Oscillation properties for Boundary-value problems with Spectral Parameter in two-points boundary Conditions

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الخلاصة

المعادلة التفاضلية التي تحكم الاهتزاز الحر في المستوي لحزم غير منشورية منحنية دائرية رقيقة هي معادلة تفاضلية من الرتبةالسادسة مع وجود معلمة القيمة الذاتية في اثنان من شروطها الحدودية, ثم تمثيل المسائلة بالمؤثر التفاضلي من الرتبة السادسة مع وجود معلمة القيم الذاتية في اثنان من شروطه الحدودية حيث تم تمثيله بتركيبة خطية لثلاث مؤثرات تفاضلية من رتب مختلفة. ثم برهنا ان هذه المؤثرات التفاضلية تكون : متناظرة, مترافقة ذاتيا ومتراصة. وتم دراسة خصائص التذبذب لنظام الدوال الذاتية في فضاء هلبرت الموسع.

الكلمات المفتاحية: مؤثر تفاضلي, قيمة ذاتية, دالة ذاتية, متناظر, متوافق ذاتيا, متراص, خواص اساسية.

Abstract

Differential equation governing free in-plane vibration of non-prismatic thin circular curved is a sixth-order differential equation with eigenvalue in two-points boundary conditions, the problem is realized by sixth-order differential operator with spectral parameter in two-points boundary conditions. It is linear combination of three differential operators of different orders. It is shown that the operators are symmetric, self- adjoint and compact .we study the oscillation properties of the system of eigenfunctions of this operators in the extended Hilbert space.

Keywords: differential operator, eigenvalue, eigenfunction, symmetric, self-adjoint, compact, basis property.

1-Introduction

Curved structural members are frequently used by civil and mechanical engineers in industrial application. Most of the literature on curve beams revolves around analysis of circular arches. The goveming differential equation of uniform inextensible Euler-Bewoulli arches is a sixth-order differential equation with constant coefficients with eigenvalue parameter in the two-point boundary conditions.

The mathematical model for beams and pipes is represented by boundary-value problems:

$$\frac{\partial 6u}{\partial x^6}+\frac{\partial 4u}{\partial x^4}=-\alpha^2\,\frac{\partial^2}{\partial t^2}\quad\left(\frac{\partial 2u}{\partial x^2}+u\right),\quad x\in(0,1), t\in(0,\infty) \tag{1.1}$$

$$\begin{aligned} &u(0,t)=u_x(0,t)=0\\ &u(1,t)=u_x(1,t)=0\\ &u_{xxx}(0,t)=\alpha u_{xx}(0,t)\\ &u_{xxx}(1,t)=\alpha u_{xx}(1,t)\end{aligned} \tag{1.2}$$

Applying the Fourier method to the boundary value problem (1.1)-(1.2) separating the variables by:

$u(x,t)=y(x)\,e^{-mt}$

We obtain the sixth-order eigenvalue problem:

$$y^{(6)}+2y^{(4)}=-\lambda^2(y^{``}+2y) \tag{1.3}$$

$y(0) = y`(0)=0$

y(1) = y\`(1)=0
y\`\`\`(0) = λy\`\`(0)   (1.4)
y\`\`\`(1) = λy\`\`(1)

where λ=$\alpha m > 0$, with a constant α depending on the geometry and the physical properties of the configuration.

The application of this boundary problem was given on [5,6,10,12] . in general, for the equation (1.3) when the boundary conditions (1.4) contain the a spectral parameter this problem can't interpreted an eigenvalue-eigenfunction problem in the Hilbert space L_2(0,1). From this point of view, in [3,4] the expression of the operator of the boundary value problems for second order differential operators with eigenvalue parameter dependent conditions have been given in the space L_2(0,1) ×C (C complex numbers).

In [1,7] this approach has been extended to a forth order differential equation describing small transversal vibrations of a homogeneous beam compressed or stretched by a force. Various aspects of a sixth-order differential operators with a spectral parameter contained in one-point boundary conditions, including spectral asymptotics and basis properties, have been investigated in [8]. Numerical methods and other techniques for the investigation of sixth-order boundary value problems can be found in [2,9,11]. This presented paper introduced a study the properties as completeness , minimality and basis prosperity are investigated for eigenfunction of the spectral problem (1.3)-(1.4) in extended Hilbert space.

2- Problem formulation

We introduce the special inner product in the Hilbert space $L_2(0,1)\times C\times C$ and we give some definition and lemmas. We denote by H= $L_2(0,1)\times C\times C$, the Hilbert space of all elements $\tilde{y}=\begin{pmatrix} y(x) \\ a \\ b \end{pmatrix}$ which is scalar product defined by:

$$<\tilde{y},\tilde{y}>=\int_0^1 y(x)\overline{y(x)}dx+a\bar{a}-b\bar{b} \qquad (2.1)$$
$$=\|y\|^2+|a|^2-|b|^2$$

We denote by A the operator is defined in the Hilbert space by:

$$A=-\lambda^2 A_1+\lambda A_2-A_3 \qquad (2.2)$$

Where A_1 is the operator which is defined in H by:

$$A_1\tilde{y}=\begin{pmatrix} -y^{``}+2y \\ 0 \\ 0 \end{pmatrix} \quad \text{for} \quad \tilde{y}\in H \qquad (2.3)$$

and A_2 is operator given by:

$$A_2=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad (2.4)$$

and A_3 the operator defined in H with domain D(A_3) by:

$$A_3\tilde{y} = \begin{pmatrix} y^{(6)} + 2y^{(4)} \\ y```(0) \\ y```(1) \end{pmatrix} \qquad \text{for } \tilde{y} \in D(A) \qquad (2.5)$$

And it's the domain D(A_3) of all elements $\tilde{y} = \begin{pmatrix} y(x) \\ a \\ b \end{pmatrix} \in H$ satisfying the conditions:

1- y(x) $\in w_6^2(0,1)$
2- y(0) = y`(0) = 0
3- y(1) = y`(1) = 0
4- a = y``(0)
5- b = y``(1)

Remark 2.1:

1- D(A) = D(A_3)
2- D(A_1) = D(A_3)

Theorem 2.2: The differential equation (1.3) and the boundary conditions (1.4) hold if and only if for $\tilde{y} \in D(A)$: for $A\tilde{y}$=0, holds.

Proof: for $\tilde{y} \in D(A)$ and

$y^{(6)} + 2y^{(4)} + \lambda^2(y`` + 2y)$=0

y(0) = y`(0) = y(1) = y`(1) = 0

y```(0)= λy``(1)

then

$$A\tilde{y} = \begin{pmatrix} y(x) \\ a \\ b \end{pmatrix} = \begin{pmatrix} y(x) \\ y``(0) \\ y``(1) \end{pmatrix} = -\lambda^2 \begin{pmatrix} -y`` + 2y \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ y``(0) \\ y``(1) \end{pmatrix} - \begin{pmatrix} y^{(6)} + 2y^{(4)} \\ y```(0) \\ y```(1) \end{pmatrix}$$

$$= \begin{pmatrix} -\lambda^2(y`` + 2y) - y^{(6)} - 2y^{(4)} \\ \lambda y``(0) - y```(0) \\ \lambda y``(1) - y```(1) \end{pmatrix}$$

Applying the equations (1.3) and (1.4) we get:

$$A\tilde{y} = \begin{pmatrix} y^{(6)} + 2y^{(4)} + \lambda^2(y`` + 2y) \\ y```(0) - \lambda y``(0) \\ y```(1) - \lambda y``(1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Then

$A\tilde{y}$=0 for $\tilde{y} \in$ D(A)

Let A $\tilde{y}$=0 for for $\tilde{y}\in$ D(A) then

$$A\ \tilde{y} = -\lambda^2 \begin{pmatrix} -y^{``} + 2y \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ y^{``}(0) \\ y^{``}(1) \end{pmatrix} - \begin{pmatrix} y^{(6)} + 2y^{(4)} \\ y^{```}(0) \\ y^{```}(1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$== \begin{pmatrix} (y^{(6)} + 2y^{(4)} + \lambda^2(y^{``} + 2y)) \\ \lambda y^{``}(0) - y^{```}(0) \\ \lambda y^{``}(1) - y^{```}(1) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Then

$$(y^{(6)} + 2y^{(4)} + \lambda^2(y^{``} + 2y)) = 0$$

y(0)=y`(0)=y(1)=y`(1)=0
y```(0)= λy``(0)
y```(1)= λy``(1)

the theorem is proved.
Remark 2.3: The operator A describes the eigenvalue problem (1.3)- (1.4).
Theorem 2.4: the domain D(A_3) is dense in the Hilbert space H.
Proof:

Let $\tilde{w} = \begin{pmatrix} w \\ c \\ d \end{pmatrix} \in H$ such that $< \tilde{y}, \tilde{w} > = 0$ for all y$\in D(A_3)$ and $c \neq d$.

$$\int_0^1 y(x)\bar{w}(x)dx + y^{``}(0)\bar{c} - y^{``}\bar{d} = 0$$

If $y \in c_0^\infty(0,1)$, then y``(0)=y``(1)=0 and $\begin{pmatrix} y \\ 0 \\ 0 \end{pmatrix} \in D(A_3)$ where

$$\int_0^1 y(x)\bar{w}(x)dx = 0 \text{ for all } y \in c_0^\infty(0,1)$$

It follows that w=0
Let y(x)=$x^2(1-x)^2$
satisfies y(0)=y`(0)=y(1)=y`(1)=0
y``(0)=2$\neq$0
y``(1)=2$\neq$0
Hence

$$\tilde{y} = \begin{pmatrix} y(x) \\ y^{``}(0) \\ y^{``}(1) \end{pmatrix} \in D(A_3)$$

Since w=0, it follows that $0 < \tilde{y}, \tilde{w} > = y``(0)\bar{c} - y``(1)\bar{d}$

$=2\bar{c} - 2\bar{d}=0$

but $\bar{c} \neq \bar{d}$ then c=d=0

showing that $\tilde{w}$=0

Hence

$D(A_3)^{\perp}=\{0\}$

The theorem is proved.

Lemma 2.5: the operator A_3 is symmetric.

Proof: from the lemma 2.4, A is densely defined. For $\tilde{y}, \tilde{z} \in D(A_3)$ we have :

$$< A_3, \tilde{y}, \tilde{z} >= \int_0^1 y^{(6)}\bar{z}(x)dx + 2\int_0^1 y^{(4)}\bar{z}(x)dx + y```(0)\overline{z``}(0) - y```(1)\overline{z``}(1)$$

Integrating by parts and observing the boundary conditions by elements in (A_3) , it follows that :

$$\int_0^1 y^{(6)}\bar{z}(x)dx = \int_0^1 y(x)\bar{z}(x)^{(6)}dx + \bar{z}``(1)y```(1) - \bar{z}``(0)y```(0) - \bar{z}```(1)y``(1) + \bar{z}```(0)y``(0)$$

$$2\int_0^1 y^{(4)}\bar{z}(x)dx = 2\int_0^1 y(x)\bar{z}^{(4)}(x)$$

Hence

$$< A_3, \tilde{y}, \tilde{z} >= \int_0^1 y(x)\bar{z}(x)^{(6)}dx + 2\int_0^1 y(x)\bar{z}^{(4)}(x) + y``(0)\bar{z}```(0) - y``(1)\bar{z}```(1) = < \tilde{y}, A, \tilde{z} >$$

The lemma is proved.

Remark 2.6: since $D(A_1) = D(A_3)$ and $D(A)= D(A_3)$ then $D(A_1)$, D(A) are dense in Hilbert space H.

Lemma 2.7: the operator $A_1$is symmetric.

Proof: the domain $D(A_1)$ is dense in H for $\tilde{y}, \tilde{z} \in D(A_1)$

We have

$$< A, \tilde{y}, \tilde{z} >= \int_0^1 y``(x)\bar{z}(x)dx + 2\int_0^1 y(x)\bar{z}(x)dx$$

Integration by parts and observing the boundary conditions by elements in $D(A_1)$, it follows that:

$$\int_0^1 y``(x)\bar{z}(x)dx = \int_0^1 y(x)\bar{z}``(x)dx$$

Hence

$$< A, \tilde{y}, \tilde{z} >= \int_0^1 y(x)\bar{z}``(x)dx + 2\int_0^1 y(x)\bar{z}(x)dx$$
$$=< \tilde{y}, A, \tilde{z} >$$

The lemma is proved.

Lemma 2.8: The operator $A_1$is positive.

Proof: [8].

Lemma 2.9: the operator A_1 is self-adjoint .

Proof: [13, 14].
Lemma 2.10: the operator A_2 is self –adjoint bounded in $L_2(0,1) \times c \times c$.
Proof: [8].
Lemma 2.11: the operator $A_1$and A_3 are semi-bounded from below in Hilbert space H.
Proof: [1,15].
Theorem 2.12: There is unboundedly increasing sequence $\{\lambda_n^2\}$ of eigenvalues of the boundary value problem (1.3) – (1.4)

$$\lambda_1^2 < \lambda_2^2 < \lambda_3^2 < \cdots < \lambda_n^2 < \cdots \quad (2.8)$$

Moreover, the eigenfunctions $y_n(x)$ corresponding to λ_n^2 has exactly n simple zeros in the interval [0,1].
Proof: [8].
Theorem 2.13: If the operator A is compact in Hilbert space H then A is bounded.
Proof:[1.15].
Theorem 2.14: the operator $A_1$and $A_3$are invertible if and only if μ_1=0 μ_1=0 are not eigenvalues of $A_1$and A_3 respectively.
Proof:[1].

3-Green's function of the operator A_3

Let $\varphi_1(x), \varphi_2(x), \varphi_3(x), \varphi_4(x), \varphi_5(x)\ and\ \varphi_6(x)$ six solutions of the equation:

$$\frac{d6y(x)}{dx^6} + 2\frac{d4y(x)}{dx^4} = \mu y(x) \quad (3.1)$$

Such that μ is a not eigenvalue of A_3 and the three solutions $\varphi_1(x), \varphi_2(x)\ and\ \varphi_3(x)$ satisfying the initial conditions:

$\varphi_1(0) = 0$ and $\varphi_2(0) = 0$ and $\varphi_3(0)$=0
$\varphi_1'(0) = 0$ $\varphi_2'(0) = 0$ $\varphi_3'(0)$=0
$\varphi_1''(0) = 1$ $\varphi_2''(0) = 1$ $\varphi_3''(0)$=-1
$\varphi_1'''(0) = 1$ $\varphi_2'''(0) = \mu$ $\varphi_3'''(0)$=μ
$\varphi_1^{(4)}(0) = 1$ $\varphi_2^{(4)}(0) = 0$ $\varphi_3^{(4)}(0)$=-1
$\varphi_1^{(5)}(0) = 0$ $\varphi_2^{(5)}(0) = 1$ $\varphi_3^{(5)}(0)$=0

Also, the three solutions $\varphi_4(x), \varphi_5(x)\ and\ \varphi_6(x)$ satisfying the initial conditions:

$\varphi_4(1) = 0$ and $\varphi_5(1) = 0$ and $\varphi_6(1)$=0
$\varphi_4'(1) = 0$ $\varphi_5'(1) = 0$ $\varphi_6'(1)$=0
$\varphi_4''(1) = 1$ $\varphi_5''(1) = 1$ $\varphi_6''(1)$=-1
$\varphi_4'''(1) = 1$ $\varphi_5'''(1) = \mu$ $\varphi_6'''(1)$=μ
$\varphi_4^{(4)}(1) = 0$ $\varphi_5^{(4)}(1) = 0$ $\varphi_6^{(4)}(1)$=-1

$\varphi_4^{(5)}(1) = -1 \quad \varphi_5^{(5)}(1) = 1 \quad \varphi_6^{(5)}(1)$=0

And let $w^0 : w(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6) \neq 0$ then
$w^x : w(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6) \neq 0$ for all x∈ [0,1], where w is the wronskian determinate and this means the solutions are linearly independent, the Green's function of the operator A_3 such that μ is a not eigenvalue is given by a function in the form:

$$G(x, t, \mu) = \begin{cases} \sum_{i=1}^{3} a_i(t)\varphi_i(x) & 0 \leq x < t \leq 1 \\ \sum_{i=4}^{6} a_i(t)\varphi_i(x) & 0 \leq t < x \leq 1 \end{cases} \quad (3.2)$$

Where

$$a_1(t) = \frac{[\varphi^2 \varphi^3 \varphi^4 \varphi^5 \varphi^6]}{w^t} \quad (3.3)$$

$$a_2(t) = \frac{(-1)[\varphi^1 \varphi^3 \varphi^4 \varphi^5 \varphi^6]}{w^t} \quad (3.4)$$

$$a_3(t) = \frac{[\varphi^1 \varphi^2 \varphi^4 \varphi^5 \varphi^6]}{w^t} \quad (3.5)$$

$$a_4(t) = \frac{(-1)[\varphi^1 \varphi^2 \varphi^3 \varphi^5 \varphi^6]}{w^t} \quad (3.6)$$

$$a_5(t) = \frac{[\varphi^1 \varphi^2 \varphi^3 \varphi^4 \varphi^6]}{w^t} \quad (3.7)$$

$$a_5(t) = \frac{(-1)[\varphi^1 \varphi^2 \varphi^3 \varphi^4 \varphi^5]}{w^t} \quad (3.8)$$

Where

$$w^t = w(\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6, t) \quad (3.9)$$

$$\varphi^j = \begin{bmatrix} \varphi_j \\ \varphi_j^{`} \\ \varphi^{``}_j \\ \varphi_j^{```} \\ \varphi_j^{(4)} \end{bmatrix} \quad (3.10)$$

Theorem 3.1: The operator A_3 is self-adjoint in the Hilbert space H.
Proof: From lemma (2.5) the operator A_3 is symmetric and to proof that :

$$(A_3 - \mu I)^{-1} H = D(A_3) \quad (3.11)$$

Where I is the unit operator.
Let $\tilde{y} = (y(x), a, b) \in D(A_3)$ and satisfying:

$$(A_3\text{-}\mu I)\, \tilde{y} = \tilde{F} \quad (3.12)$$

Where F= $(f_1(x), f_2, f_3) \in H$ and μ is a not eigenvalue of A_3.
The equation (3.12) is a non homogeneous differential equation has a solution is given by a function in the form:

$$y(x) = \sum_{i=1}^{6} k_i \varphi_i(x) - \int_0^1 G(x, t, \mu) f_1(x) dx \quad (3.13)$$

a=y``(0)

b=y``(1) (3.14)

where k_i, i=1, … 6 constants and G(x, t, μ) defined in (3.2).

from the (theorem (2.1) [7]) we get:

$$\tilde{y} = (A_3 - \mu I)^{-1}\tilde{F}$$

So

$$\tilde{y} \in (A_3 - \mu I)^{-1}H$$

Then

$$D(A_3) \subseteq (A_3 - \mu I)^{-1}H \qquad (3.15)$$

Since μ is a not eigenvalue of A_3, for all

$\tilde{F} = (f_1(x), f_2, f_3)$ in H, there exist $\tilde{y}$=(y(x), a, b) such that:

$$(A_3 - \mu I)\ \ \tilde{y} = \tilde{F} \qquad (3.16)$$

We obtain $y \in w_6^2(0,1)$ and y(0)=y`(0)=y(1)=y`(1)=0

Then

$\tilde{y}$=(y(x), a, b) $\in D(A_3)$

From [15]

$$\tilde{y} = (A_3 - \mu I)^{-1}\tilde{F} \qquad (3.17)$$

Then

$$(A_3 - \mu I)^{-1}\tilde{y} \in D(A_3) \qquad (3.18)$$

So

$$(A_3 - \mu I)^{-1}H \subseteq D(A_3) \qquad (3.19)$$

From (3.15) and (3.19) we obtain:

$$(A_3 - \mu I)^{-1}H = D(A) \qquad (3.20)$$

The theorem is proved.

Theorem 3.2: The operator $(A_3 - \mu I)$ is compact if μ is a not eigenvalue of A_3.

Proof: From equation (3.12) we obtain

$$(A_3 - \mu I)^{-1}(f_1(x), f_2, f_3) = (\sum_{i=1}^{6} k_i\varphi_i(x) - \int_0^1 G(x,t,\mu)f_1(t)dt, y``(0), y``(1)) \qquad (3.21)$$

is a linear compact operator in H such that μ is a not eigenvalue of A. [15]

Remark 3.3:The operator A is : symmetric, self-adjoint and compact in H.

4-Oscillation properties of Eigenfunction of the operator A

Remark 4.1: The solution of the boundary problem (1.3) – (1.4) is given by a function in the form:

$$y(x) = c_1(\mu_1)\cos\sqrt{2}\,x + c_2(\mu_1)\sin\sqrt{2}\,x + c_3(\mu_1)\,e^{\frac{\mu_1}{\sqrt{2}}x}\cos\frac{\mu_1}{\sqrt{2}}x + c_4(\mu_1)e^{\frac{\mu_1}{\sqrt{2}}x}\sin\frac{\mu_1}{\sqrt{2}}x + c_5(\mu_1)e^{-\frac{\mu_1}{\sqrt{2}}x}\cos\frac{\mu_1}{\sqrt{2}}x + c_6(\mu_1)e^{-\frac{\mu_1}{\sqrt{2}}x}\sin\frac{\mu_1}{\sqrt{2}}x \qquad (4.1)$$

where $\lambda^2 = \mu_1^6$ and $c_i(\mu_1)$, i=1, …, 6 are functions of μ_1.

Theorem 4.2: The eigenfunction of the operator A form orthonormal basis in the space H.

Proof: The operator A has at most countable eigenvalues λ_n^2 and eigenfunction $\tilde{y}_n(x)$ which have the asymptotic form:

$$\lambda_n^2 = \mu_{1n}^2 + 0(\frac{1}{n}) \qquad (4.2)$$

$$\tilde{y}_n(x) = \begin{pmatrix} y_n(x) \\ y_n^{``}(0) \\ y_n^{``}(1) \end{pmatrix} \tag{4.3}$$

$$\tilde{y}_n(x) = \begin{pmatrix} c_1(\mu_{1n})\cos\sqrt{2}\,x + c_2(\mu_{1n})\sin\sqrt{2}\,x + c_3(\mu_{1n})\,e^{\frac{\mu_{1n}}{\sqrt{2}}x}\cos\frac{\mu_{1n}}{\sqrt{2}}x + \\ c_4(\mu_{1n})e^{\frac{\mu_{1n}}{\sqrt{2}}x}\sin\frac{\mu_{1n}}{\sqrt{2}}x + c_5(\mu_{1n})e^{-\frac{\mu_{1n}}{\sqrt{2}}x}\cos\frac{\mu_1}{\sqrt{2}}x + c_6(\mu_{1n})e^{-\frac{\mu_1}{\sqrt{2}}x}\sin\frac{\mu_1}{\sqrt{2}}x \\ y_n^{``}(0) \\ y_n^{``}(1) \end{pmatrix} \tag{4.4}$$

Since the operator A: compact, self-adjoint and bounded, Applying the Hilbert-Schmidt theorem [16] to the operator A, we obtain that the eigenfunctions of the operator A from an orthonormal basis in the Hilbert space H.

Theorem 4.3: the system of eigenfunctions $\{\tilde{y}_n(x)\}_0^\infty$ $(n \neq n_0)$ (where no be an arbitrary fixed nonnegative integer), of the boundary problem (1.3) - (1.4) is a compact and minimal system.

Proof: From the theorem (4.2) the eigenfunctions

$$\tilde{y}_n(x) = \begin{pmatrix} y_n(x) \\ y_n^{``}(0) \\ y_n^{``}(1) \end{pmatrix}$$

(where $\tilde{y}_n(x)$ defined in (4.4)), of the boundary problem (1.3) – (1.4) from a basis in H=$L_2(0,1) \times c \times c$.

So, the system $\{y_n(x)\}_0^\infty$ is complete and minimal in H, we denote by P the orthoprojection which is define by the formula:

P $\tilde{y}_n(x) = y_n(x)$ in H.

Thus codimp=1 . Then by (3.2) [7] the system:

$$\{\mathrm{P}\tilde{y}_n(x)\}_0^\infty = \{y_n(x)\}_0^\infty$$
$$=$$
$$\left\{ \begin{matrix} c_1(\mu_{1n})\cos\sqrt{2}\,x + c_2(\mu_{1n})\sin\sqrt{2}\,x + c_3(\mu_{1n})\,e^{\frac{\mu_{1n}}{\sqrt{2}}x}\cos\frac{\mu_{1n}}{\sqrt{2}}x + \\ c_4(\mu_{1n})e^{\frac{\mu_{1n}}{\sqrt{2}}x}\sin\frac{\mu_{1n}}{\sqrt{2}}x + c_5(\mu_{1n})e^{-\frac{\mu_{1n}}{\sqrt{2}}x}\cos\frac{\mu_1}{\sqrt{2}}x + c_6(\mu_{1n})e^{-\frac{\mu_1}{\sqrt{2}}x}\sin\frac{\mu_1}{\sqrt{2}}x \end{matrix} \right\}$$

Whose one element is omitted from faroms a complete and minimal system in $H_p = P(H) = L_2(0,1)$.

Hence, the eigenfunctions $\{y_n(x)\}_0^\infty$ of the boundary problem (1.3) – (1.4) are complete and minimal in $L_2(0,1)$.

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