{y} \int_0^\infty P(x, y) \theta(x) dx + \alpha\Lambda \int_0^\infty P(x, y) dx + \alpha\Lambda P_0 \tag{37}$$

$$N_b(0, y) = \int_0^\infty M_b(x, y) \mu_b(x) dx \tag{38}$$

$$G_{v,n}(0) = G_{0,v}(0) \tag{39}$$

$$U(0, y) = \bar{\alpha}\Lambda y \int_0^\infty P(x, y) dx + \bar{\alpha} \int_0^\infty P(x, y) \theta(x) dx + \bar{\alpha}\Lambda y P_0 \tag{40}$$

$$G_{0,v}(0) = \frac{\Lambda P_0}{W_v^*(\Lambda)} \tag{41}$$

$$G_v(0, y) = \frac{\Lambda P_0}{W_v^*(\Lambda)} \tag{42}$$

$$M_b(0, y) = \alpha\Lambda P_0 + \frac{\alpha}{y}\left[y + (1 - y) A^*(\Lambda)\right] P(0, y) \tag{43}$$

$$N_b(0,y) = M_b(0,y) L_b^*(\Lambda(1-y)) \tag{44}$$

$$U(0,y) = \bar{\alpha}\Lambda y P_0 + \bar{\alpha}\left[y + (1-y)A^*(\Lambda)\right] P(0,y) \tag{45}$$

Substitute Eqs. (33) to (35) in Eq. (36) and then Eqs. (42), (43) and (45) so the required equation is

$$P(0,y) = \frac{\Lambda y P_0}{W_v^*(\Lambda)} \times \left\{ \frac{\left(1 - W_v^*((1-y)\Lambda)\right) + W_v^*(\Lambda)\left[1 - L_b^*((1-y)\Lambda)\alpha R_b^*((1-y)\Lambda) - \bar{\alpha} y S_f^*(\Lambda(1-y))\right]}{\left(\alpha L_b^*((1-y)\Lambda) R_b^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*((1-y)\Lambda)\right)(y + (1-y)A^*(\Lambda)) - y} \right\} \tag{46}$$

Also substitute Eqs. (31) and (46) into Eq. (37), getting a new equation.

$$M_b(0,y) = \frac{\alpha\Lambda P_0}{W_v^*(\Lambda)} \left\{ \frac{\left(1 - W_v^*((1-y)\Lambda)\right)(y + (1-y)A^*(\Lambda)) + A^*(\Lambda)(1-y)W_v^*(\Lambda)}{\left(\alpha L_b^*((1-y)\Lambda) R_b^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*((1-y)\Lambda)\right)(y + (1-y)A^*(\Lambda)) - y} \right\} \tag{47}$$

Also substitute Eqs. (32) and (47) into Eq. (38), to get a new equation.

$$N_b(0,y) = \frac{\alpha\Lambda P_0 L_b^*((1-y)\Lambda)}{W_v^*(\Lambda)} \times \left\{ \frac{\left(1 - W_v^*((1-y)\Lambda)\right)(y + (1-y)A^*(\Lambda)) + A^*(\Lambda)(1-y)W_v^*(\Lambda)}{\left(\alpha L_b^*((1-y)\Lambda) R_b^*((1-y)\Lambda) + \bar{\alpha} y S_f^*((1-y)\Lambda)\right)(y + (1-y)A^*(\Lambda)) - y} \right\} \tag{48}$$

By taking the Eqs. (31) and (46) into Eq. (40), getting a new equation

$$U(0,y) = \frac{\bar{\alpha} y \Lambda P_0}{W_v^*(\Lambda)} \left\{ \frac{\left(1 - W_v^*(\Lambda(1-y))\right)(y + (1-y)A^*(\Lambda)) + A^*(\Lambda)(1-y)W_v^*(\Lambda)}{\left(\alpha L_b^*((1-y)\Lambda) R_b^*((1-y)\Lambda) + \bar{\alpha} y S_f^*((1-y)\Lambda)\right)(y + (1-y)A^*(\Lambda)) - y} \right\} \tag{49}$$

Substitute Eq. (46) in Eq. (31)

$$P(x,y) = \frac{\Lambda y P_0}{W_v^*(\Lambda)} \times \left\{ \frac{\left(1 - W_v^*(\Lambda(1-y))\right) + W_v^*(\Lambda)\left[1 - \alpha L_b^*(\Lambda(1-y)) R_b^*(\Lambda(1-y)) - \bar{\alpha} y S_f^*(\Lambda(1-y))\right]}{\left(\alpha L_b^*(\Lambda(1-y)) R_b^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y + (1-y)A^*(\Lambda)) - y} \right\} e^{-\Lambda x}(1 - A(x)) \tag{50}$$

Substitute Eq. (47) in Eq. (32) arriving to

$$M_b(x,y) = \frac{\alpha\Lambda P_0}{W_v^*(\lambda)} \times \left\{ \frac{\left(1 - W_v^*(\Lambda(1-y))\right)(y + (1-y)A^*(\Lambda)) + (1-y)A^*(\Lambda)W_v^*(\Lambda)}{\left(\alpha L_b^*(\lambda(1-y)) R_b^*(\lambda(1-y)) + \bar{\alpha} y S_f^*(\lambda(1-y))\right)(y + (1-y)A^*(\lambda)) - y} \right\} e^{-\Lambda(1-y)x}(1 - N_b(x)) \tag{51}$$

Substitute Eq. (48) in Eq. (33)

$$N_b(x,y) = \frac{\alpha\lambda P_0 N_b{}^*(\Lambda(1-y))}{W_v^*(\Lambda)} \times \left\{ \frac{\left\{\left(1-W_v^*((1-y)\Lambda)\right)(y+(1-y)A^*(\Lambda)) + (1-y)A^*(\Lambda)W_v^*(\Lambda)\right\}\left(e^{-\Lambda(1-y)x}\right)(1-R_b(x))}{\left(\alpha L_b{}^*(\Lambda(1-y))R_b{}^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y+(1-y)A^*(\Lambda)) - y} \right\} \quad (52)$$

Substitute Eq. (42) in Eq. (34)

$$G_v(x,y) = \frac{\Lambda P_0}{W_v^*(\Lambda)} e^{-\Lambda(1-y)x}(1-W_v(x)) \quad (53)$$

When replacing Eq. (49) in Eq. (35), acquiring the resulting equation

$$U(x,y) = \frac{\bar{\alpha} y \Lambda P_0}{W_v^*(\Lambda)} \times \left\{ \frac{\left(1-W_v^*(\Lambda(1-y))\right)(y+(1-y)A^*(\Lambda)) + (1-y)A^*(\Lambda)W_v^*(\Lambda)}{\left(\alpha L_b{}^*(\Lambda(1-y))R_b{}^*(\Lambda(1-y)) + \bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y+(1-y)A^*(\Lambda)) - y} \right\} e^{-\Lambda(1-y)x}\left(1-S_f(x)\right) \quad (54)$$

For the limiting PGF $(x,\ y), M_b(x,\ y), N_b(x,\ y),\ G_v(x,\ y), U(x,\ y)$ the PGF was defined as

$$P(y) = \int_0^\infty P(x,y)\,dx;\ M_b(y) = \int_0^\infty M_b(x,y)\,dx;\ N_b(y) = \int_0^\infty N_b(x,y)\,dx;\ G_b(y) = \int_0^\infty G_b(x,y)\,dx;$$

$$U(y) = \int_0^\infty U(x,y)\,dx.$$

Here, $(y),\ M_b(y),\ N_b(y),\ G_v(y)$, $U(y)$ indicates the PGF whenever the server is vacant, FPS, SPS, WV, and repair respectively. Now, the above Eqs. (50) to (54) were integrated towards x with limit 0 to ∞, to obtain the required Eqs. (20) to (24).

By using the above equation, the only unknown P_0 was eliminated as follows,

$$P_0 + P(1) + M_b(1) + N_b(1) + G_v(1) + U(1) = 1$$

$$P_0 = \frac{W_v^*(\Lambda)\left\{A^*(\Lambda) - \alpha(E(L_b)+E(R_b))\Lambda - \bar{\alpha}\left(\Lambda E\left(S_f\right)+1\right)\right\}}{\alpha\left(A^*(\Lambda)W_v^*(\Lambda) + \Lambda E(W_v)\right)} \quad (55)$$

Theorem 3: *When the model meets the stability criterion* $\alpha\Lambda(E(L_b)+E(R_b)) + \bar{\alpha}(1+\Lambda E(S_f)) < A^*(\lambda)$, *then* $K(y) = P_0 + P(y) + yM_b(y) + yN_b(y) + yG_v(y) + U(y)$ *and* $H(y) = P_0 + P(y) + M_b(y) + N_b(y) + G_v(y) + U(y)$ *offer the stationary distribution of the overall number of customers within the system and orbit, once for idleness of service, normal working on both the phases, and a slower service period.*

Proof: The PGF of the customer's number within the system and in orbit are resulted by substituting the Eqs. (55) to (59) in $K(y) = P_0 + P(y) + yM_b(y) + yN_b(y) + yG_v(y) + U(y)$ and in $H(y) = P_0 + P(y) + M_b(y) + N_b(y) + G_v(y) + U(y)$ and the results are as follows

$$H(y) = P_0 \frac{\alpha\left(1-W_v^*((1-y)\Lambda)\right)(y+(1-y)A^*(\Lambda)) + W_v^*(\Lambda)\alpha A^*(\Lambda)(1-y)}{W_v^*(\Lambda)\left(\left(\alpha L_b{}^*((1-y)\Lambda)R_b{}^*((1-y)\Lambda) + \bar{\alpha} y S_f^*((1-y)\Lambda)\right)(y+(1-y)A^*(\Lambda)) - y\right)} \quad (56)$$

and

$$K(y) = P_0 \frac{\alpha(1-y)A^*(\Lambda)W_v^*(\Lambda)L_b{}^*(\Lambda(1-y))R_b{}^*(\Lambda(1-y))\, y\left(1-W_v^*(\Lambda(1-y))\right) - \bar{\alpha} y\left(1-W_v^*(\Lambda(1-y))\right)(y+(1-y)A^*(\Lambda))S_f^*(\Lambda(1-y))}{W_v^*(\Lambda)\left(\left(\alpha L_b{}^*(\Lambda(1-y))R_b{}^*(\Lambda(1-y))+\bar{\alpha} y S_f^*(\Lambda(1-y))\right)(y+(1-y)A^*(\Lambda))-y\right)} \tag{57}$$

Results and discussion

Performance measures

The mean length of the system and orbit

While the entire model is in steady state,

(i) The estimated customer's quantity toward orbit (L_q) was calculated by differentiating Eq. (56) by y and assessing at $y = 1$, getting L_q as

$$L_q = \frac{2\bar{\alpha}\Lambda E(S_f) + 2\bar{\alpha}(1 - A^*(\Lambda)) + 2\bar{\alpha}\Lambda E(S_f)(1-A^*(\Lambda))}{2\left(A^*(\Lambda) - \alpha\Lambda(E(L_b)+E(R_b)) - \bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)} + \frac{2\alpha\Lambda E(W_v)(1-A^*(\Lambda)) + \alpha\Lambda^2 E\left(w_v^2\right)}{2\left(\alpha\Lambda E(W_v) + \alpha A^*(\Lambda)W_v^*(\Lambda)\right)}$$
$$+\frac{\alpha\Lambda^2\left(E\left(L_b^2\right)+2E(L_b)E(R_b)+E\left(R_b^2\right)\right)+\bar{\alpha}\Lambda^2 E\left(S_f^2\right)+2\alpha\Lambda(E(L_b)+E(R_b))(1-A^*(\Lambda))}{2\left(A^*(\Lambda)-\alpha\Lambda(E(L_b)+E(R_b))-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)} \tag{58}$$

(ii) The estimated customer number of the system (L_s) is calculated by differentiating Eq. (62) w.r.t y, and also assessing $y = 1$, changing to L_s.

$$L_s = \frac{2\alpha\lambda A^*(\Lambda)W_v^*(\Lambda)(E(L_b)+E(R_b)) + 2\Lambda E(W_v) - \bar{\alpha}\Lambda^2 E\left(w_v^2\right) + \Lambda^2 E\left(w_v^2\right)}{2\left(\alpha A^*(\Lambda)W_v^*(\Lambda)+\alpha\Lambda E(W_v)\right)}$$
$$-\frac{2\left\{\bar{\alpha}\Lambda^2 E(W_v)E(S_f)+\bar{\alpha}\Lambda E(W_v)+\bar{\alpha}\Lambda E(W_v)(1-A^*(\Lambda))\right\}}{2\left(\alpha A^*(\Lambda)W_v^*(\Lambda)+\alpha\Lambda E(W_v)\right)}$$
$$+\frac{\alpha\Lambda^2\left(E\left(L_b^2\right)+E\left(R_b^2\right)\right)+\bar{\alpha}\Lambda^2 E\left(S_f^2\right)+2\alpha\Lambda(E(L_b)+E(R_b))(1-A^*(\Lambda))+2\alpha\Lambda^2 E(L_b)E(R_b)}{2\left(A^*(\Lambda)-\alpha\Lambda(E(L_b)+E(R_b))-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)}$$
$$+\frac{2\bar{\alpha}\Lambda E(S_f)+2\bar{\alpha}(1-A^*(\Lambda))+2\bar{\alpha}\Lambda E(S_f)(1-A^*(\Lambda))}{2\left(A^*(\Lambda)-\alpha\Lambda(E(L_b)+E(R_b))-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)} \tag{59}$$

The system's state probabilities

This section reveals certain performance metrics of this system's $M/G/1$ retrials queue, which are associated with two phases of basic servicing using starting failure with WV under steady state, as well as reliability analysis, average busy with average cycle period, orbit distribution of size at a departing period of these suggested design. Now, the steady state probability was given by

1. The likelihood of the server staying idle during the retrial in a stable state was accepted through

$$\mathbb{P} = P(1) = \frac{P_0(1-A^*(\Lambda))}{W_v^*(\Lambda)}\left\{\frac{\Lambda E(W_v)+W_v^*(\Lambda)\left[\bar{\alpha}+\bar{\alpha}\Lambda E(S_f)+\alpha\Lambda E(L_b)+\alpha\Lambda E(R_b)\right]}{\left\{A^*(\Lambda)-\alpha\Lambda(E(L_b)+E(R_b))-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right\}}\right\} \tag{60}$$

2. The likelihood of server existence busy at normal FPS under steady state was agreed by

$$\mathbb{M} = M_b(1) = \frac{\alpha P_0\left(\Lambda E(L_b)\left(\Lambda E(W_v)+W_v^*(\Lambda)\ A^*(\Lambda)\right)\right)}{W_v^*(\Lambda)\ \left(A^*(\Lambda)-\alpha\Lambda(E(L_b)+E(R_b))-\bar{\alpha}\left(1+E(S_f)\Lambda\right)\right)} \tag{61}$$

3. The likelihood of the server being busy at normal SPS under a steady state was agreed upon by

$$\mathbb{N} = N_b(1) = \frac{P_0\alpha}{W_v^*(\Lambda)}\left\{\frac{\Lambda E(R_b)\left[\left(\Lambda E(W_v) + W_v^*(\Lambda)A^*(\Lambda)\right)\right]}{\left\{A^*(\Lambda) - \alpha\Lambda\left(E(L_b) + E(R_b)\right) - \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right)\right\}}\right\} \tag{62}$$

4. The likelihood of the server being busy on WV under a steady state was agreed by

$$\mathbb{G} = G_v(1) = \frac{P_0\Lambda E(W_v)}{W_v^*(\Lambda)} \tag{63}$$

5. The likelihood of a server being on repair under steady state was agreed upon by

$$\mathbb{U} = U(1) = \frac{P_0\bar{\alpha}}{W_v^*(\Lambda)}\left\{\frac{\Lambda E\left(S_f\right)\left[\left(\Lambda E(W_v) + W_v^*(\Lambda)\ A^*(\Lambda)\right)\right]}{\left\{A^*(\Lambda) - \alpha\Lambda\left(E(L_b) + E(R_b)\right) - \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right)\right\}}\right\} \tag{64}$$

6. The mean time customers spend in the system (W_s) as well as in the orbit (W_q) during steady state by the effect of Little's formula follows,

$$(W_s) = \frac{(L_s)}{\lambda} \quad \text{and} \quad (W_q) = \frac{(L_q)}{\lambda}$$

Measures of reliability

Reliability Under Heavy Traffic: Under high traffic conditions, which are defined by a high Poisson arrival rate (Λ) compared to the service rate (μ), the model's reliability can be compromised. Under such circumstances, the queue length and waiting time tend to rise significantly, resulting in system congestion and possible instability. If the retrial rate (θ) is high, the retrial mechanism, which enables consumers to reattempt service after being blocked, may worsen the problem by imposing unnecessary strain on the system.

To mitigate these consequences, our model incorporates a two-phase service mechanism during working vacations, in which the server functions at a reduced capacity while still handling clients. This reduces a portion of the strain during periods of high demand, ensuring a consistent level of service even in high system load conditions. Still, if the traffic intensity, which is the ratio of arrival rate to service rate, gets close to or goes over one, the system may become unstable. This could cause the wait time to get longer and worsen performance.

Reliability Under High Failure Rates: In scenarios characterized by a high likelihood of first failure, the system encounters frequent interruptions that necessitate repair before service can resume. Elevated failure rates can have a significant impact on the system's dependability because frequent failures result in longer periods of inactivity and longer repair times. Inefficient repair procedures, characterized by a poor repair rate, may result in rapid queue growth, causing extended waiting periods and consequent consumer dissatisfaction.

In such situations, our model's reliability depends on the equilibrium between failure rates and repair procedure efficiency. Implementing a two-phase service methodology during working vacation improves durability by allowing for a certain degree of service continuity, even in the event of failures. Nevertheless, if the rate of failure exceeds a certain threshold, the system may encounter difficulties in its recovery process, resulting in possible bottlenecks and hindering overall system performance.

The interplay among the arrival rate, service rate, and retrial rate primarily influences system stability and bottlenecks. Achieving system stability necessitates ensuring that the effective service rate is sufficient to accommodate both direct arrivals and repeat customers. High arrival rates, coupled with frequent starting failures, provide a considerable danger of instability to the system. At the point of service initiation, bottlenecks are prone to arise, causing delays in the processing of new clients, and resulting in longer queue lengths and waiting periods.

Our model specifically manages traffic and failure rate fluctuations, but its dependability and consistency rely on maintaining a satisfactory equilibrium between these factors. By conducting an analysis of the system in various circumstances, it may detect possible obstacles and implement measures to guarantee ongoing dependability, even in situations of high traffic or frequent failures.

Queueing mechanism through an unstable server, reliability metrics deliver few details needed to enhance the system. For justification and validate the analytical findings for the above framed model, also level of availability measure along with frequency towards failure was calculated as follows:

Λ_v represents steady-state accessibility, which refers to the possibility when a server is functioning either towards positive customers or idle.

$$(\lambda_v) = 1 - \lim_{\mathcal{y}\to 1} U(\mathcal{y}) = 1 - \mathbb{U}(1)$$

$$(\lambda_v) = 1 - \frac{P_0\bar{\alpha}}{W_v^*(\Lambda)}\left\{\frac{\Lambda E(S_f)\left[(\Lambda E(W_v) + W_v^*(\Lambda)A^*(\Lambda))\right]}{\{A^*(\Lambda) - \alpha\Lambda(E(L_b) + E(R_b)) - \bar{\alpha}\left(1 + \Lambda E(S_f)\right)\}}\right\} \tag{65}$$

7. The steady state for the failure frequency is attained using $F_{\mathcal{F}} = \bar{\alpha}(M_b(1))$.

$$F_{\mathcal{F}} = \bar{\alpha}\left\{\frac{\alpha P_0(\Lambda E(L_b)(\Lambda E(W_v) + A^*(\Lambda)W_v^*(\Lambda)))}{W_v^*(\Lambda)(A^*(\Lambda) - \alpha\Lambda(E(L_b) + E(R_b)) - \bar{\alpha}(1 + E(S_f)\Lambda))}\right\} \tag{66}$$

Average busy period also busy cycle

Assume that $E(T_b)$ and $E(T_C)$ reflect on the planned extent of the busy period with a busy cycle beneath a steady-state scenario. By smearing a few arguments of the discontinuous process of renewal and then arriving at the outcomes directly, which leads to

$$P_0 = \frac{E(T_0)}{E(T_b) + E(T_0)};\ E(T_b) = \frac{1}{\Lambda}\left(\frac{1}{P_0} - 1\right) \quad \text{and} \quad E(T_c) = \frac{1}{\Lambda P_0} = E(T_b) + E(T_0)$$

Where T_0 denotes the time length of the model during the vacant state. Meanwhile when the inter-arrival time of each client approaches a distribution that grows exponentially with a parameter Λ, obtaining $E(T_0) = \frac{1}{\Lambda}$. By using the above-required results, getting the following

$$E(T_b) = \frac{1}{\Lambda}\frac{\alpha\lambda\left(E(W_v) + W_v^*(\Lambda)(E(L_b) + E(R_b))\right) + \bar{\alpha}W_v^*(\Lambda)\left(1 - E(S_f) - A^*(\Lambda)\right)}{W_v^*(\Lambda)\left\{A^*(\Lambda) - \alpha\Lambda(E(L_b) + E(R_b)) - \bar{\alpha}\left(1 + \Lambda E(S_f)\right)\right\}} \tag{67}$$

$$E(T_c) = \frac{1}{\Lambda P_0} = \frac{1}{\Lambda}\left\{\frac{\alpha A^*(\Lambda)W_v^*(\Lambda) + \alpha\Lambda E(W_v)}{W_v^*(\Lambda)\left\{A^*(\Lambda) - \alpha\Lambda(E(L_b)) - \bar{\alpha}\left(1 + \Lambda E(S_f)\right)\right\}}\right\} \tag{68}$$

Distribution of orbit sizes at departing epoch

In accordance with PASTA: Poisson Arrival See Time Average reasoning, an inactive customer sees n consumers in an orbit, immediately after disappearance if and only if the system had $n + 1$ customers just before disappearance. Uncertainty of denoting the likelihood for $\{L_n^+; n \geq 0\}$; n units of customers are in orbit during departure time, getting into

$$L_n^+ = K_0\int_0^\infty G_{v,n}(x)\mu_v(x)\,dx + K_0\int_0^\infty N_{b,n}(x)\mu_{sb}(x)\,dx + K_0\int_0^\infty U_n(x)\vartheta(x)\,dx;\ n \geq 1 \tag{69}$$

where K_0 denotes the normalizing constant.

Corollary 1: *PGF of customer quantity inside the orbit at the departing epoch was calculated by*

$$L_D(\mathcal{y}) = \frac{K_0\Lambda P_0\{(\mathcal{y} + (1-\mathcal{y})A^*(\Lambda)) + (1-\mathcal{y})W_v^*(\Lambda)A^*(\Lambda)\}\left(\left(\alpha L_b^*((1-\mathcal{y})\Lambda)R_b^*(\Lambda(1-\mathcal{y})) + \bar{\alpha}\mathcal{y}S_f^*((1-\mathcal{y})\Lambda)\right)\right)}{W_v^*(\Lambda)\left\{\left(\alpha L_b^*(\Lambda(1-\mathcal{y}))R_b^*((1-\mathcal{y})\Lambda) + \bar{\alpha}\mathcal{y}S_f^*((1-\mathcal{y})\Lambda)\right)(\mathcal{y} + A^*(\Lambda)(1-\mathcal{y})) - \mathcal{y}\right\}}$$
$$+ \frac{K_0\Lambda P_0\mathcal{y}W_v^*((1-\mathcal{y})\Lambda)}{W_v^*(\Lambda)\left\{\left(\alpha L_b^*((1-\mathcal{y})\Lambda)R_b^*(\Lambda(1-\mathcal{y})) + \bar{\alpha}\mathcal{y}S_f^*((1-\mathcal{y})\Lambda)\right)(\mathcal{y} + (1-\mathcal{y})A^*(\Lambda)) - \mathcal{y}\right\}} \tag{70}$$

Proof: Multiplying Eq. (60) using essential factors of z followed by adding over $n = 0, 1, 2, 3, \ldots$

$$L_D(y) = K_0 \int_0^\infty G_v(x, y)\, \mu_v(x)\, dx + K_0 \int_0^\infty N_b(x, y)\, \mu_{sb}(x)\, dx + K_0 \int_0^\infty U(x, y)\, \vartheta(x)\, dx \tag{71}$$

Using the Eqs. (52) and (54) in Eq. (62), arriving at the PGF for orbit's customer number at a departing epoch (i.e.,) Eq. (61) is obtained. The limiting case $y \to 1$ is used to calculate the constant in Eq. (61). Where K_0 is given as

$$K_0 = \frac{\alpha \Lambda A^*(\Lambda)\, W_v^*(\Lambda) + \alpha \Lambda E(W_v)}{\left\{(A^*(\Lambda) - \alpha \Lambda (E(L_b) + E(R_b))) - \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right) + \left(\Lambda E(W_v) + A^*(\Lambda)\, W_v^*(\Lambda)\right)\right\}} \tag{72}$$

Stochastic decomposition

Stochastic decomposition broadly detected the $M/G/1$ retrial model which includes the server's vacation. In these analyses, the most important finding is customer's quantity in the system at an indeterminate epoch becomes dispersed as a combination of two distinct RVs, one among is the customer's quantity in a comparable regular queueing model, together with the model's stable state at an indeterminate point in time. In some circumstances, depending on the anticipated vacation, an additional random variable might provide various probabilistic clarifications. In addition, stochastic decomposition holds for various M/G/1 retrial queues.

Let $\chi(y)$ denotes the PGF of the customer's number in two compulsory phase queueing models under a steady state throughout a random value, whereas $\phi(y)$ represents the PGF of the customer's number present in orbit either on the server's idealness', on vacation, or server under repair during a random point. Whereas $Z_s(y)$ signifies the PGF of the customer's quantity in the framed model, which was decomposed into two random variables. Finally, the mathematical representation of the stochastic decomposition for two random variables is given by $Z_s(y) = \phi(y)\chi(y)$. Now, this law was verified and the same was implemented in the retrial model also analyses were made in this paper. For the two-phase queueing model, getting as,

$$\chi(y) = \frac{M_b(y) + N_b(y) + G_v(y)}{M_b(1) + N_b(1) + G_v(1)}; \quad \phi(y) = \frac{P_0 + P(y) + U(y)}{P_0 + P(1) + U(1)}$$

$$\chi(y) = \frac{\begin{gathered}\left(1 - W_v^*(\Lambda(1-y))\right)(y + (1-y)A^*(\Lambda)) \\ \left\{\times 2\alpha - \left(L_b{}^*(\Lambda(1-y)) + R_b{}^*((1-y)\Lambda)\right) + \alpha L_b{}^*(\Lambda(1-y))\, R_b{}^*(\Lambda(1-y)) + \overline{\alpha} y S_f^*(\Lambda(1-y))\right\} \\ + \ W_v^*(\Lambda)A^*(\Lambda)(1-y)\left\{2\alpha - \left(L_b{}^*((1-y)\Lambda) + R_b{}^*((1-y)\Lambda)\right)\right\} - y\left(1 - W_v^*(\Lambda(1-y))\right)\end{gathered}}{(1-y)\left\{\left[\alpha L_b{}^*(\Lambda(1-y))\, R_b{}^*(\Lambda(1-y)) + \overline{\alpha} y S_f^*(\Lambda(1-y))\right] - y\right\}}$$

$$\times \frac{\alpha \Lambda A^*(\Lambda)\, W_v^*(\Lambda)(E(L_b) + E(R_b)) + \Lambda E(W_v)\left(A^*(\Lambda) - \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right)\right)}{A^*(\Lambda) - \alpha \Lambda (E(L_b) + E(R_b)) - \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right)} \tag{73}$$

$$\phi(y) =$$

$$\frac{\begin{gathered} W_v^*(\Lambda)(1-y)\left[\alpha L_b{}^*(\Lambda(1-y))\, R_b{}^*((1-y)\Lambda) + \overline{\alpha} y S_f^*(\Lambda(1-y))\right] \\ + y\left(1 - W_v^*(\Lambda(1-y))\right)(1 - \alpha y) - \overline{\alpha} y S_f^*((1-y)\Lambda)\left(1 - W_v^*(\Lambda(1-y))\right)(y + (1-y)A^*(\Lambda)) \\ -\overline{\alpha} y (1-y)\, W_v^*(\Lambda)A^*(\Lambda)S_f^*((1-y)\Lambda) - \alpha y (1-y)A^*(\Lambda)W_v^*(\Lambda) - \alpha y(1-y)A^*(\lambda)(1 - W_v^*(\Lambda(1-y))\end{gathered}}{(1-y)\left(\left(\alpha L_b{}^*((1-y)\Lambda)\, R_b{}^*(\Lambda(1-y)) + \overline{\alpha} y S_f^*((1-y)\Lambda)\right)(y + A^*(\Lambda)(1-y)) - y\right)}$$

$$\times \frac{\left\{A^*(\Lambda) - \alpha \Lambda (E(L_b) + E(R_b)) - \bar{\alpha}\left(1 + \Lambda E\left(S_f\right)\right)\right\}}{\Lambda E(W_v)(1 - A^*(\Lambda)) - \alpha \Lambda A^*(\Lambda)(E(L_b) + E(R_b)) + \bar{\alpha} \Lambda^2 E\left(S_f\right) E(W_v) + \alpha A^*(\Lambda)\, W_v^*(\Lambda)} \tag{74}$$

Now,

$$\phi(\mathcal{y}) \times \chi(\mathcal{y})$$

$$= \frac{\begin{array}{c} W_v^*(\Lambda)(1-\mathcal{y})\left[\alpha L_b^*(\Lambda(1-\mathcal{y}))R_b^*((1-\mathcal{y})\Lambda)+\bar{\alpha}S_f^*((1-\mathcal{y})\Lambda)\right]+\mathcal{y}\left(1-W_v^*((1-\mathcal{y})\right. \\ \times\ \Lambda))(1-\alpha\mathcal{y})-\bar{\alpha}\mathcal{y}S_f^*((1-\mathcal{y})\Lambda)\left(1-W_v^*(\Lambda(1-\mathcal{y}))\right)(\mathcal{y}+(1-\mathcal{y})A^*(\Lambda))-\bar{\alpha}\mathcal{y}(1-\mathcal{y}) \\ \times W_v^*(\Lambda)A^*(\Lambda)S_f^*(\Lambda(1-\mathcal{y}))-\alpha\mathcal{y}(1-\mathcal{y})A^*(\Lambda)W_v^*(\Lambda)-\alpha\mathcal{y}(1-\mathcal{y})A^*(\Lambda)\left(1-W_v^*(\Lambda(1-\mathcal{y}))\right) \end{array}}{(1-\mathcal{y})\left(\left(\alpha L_b^*((1-\mathcal{y})\Lambda)R_b^*((1-\mathcal{y})\Lambda)+\bar{\alpha}\mathcal{y}S_f^*((1-\mathcal{y})\Lambda)\right)(\mathcal{y}+(1-\mathcal{y})A^*(\Lambda))-\mathcal{y}\right)}.$$

$$\times \frac{\{\alpha\Lambda A^*(\Lambda)W_v^*(\Lambda)(E(L_b)+E(R_b))+\Lambda E(W_v)\left(A^*(\Lambda)-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)\}}{\Lambda E(W_v)(1-A^*(\Lambda))-\alpha\Lambda A^*(\Lambda)(E(L_b)+E(R_b))+\bar{\alpha}\Lambda^2E(S_f)E(W_v)+\alpha W_v^*(\Lambda)A^*(\Lambda)}$$

$$\times \frac{\begin{array}{c} \left(1-W_v^*((1-\mathcal{y})\Lambda)\right)\{2\alpha-\left(L_b^*((1-\mathcal{y})\Lambda)+R_b^*((1-\mathcal{y})\Lambda)\right)+\alpha L_b^*((1-\mathcal{y})\Lambda)R_b^*(\Lambda(1-\mathcal{y})) \\ +\bar{\alpha}S_f^*((1-\mathcal{y})\Lambda)\}(\mathcal{y}+(1-\mathcal{y})A^*(\Lambda))W_v^*(\Lambda)A^*(\Lambda)(1-\mathcal{y})\{2\alpha-\left(L_b^*(\Lambda(1-\mathcal{y}))+R_b^*(\Lambda(1-\mathcal{y}))\right)\} \\ -\mathcal{y}\left(1-W_v^*((1-\mathcal{y})\Lambda)\right) \end{array}}{(1-\mathcal{y})\left\{\left[\alpha L_b^*((1-\mathcal{y})\Lambda)R_b^*(\Lambda(1-\mathcal{y}))+\bar{\alpha}S_f^*((1-\mathcal{y})\Lambda)\right]-\mathcal{y}\right\}}$$

$$\times \frac{\{\alpha\Lambda A^*(\Lambda)W_v^*(\Lambda)(E(L_b)+E(R_b))+\Lambda E(W_v)\left(A^*(\Lambda)-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)\}}{\Lambda E(W_v)(1-A^*(\Lambda))-\alpha\Lambda A^*(\Lambda)(E(L_b)+E(R_b))+\bar{\alpha}\Lambda^2E(S_f)E(W_v)+\alpha W_v^*(\Lambda)A^*(\Lambda)}$$

Special cases

This section looked at some of our concept's extraordinary applications.

Case 1: When $E(R_b) = 0$, the simplest version of the framed model through single server retrial queuing having single phase customer service, WV, plus commencing disaster. During this instance, results have a decent contract through Gowsalya and Arivudainambi [24] when the PGF for the number of system's customers $K(\mathcal{y})$, also for idle probability P_0, PGF for orbit's customer quantity $H(\mathcal{y})$, the average system length Ls, as well as the average orbit length Lq are as follows.

$$P_0 = \frac{W_v^*(\Lambda)\{A^*(\Lambda)-\alpha\Lambda(E(L_b))-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\}}{\alpha A^*(\Lambda)W_v^*(\Lambda)+\alpha\Lambda E(W_v)}$$

$$K(\mathcal{y}) = P_0\frac{\begin{array}{c}\alpha(1-\mathcal{y})A^*(\Lambda)W_v^*(\Lambda)L_b^*(\Lambda(1-\mathcal{y}))+\left(1-W_v^*(\Lambda(1-\mathcal{y}))\mathcal{y}\right)-\bar{\alpha}\mathcal{y}\left(1-W_v^*((1-\mathcal{y})\Lambda)\right) \\ \times(\mathcal{y}+A^*(\Lambda)(1-\mathcal{y}))S_f^*(\Lambda(1-\mathcal{y}))\end{array}}{W_v^*(\Lambda)\left(\left(\alpha L_b^*(\Lambda(1-\mathcal{y}))+\bar{\alpha}\mathcal{y}S_f^*(\Lambda(1-\mathcal{y}))\right)(\mathcal{y}+A^*(\Lambda)(1-\mathcal{y}))-\mathcal{y}\right)}$$

$$L_q = \frac{2\alpha\Lambda E(W_v)(1-A^*(\Lambda))+\alpha\Lambda^2E(w_v^2)}{2(\alpha\Lambda E(W_v)+\alpha A^*(\Lambda)W_v^*(\Lambda))} + \frac{\alpha\Lambda^2(E(L_b^2))+\bar{\alpha}\Lambda^2E\left(S_f^2\right)+2\alpha\Lambda(E(L_b))(1-A^*(\Lambda))}{2\left(A^*(\Lambda)-\alpha\Lambda(E(L_b))-\bar{\alpha}\left(1+\Lambda E(S_f)\right)\right)}$$

$$+\frac{2\bar{\alpha}\Lambda E(S_f)+2\bar{\alpha}(1-A^*(\Lambda))+2\bar{\alpha}\Lambda E(S_f)(1-A^*(\Lambda))}{2\ \left(A^*(\Lambda)-\alpha\Lambda(E(L_b))-\bar{\alpha}\left(1+E(S_f)\Lambda\right)\right)}$$

$$L_s = \frac{\begin{array}{c}\left(2\alpha\Lambda A^*(\Lambda)W_v^*(\Lambda)(E(L_b))+2\Lambda E(W_v)-\bar{\alpha}\Lambda^2E(w_v^2)+\Lambda^2E(w_v^2)\right) \\ -2\{\bar{\alpha}\Lambda^2E(W_v)E(S_f)+\bar{\alpha}\Lambda E(W_v)+\bar{\alpha}\Lambda E(W_v)(1-A^*(\Lambda))\}\end{array}}{2(\alpha A^*(\lambda)W_v^*(\lambda)+\alpha\Lambda E(W_v))}$$

$$+\frac{\alpha\Lambda^2(E(L_b^2))+\bar{\alpha}\Lambda^2E\left(S_f^2\right)+2\alpha\Lambda(E(L_b))(1-A^*(\Lambda))2\bar{\alpha}\Lambda E(S_f)+\bar{\alpha}(1-A^*(\Lambda))+\bar{\alpha}\Lambda E(S_f)(1-A^*(\Lambda))}{2\ \left(A^*(\Lambda)-\alpha\Lambda(E(L_b))-\bar{\alpha}\left(1+E(S_f)\Lambda\right)\right)}$$

Case 2: If $E(R_b) = 0,\ \alpha = 1$, and the simplest version of our approach is a single server comprising a single-phase service and WV. In this instance, the mean system capacity Ls, the mean orbit capacity Lq, an inactive probability P_0, the PGF of the system's actual customer number $K(\mathcal{y})$ with the PGF of the orbit's actual $H(\mathcal{y})$, and the findings are in good accordance with Arivudainambi.[25]

$$P_0 = \frac{W_v^*(\Lambda)\{A^*(\Lambda) - \Lambda(E(L_b))\}}{A^*(\Lambda)W_v^*(\Lambda) + \Lambda E(W_v)}; K(\mathcal{y}) = P_0 \frac{(1-\mathcal{y})A^*(\Lambda)W_v^*(\Lambda)L_b{}^*(\Lambda(1-\mathcal{y})) + \mathcal{y}\left(1 - W_v^*(\Lambda(1-\mathcal{y}))\right)}{W_v^*(\Lambda)\left(\left(L_b{}^*(\Lambda(1-\mathcal{y}))\right)(\mathcal{y} + (1-\mathcal{y})A^*(\Lambda)) - \mathcal{y}\right)}$$

$$L_q = \frac{2\Lambda E(W_v)(1 - A^*(\Lambda)) + \Lambda^2 E\left(w_v^2\right)}{2\left(\Lambda E(W_v) + A^*(\Lambda)W_v^*(\Lambda)\right)} + \frac{\Lambda^2\left(E\left(L_b^2\right)\right) + 2\Lambda(E(L_b))(1 - A^*(\Lambda))}{2(A^*(\Lambda) - \Lambda E(L_b))}$$

$$L_s = \frac{2\Lambda A^*(\Lambda)W_v^*(\Lambda)(E(L_b)) + 2\Lambda E(W_v) + \Lambda^2 E\left(w_v^2\right)}{2\left(W_v^*(\Lambda)A^*(\Lambda) + \Lambda E(W_v)\right)} + \frac{\Lambda^2\left(E\left(L_b^2\right)\right) + 2\Lambda(E(L_b))(1 - A^*(\Lambda))}{2(A^*(\Lambda) - (E(L_b))\Lambda)}$$

Case 3: If $E(R_b) = 0,\ A^*(\Lambda) = 1, \alpha = 1$, the framed structure was simplified as M/G/1 queues include service provided in one phase and a working holiday. Under this instance, an inactive probability P_0, mean system capacity Ls, also no. of tasks in the system K $(\mathcal{y})$ have prob generating functions that are given, and the findings show strong agreement towards Zang and Hou.[26]

$$P_0 = \frac{W_v^*(\Lambda)\{1 - \Lambda(E(L_b))\}}{W_v^*(\Lambda) + \Lambda E(W_v)}; \quad K(\mathcal{y}) = P_0 \frac{(1-\mathcal{y})W_v^*(\Lambda)L_b{}^*(\Lambda(1-\mathcal{y})) + \mathcal{y}\left(1 - W_v^*(\Lambda(1-\mathcal{y}))\right)}{W_v^*(\Lambda)\left(\left(L_b{}^*(\Lambda(1-\mathcal{y}))\right) - \mathcal{y}\right)}$$

$$L_s = \frac{2\Lambda W_v^*(\Lambda)(E(L_b)) + 2\Lambda E(W_v) + \Lambda^2 E\left(w_v^2\right)}{2\left(W_v^*(\Lambda) + \Lambda E(W_v)\right)} + \frac{\Lambda^2\left(E\left(L_b^2\right)\right)}{2(1 - \Lambda(E(L_b)))}.$$

Case 4: If $(R_b) = 0,\ A^*(\Lambda) = 1, \alpha = 1, W_v^*(\Lambda) = 1$, our model was simplified to the $M/G/1$queueing system through one phase service. In this instance, the standard P-K equation in Gross and Harris[27] holds, and the PGF of the total no. of customers present in the system $K(\mathcal{y})$, the idle probability P_0, besides average system size L_s remains as follows.

$$P_0 = \{1 - \Lambda(E(L_b))\}; \quad L_s = \Lambda(E(L_b)) + \frac{\Lambda^2(E(L_b^2))}{2(1 - (E(L_b)))}$$

Numerical illustrations

The outcomes of our framed model were examined using MATLAB software program by fluctuating values of parameters $\Lambda;\ \theta;\ \mu_b;\ \mu_{sb};\ \mu_v;\ \vartheta;\ \alpha;\ \bar{\alpha}$ on the model. It was anticipated that exponential distribution whose density function $f(\xi) = \nu e^{-\nu\xi},\ \xi > 0$, have been executed over the times on retrial, FPS and SPS, WV, and on repair. The characteristics of probability were discussed using tables and graphs. Table 3 reveals that by gradually increasing the values for FPS (μ_b), idle for the system (P_0) goes on increase, whereas $L_s,\ L_q,\ \ W_s,\ W_q$ falls for the parameters $\theta = 3;\ \mu_{sb} = 35;\ \mu_v = 2;\ \vartheta = 10;\ \Lambda = 0.2;\ \alpha = 0.8;\ \bar{\alpha} = 0.2$. Table 4 reveals that gradually increasing the values for arrival rate (Λ), idle for the system (P_0) goes on increase, whereas $L_s,\ L_q,\ \ W_s,\ W_q$ goes on increases for the parameters $\theta = 3;\ \mu_b = 35;\ \mu_{sb} = 35;\ \mu_v = 2;\ \vartheta = 10;\ \alpha = 0.8;\ \bar{\alpha} = 0.2$. Table 5 reveals that gradually increasing the values for repair rate (ϑ), idle for the system (P_0) goes on increase, whereas $L_s,\ L_q,\ \ W_s,\ W_q$ goes on decreases for the parameters $\theta = 3;\ \mu_b = 35;\ \mu_{sb} = 35;\ \mu_v = 2;\ \vartheta = 10;\ \alpha = 0.8;\ \bar{\alpha} = 0.2$. Table 6 reveals that on gradually increasing the values for retrial rate (θ), idle for the system (P_0) goes on increase, whereas $L_s,\ L_q,\ \ W_s,\ W_q$ goes on decreases for the parameter $\mu_b = 35;\ \mu_{sb} = 35;\ \mu_v = 2;\ \vartheta = 10;\ \Lambda = 0.2;\ \alpha = 0.8;\ \bar{\alpha} = 0.2$.

For the system performance measures, $P_0,\ L_s,\ L_q,\ \ W_s,\ W_q$ the consequence of the parameters such as $\Lambda;\ \theta;\ \mu_b;\ \mu_{sb};\ \mu_v;\ \vartheta;\ \alpha;\ \bar{\alpha}$ on it was explored in three and two-dimensional graphs which are exemplified in Figs. 2 to 5 and in Figs. 6 and 7 respectively.

Fig. 2 analyzes the average queue duration of the arrival rate and repair rate. An increase in both the arrival rate and repair rate leads to an initial drop in the average queue length. For the parameters $\theta = 0.5;\ \mu_b = 35;\ \mu_{sb} = 35;\ \mu_v = 0.1;\ \alpha = 0.9$ and $\bar{\alpha} = 0.1$.

Fig. 3 illustrates the influence of FPS rate and repair rate on average wait length. It shows how the FPS rate, repair rate, and average queue duration are related. As the frame rate per second (FPS) and repair rate

Table 3. Efforts of FPS (μ_b) on P_0, L_s, L_q, W_s, W_q.

μ_b	P_0	L_s	L_q	W_s	W_q
20.00	0.86030	0.15206	0.04171	0.76028	0.76028
21.67	0.86103	0.15127	0.04161	0.75636	0.75636
23.33	0.86166	0.15060	0.04153	0.75301	0.75301
25.00	0.86221	0.15002	0.04146	0.75011	0.75011
26.67	0.86268	0.14951	0.04140	0.74757	0.74757
28.33	0.86310	0.14907	0.04135	0.74533	0.74533
30.00	0.86348	0.14867	0.04131	0.74335	0.74335

Table 4. Efforts of arrival rate (Λ) on P_0, L_s, L_q, W_s, W_q.

Λ	P_0	L_s	L_q	W_s	W_q
10.00	0.86677	3.51562	3.72118	0.35156	0.37212
10.83	0.85696	3.90924	4.15360	0.36085	0.38341
11.67	0.85312	4.29162	4.57275	0.36785	0.39195
12.50	0.85025	4.66607	4.98224	0.37329	0.39858
13.33	0.84737	5.03473	5.38448	0.37760	0.40384
14.17	0.84348	5.39906	5.78114	0.38111	0.40808
15.00	0.84317	5.76007	6.17342	0.38400	0.41156

Table 5. Efforts of repair rate (ϑ) on P_0, L_s, L_q, W_s, W_q.

ϑ	P_0	L_s	L_q	W_s	W_q
20.00	0.86677	0.14490	0.03812	0.72448	0.19061
21.67	0.86696	0.14468	0.03789	0.72341	0.18944
23.33	0.86712	0.14450	0.03769	0.72250	0.18844
25.00	0.86725	0.14434	0.03752	0.72171	0.18758
26.67	0.86737	0.14420	0.03736	0.72102	0.18682
28.33	0.86748	0.14408	0.03723	0.72041	0.18615
30.00	0.86757	0.14397	0.03711	0.71987	0.18556

Table 6. Efforts at retrial rate (θ) on P_0, L_s, L_q, W_s, W_q.

θ	P_0	L_s	L_q	W_s	W_q
10.00	0.87956	0.13111	0.02293	0.65556	0.11467
10.83	0.88006	0.13057	0.02234	0.65286	0.11168
11.67	0.88049	0.13011	0.02182	0.65053	0.10912
12.50	0.88086	0.12970	0.02138	0.64852	0.10691
13.33	0.88119	0.12935	0.02099	0.64677	0.10497
14.17	0.88148	0.12904	0.02065	0.64521	0.10326
15.00	0.88173	0.12877	0.02035	0.64384	0.10174

rise, the average queue length initially falls for the values $\theta = 0.5$; $\mu_{sb} = 20$; $\mu_v = 5$; $\Lambda = 0.2$; $\alpha = 0.8$ and $\bar{\alpha} = 0.2$.

Fig. 4 shows how the FPS and repair rates affect P_0. It shows that the surface has a positive correlation when the repair rate and SPS rate go up compared to P_0 for the given parameters $\theta = 0.5$; $\mu_{sb} = 20$; $\mu_v = 5$; $\Lambda = 0.2$; $\alpha = 0.8$ and $\bar{\alpha} = 0.2$.

Fig. 5 illustrates the influence of retrial rate and repair rate on P_0. It demonstrates that the surface displays a positive trend, as expected when the retrial rate and repair rate rise around P_0 for the given parameters $\mu_b = 5$; $\mu_{sb} = 20$; $\mu_v = 5$; $\Lambda = 0.2$; $\alpha = 0.8$ and $\bar{\alpha} = 0.2$.

Fig. 6 illustrates the impact of FPS on P_0, L_s, L_q, and W_s. This figure analyzes the effects of increasing values of the FPS on the mean system and orbit size L_s, L_q as well as the mean system's waiting time W_s. The parameter values $\theta = 3$; $\mu_{sb} = 35$; $\mu_v = 2$; v = 10; $\Lambda = 0.2$; $\alpha = 0.8$ and $\bar{\alpha} = 0.2$ is considered within this analysis.

Fig. 7 illustrates how the repair rate (ϑ) affects P_0, L_s, L_q, and W_s. Fig. 7 shows an analysis that looks at how the mean system and orbit size L_s, L_q, and the mean system waiting time W_s change as the repair rate (Υ) rises. The parameter values $\theta = 3$; $\mu_b = 35$; $\mu_{sb} = 35$; $\mu_v = 5$; $\Lambda = 0.2$; $\alpha = 0.7$ and $\bar{\alpha} = 0.3$ are considered.

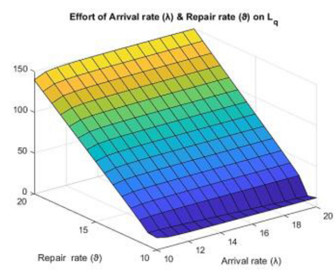

Fig. 2. Effort of arrival rate (Λ) & repair rate (ϑ) on Lq.

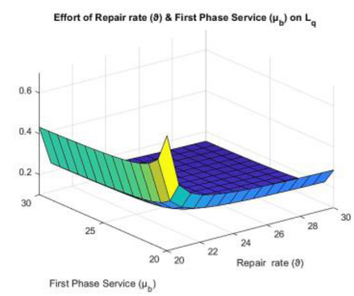

Fig. 3. Effort of FPS & repair rate (ϑ) on Lq.

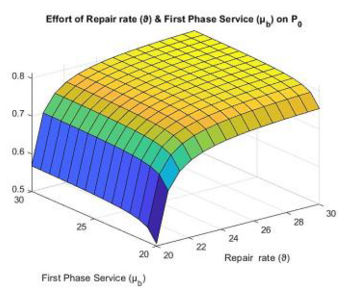

Fig. 4. Effort of repair rate (ϑ) & FPS on P_0.

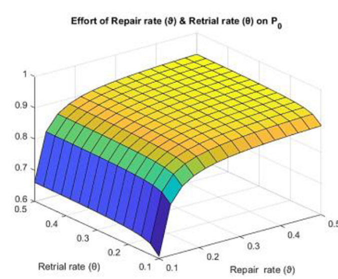

Fig. 5. Efforts at retrial rate (θ) & repair rate.

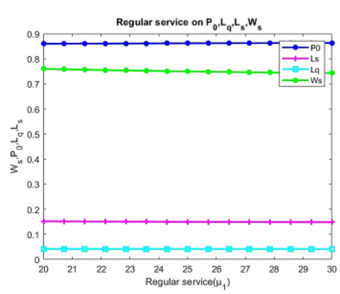

Fig. 6. Effort of Regular service (FPS) in P_0, L_s, L_q, W_s.

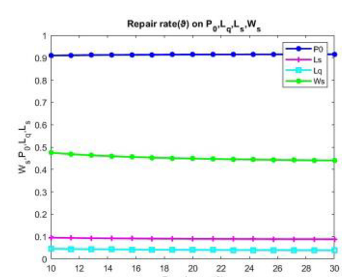

Fig. 7. Effort of Repair rate (ϑ) on P_0, L_s, L_q, W_s.

The methodology for sensitivity analysis

The examination of sensitivity reveals that the M/G/1 retrial queue model is very responsive to variations in its fundamental characteristics. These findings provide valuable insights into the factors influencing the system's performance, which can inform decision-making and optimization efforts.

Results

The sensitivity analysis produced the following significant findings:

Arrival rate: As anticipated, the system experiences additional overcrowding and longer waiting times as the arrival rates rise.

Service rate: Increasing the service rate reduces waiting times and increases system throughput.

Failure rate: Elevated failure rates lead to a greater number of retrial attempts and extended waiting periods, especially with low repair rates.

Repair rate: Accelerated repairs can greatly mitigate the consequences of failures on system performance.

Working vacation rate: While vacations can effectively distribute workload and mitigate server congestion, an excessive amount of vacation time can result in extended waiting times since service is given at a slow rate.

The result of the system's performance measures

Parameters such as arrival rate, service rate, and retrial rate have a significant impact on queue length and waiting time. Below is a summary of the expected results when it was altered with these crucial parameters:

Greater arrival rate: An increase in the number of arrivals might cause system congestion to rise, leading to longer queues and prolonged waiting periods. If the arrival rate exceeds the system's capacity, it has the potential to become dynamically unstable. These factors can result in a significant increase in queue length and waiting time, ultimately leading to customer dissatisfaction and a decline in business.

Reduced arrival rate: A decrease in the number of arrivals can help to alleviate congestion, resulting in shorter queues and more efficient waiting times. Nevertheless, the system may still encounter inefficiencies if the service rate is insufficient or if other parameters are not perfectly tuned. For instance, if the service rate stays high while the arrival rate decreases, the system might underutilize, leading to an inefficient use of resources.

Increasing the service rate: Increasing the service rate can alleviate congestion by reducing queues and minimizing waiting times. Adopting this approach can enhance customer happiness and optimize system performance. However, if the arrival rate continues at a high level, the system might still face challenges unless it significantly accelerates the service rate.

Reduction in service rate: The reduction in service rate might worsen congestion, resulting in extended queues and longer waiting periods. Instances of this nature might have adverse effects on consumer satisfaction and perhaps result in system instability. Under exceptional circumstances, an inundation may cause the system to fail or collapse.

Intensified Retrial Rate: An elevated retrial rate can amplify the burden on the system, resulting in extended lines and prolonged waiting periods. However, it can also enhance system efficiency by incentivizing loyal consumers to make yet another attempt. The ideal rate of retrial is determined by the system's explicit attributes and the intended level of customer service.

Reduced Retrial Rate: A lower retrial rate can decrease the load on the system, resulting in shorter queues and reduced waiting times. Nevertheless, the system's inability to meet demand could lead to a significant loss of clients. In cases where the system is already functioning close to its maximum capacity or when the cost of retrials exceeds the possible benefits, a lower retrial rate can be advantageous.

Researchers and practitioners can gain significant insights into the behavior of M/G/1 retrial queue systems and make educated decisions regarding system design, optimization, and performance enhancement by meticulously analyzing the interaction of these crucial elements.

Computational complexity

Our model, which includes Poisson arrivals, initial failures, repair procedures, two-phase service mechanisms, and working vacations, does involve intricate mathematical structures, particularly when examining performance metrics such as queue length and waiting time. Trials and implementing the service in two stages during working vacations make things more complicated because you have to keep an eye on and figure out a lot of different random events that are all connected.

As the system's magnitude increases, characterized by higher arrival rates, larger retry populations, or more frequent failures, the computational load can see significant growth. This statement is especially true when attempting to solve the model analytically, since the state space can grow significantly large, posing difficulties in discovering precise answers.

Although our M/G/1 retrial queue model presents computational complexity issues for large-scale systems, there are numerous approaches to overcome these obstacles. These include employing approximation methods, numerical simulations, and parallel computing. Through meticulous evaluation of these methodologies, it becomes viable to implement the model and get solutions for extensive applications, guaranteeing its realism and practicality in real-life situations.

Conclusion

This paper studied the M/G (P1, P2)/1 retrial two-phase service queueing system, which involves starting failures and working vacations due to server maintenance issues. The PGF and SVs methodology was used to determine the PGF for the system's customer numbers and orbital customer numbers.

The goal of this project is to integrate these advanced queuing mechanisms into customer support operations to improve their efficiency and dependability. The research aids in the development of strategies for lowering response times, eliminating service disruptions, and optimizing resource utilization in customer support operations by examining the effects of various factors, such as starting failures, repair strategies, and working vacations, on the system's overall efficiency. The analytical findings were validated with numerical examples. Additionally, it finds application across numerous sectors. For example, in flexible manufacturing systems, there are multipurpose, adaptable machines that are capable of doing a variety of tasks, such as drilling, milling, and lathing. This paradigm has potential use in the Simple Mail Transfer Protocol mail system, which transmits packets among email hosts as well as wired connections to choose routes from the routing table.

Although the M/G/1 retrial queue paradigm provides a robust foundation for handling intricate systems, its implementation in a practical setting necessitates meticulous design, adaptation, and continuous evaluation. By effectively tackling the probable obstacles and consistently improving the model using actual data, it is feasible to develop a dependable and effective system that perfectly suits the requirements of a constantly changing environment.

Limitations: Although our M/G/1 retrial queue model includes Poisson arrivals, starting failure, repair, two-phase service, and working vacation, it relies on some simplifying assumptions that limit its usefulness. Consider, for example, the assumption that the arrival process is Poisson, that the service times follow a common distribution, and that the retry times follow an exponential distribution. The Poisson arrival assumption might not fully account for certain real-world situations, especially those involving bursty or linked arrivals. Even though exponential service times are easy to work with in math, they might not be able to fully capture the range of service times in some situations, especially when the distributions are heavy-tailed. The model limits its scope to a single server, potentially ignoring the complexities of multi-server systems that integrate load balancing and queue prioritization. In addition, the model assumes that the system works with a two-phase service mechanism when people are on vacation. This may not fully reflect the complexity of real systems, where service dynamics can be more varied and unpredictable.

Future Directions: Potential possibilities for further investigation may include the relaxation of some assumptions to enhance the model's applicability to a broader spectrum of businesses. There was a plan to achieve a more accurate representation of telecommunications, manufacturing, or healthcare systems by considering non-Poisson arrival processes or alternative distributions for retry times.

Also, exploring alternative arrival processes, like batch arrivals or modified Poisson processes, to identify more intricate arrival patterns. This study aims to investigate the influence of non-exponential service distributions, namely phase-type or heavy-tailed distributions, on system performance. Also, the model will be extended to encompass multiple servers, considering various server allocation methods and load-balancing strategies. Additionally, the authors intend to look at problems such as retrial queues with orbital search, impatience, priority customers, intermissions, setup time, catastrophes, mending, and cost analytics for enhancing this system shortly.

Also, the concept of extending a broader spectrum of industries and applications, such as healthcare, transportation, and finance, to address the unique obstacles they present.

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Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the figures and tables in the manuscript are ours. Furthermore, any figures and images that are not ours have been included with the necessary permission for re-publication, which is attached to the manuscript.

- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at Vellore Institute of Technology, India.

Authors' contribution statement

M.C.S created the suggested concept. The calculations, besides theoretical development, were done by S.B. The analytical techniques were confirmed by S.B. was advised to look into [a certain element] by M.C.S. The results of this work were overseen by M.C.S., and S.B. was the main author of the manuscript. The software program was created by SB. M.C.S verified the numerical results for the suggested experiment of the S.B. by an independent implementation. M.C.S. and S.B. offered constructive criticism and assisted in shaping the research, analysis, and writing.

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الوصول إلى الخدمة على مرحلتين بجانب طوابير إعادة المحاكمة، وآلية المدار، وسياسات إجازة العمل لبيئات الخادم الفردي

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الخلاصة

تحلل هذه الورقة قائمة انتظار M/G (P1, P2)/1 ،1/(P2 باستخدام خادم واحد يوفر الخدمات الأساسية في كل من المرحلة الأولى (FPS) والمرحلة الثانية (SPS). كان من المفترض أن الخادم يتعهد بعدم الموثوقية بسبب فشل البدء (SF) خلال بداية المرحلة الأولى مع احتمال (α) أو تبدأ الخدمة عن طريق الخادم العاطل بنجاح مع احتمال (1- α) للعميل القادم أو إعادة المحاكمة. أخذنا الخادم الفاشل على الفور للإصلاح وأطلقنا العميل في المدار. أيضًا، يعمل الخادم بأسعار متفاوتة بدلاً من التوقف التام عن الخدمة أثناء إجازته. بعد العودة من الإجازة، يستأنف الخادم العمليات العادية للعملاء. ناقشنا النظام وحجم المدار والمقاييس المهمة الأخرى، باستخدام تقنية المتغير التكميلي (SVT) ودالة توليد الاحتمالات (PGF). نقوم ببناء تطبيقات لنظام إعادة المحاكمة هذا الذي يتضمن النموذج وقانون التحلل العام. نجري أيضًا تحليلًا رقميًا لفحص تأثير العوامل المختلفة على كفاءة النظام. استخدمنا برنامج MATLAB لإنشاء العديد من التأثيرات على المقاييس السلوكية للنظام.

الكلمات المفتاحية: المراحل المزدوجة، مقاييس الأداء، طوابير إعادة المحاكمة (RQ)، فشل البدء، إجازة العمل (WV).