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RESEARCH ARTICLE

Effect of Surface Stress on Love Wave Propagation in a Rotating Initially Stressed Orthotropic Elastic Solid Half-Space with Impedance Boundary Conditions

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ABSTRACT

This article aims to investigate the effect of material surface stress on the propagation of Love waves in a rotating, homogeneous, initially stressed orthotropic elastic solid with impedance-type boundary conditions. For deriving Love waves, the basic equations are solved with the help of the traditional wave solution method. Secular equations about Love waves are computed as a function of surface stress, rotation, and initial stress. The effects of initial compression and surface stress have been discussed numerically using MATLAB for a particular model.

Keywords: Impedance boundary conditions, Initial stresses, Love waves, Orthotropic elastic solid, Surface stresses

Introduction

A material boundary is a boundary that supports surface stresses. The surface tension in liquids is treated as a surface stress. Surface stresses on the boundaries of the body have been detected in some special types of crystals where the order of magnitude agrees with the predictions made by the “molecular theory” of Gurtin and Murdoch.¹ Rayleigh-type surface wave frequencies at the interface of viscous liquid and micropolar micro-stretch elastic solids are derived by Somaiah.² The effect of material surface stress on the propagation of waves at different angles is studied by Abd-Alla³ and Jihyun et al.⁴ The roughness effect on non-linear Rayleigh wave propagation was studied by Chaitanya et al.⁵ The knowledge on wave propagation in elastic materials having non-parallel boundaries was developed by the seismic behaviour at the roots of mountains

and at the margins of continentals. Crustal actuation switching by crystal thickness was investigated by Shodai Hasebe et al.⁶ Shayakhmetov⁷ studied the Rayleigh and Love waves about the seismic stress state of the earth bed. The initial stress and gravity effects on Love wave propagation are studied on the different types of boundaries by Gupta et al.⁸ Uma Bharati⁹ explained the effects of rigid and soft mountain surfaces on the phase velocities of Love waves. Sharma et al.¹⁰ derived two types of waves namely: quasi-longitudinal and transverse waves by using the theories of Green-Naghdi Type-II and Type-III in an anisotropic thermoelastic medium. Sharma and Bhargava¹¹ investigated the plane wave propagation, and they derived two coupled LD waves, LM waves, and CD-I, CD-II waves. The phase velocities, attenuation coefficients, specific loss, and penetration depth are computed analytically and numerically. Saurav Sharma et al.¹² derived a novel mathematical

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formulation of temperature dependent thermoelastic diffusion with multi-phase delays (TDMT). In this study, they calculated wave properties analytically and visually.

Imperfect earth surfaces play a very important role in maintaining the environmental requirements to study the Love wave propagation impact. From the available literature, it is noticed that the mountains have soft as well as rigid surfaces. The earth has a spherical shape with composite materials, irregular surfaces, and high initial stresses. Love waves are a horizontally polarized type of surface wave, and they also propagate in the finite thickness layers, which are overlying half-space. Most Love-type waves are caused by building damage from an earthquake. The initial compression is the hydrostatic stress that is generated in the interior of earth layers due to high temperatures, pressure, and gravitating pulls. The speed of propagation of Love waves in the medium is affected by initial stress. Singh,¹³ studied the Rayleigh type surface wave propagation in orthotropic elastic material with two temperature contexts of thermoelasticity. Rayleigh wave propagation is considered in a transversely isotropic, thermo-elastic medium by Kaur¹⁴ and the effects of three-phase heat transfer and another type of heat transfer are derived. V. T. N. Anh¹⁵ investigated the effects of non-locality and incompressibility on Rayleigh type surface waves by considering impedance type boundary conditions. The effects of non-locality on waves were also studied by Baljeet Singh¹⁶ by considering Rayleigh-type surface waves in a homogeneous micropolar piezoelectric media. The impedance type boundary conditions are considered by Santanu Manna¹⁷ to investigate the reflection phenomena of plane waves at the free boundary of the surface.

A particular type of mixed boundary condition is named the impedance boundary condition, which is also a linear combination of unknown functions and their derivatives. These types of boundary conditions are applicable in different fields like physics, acoustics, electromagnetism, and seismology. The investigations of surface waves under impedance boundary conditions are very limited in the literature. The effects of rotation and initial compression on the propagation of Love waves in an orthotropic elastic solid half-space under impedance boundary conditions were studied in recent years by Somaiah.¹⁸ Maan and Suad Naji Kadhim¹⁹ considered homogeneous Dirichlet boundary conditions to derive the Crank-Nicolson finite difference equation. A numerical scheme is developed by Hussain et al.²⁰ for solving a boundary value problem involving singular perturbation. Kumar²¹ studied the effect of impedance

boundary parameters on the propagation of Raleigh surface waves in a micropolar thermoelastic half-space and they compared the results with the theory of classical elasticity.

Numerous authors have investigated the effects of initial stress on SH-type Love wave propagation in different elastic media by applying impedance boundary conditions. But in this article, the impedance boundary conditions are considered to study the effect of “material surface stress” by simultaneously including the effects of initial stress and angular rotation of the solid. This article is motivated from the studies of authors Anh et al.¹⁵ and Somaiah¹⁸ to investigate the effect of material surface stress.

Formulation

For SH-type Love wave propagation along x -axis in a homogeneous orthotropic elastic solid of finite thickness $2L$ under an initial stress P as shown in the Fig. 1. Let us consider any point on the plane surface as the origin of the coordinate system (x, y, z) . Choose the z -axis as pointing vertically downwards into the half-space. Let the half-space $z = 0$ be an impedance boundary plane and $z > 0$ and $z < 0$ be stress free surface and material surface planes, respectively. After choosing that the continuum is rotating about z -axis with the angular rotation speed Ω , one can define the angular rotation vector $\vec{\Omega}$ as $\vec{\Omega} = (0, 0, \Omega)$ with Centripetal acceleration $\vec{\Omega} \times (\vec{\Omega} \times \vec{u})$ and Coriolis acceleration $2(\vec{\Omega} \times \vec{u})$, where $\vec{u}$ is the macro-displacement vector of the solid.

The surface stress tensor $\Sigma_{i\alpha}$ given by Gurtin and Murdoch¹ law is as follows:

$$\left. \begin{aligned} \Sigma_{i\alpha} &= [\sigma + (\lambda_o + \sigma) u_{r,r}] \delta_{i\alpha} + \mu_o u_{i,\alpha} \\ &\quad + (\mu_o - \sigma) u_{\alpha,i}; \text{ for } 1 \leq i, \alpha, r \leq 2 \\ \Sigma_{i\alpha} &= \sigma u_{3,\alpha}; \text{ for } i = 3 \end{aligned} \right\} \quad (1)$$

When the boundary (the material surface) is free of external loads, the balance of linear momentum is in the form

$$\Sigma_{i\alpha,\alpha} + \tau_{i3} = \rho_0 \ddot{u}_i \text{ on } z = 0 \quad (2)$$

Where λ_o , μ_o are Lamé's parameters of the surface material, ρ_0 is mass per unit surface and σ represents the residual surface tension of material boundary.

Dutta²² formulated the equations of motion of an initially stressed with initial compression P , and rotating orthotropic elastic solid as follows:

$$\begin{aligned} \tau_{11,1} + \tau_{12,2} + \tau_{13,3} + P [\phi_{12,2} - \phi_{13,3}] \\ = \rho [\ddot{u} - \Omega^2 u - 2\Omega v] \end{aligned} \quad (3)$$

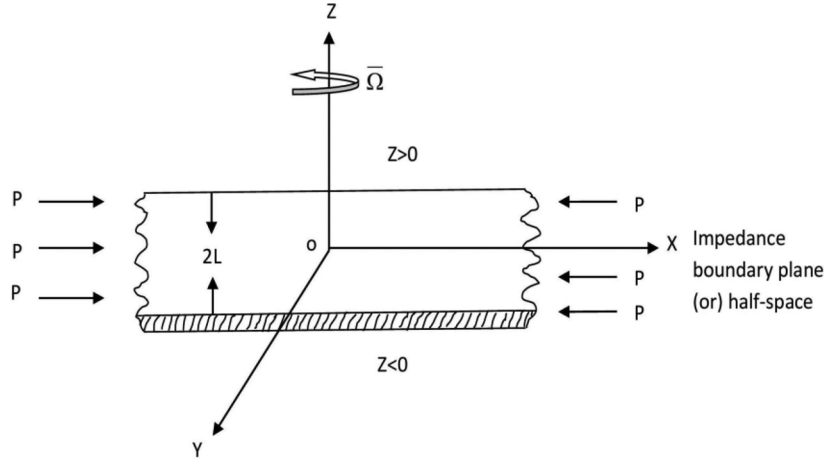


Fig. 1. Model of the problem.

$$\tau_{12,1} + \tau_{22,2} + \tau_{23,3} - P\phi_{12,1} = \rho [\ddot{v} - \Omega^2 v + 2\Omega \dot{u}] \quad (4)$$

$$\tau_{33,3} + \tau_{31,1} + \tau_{32,2} - P\phi_{13,1} = \rho \ddot{w} \quad (5)$$

where $\vec{u} = (u, v, w)$ represents the macro displacement vector, and the mass density of the solid is represented by ρ . The micro-rotational vector components in x, y, z directions are respectively given by ϕ_1, ϕ_2, ϕ_3 with $\phi_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)$; $i, j = 1, 2, 3$. The partial derivative with respect to corresponding coordinate axes represents “followed by a comma”. The “superposed dot” indicates the partial derivative with respect to time variable t .

The stresses τ_{ij} ; $1 \leq i, j \leq 3$ are given by

$$\tau_{11} = (c_{11} + P) \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} + c_{32} \frac{\partial w}{\partial z} \quad (6)$$

$$\tau_{22} = c_{12} \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} + c_{32} \frac{\partial w}{\partial z} \quad (7)$$

$$\tau_{33} = c_{13} \frac{\partial u}{\partial x} + c_{23} \frac{\partial v}{\partial y} + c_{33} \frac{\partial w}{\partial z} \quad (8)$$

$$\tau_{12} = c_{44} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (9)$$

$$\tau_{13} = c_{55} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \quad (10)$$

$$\tau_{23} = c_{66} \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \quad (11)$$

The stiffness tensor components c_{ij} ; $1 \leq i, j \leq 6$ in the contraction notation are given by

$$\begin{aligned} c_{44} &= \frac{1}{2} (c_{11} - c_{12}); \quad c_{55} = \frac{1}{2} (c_{11} - c_{13}); \\ c_{66} &= \frac{1}{2} (c_{22} - c_{23}) \end{aligned} \quad (12)$$

Love wave propagation

Because of Love waves are SH-type waves, the components (u, v, w) of displacements in xz -plane, along x -direction are taken in the form $v = v(x, z, t)$; $u = w = c_{11} = c_{33} = c_{13} = 0$; $\frac{\partial}{\partial y} \equiv 0$.

From the above assumptions and from Eqs. (6) to (12), the Eqs. (3) to (5) reduces to

$$(P - 2c_{12})v_{,11} + 2(c_{22} - c_{23})v_{,33} = 4\rho [\ddot{v} - \Omega^2 v] \quad (13)$$

with the method of plane harmonic wave solution, the solution of Eq. (13) is of the form

$$v(x, z, t) = V(z) \exp[ik(x - ct)] \quad (14)$$

Where k is wave number, $V(z)$ is an amplitude decay and ω is circular frequency related with wave velocity c as $c = \omega k$.

On using Eq. (14) in Eq. (13), it obtain

$$(D^2 - \eta^2)V = 0 \quad (15)$$

where

$$D^2 \equiv \frac{d^2}{dz^2} \quad \text{and} \quad \eta^2 = \frac{k^2 (P - 2c_{12} - 4\rho c^2) - 4\rho \Omega^2}{2(c_{22} - c_{23})} \quad (16)$$

The solution of Eq. (15) is given by

$$V(z) = A \cosh(\eta z) + B \sinh(\eta z) \quad (17)$$

From Eq. (14), the displacement component $v(x, z, t)$ reduces to

$$v(x, z, t) = [A \cosh(\eta z) + B \sinh(\eta z)] \exp[ik(x - ct)] \quad (18)$$

Derivation of secular equations

Displacements and stresses of the impedance boundary conditions are given Malischewsky²³ as under $\tau_{j2} + \varepsilon_j u_j = 0$, for $z = 0$, where impedance parameters ε_j having dimensions “stress/length”. For elastic half-space Goday²⁴ formulated ε_j as $\varepsilon_j = \omega Z_j$, with Z_j are impedance real-valued parameters having dimensions “stress/velocity”. Circular frequency ω , wave number k and phase velocity c are related as $\omega = ck$. The impedance boundary conditions stated at the surface $z = 0$ as

$$\tau_{i3} + \omega Z_i u_i = 0 \quad \text{at } z = 0; \quad 1 \leq i \leq 3 \quad (19)$$

For Love wave propagation, Eq. (19) and Eq. (2) can be expressed as: Impedance boundary conditions:

$$\tau_{23} + \omega Z_2 v = 0 \quad \text{at } z = 0 \quad (20)$$

Material surface stress boundary conditions:

$$\Sigma_{21,1} + \tau_{23} = \rho \ddot{v} \quad \text{at } z = -L \quad (21)$$

After using Eqs. (11) and (17), the Eqs. (19) and (20) becomes

$$\left. \begin{aligned} 2\omega Z_2 A + a\eta B &= 0 \\ [-a\eta \sinh(\eta L) + b \cosh(\eta L)] A \\ &+ [a\eta \cosh(\eta L) - b \sinh(\eta L)] B = 0 \end{aligned} \right\} \quad (22)$$

where $a = c_{22} - c_{23}$ and $b = 2(\rho_0 \omega^2 - \mu_0 k^2)$ After eliminating A and B , the Eq. (22) reduces

$$\tan h(\eta L) = \frac{a\eta (2\omega Z_2 - b)}{2\omega Z_2 b - (a\eta)^2} \quad (23)$$

Eq. (23) is the secular equation of Love wave propagation in an initially stressed, rotating orthotropic elastic solid half-space with the effect of surface stress of material surface and subject to the impedance boundary conditions. When material surface stress is neglected, the results coincide with the initial stress, rotation, and impedance parameter-dependent dispersion relations of research article.¹⁸

Special cases

- (i) When initial compression is absent (i.e., $P = 0$), the Eq. (23) becomes

$$\tan h(\zeta L) = \frac{a\zeta (2\omega Z_2 - b)}{2b\omega Z_2 - (a\zeta)^2} \quad (24)$$

where

$$\zeta^2 = \frac{k^2 c_{12} + 2\rho (\omega^2 + \Omega^2)}{c_{23} - c_{22}} \quad (25)$$

Eq. (24) is a surface stress dependent secular equation of Love wave propagation in a rotating orthotropic elastic solid half-space with the effect of surface stress and subject to impedance boundary conditions. The outcomes are consistent with those presented in research paper,¹⁸ when there is no material surface.

- (ii) When surface stress is absent (i.e., $\rho_0 = 0$, $\mu_0 = 0$ or $b = 0$), Eq. (23) reduces

$$\tan h(\eta L) = \frac{2\omega Z_2}{\eta (c_{23} - c_{22})} \quad (26)$$

known as the frequency equation of Love waves in a rotating, initially compressed, orthotropic elastic solid half-space subject to impedance-type boundary conditions. These are well-known Eq. (17) results, as shown in research paper,¹⁸ after minor adjustments.

- (iii) When surface stress and initial stress are absent (i.e., $b = 0$, $P = 0$), Eq. (24) becomes

$$\tan h(\zeta L) = \frac{2\omega Z_2}{\zeta (c_{23} - c_{22})} \quad (27)$$

known as the frequency equation of Love waves in a rotating orthotropic elastic solid half-space subject to the impedance type boundary conditions. Following minor adjustments, these are the widely recognized outcomes of Eq. (18) as given in research article.¹⁸

- (iv) Under the absence of impedance boundary (i.e., $Z_2 = 0$), Eq. (23) reduces

$$\tan h(\eta L) = \frac{b}{a\eta} \quad \text{or} \quad \tan h(\eta L) = \frac{2(\rho_0 \omega^2 - \mu_0 k^2)}{\eta (c_{22} - c_{23})} \quad (28)$$

which is known as the Love wave dispersion relation in an initially stressed and rotating orthotropic elastic solid half-space with the effect of surface stress. These are the well-known outcomes

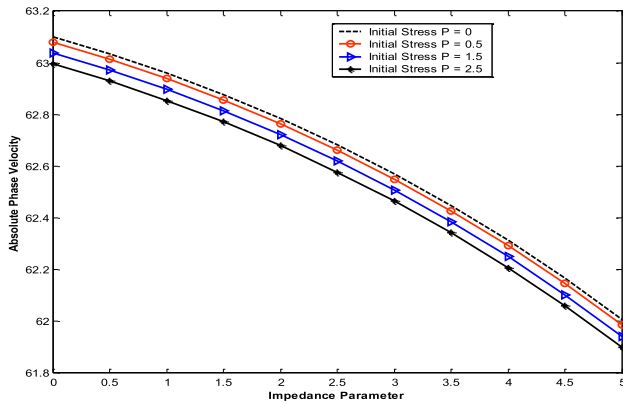


Fig. 2. Phase velocity vs. Impedance Parameter Z_2 for different initial stresses.

of Eq. (20), as shown in research paper,¹⁸ after some adjustments.

Results and discussion

For a detailed study of the effects of the impedance type boundary, the speed of angular rotation, the surface stress of the material surface, and initial compression on the behavior of Love waves in the type of orthotropic elastic solid, adopt the material values of the orthotropic medium as follows:

$$c_{12} = 0.661; c_{22} = 2.363; c_{23} = 2.694$$

and density $\rho = 6$.

Gurtin¹ presented the physical parameters of the surface medium of iron film as:

$$\sigma = 1.7 \text{ N/m}; \mu_0 = 2.5 \text{ N/m}; \lambda_0 = -8 \text{ N/m};$$

$$\rho_0 = 7 \times 10^{-4} \text{ kg/m}^3.$$

At time $T = 30$ sec, wave length $l = 10$ m, angular wave number k taken as $k = \frac{2\pi}{l} = \frac{2\pi}{10}$, angular frequency ω taken as $\omega = \frac{2\pi}{T} = \frac{2\pi}{30}$ and thickness of the solid L taken as $L = 0.1$ m.

Phase velocity profiles of Love waves versus the impedance boundary parameter Z_2 with $0 \leq Z_2 \leq 5$ are shown in Figs. 2 to 7. In Fig. 2 the effects of initial compressions are explained, Wave phase velocities are decreasing in the given range of Z_2 and they are inversely proportional to initial compression.

The effects of angular rotations of the solid on Love waves are presented in Fig. 3. From this figure, it is noticed that the phase velocity increases as angular rotation of the solid decreases in the range of Z_2 with $2 \leq Z_2 \leq 5$ and constant at $Z_2 = 1$.

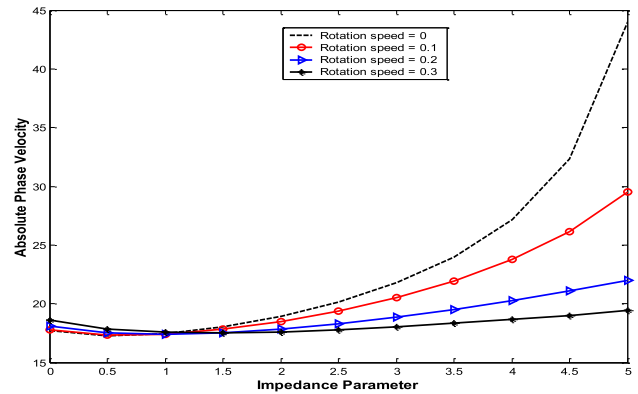


Fig. 3. Phase velocity vs. Impedance Parameter Z_2 for different angular rotation speed.

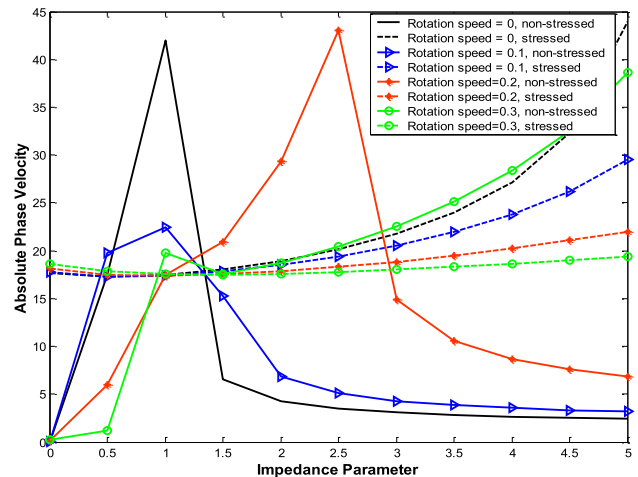


Fig. 4. Phase velocity vs. Impedance Parameter Z_2 for different angular rotation speed in surface stressed and non-stressed solid.

The phase velocity curves for different angular rotations in material surface and non-material surface boundaries are presented in Fig. 4. From this figure, it is observed that the phase velocities of Love waves in material surfaces are faster than in non-material surfaces in the range of the impedance parameter Z_2 with $3 \leq Z_2 \leq 5$.

From Fig. 5, it is noticed that phase velocities in rotating, initially compressed material surfaces are faster than in rotating, initially compressed non-material surfaces in the range of Z_2 with $2 \leq Z_2 \leq 5$.

From Fig. 6, it is observed that the phase velocities in rotating, non-compressed material surface are faster than a rotating, non-compressed non-material surface in the given range of Z_2 with $2 \leq Z_2 \leq 5$.

From Fig. 7, observed that the phase velocities in non-rotating, initially compressed material surfaces are faster than in non-rotating, initially compressed, non-material surfaces in the range of Z_2 with $1.5 \leq Z_2 \leq 5$.

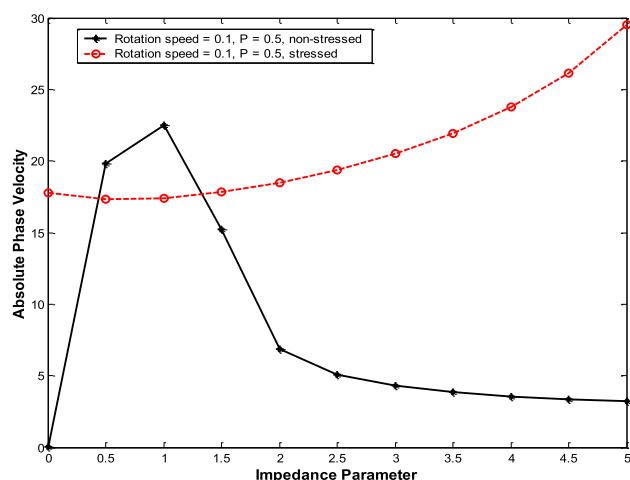


Fig. 5. Phase velocity vs. Impedance Parameter Z_2 for $\Omega = 0.1$, $P = 0.5$ in surface stressed and non-stressed solid.

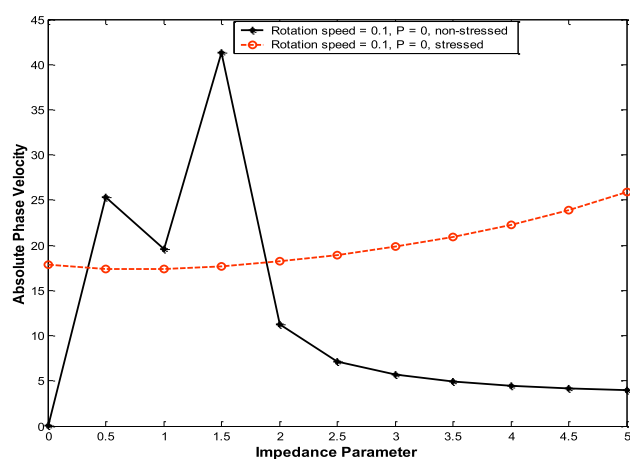


Fig. 6. Phase velocity vs. Impedance Parameter Z_2 for $\Omega = 0.1$, $P = 0$ in surface stressed and non-stressed solid.

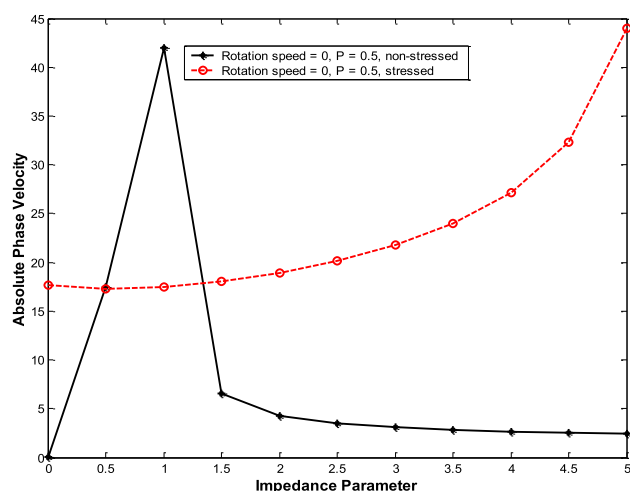


Fig. 7. Phase velocity vs. Impedance Parameter Z_2 for $\Omega = 0$, $P = 0.5$ in surface stressed and non-stressed solid.

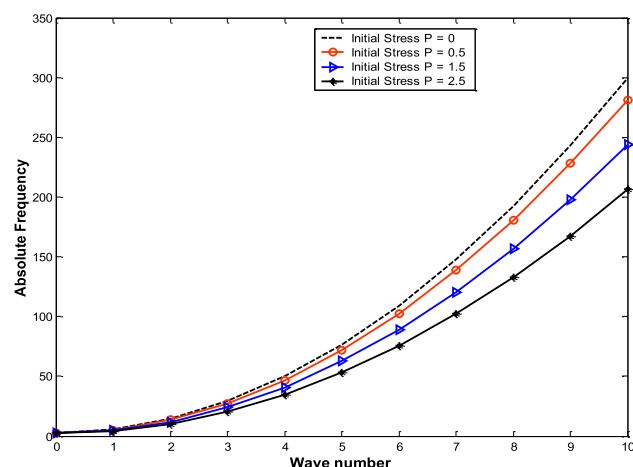


Fig. 8. Absolute frequency vs. Wave number for different initial stresses.

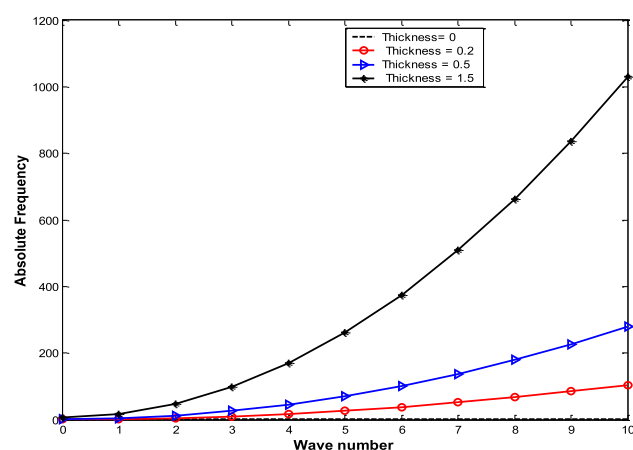


Fig. 9. Absolute frequency vs. Wave number for different thickness.

Frequency profiles of Love waves versus wave number are shown in Figs. 8 and 9. From Fig. 8, it is observed that frequencies are increasing in the given range of wave numbers and these frequencies are inversely proportional to the initial compression.

From Fig. 9, it is noticed that the frequencies of Love waves are increasing with the increasing thickness L of the solid.

Conclusion

The basic governing equations of initially compressed and rotating orthotropic elastic material half-space are solved by using the method of plane harmonic solution for Love wave propagation under the impedance boundary conditions. The effects of surface stress on the material surface and the initial compression of orthotropic elastic solids are discussed theoretically and numerically. From theoretical

illustrations and a particular numerical example, it is observed that:

1. Material surface stress and impedance parameter dependent dispersion relations have been derived for Love waves in an orthotropic elastic solid.
2. The phase velocities of the Love wave are increasing with the increasing angular rotation and initial compression of the solid.
3. The phase velocities of the Love wave have been observed to be increasing in a rotating solid and decreasing in a non-rotating orthotropic elastic solid.
4. Love wave phase velocities in initially compressed material surfaces are faster than in initially compressed non-material surfaces in the range of Z_2 with $2 \leq Z_2 \leq 5$.
5. Love wave frequencies are increasing with the increasing thickness of the solid.

The present investigation can be useful for researchers in the fields of seismology, geophysics, and engineering for their experimental and theoretical investigations on Rayleigh and Stoneley type surface waves in half spaces and multilayered elastic media. The consideration of the impedance type boundary, angular rotation, and initial compression of this theoretical illustration with application is nearly closer to reality.

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Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures and Tables in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for republication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at the Kakatiya University.

Authors' contribution statement

S.K., V.R.B and S.R., contributed to the research design and implementation, S.K. and V.R.B, analysis

of the results and visualization, S.K. and S.R., revision of the results and proof reading. All authors have read and agreed to the published version of the manuscript.

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تأثير الإجهاد السطحي على انتشار موجة الحب في نصف فضاء صلب مرن متعامد الشكل مجهد في البداية مع شروط حدود المعاوقة

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الخلاصة

تهدف هذه المقالة إلى دراسة تأثير إجهاد سطح المادة على انتشار موجات لوف في مادة صلبة مرنة متجانسة دوارة ومجهد في البداية مع ظروف حدودية من نوع المعاوقة. لاستخلاص موجات لوف، يتم حل المعادلات الأساسية بمساعدة طريقة حل الموجة التقليدية. يتم حساب المعادلات العلمانية حول موجات لوف كدالة لإجهاد السطح والدوران والإجهاد الأولي. تمت مناقشة تأثيرات الضغط الأولي والإجهاد السطحي عددياً باستخدام MATLAB لنموذج معين.

الكلمات المفتاحية: شروط حدود المعاوقة، الإجهادات الأولية، موجات الحب، المواد الصلبة المرنة المتعامدة، الإجهادات السطحية.