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Derivation of a Novel Conjugate Gradient Parameter and Hybridization of Hummingbird AHA and Tunicate Swarm TSA Algorithms with the Conjugate Gradient Method for Enhanced Optimization Efficiency

Salih Awad Khudhur[1], Ban Ahmed Mitras [2]
Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Iraq
salih.23csp151@student.uomosul.edu.iq
banah.mitras@uomosul.edu.iq
+09647709518220

Abstract:

Every living creature on Earth has specific behaviors during its search for food. These methods vary from one organism to another. Researchers have sought to mathematically model these methods and use them to solve some of the complex problems that traditional methods have failed to solve. Since each algorithm has its strengths and weaknesses, and no algorithm is capable of solving all problems, researchers have turned to finding more efficient hybrid algorithms. In this paper, we derived a new conjugate parameter for the conjugate gradient method and combined it with both algorithms to improve the results. We also hybridized the AHA algorithm with TSA using a novel approach based on arranging the populations in each iteration in ascending order according to the objective function values, with the first half of the population optimization allocated to one algorithm and the other half to the other based on the strength of exploration and exploitation. The results showed that the hybrid algorithm outperformed both the original algorithms, as well as the one improved by the conjugate gradient method, on most test functions.

Keywords: Conjugate gradient method, Tunicate Swarm Algorithm, Hummingbird algorithm.

اشتقاق معامل جديد لطريقة التدرج المترافق وتهجين خوارزميتي الطائر الطنان AHA وسرب التونيكيت TSA مع طريقة التدرج المترافق لتعزيز كفاءة الامثلية

صالح عواد خضر [1] ، بان احمد متراس [2]
قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة الموصل، العراق
salih.23csp151@student.uomosul.edu.iq
banah.mitras@uomosul.edu.iq
+09647709518220

المستخلص

لكل كائن حي على الأرض سلوكيات محددة أثناء بحثه عن الطعام. وتختلف هذه الطرق من كائن حي لآخر. وقد سعى الباحثون إلى نمذجة هذه الطرق رياضيًا واستخدامها لحل بعض المشكلات المعقدة التي فشلت الطرق التقليدية في حلها. ونظرًا لأن لكل خوارزمية نقاط قوة ونقاط ضعف، ولا توجد خوارزمية

قادرة على حل جميع المشكلات، فقد لجأ الباحثون إلى إيجاد خوارزميات هجينة أكثر كفاءة. في هذه الورقة البحثية، اشتققنا معاملًا مترافقًا جديدًا لطريقة التدرج المترافق ودمجناه مع كلتا الخوارزميتين لتحسين النتائج. كما قمنا بتهجين خوارزمية AHA مع خوارزمية TSA باستخدام نهج جديد يعتمد على ترتيب المجموعات السكانية في كل تكرار بترتيب تصاعدي وفقًا لقيم دالة الهدف، مع تخصيص النصف الأول من تحسين المجموعة السكانية لخوارزمية واحدة والنصف الآخر للأخرى بناءً على قوة الاستكشاف والاستغلال. أظهرت النتائج أن الخوارزمية الهجينة تفوقت على كل من الخوارزميتين الأصلية، وكذلك على الخوارزمية المُحسّنة بطريقة التدرج المترافق، في معظم دوال الاختبار.

الكلمات المفتاحية: طريقة التدرج المترافق، خوارزمية السرب TSA، خوارزمية الطائر الطنانAHA

1. Introduction

Recently, researchers have become increasingly interested in intelligent search and optimization methods, due to the tremendous development of applied sciences and the emergence of more complex problems in various fields, such as economics, energy, artificial intelligence, and others. These algorithms provide solutions in a wide search range., in addition to their speed and accuracy, especially in complex problems that traditional methods are unable to solve (Abdullah & Mitras, 2025; Yahia et al., 2021). Meta-heuristic algorithms have been on the minds of researchers and developers, as they are an important type capable of handling difficult and complex problems. Studies have addressed the use of these algorithms to solve life-related problems in many areas, including determining important parameters for solar cells (El-Sehiemy et al., 2023). and improving electrical power systems (Sarhana et al., 2023). And finding the parameters of the state of charge in batteries used in electric cars using the artificial hummingbird algorithm (AHA) (Hamida et al., 2022) . And many other fields in design, machine learning, artificial intelligence, economics and energy. Researchers have also hybridized many of these algorithms with each other and with other traditional algorithms to achieve a balance in the search methods for the new hybrid algorithm, such as exploration and exploitation skills. An example of hybridization is the hybridization of the Harris hawk optimization algorithm reinforced with opposition learning (HHOA-OBL) (Ismael et al., 2020) . And the sand cat swarm algorithm (SCSO) with the artificial rabbit algorithm (ARO) (Shalal & Mitras, 2024) , And linking the (AHA) algorithm with the (K-means) algorithm[8], Some researchers have also turned to combining traditional methods such as conjugate gradient with intelligent algorithms by deriving new conjugate coefficients (Hestenes & Stiefel, 1952).

2.Derivation of a new parameter for the conjugate gradient technique.

In 2024, Ibrahim & Salihu published a paper in which they proposed a standard formula for the conjugate gradient vector as follows:

$$d_{k+1}^{AMIL} = -\theta_{k+1} g_{k+1} + \beta_k^{AMIL+} d_k \qquad (1)$$

Whereas $\beta_k^{AMIL+} = \frac{\|g_{k+1}\|^2}{\|d_{k+1}\|^2}$

θ_k was defined by them for several cases, including:

$$\theta_k^1 = \rho_1 + \frac{\|y_k\|^2}{\|d_k\|} \quad , \quad \theta_k^2 = \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|} \quad , \quad \theta_k^3 = \rho_3 + \frac{\|y_k\|}{\|d_k\|} + 1$$

Where they considered $\rho_1 \ , \ \rho_2 \ , \ \rho_3 > 0$

We have the conjugate gradient vector DY as follows (Yabe & Sakaiwa, 2005) :

$$d_{k+1} = -g_{k+1} + \beta_K^{Dy} d_k \qquad s.t \qquad DY = \frac{\|g_{k+1}\|^2}{y_k^T d_k} \qquad (2)$$

We equate equations (1) and (2) as follows:

$$d_{k+1}^{DY} = d_{k+1}^{AMIL}$$

$-g_{k+1} + \beta_K^{DY} d_k = -\theta_{k+1} g_{k+1} + \beta_k^{AMIL+} d_k$ We multiply both sides by y_k^T

$$-y_k^T g_{k+1} + \beta_K^{DY} y_k^T d_k = -\theta_{k+1} y_k^T g_{k+1} + \beta_k^{AMIL+} y_k^T d_k$$

$$-y_k^T g_{k+1} + \frac{\|g_{k+1}\|^2}{y_k^T d_k} y_k^T d_k = -\theta_{k+1} y_k^T g_{k+1} + \beta_k^{AMIL+} y_k^T d_k$$

$\|g_{k+1}\|^2 = \beta\tau$ (Dai & Liao, 2001), Substitute for the formula for θ

$$\theta_k^2 = \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|}$$

We get $\beta\tau = \left[1 - \rho_2 - \frac{\|g_{k+1}\|}{\|d_k\|}\right] y_k^T g_{k+1} + \frac{\|g_{k+1}\|^2}{\|d_k\|^2} y_k^T d_k$

Assuming that $\eta = \frac{\|g_{k+1}\|}{\|d_k\|}$

$$\beta\tau = [1 - \rho_2 - \eta] y_k^T g_{k+1} + \eta^2 y_k^T d_k$$

$$\beta_k^{new} = \frac{[1 - \rho_2 - \eta] y_k^T g_{k+1} + \eta^2 y_k^T d_k}{\tau} \quad , \quad \tau > 0$$

It is possible to take other forms of θ to find (β) in other forms.

2.1 We will prove the sufficient gradient of our algorithm, If k=1

$$d_1 = -g_1 \quad \Rightarrow \quad d_1^T g_1 = -\|g_1\|^2$$

let $d_k^T g_k \le -c\|g_k\|^2 \qquad s.t \qquad c = \frac{\sigma}{1-\sigma} \quad , \quad \forall \ 0 < \sigma < 1$

Now, we prove when k+1

$d_{k+1} = -g_{k+1} + \beta_K^{new} d_k$, Multiply both sides by g_{k+1}^T

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_K^{new} g_{k+1}^T d_k \quad (3)$$

We take into consideration the following two scenarios based on the sign of $g_{k+1}^T d_k$:

(1) *The case* $g_{k+1}^T d_k \le 0$ and $\beta_K^{new} > 0$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_K^{new} g_{k+1}^T d_k$$

$$g_{k+1}^T d_{k+1} \le -\|g_{k+1}\|^2$$

$$g_{k+1}^T d_{k+1} \le -\frac{\sigma}{1-\sigma}\|g_{k+1}\|^2$$

$$\therefore \quad g_{k+1}^T d_{k+1} \le -c\|g_{k+1}\|^2 \quad , \quad c = \frac{\sigma}{1-\sigma}$$

(2) The case $\mathbf{g_{k+1}^T d_k > 0}$

$$\beta_k^{\text{new}} = \frac{[1-\rho_2-\eta]y_k^T g_{k+1} + \eta^2 y_k^T d_k}{\tau}$$

$$y_k^T g_{k+1} = \|g_{k+1}\|^2 - g_{k+1}^T g_k \;, \because g_{k+1}^T g_k > 0 \Rightarrow y_k^T g_{k+1} \le \|g_{k+1}\|^2$$

$$\mu\|s_k\|^2 \le y_k^T s_k \le L\|s_k\|^2 \quad \xRightarrow{s_k=\lambda d_k} \quad \mu\lambda^2\|d_k\|^2 \le \lambda y_k^T d_k \le L\lambda^2\|d_k\|^2$$

$$\mu\lambda\|d_k\|^2 \le y_k^T d_k \le L\lambda\|d_k\|^2$$

$$\beta_k^{\text{new}} \le \frac{\left[1-\rho_2-\frac{\|g_{k+1}\|}{\|\mathrm{d}_k\|}\right]\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\|\mathrm{d}_k\|^2}L\lambda\|d_k\|^2}{\tau}$$

$$\beta_k^{\text{new}} \le \frac{[1-\rho_2]\|g_{k+1}\|^2 + \|g_{k+1}\|^2 L\lambda}{\tau}$$

$$\beta_k^{\text{new}} \le \frac{(1-\rho_2+L\lambda)\|g_{k+1}\|^2}{\tau}, let \;\; \rho_2 = L\lambda \Rightarrow \beta_k^{\text{new}} \le \frac{\|g_{k+1}\|^2}{\tau} \quad (4)$$

We substitute equation (4) in equation (3).

$$g_{k+1}^T d_{k+1} \le -\|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\tau} g_{k+1}^T d_k$$

$$g_{k+1}^T d_{k+1} \le \left(-1 + \frac{g_{k+1}^T d_k}{\tau}\right)\|g_{k+1}\|^2 \Rightarrow g_{k+1}^T d_{k+1} \le -\left(1 - \frac{g_{k+1}^T d_k}{\tau}\right)\|g_{k+1}\|^2$$

$$\because \frac{g_{k+1}^T d_k}{\tau} < 1 \quad \Rightarrow \quad g_{k+1}^T d_{k+1} \le -c\|g_{k+1}\|^2$$

2.2 Comprehensive convergence study of the proposed algorithm:

$$d_{k+1} = -g_{k+1} + \beta_K^{new} d_k \quad \Rightarrow \quad \|d_{k+1}\| \le \|g_{k+1}\| + |\beta_K^{new}|\|d_k\| \quad (5)$$

$$|\beta_K^{new}| \leq \left| \frac{\left[1 - \rho_2 - \frac{\|g_{k+1}\|}{\|d_k\|}\right] \|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\|d_k\|^2} L\lambda \|d_k\|^2}{\tau} \right|$$

$$|\beta_K^{new}| \leq \frac{1}{\tau}\left(\left[1 + \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|}\right] \|g_{k+1}\|^2 + \frac{\|g_{k+1}\|^2}{\|d_k\|^2} L\lambda \|d_k\|^2\right)$$

$$|\beta_K^{new}| \leq \frac{\|g_{k+1}\|^2}{\tau}\left(1 + \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|} + L\lambda\right) \qquad (6)$$

We substitute equation (6) in equation (5)

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \frac{\|g_{k+1}\|^2}{\tau}\left(1 + \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|} + L\lambda\right)\|d_k\|$$

$$\because\ \tau > y_k^T d_k \quad and \quad \mu\lambda\|d_k\|^2 \leq y_k^T d_k \leq L\lambda\|d_k\|^2\ ,\therefore\ \tau > \mu\lambda\|d_k\|^2$$

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \frac{\|g_{k+1}\|^2\|d_k\|}{\mu\lambda\|d_k\|^2}\left(1 + \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|} + L\lambda\right)$$

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \frac{\|g_{k+1}\|^2}{\mu\lambda\|d_k\|}\left(1 + \rho_2 + \frac{\|g_{k+1}\|}{\|d_k\|} + L\lambda\right)$$

$$\|d_{k+1}\| \leq \zeta + \frac{\zeta^2}{\mu\lambda\rho_2 m}\left(1 + \rho_2 + \frac{\zeta}{\rho_2 m} + L\lambda\right)$$

let $\zeta + \frac{\zeta^2}{\mu\lambda\rho_2 m}\left(1 + \rho_2 + \frac{\zeta}{\rho_2 m} + L\lambda\right) = \Gamma$

$$\|d_{k+1}\| \leq \Gamma \quad \Rightarrow \quad \frac{1}{\|d_{k+1}\|} \geq \frac{1}{\Gamma}$$

$$\sum_{k=1}^{\infty} \frac{1}{\|d_{k+1}\|^2} \geq \frac{1}{\Gamma^2}\sum_{k=1}^{\infty} 1 = \infty \qquad \Rightarrow \qquad \lim_{k\to\infty} inf\ \|g_k\| = 0$$

3.1 The Artificial Hummingbird Algorithm (AHA):

Hummingbirds are among the smallest and most intelligent birds in the world, with over 360 species, the smallest of which is the bee hummingbird, which is only 5.5 cm long. These birds feed on insects and flower nectar. This bird has unique and distinctive flight patterns, outperforming all other birds. It can fly in all directions, including forward, backward, up, and down. It can also remain stationary for periods of time, similar to helicopter flight. In addition, hummingbirds can remember flowers and the number of times they have visited

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each flower within a region. Hummingbirds are also migratory birds, traveling long distances in search of the best food sources. The Artificial Hummingbird Algorithm (AHA) is a biologically inspired optimization method that mimics hummingbird foraging behavior through three search methods: directed search, regional search, and migratory search. Hummingbirds also use three flight patterns: diagonal, Axial, and omnidirectional flight (Zhao et al., 2022; Ramadan et al., 2022).

3.2 Mathematical model and algorithm:

A population of **n** hummingbirds is placed on **n** food sources, initialized randomly using the formula:

$$x_i = \text{Low} + r \cdot (\text{Up} - \text{Low}), \quad i = 1, \ldots, n$$

where **Low** and **Up** are the lower and upper boundaries of a **d**-dimensional problem, **r** is a random vector in **[0,1]**, and x_i represents the position of the i_th food source, which is the solution to the given problem.

3.2.1 visit table

The visitation table is a key component of the algorithm, recording the number of cycles in which a particular source was not visited. The hummingbird selects the most frequently visited source, and if sources have the same visitation level, it selects the one with the best nectar fill rate. During updates, the increment levels of other sources are increased by 1, while the visited source is reset to 0.

The **visit table** for food sources is initialized as follows:

$$VT_{i,j} = \begin{cases} 0 & \text{if } i \neq j \\ \text{null} & \text{if } i = j \end{cases} \quad i = 1, \ldots, n \quad ; \quad j = 1, \ldots, n$$

where $VT_{i,j} = $ **null** for $i = j$ indicates that a hummingbird is feeding at its specific food source, and $VT_{i,j} = 0$ for $i \neq$ j means that the j_th food source has just been visited by the i_th hummingbird in the current iteration.

3.2.2 Flight in d-D space:

Axial Flight:

$$D(i) = \begin{cases} 1 & \text{if } i = \text{randi}([1, d]) \\ 0 & \text{otherwise} \end{cases}$$

Diagonal Flight:

$$D(i) = \begin{cases} 1 & if \;\; i = p(j), j = [1, k]\,, p = randprem(k)\,, k \in [2, \lceil r1.(d-2) \rceil + 1] \\ 0 & otherwise \end{cases}$$

Omnidirectional Flight:

$D(i) = 1, \quad i = 1, \ldots, d$

3.2.3 Guided foraging

At this stage, the hummingbird targets flowers that it has not visited for a long time. If the intervals are equal, it targets the source that contains the highest percentage of nectar. It heads towards the targeted food source according to the equation:

$$v_i(t+1) = x_{i,tar}(t) + a \cdot D \cdot \left(x_i(t) - x_{i,tar}(t)\right)$$

$x_i(t)$ represents the current location of the food source for hummingbird i at time t, $x_{i,tar}(t)$ represents the location of the targeted food source, $a \sim N(0,1)$,D represents the Flight pattern vector, $v_i(t+1)$represents the new site for hummingbird i at time $t+1$.

The food source location is updated as follows:

$$x_i(t+1) = \begin{cases} x_i(t), & \text{if } f\left(x_i(t)\right) \le f\left(v_i(t+1)\right) \\ v_i(t+1), & \text{if } f\left(x_i(t)\right) > f\left(v_i(t+1)\right) \end{cases}$$

3.2.4 Territorial Foraging

Territorial foraging is the search for new solutions within the neighboring area instead of searching in distant regions, according to the following equation:
$v_i(t+1) = x_i(t) + b \cdot D \cdot x_i(t)$
where $b \sim N(0,1)$.

3.2.5 Migrating Foraging

Migrating foraging is the search for a food source far from the territorial area, which occurs after every 2n repetitions, according to the following equation:
$x_{wor}(t+1) = Low + r \cdot (Up - Low)$
where x_{wor} is the food source with the worst nectar filling rate in the group.

Note the drawing in Figure (1) showing the steps of the algorithm.

4.1 TSA Swarm Algorithm:

It is a new Metaheuristic Optimization algorithm inspired by the style of a marine organism called tunicates, which are organisms from the chordate's phylum. These organisms live in salt water and are characterized by a special style of movement while escaping from danger and searching for food called jet propulsion, a distinctive mechanism for absorbing and pumping water. They also move in groups called swarms (Kaur et al., 2020).

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Iraqi Journal of Humanitarian, Social and Scientific Research
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4.2 The three stages of an organism's movement are:

Jet propulsion: It is a random movement that helps an organism explore a new place.

Swarm Behavior: It is an interactive movement between members of the swarm to obtain the best resource.

Best Position Attraction: When a good food source is discovered, everyone heads towards it.

4.3 Basic equations of the algorithm:

Avoid collisions between swarm elements through equations:

$$\vec{A} = \frac{\vec{G}}{\vec{M}} \quad (1)$$

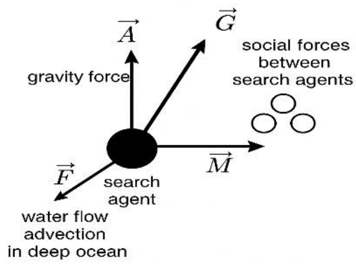
$$\vec{G} = c_2 + c_3 - \vec{F} \quad (2)$$

$$\vec{F} = 2 \cdot c_1 \quad (3)$$

$$\vec{M} = \lfloor P_{\min} + c_1 \cdot (P_{\max} - P_{\min}) \rfloor \quad (4)$$

where c1, c2, c3 are random values between [0, 1], A is a vector representing the acceleration of the particle. G aviator representing the force of gravity, which is the drive to attract the better a vector representing the flow of water in the deep ocean simulating a natural turbulence .M Represents the interaction of the object with the rest of the swarm . $P_{\min}$ The initial velocity represents and $P_{\max}$ represents the maximum velocity, The following figure shows the interconnection of these vectors.

4.4 Equation for moving towards the best food source:



$$\overrightarrow{PD} = | FS - rand \cdot \vec{P}_p(x) | \quad (5)$$

where PD represents the distance vector between the particle and the best food source. FS represents the best source. $\vec{P}_p(x)$ Represents the particle's position vector. Where navigation to the best source is done in a non-linear manner.

4.5 The equation of centering around the best particle:

$$\vec{P}_p(x) = \begin{cases} FS + A \cdot \overrightarrow{PD}, & \text{if } rand \geq 0.5 \\ FS - A \cdot \overrightarrow{PD}, & \text{if } rand < 0.5 \end{cases} \quad (6)$$

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4.6 Position update equation according to swarm behavior:

$$\vec{P}_p(x+1) = \frac{\vec{P}_p(x)+\vec{P}_p(x+1)}{2+c_1} \qquad (7)$$

5.1 Hybrid algorithms

- AHA-TSA hybrid algorithm method, Hybridization was performed by rearranging the set after each iteration according to the objective function values. The set was partitioned according to its proximity to the target. The optimization of the set near the target was assigned to the Hummingbird algorithm, while the optimization of the set in the other half was assigned to the TSA algorithm. Note the flowchart (1) which illustrates the hybridization method
- By hybridizing the swarm algorithms TSA and AHA with the conjugate gradient method using the new derivative operator beta, we obtain two hybrid algorithms AHA-CG-S and TSA-CG-S, where the population members are optimized by the gradient method at each iteration.

6. Results

From the results obtained using MATLAB 2022, where six different basic test functions were selected, as shown in Table (1), it was found that the hybrid algorithm (TSA-AHA) showed improved results, outperforming both the TSA and AHA algorithms after averaging the results for 30 different initial population groups, with 200 iterations for F1,F2,F3,F4 and 500 iterations for F5 We also found the standard deviation to demonstrate the stability of the results around the mean, As shown in Table (2) and Figures{ Fig 1, Fig 2, Fig 3, Fig 4, Fig 5}.

Flowchart of hybridizing AHA algorithm with TSA algorithm flowchart (1)

TABLE (1)

F_i	Nam	Objective function	D	Range	F_{min}
F_1	Sphere	$f(x)=\sum_{i=1}^{n} x_i^2$	30	[-100,100]	0
F_2	Schwefel 2.22	$f(x)=\sum_{i=1}^{n} \lvert x_i\rvert + \prod_{i=1}^{n} \lvert x_i\rvert$	30	[-10,10]	0
F_3	Schwefel 1.2	$f(x)=\sum_{i=1}^{n}\left(\sum_{j-1}^{i} x_j\right)^2$	30	[-100,100]	0
F_4	Schwefel 2.21	$maxi\{\lvert xi\rvert .1 \le i \le n\}$	30	[-100,100]	
F_5	Rosenbrock	$f(x)=\sum_{i=1}^{n-1}(100(x_{i+1}-x_i)^2 + (x_i-1)^2)$	30	[-30,30]	0

TABLE (2). The calculated average and standard deviation were for 30 restart attempts.

F_i	It		AHA	TSA	AHA-CG-S	TSA-CG-S	TSA-AHA
F_1	200	Mean	2.5385e-64	1.4415e-75	2.2434e-79	1.0112e-302	0
		std	0	2.4965e-91	1.219e-94	0	0
F_2	200	Mean	3.3569e-36	2.3049e-39	9.4634e-34	2.2389e-147	5.0834e-195
		std	06.7961e-52	9.9553e-55	1.7398e-49	0	0
F_3	200	Mean	5.93e-57	1.352e-71	3.5576e-64	4.84e-275	2.3599e-317
		Std	2.3026e-72	2.0451e-87	2.0587e-79	0	0
F_4	200	Mean	1.2673e-35	7.9536e-37	3.1925e-35	6.5503e-143	3.1264e-172
		Std	1.0874e-50	5.0971e-52	0	1.852e-158	0
F_5	500	Mean	1.228e-06	28.8963	26.3045	28.9605	25.3482

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		Std	6.4613e-22	1.4454e-14	1.084e-14	2.1681e-14	0

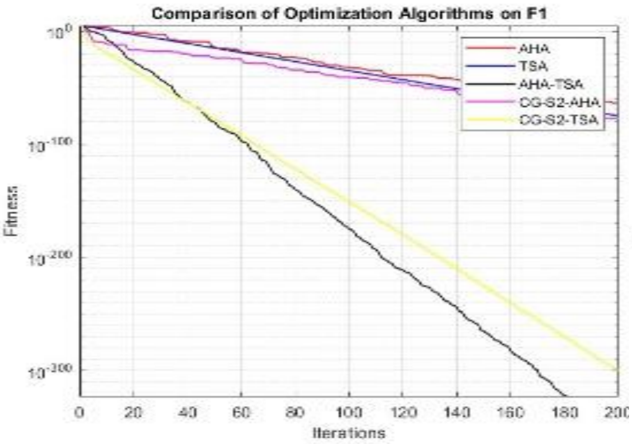


Fig1

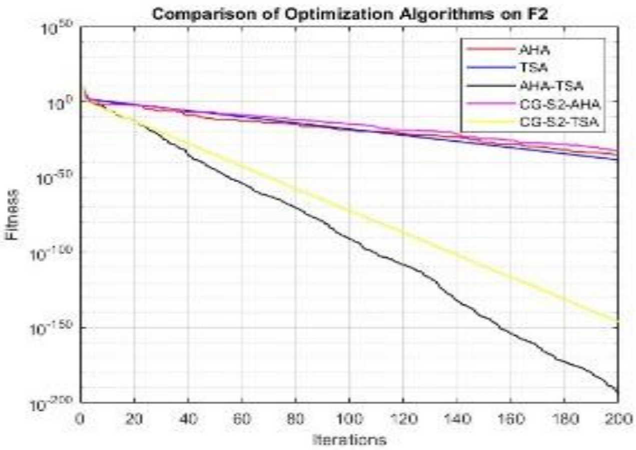


Fig2

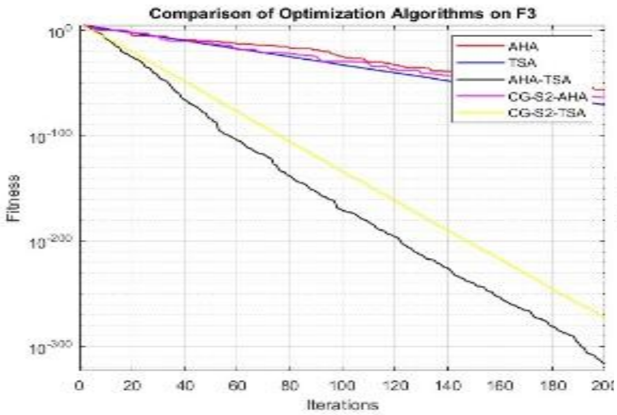
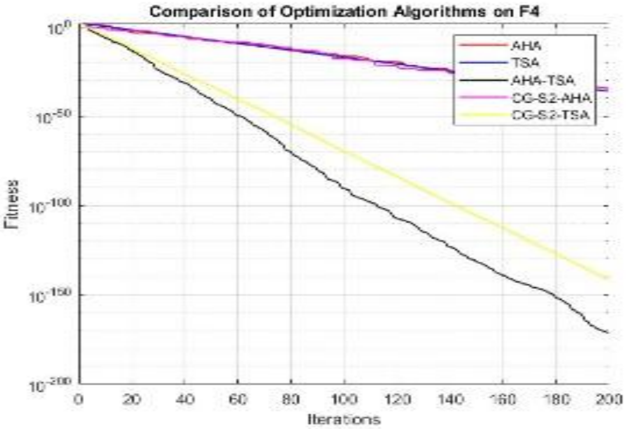
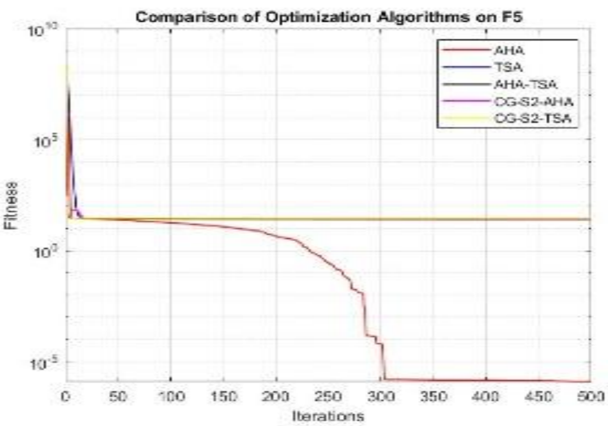


Fig3





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