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Analysis of the COVID-19 epidemic utilizing Ornstein-Uhlenbeck processes in Iraq

Zaineb Abdul AL-Ameer*, Hussein Kazem Asker

Department of Mathematics, Faculty of Computer Science and Mathematics, University of Kufa

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ABSTRACT

The swift and unforeseen global spread of COVID-19 has intensified the focus on mathematical modeling of the disease worldwide. This study presents a stochastic differential equation, specifically the Ornstein-Uhlenbeck process, representing Iraq's COVID-19 time series data. This dataset includes the number of infected individuals, fatalities, and vaccinated cases. We estimated the process parameters using the maximum likelihood estimator for the counts of infected, deceased, and vaccinated individuals.
We utilized the Euler approximation and the Milstein method within MATLAB to simulate the epidemic's time series based on estimated and actual data. Finally, we compared the effectiveness of the Euler and Milstein methods in approximating the processing of the Ornstein-Uhlenbeck process concerning the numbers of infected, deceased, and vaccinated individuals in Iraq. We evaluated actual and estimated cases, highlighting the reliability and precision of the two numerical methods.

1. Introduction

Infectious illnesses have always posed a significant threat to human life. The notorious Black Death swept through Medieval Europe over six years. This devastating epidemic, marked by rapid transmission, led to the immediate loss of approximately 25 million lives, which accounted for about one-third of Europe's total population at that time. More recently, the COVID-19 pandemic, which began in 2019, spread worldwide and caused widespread harm [3].

Coronaviruses constitute a vast viral family that generally causes respiratory ailments. COVID-19 is a zoonotic coronavirus, indicating its ability to transfer between people and animals

Coronaviruses are frequently spread via exudates or through contact with the surfaces of infected individuals and are commonly linked to colds, particularly during the winter [6].

Before 2019, two varieties of coronavirus precipitated a significant outbreak. The initial outbreak occurred from 2002 to 2004, involving acute respiratory syndrome with severe consequences, attributed to the SARS-COV strain, which affected 8,000 individuals and resulted in 774 fatalities across 29 countries. The second significant concern was attributed to Middle East Respiratory Syndrome Coronavirus (MERS-COV), initially identified within the Kingdom of Saudi Arabia; during the first outbreak, 1,841 individuals were affected, resulting in 652 fatalities [6].

*Corresponding author email : zainaba.albusaisi@student.uokufa.edu.iq

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In December 2019, instances of severe pneumonia were identified in a wet market in Wuhan, the capital of Hubei Province, which has a population of 11 million. On January 7, 2020, a novel coronavirus, subsequently designated COVID-19, was found, and the initial fatality was documented [6].

The novel strain is part of the broader coronavirus family, with six strains affecting the human respiratory apparatus. The time for this virus is estimated to range from two to fourteen days [6].

COVID-19 primarily endangers individuals with chronic respiratory conditions and compromised immunity, as well as the elderly population. Transmission occurs via intimate association with an infected individual. The ailment manifests with symptoms such as dyspnoea, coughing, cephalalgia, acute pneumonia, and fever, the latter being one of the most prevalent indicators. [7, 8] By mid-May 2020, global discussions centered on 4.5 million infections and approximately 500,000 fatalities; nevertheless, there were around 2 million recoveries, with the highest quantity of infections reported within the United States of America [5].

In several countries, the number of infections and deaths is increasing due to COVID-19. Effectively managing the risks associated with the spread of the virus requires specific protocols and guidelines. Therefore, predicting future confirmed cases is essential. Data from various nations indicate that the patterns of infections, recoveries, and fatalities are unpredictable. Consequently, a stochastic model is suitable for capturing this behavior [9, 10].

We plan to investigate stochastic mathematical models for dynamic and time series data related to this outbreak. The processes of the Ornstein-Uhlenbeck (OU) are a valuable and active class of stochastic differential equations for simulating, modelling, and forecasting actual datasets.

The Barndorff-Nielsen-Shephard stochastic volatility model is mostly based on the Ornstein-Uhlenbeck process (OU process). [1٥], [4]. Recent researche has revealed a growing interest in models using diffusion-type MROU processes. Larribi et al. investigated a new Stochastic SIRS model based on the MROU framework in epidemiology. [1]. Wang et al. investigated a SIS model with the OU process and estimated the reproductive number to assess the likehood of stochastic extinction and survival. [11]. Zhiming et al. investigated a stochastic SIRC model based on the Ornstein-Uhlenbeck process [2].

One practical strategy to deal with stochastic differential equations (SDEs) with non-global Lipschitz coefficients is to alter the drift and diffusion terms in numerical methods. This improvement enables the creation of explicit approaches that successfully converge when dealing with SDEs with super-linear growth. The Euler and Milstein algorithms are the most often utilised for solving superlinear SDEs. [13-12].

In this study, we use MATLAB to construct the Euler and Milstein methods for modelling epidemic time series, taking into account both estimated and real-world data. Our research focusses on modelling the dynamics of Iraq's sick, deceased, and vaccinated populations. We compare the accuracy and reliability of several numerical approaches for estimating the Ornstein-Uhlenbeck process, showing how well each represents pandemic tendencies.

The rest of the paper is structured as follows: Section 2 provides an overview of the Ornstein-Uhlenbeck process, along with its formulation using the Euler-Maruyama and Milstein methods. In the section (3) discusses the model's parameter estimates. Section (4) includes numerical simulations. Finally, Section (5) briefs our findings and conclusions.

2. Methodology

The processing of the Ornstein-Uhlenbeck is a mathematical model utilised to draw stochastic processes in engineering and physics. The processing of Markov reproduces a system's development over time while taking into account both deterministic and stochastic elements.

The relaxation period, which is an explaining element of the process, rules how quickly the system returns to balance after being interrupted. It is utilized in a broad diversity of applications and encompasses population increase, financial modelling, and material research. The mathematical formula is given below:

$$dX_t = \beta(\alpha - X_t)dt + \eta dB_t,\ X_0 > 0, \tag{2.1}$$

The mean reversion rate is β, the mean is α, the volatility distraction is η, and a Brownian motion is B_t. The analytical solution of equation (2.1) is,

$$X_t = X_0 e^{-\beta t} + \alpha\left(1 - e^{-\beta t}\right) + \eta e^{-\beta t}\int_0^t e^{\beta s} dB_s \tag{2.2}$$

With some algebraic manipulation, we can obtain the mean and variance of X(t).

$$E(X_t) = E\left[X_0 e^{-\beta t} + \alpha\left(1 - e^{-\beta t}\right)\right], \tag{2.3}$$

$$\mathrm{var}(X_t) = \frac{\eta^2}{2\beta} + \left[\mathrm{var}\left(X_0 - \frac{\eta^2}{2\beta} e^{-2\beta t}\right.\right], \tag{2.4}$$

respectively.

Let $\prod_n = \{0, \Delta t, 2\Delta t, \dots, n\Delta t\}$ be a partition of a temporal interval $[0, T]$, where $\Delta t = \frac{T}{n}$.

The Euler Maruyama scheme for model (2.1) is as follows:

$$X(t + \Delta t) = X(t) + \beta(\alpha - X(t)\Delta t + \eta \Delta B_t, \tag{2.5}$$

where $\Delta B_t \sim N(0, \Delta t)$ [14].

And the Milstein formula for model (2.1) uses the term:

$$X_{t+\Delta t} = X_t + \beta(\alpha - X_t)\Delta t + \eta \Delta W_t + \tfrac{1}{2}\eta^2(\Delta W_t^2 - \Delta t), \tag{2.6}$$

where Δt is a time step and ΔW_t is the change in the Wiener frequency over the time interval Δt [1٦].

3. Estimating the parameters

Parameter estimation refers to the process of determining the values of parameters within a statistical model, which is a significant challenge in any study based on mathematical models. In statistics, maximum likelihood estimate (MLE) is a method for estimating unidentified parameters by maximizing a likelihood function based on the most probable empirical data. Statistically, a specific collection of observations constitutes a random sample from an unidentified population. The Euler scheme specifies the distribution of $Xt + \Delta t$ as follows:

$$f(x_{t+\Delta t} \setminus x_t) = \frac{1}{\sqrt{2\pi\eta^2\Delta t}} e^{\frac{-[(x_{t+\Delta t} - (x_t + \beta(\alpha - x_t)\Delta t)]^2}{2\eta^2\Delta t}} \tag{3.1}$$

The log-likelihood is as follows:

$$L(\alpha, \beta, \eta) = \prod_{i=1}^{n} f(x_i \setminus x_{i-1}, \alpha, \beta, \eta)$$

Maximum likelihood estimation seeks to identify the parameter values that optimize the likelihood function within the parameters space, that is Θ=argmax(L(T,x)). The logarithm is a monotonic function, with the maximum of L(Θ;X) reaching its maximum at the same value of Θ as the maximum of Ln. This study will empoly the MLE strategy to estimate the parameters α,β and η. The parameters are described as follows:

$$\hat{\alpha} = \frac{1}{n}\frac{\sum_{i=1}^{n}\left[x_i - x_{i-1}(1 - \hat{\beta}\Delta t)\right]}{\hat{\beta}\Delta t}. \tag{3.2}$$

$$\hat{\beta} = \frac{\sum_{i=1}^{n}(x_i - x_{i-1})}{\Delta t \sum_{i=1}^{n}(\hat{\alpha} - x_{i-1})^2}. \tag{3.3}$$

$$\hat{\eta} = \frac{1}{n}\sum_{i=1}^{n}\frac{\left[x_i - \left(x_{i-1} + \hat{\beta}(\hat{\alpha} - x_{i-1})\Delta t\right)\right]^2}{\Delta t}. \tag{3.4}$$

$$\hat{\alpha} = \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_{i-1}^2 - \sum_{i=1}^{n} x_i x_{i-1} \sum_{i=1}^{n} x_{i-1}}{\left[\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_{i-1} - \left(\sum_{i=1}^{n} x_{i-1}\right)^2 - n\left(\sum_{i=1}^{n} x_i x_{i-1} + \sum_{i=1}^{n} x_{i-1}^2\right)\right]} \tag{3.5}$$

The model parameters were developed and determined using the initial population magnitude, and the model simulation was aligned with the recorded COVID-19 cases.

4. Numerical simulation

In this section, we obtained the actual and estimated values of the mathematical model's parameters (2.1) using the data from the source [https://github.com/owid/covid-19-data/tree/master/public/data] and external parameter capabilities using MATLAB programming for the number of infected cases, deceased individuals, and vaccinated individuals in Iraq, as shown in Tables 1 and 2.

Table 1. The value of estimated parameters of the stochastic model (2.1).

Country	Infected people	Protected people	Death people
Iraq	$\hat{\beta}_I = 4.2919e^{-8}$	$\hat{\beta}_P = 2.8311e^{-10}$	$\hat{\beta}_D = 0$
	$\hat{\alpha}_I = 2.6862e^4$	$\hat{\alpha}_P = 2.9076e^4$	$\hat{\alpha}_D = 22.2393$
	$\hat{\eta}_I = 897.6922626$	$\hat{\eta}_P = 21.44795204$	$\hat{\eta}_D = 0$

Table 2. The real value of the parameters of the stochastic model (2.1)

Country	Infected people	Protected people	Death people
Iraq	$\beta_I = 4.2919e^{-8}$	$\beta_P = 2.8311e^{-10}$	$\beta_D = 0$
	$\alpha_I = 26149.5996$	$\alpha_P = 29147.65723$	$\alpha_D = 22.25877193$
	$\eta_I = 17983.59891$	$\eta_P = 45576.39625$	$\eta_D = 26.92417329$

We generated plots for the random model (2.1) representing each stage of the epidemic, infected, deceased, and vaccinated, based on both the estimated and actual parameter values. These plots enabled us to compare the behavior of the COVID-19 epidemic over time. By utilizing numerical simulations conducted in MATLAB and calculating the exact values in Excel, we modeled the system of equations (2.1) for the COVID-19 pandemic in Iraq. In this table, we obtained the following three charts along with the parameter values:

In the next figure (Figure 1) compares the Euler and Milstein approach to estimate the parameters of infected individuals in transit with time. It includes real and estimated parameters and their respective errors. The larger deviations and higher error values for the Euler Method are evident as compared to the evaluations for the Milstein Method which yields values significantly closer in range of the exact solution while also having lower error values. These results demonstrate the improved precision of the Milstein method in simulating the infection-course dynamics.

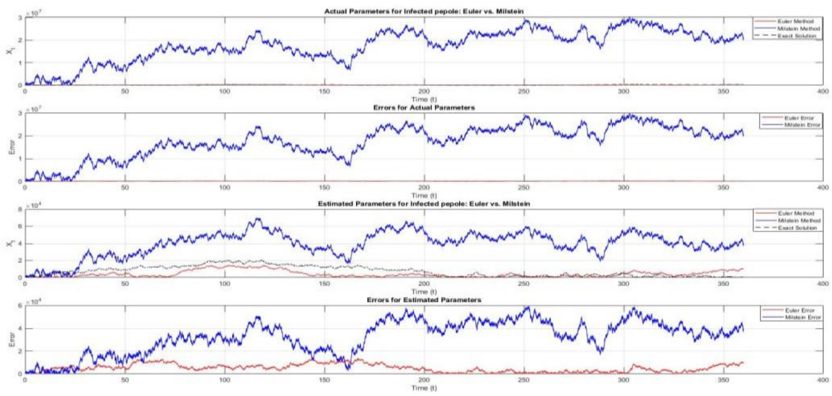

Fig. 1. Ornstein-Uhlenbeck processes time series using the Euler and Milstein methods with estimated and actual parameters of infected individuals.

Figure 2 compares the two techniques for concluding the parameters of the vaccinated. As expected, both methods are away from the exact solution; however, the Euler method tends to introduce greater errors for actual and estimated parameters. In other words, in general, we can say that the Milstein method is more accurate, as can be seen by comparing its error values with the data and its lower error values.

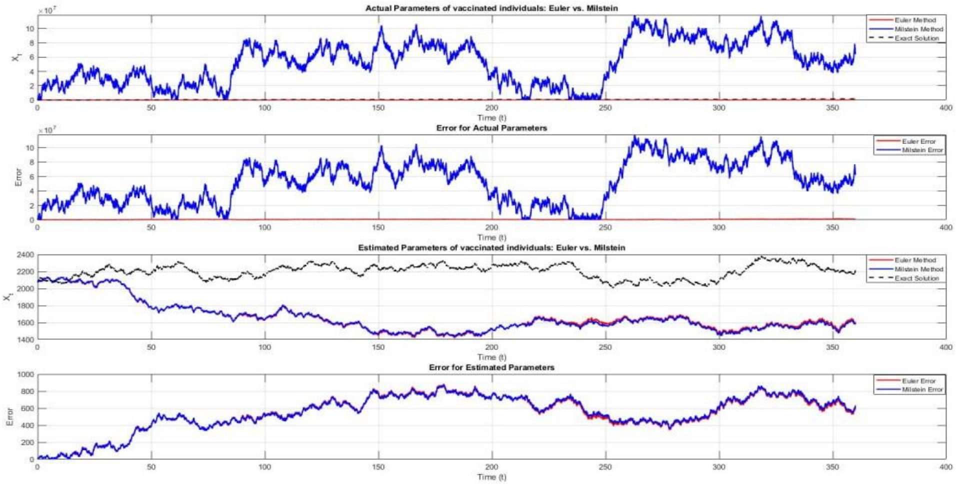

Fig. 2. Ornstein-Uhlenbeck processes time series using the Euler and Milstein methods with estimated and actual parameters of vaccinated individuals.

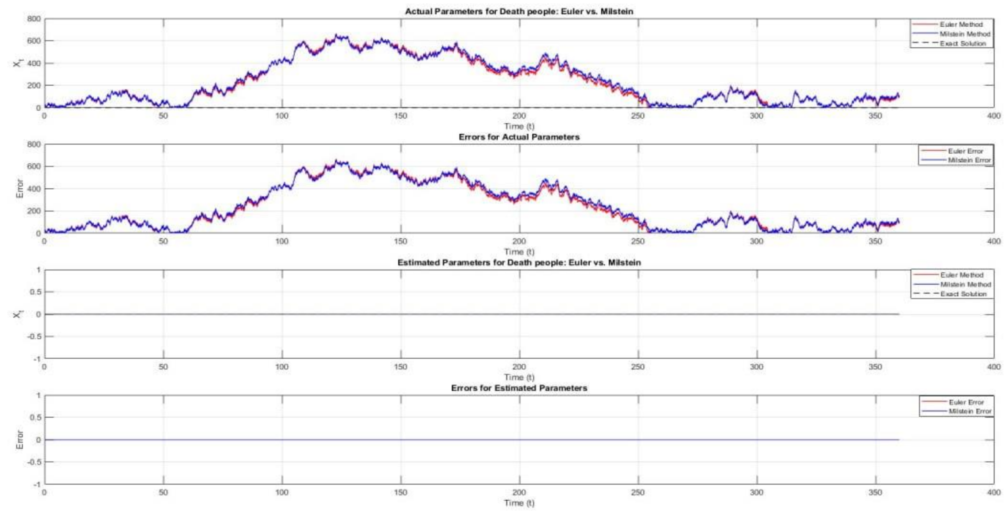

Fig.3. Ornstein-Uhlenbeck processes time series using the Euler and Milstein methods with estimated and actual parameters of death individuals.

Figure 3 presents a comparative analysis of the Euler and Milstein methods for modeling parameters over time, specifically focusing on death cases. It is divided into four subplots: real parameters, errors for actual parameters, estimated parameters, and errors for estimated parameters. Both models are compared versus the exact solution; on the one hand, the Euler model consistently explains larger deviations and higher errors. On the other hand, the Milstein modle follows the exact solution and maintains lower error values. This orientation further emphasizes the superior accuracy of the Milstein model in fixing the processing of the Ornstein-Uhlenbeck.In other words, the Euler model contrasts the Milstein model.

The Euler model consistently produces larger deviations and higher errors from the real answer, but the Milstein model delivers more accurate approximations and lower error values in all cases. This finding was introduced when we compared the Euler and Milstein approaches for estimating

parameters over time for three figures—infected individuals (Figure 1), vaccinated persons (Figure 2), and death cases (Figure 3).
Finally, when modelling the number of infected people, vaccine parameters, or mortality over time, we find that the Milstein technique outperformed the Ornstein-Uhlenbeck process.

4. Conclusion

In this paper, we investigate the processing of the Ornstein-Uhlenbeck as a stochastic model for COVID-19. We studied the mean and variance statistics of this model. Furthermore, we employed maximum likelihood estimation (MLE), and utilized Maximum Likelihood Estimation (MLE) to estimate the model parameters α, β, and η. Finaly, We compared the estimated values with the real values for the epidemic, including the number of infected, deceased, and vaccinated individuals in Iraq.

To derive the numerical solutions, we used both the Euler-Maruyama and Milstein methods. Additionally, by using both the real and estimated parameter values, we analyzed the time series data for the infected, deceased, and vaccinated populations, which are represented in Figures 1, 2, and 3, respectively.

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تحليل وباء كوفيد-١٩ باستخدام عمليات أورنشتاين-أولينبيك في العراق.

زينب عبد الأمير محسن*, حسين كاظم عسكر

قسم الرياضيات، كلية علوم الحاسوب والرياضيات، جامعة الكوفة

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كوفيد-١٩، عملية أورنشتاين-أولينبيك، المحاكاة العددية، التحليل العشوائي، تقريب أويلر، طريقة ميلستين، مقدر الاحتمالية القصوى.

الملخص

لقد أدى الانتشار العالمي السريع وغير المتوقع لفيروس كورونا المستجد إلى تكثيف التركيز على النمذجة الرياضية للمرض في جميع أنحاء العالم. تقدم هذه الدراسة معادلة تفاضلية عشوائية، وتحديدًا عملية أورنشتاين-أولينبيك، تمثل بيانات سلسلة زمنية لفيروس كورونا المستجد في العراق. تتضمن مجموعة البيانات هذه عدد الأفراد المصابين والوفيات والحالات المطعمة. لقد قمنا بتقدير معلمات العملية .باستخدام مقدر أقصى احتمالية لعدد الأفراد المصابين والمتوفين والملقحين

لقد استخدمنا تقريب أويلر وطريقة ميلستين داخل MATLAB لمحاكاة السلسلة الزمنية للوباء بناءً على البيانات المقدرة والفعلية. أخيرًا، قارنا فعالية طريقتي أويلر وميلستين في تقريب عملية أورنشتاين-أولينبيك فيما يتعلق بأعداد الأفراد المصابين والمتوفين والملقحين في العراق. لقد قمنا بتقييم الحالات الفعلية والمقدرة، مسلطين الضوء على موثوقية ودقة الطريقتين العدديتين..

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***Corresponding author email :** zainaba.albusaisi@student.uokufa.edu.iq

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