

**A comparison of Bayesian and
non-Bayesian Methods for Estimation of
Reliability function and Hazard function
for Lomax Distribution with two
parameters**

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المستخلص

يعتبر توزيع لوماكس Lomax او باريتو من النوع الثاني من التوزيعات المهمة التي تستخدم بكثرة في اختبارات الحياة (Life testing) والمعولية (Reliability) ولها تطبيقات واسعة في مجالات عدة منها العلوم الاقتصادية والتأمين (Actuarial) وبناء النماذج (Modelling) ومسائل صفوف الانتظار (Queuing problems) والعلوم البايولوجية . هناك العديد من الدراسات والبحوث التي استخدمت هذا التوزيع وتقدير بعض دواله ومعالمه بالاعتماد على طرق تقدير اعتيادية او طرق بيزية . ففي عام 2011 قام كل من الباحثين (H.Panahi) و (S.Asadi) بتقدير المعلمة $R=P(y<x)$ عندما x و y مستقلان وذو توزيع لوماكس لمعلمة قياس واحدة ولكن بمعالم شكل مختلفة . وفي عام 2011 ايضا قام كل من الباحثين (Abd.Elftah) و (Aloboud.F,M) و (Alharby.A.H) بالاعتماد على توزيع لوماكس وباستخدام طرائق بيزية واعتيادية بتقدير حجم العينة لبيانات تحت المراقبة من النوع الاول مع تطبيق عددي . وفي هذا البحث تم استخدام اربع طرائق لتقدير دالة المعولية ودالة المخاطرة هي طريقة الامكان الاعظم , طريقة المربعات الصغرى , طريقة العزوم وطريقة بيز وتمت المقارنة بينهما من خلال مؤشري MSE و MAPE باستخدام المحاكاة , وقداظهرت النتائج تفوق طريقة بيز للقيم الصغيرة للمعلمة α وطريقة المربعات الصغرى للقيم الكبيرة للمعلمة α ولجميع حجوم العينات وقيم المعلمات الاخرى . كما اعطت طريقتي الامكان الاعظم وطريقة العزوم نفس نتائج MSE و MAPE ولجميع حجوم العينات وقيم المعالم .

Introduction:

Lomax distribution also called the pareto type II distribution is a heavy-tail probability often used in business and life testing, Reliability, with extended application in economic science and actuarial modeling , queuing problems , and biological science. Panahi and Asadi (2011) studied estimation of the stress-strength parameter $R=P(Y<X)$ when X,Y are independent and both are Lomax distribution with common scale parameters but different shape , and derived a maximum likelihood estimation. Bindu (2011) studied estimation of $R=P(X>Y)$ when X,Y are independent random variables , and introduce a new family of distribution referred to as the double Lomax distribution and derive the p.d.f and the expression for the reliability R for the double Lomax distribution truncated below zero. David ,Hui ,and Ryan (2011) analyzed the second-order bias of the maximum likelihood estimators for Lomax (Pareto II) distribution for finite sample size and show that this bias is positive . Asghar zadeh and Valiollahi (2011) derived explicit estimator for Lomax distribution by approximating the likelihood function . Morteza , Farhad and Manoochehr (2012) they use the Bayesian estimators for Lomax distribution obtained thought conjugate prior for the shape and scale parameters under generalized order statistics.

In this study we explore and compare the performance of the maximum likelihood and least square and moment estimates with the Bayesian of Reliability and Hazard function for the two parameter Lomax distribution.

The probability density function of two parameter Lomax distribution becomes :

$$f(x) = \frac{\alpha}{\lambda} \left(1 + \frac{x}{\lambda}\right)^{-(\alpha+1)} \dots\dots x > 0, \alpha, \lambda > 0 \dots\dots\dots (1)$$

Where α is shape parameter , λ is the scatter parameter of the distribution.
The reliability function is given by :

$$R(x) = Pr(X > x) = 1 - F(x) \\ = \int_0^x f(u; \alpha, \lambda) du = (1 + \frac{x}{\lambda})^{-\alpha} \dots \dots \dots (2)$$

And the Hazard function become :

$$H(x) = \frac{f(x)}{R(x)} = \frac{\alpha}{x+\lambda} \dots \dots \dots (3)$$

The rest of the paper is arranged as follows . The methods , Maximum likelihood estimator, least square estimator , Bayes estimator , estimator of moment method . In results , Simulation study is discussed and the results are presented and followed by conclusion.

Materials and Methods:

Maximum likelihood Estimation(MLE): we introduce the concept of maximum likelihood estimation with two parameters Lomax distribution. We have set of random failure times $(x_1, x_2, \dots, x_n)$ and vector of unknown parameters $\theta = (\theta_1, \theta_2, \dots, \theta_k)$ then the likelihood function is :

$$L(x; \theta) = \prod_{i=1}^n f(x_i, \theta) \quad (4)$$

$$U_i(\theta) = \frac{\partial \ln L(x_i, \theta)}{\partial \theta_i} \quad \text{is The } i\text{th element of the score vector with respect to } \theta \quad (5)$$

Now we can find the maximum likelihood estimator by using two parameter Lomax distribution with parameters α, λ . The probability density function is given by :

$$f(x) = \frac{\alpha}{\lambda} (1 + \frac{x}{\lambda})^{-(\alpha+1)} \dots \dots x > 0, \alpha, \lambda > 0 \dots \dots \dots (6)$$

The likelihood function is :

$$L(x_1, x_2, \dots, x_n; \alpha, \lambda) = \prod_{i=1}^n f(x_i, \alpha, \lambda) \quad (7)$$

$$= (\frac{\alpha}{\lambda})^n \prod_{i=1}^n (1 + \frac{x_i}{\lambda})^{-(\alpha+1)} \quad (8)$$

The score vector is :

$$U(\theta) = \frac{\partial \ln(x; \alpha, \lambda)}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^n (1 + \frac{x_i}{\lambda}) \quad (9)$$

Let $U(\theta) = 0$, Then the maximum likelihood estimator is Eq. 10 :

$$\hat{\alpha}_{\text{M.L.E}} = \frac{n}{\sum(1 + \frac{x_i}{\lambda})} \quad (10)$$

The Scale parameter λ is taken to be constant. Since the maximum likelihood estimator is invariant and one to one mapping . The maximum likelihood estimator of Reliability function is Eq. 11 :

$$\hat{R}_{\text{m.l.e}}(x) = (1 + \frac{x}{\lambda})^{-\hat{\alpha}_{\text{m.l.e}}} \quad (11)$$

And the maximum likelihood estimator of Hazard function is Eq. 12 :

$$\hat{H}_{\text{m.l.e}}(x) = \frac{\hat{\alpha}_{\text{m.l.e}}}{x + \lambda} \quad (12)$$

Least Square Method(LSM): We can found quantiles of parametric distribution by least square method as follows:

The Mean square error is given by :

$$K = \sum e_i^2 = \sum (y_i - \beta_0 - \beta_1 x_i)^2 \quad (13)$$

$$\text{And } V2 = \frac{\partial K}{\partial \beta_1} \quad \text{Let } V1 = \frac{\partial K}{\partial \beta_0}$$

And when $V1=0$, $V2=0$ we find that :

$$\hat{\beta}_1 = \frac{n \sum z_i y_i - \sum z_i \sum y_i}{n \sum z_i^2 - (\sum z_i)^2}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Now we can find the Least square estimator ,The cumulative function for Lomax distribution is :

$$F(x_i) = 1 - \left(1 + \frac{x_i}{\lambda}\right)^{-\alpha} \quad \alpha, \lambda > 0 \quad (14)$$

$$1 - F(x_i) = \left(1 + \frac{x_i}{\lambda}\right)^{-\alpha} \quad (15)$$

$$\text{Ln}[1 - F(x_i)] = -\alpha \text{Ln}\left(1 + \frac{x_i}{\lambda}\right) \quad (16)$$

We can put Eq.16 in the form

$$y_i = \beta_0 + \beta_1 z_i \quad (17)$$

Where $\text{Ln}[1 - F(x_i)] = y_i$ and $\beta_0 = 0$ and $\beta_1 = -\alpha$ and

$z_i = \text{Ln}\left(1 + \frac{x_i}{\lambda}\right)$ then the Least square estimator of α is Eq. 18:

$$\hat{\beta}_1 = -\hat{\alpha}_{\text{l.s.m}} = \frac{n \sum z_i y_i - \sum z_i \sum y_i}{n \sum z_i^2 - (\sum z_i)^2} \quad (18)$$

The Least square estimator of Reliability function is Eq.19

$$\hat{R}_{l.s.m}(x) = (1 + \frac{x}{\lambda})^{-\alpha_{l.s.m}} \quad (19)$$

And the Least square estimator of Hazard function is Eq.20

$$\hat{H}_{l.s.m}(x) = \frac{\hat{\alpha}_{l.s.m}}{x + \lambda} \quad (20)$$

Bayes Method(BYS) : Let $x_1, x_2, \dots, x_n$ be a random sample of size n with p.d.f $f(x; \alpha, \lambda)$ In the two parameters Lomax distribution is given by :

$$f(x; \alpha, \lambda) = \frac{\alpha}{\lambda} (1 + \frac{x}{\lambda})^{-(\alpha+1)} \quad (21)$$

The prior distribution for α is

$$g(\alpha) = \frac{1}{\alpha} \quad (22)$$

We can found Bayes estimator with prior by using posterior distribution which equal to conditional distribution ,depend on joint probability density function and marginal probability density function , so the posterior distribution is given by :

$$h(\alpha \setminus x_1, x_2, \dots, x_n) = \frac{L(x_1, x_2, \dots, x_n, \alpha, \lambda) g(\alpha)}{\int_0^{\infty} L(x_1, x_2, \dots, x_n, \alpha, \lambda) g(\alpha) d\alpha} \quad (23)$$

After we evaluate the integral :

$$\begin{aligned} \int_0^{\infty} L(x_1, x_2, \dots, x_n, \alpha, \lambda) g(\alpha) d\alpha &= I \quad (24) \\ I &= \int_0^{\infty} \frac{\alpha^{n-1}}{\lambda^n} e^{-(\alpha+1) \sum \ln(1+\frac{x}{\lambda})} \frac{1}{\alpha} d\alpha \\ I &= \frac{e^{-\sum \ln(1+\frac{x}{\lambda})}}{\lambda^n} \int_0^{\infty} \alpha^{n-1} e^{-\sum \ln(1+\frac{x}{\lambda})} d\alpha \quad (25) \end{aligned}$$

Let $U = \alpha \sum \ln(1 + \frac{x}{\lambda})$, $d\alpha = \frac{du}{\sum \ln(1+\frac{x}{\lambda})}$ Then:

$$I = \frac{e^{-\sum \ln(1+\frac{x}{\lambda})} \Gamma n}{\lambda^n [\alpha \sum \ln(1 + \frac{x}{\lambda})]^n} \quad (26)$$

Where Γ is gamma function

And

$$h(\alpha \setminus x_1, x_2, \dots, x_n) = \frac{\frac{\alpha^{n-1}}{\lambda^n} e^{-(\alpha+1) \sum \ln(1+\frac{x}{\lambda})}}{\frac{e^{-\sum \ln(1+\frac{x}{\lambda})} \Gamma n}{\lambda^n [\alpha \sum \ln(1 + \frac{x}{\lambda})]^n}} \quad (27)$$

$$g(\alpha \backslash x) = \frac{1}{\Gamma n} \left[\sum L n \left(1 + \frac{x}{\lambda} \right) \right]^n \cdot \lambda^{n-1} \cdot e^{-\sum L n \left(1 + \frac{x}{\lambda} \right)} \quad (28)$$

The above posterior distribution is gamma distribution with parameters $[\sum L n \left(1 + \frac{x}{\lambda} \right), n]$. Then the Bayes estimator of α is Eq. 29:

$$\hat{\alpha}_{bys} = \frac{n}{\sum L n \left(1 + \frac{x}{\lambda} \right)} \quad (29)$$

The Bayes estimator of Reliability function is Eq. 30:

$$\hat{R}_{bys}(x) = \left(1 + \frac{x}{\lambda} \right)^{-\hat{\alpha}_{bys}} \quad (30)$$

And the Bayes estimator of Hazard function is Eq. 31:

$$\hat{H}_{bys}(x) = \frac{\hat{\alpha}_{bys}}{x + \lambda} \quad (31)$$

Method of Moment (MOM): We can found quantiles of parametric distribution by using the hypothesis of this method that is (moment of population equal moment of sample) for that estimator .

We have

$$\begin{aligned} \beta(m, n) &= \int_0^1 x^{m-1} (1-x)^{n-1} dx \\ &= \int_0^{\infty} x^{m-1} (1+x)^{-(m+n)} dx \end{aligned}$$

Where $\beta(m, n)$ is Beta function , and for Lomax distribution

$$Ex = \int_0^{\infty} x \frac{\alpha}{\lambda} \left(1 + \frac{x}{\lambda} \right)^{-(\alpha+1)} dx \quad (32)$$

And after we evaluate Eq. 32 we find :

$$\begin{aligned} Ex &= \alpha \lambda \beta(2, \alpha - 1) = \alpha \lambda \frac{\Gamma 2 \Gamma \alpha - 1}{\Gamma \alpha + 1} \\ Ex &= \frac{\alpha \lambda}{\alpha(\alpha - 1)} = \frac{\lambda}{\alpha - 1} \end{aligned} \quad (33)$$

Then the Moment estimator of α is Eq. 34:

$$\hat{\alpha}_{mom} = \frac{\lambda}{\bar{x}} + 1 \quad (34)$$

And the Moment estimator of Reliability function and Hazard function is Eq. 35 and 36 respectively .

$$\hat{R}_{mom}(x) = \left(1 + \frac{x}{\lambda} \right)^{-\hat{\alpha}_{mom}} \quad (35)$$

$$\hat{H}_{mom}(x) = \frac{\hat{\alpha}_{mom}}{x + \lambda} \quad (36)$$

We can reach to the same estimator $\hat{\alpha}_{mom}$ by using Integral 3.241(4) in (Ryhiik 1965) as follows :

$$Ex^r = \alpha \lambda^r \frac{\Gamma(r+1)\Gamma(\alpha-r)}{\Gamma(\alpha+1)} \quad (37)$$

Then

$$Ex = \frac{\lambda}{\alpha - 1} \quad (38)$$

Results

In this simulation study we have chosen $n=10,20,50$ and 100 to represent small , moderate and large sample size , Several values of parameter $\alpha=0.5, 1, 1.5$ and $\lambda=0.5, 1, 1.5$. The number of replication used was ($L=1000$) . The simulation program was written by using Matlab-R2011b program. After the Reliability and Hazard function are estimated ,Integrated Mean Square Error (IMSE) and Integrated Mean Absolute Percentage Error (IMAPE) was calculated to compare the methods of estimation, where:

$$IMSE(\hat{R}(x)) = \frac{1}{L} \sum_{i=1}^L \left\{ \frac{1}{n} \sum_{j=1}^n [R_i(x_j) - \hat{R}_i(x_j)]^2 \right\}$$

$$= \frac{1}{L} \sum_{i=1}^L IMSE(\hat{R}(x))$$

$$IMAPE(\hat{R}(x)) = \frac{1}{L} \sum_{i=1}^L \left\{ \frac{1}{n} \sum_{j=1}^n \left| \frac{R_i(x_j) - \hat{R}_i(x_j)}{R_i(x_j)} \right| \right\}$$

$$= \frac{1}{L} \sum_{i=1}^L IMAPE(\hat{R}(x))$$

And so

$$IMSE(\hat{H}(x)) = \frac{1}{L} \sum_{i=1}^L \left\{ \frac{1}{n} \sum_{j=1}^n [H_i(x_j) - \hat{H}_i(x_j)]^2 \right\}$$

$$= \frac{1}{L} \sum_{i=1}^L IMSE(\hat{H}(x))$$

$$IMAPE(\hat{H}(x)) = \frac{1}{L} \sum_{i=1}^L \left\{ \frac{1}{n} \sum_{j=1}^n \left| \frac{H_i(x_j) - \hat{H}_i(x_j)}{H_i(x_j)} \right| \right\}$$

$$= \frac{1}{L} \sum_{i=1}^L IMAPE(\hat{H}(x))$$

Table 1 : IMSE for different estimators of Reliability and Hazard function for lomax distribution

N	α	λ	Reliability function				Hazard function			
			MLE	LSM	BYS	MOM	MLE	LSM	BYS	MOM
10	0.5	0.5	0.0525	0.0404	0.0382	0.0525	0.0525	0.0404	0.0382	0.0525
		1	0.0000	0.0522	0.0021	0.0000	0.0000	0.0005	0.0005	0.0000
		1.5	0.0190	0.0675	0.0004	0.0190	0.0068	0.0320	0.0001	0.0068
	1	0.5	0.0137	0.0007	0.0193	0.0137	0.0499	0.0080	0.0601	0.0499
		1	0.0009	0.0065	0.0053	0.0009	0.0033	0.0246	0.0036	0.0033
		1.5	0.0173	0.0277	0.0112	0.0173	0.0436	0.0831	0.0239	0.0436
	1.5	0.5	0.0093	0.0003	0.0087	0.0093	0.1312	0.0067	0.0857	0.1312
		1	0.0001	0.0003	0.0076	0.0001	0.0000	0.0053	0.0232	0.0000
		1.5	0.0154	0.0161	0.0054	0.0154	0.2215	0.2515	0.0600	0.2215
20	0.5	0.5	0.0226	0.0401	0.0345	0.0226	0.0141	0.1104	0.0189	0.0141
		1	0.0021	0.0530	0.0001	0.0021	0.0009	0.0410	0.0000	0.0009
		1.5	0.0244	0.0646	0.0085	0.0244	0.0075	0.0285	0.0021	0.0075
	1	0.5	0.0246	0.0005	0.0072	0.0246	0.0799	0.0056	0.0248	0.0799
		1	0.0049	0.0010	0.0113	0.0049	0.0058	0.0021	0.0122	0.0058
		1.5	0.0223	0.0260	0.0075	0.0223	0.0710	0.0839	0.0200	0.0710
	1.5	0.5	0.0161	0.0016	0.0001	0.0161	0.1997	0.0430	0.2482	0.1997
		1	0.0016	0.0025	0.0251	0.0016	0.0201	0.0355	0.0006	0.0201
		1.5	0.0186	0.0162	0.0061	0.0186	0.4060	0.3666	0.1287	0.4060
50	0.5	0.5	0.0238	0.0402	0.0096	0.0238	0.0149	0.1110	0.0068	0.0149
		1	0.0000	0.0489	0.0025	0.0000	0.0000	0.0360	0.0007	0.000
		1.5	0.0047	0.0534	0.0008	0.0047	0.0013	0.0212	0.0002	0.0013
	1	0.5	0.0221	0.0003	0.0054	0.0221	0.0721	0.0028	0.0258	0.0721
		1	0.0000	0.0027	0.0008	0.0000	0.0000	0.0070	0.0011	0.0000
		1.5	0.0170	0.0088	0.0002	0.0170	0.0312	0.0156	0.0007	0.0312
	1.5	0.5	0.0217	0.0014	0.0052	0.0217	0.2482	0.0407	0.1030	0.2482
		1	0.0000	0.0001	0.0032	0.0000	0.0000	0.0001	0.0132	0.0000
		1.5	0.018	0.0159	0.0086	0.0186	0.2905	0.2297	0.0840	0.2908
100	0.5	0.5	0.0173	0.0403	0.0039	0.0172	0.0113	0.1115	0.0029	0.0113
		1	0.0001	0.0492	0.0000	0.0001	0.0000	0.0364	0.0000	0.0000
		1.5	0.0092	0.0527	0.0006	0.0092	0.0023	0.0208	0.0001	0.0023
	1	0.5	0.0314	0.0002	0.0069	0.0314	0.0944	0.0022	0.0319	0.0944
		1	0.0003	0.0014	0.0019	0.0003	0.0003	0.0037	0.0016	0.0003
		1.5	0.0140	0.0104	0.0023	0.0140	0.0231	0.0155	0.0027	0.0231
	1.5	0.5	0.0263	0.0022	0.085	0.0263	0.2738	0.0576	0.0085	0.0263
		1	0.1691	0.0016	0.9192	0.1691	0.0010	0.0000	0.0048	0.0010
		.1	0.0107	0.0085	0.0039	0.0107	0.0964	0.0718	0.0256	0.0964

Table 2 : IMAPE for different estimator of reliability and Hazard function for Lomax distribution .

N	α	λ	Reliability function				Hazard function			
			MLE	LSM	BYS	MOM	MLE	LSM	BYS	MOM
10	0.5	0.5	0.7425	0.6146	0.6326	0.7425	0.0525	0.0404	0.0382	0.0525
		1	0.0124	0.5352	0.1094	0.012	0.0134	1.0626	0.1200	0.0134
		1.5	0.2791	0.5235	0.0404	0.2791	0.5942	1.2860	0.0559	0.5942
	1	0.5	1.2171	0.2336	1.4650	1.2171	0.3368	0.1349	0.3693	0.3368
		1	0.1506	0.4208	0.4414	0.1506	0.1535	0.4165	0.1594	0.1535
		1.5	0.5117	0.6358	0.4177	0.5117	0.7501	1.0361	0.5562	0.7501
	1.5	0.5	3.0664	0.4566	3.1979	3.0664	0.3639	0.0821	0.2941	0.3639
		1	0.1390	0.1721	1.2303	0.1390	0.0057	0.1285	0.2697	0.0057
		1.5	0.7495	0.7575	0.4514	0.7495	1.1277	1.2018	0.5871	1.1277
20	0.5	0.5	0.4856	0.6129	0.6013	0.4856	0.3577	1.0014	0.4148	0.3577
		1	0.1087	0.5390	0.0195	0.1087	0.1559	1.0764	0.0296	0.1559
		1.5	0.3175	0.5129	0.1878	0.3175	0.6235	1.2144	0.3264	0.6235
	1	0.5	1.6462	0.1994	0.8918	1.6462	0.4260	0.1125	0.2372	0.4260
		1	0.4102	0.1743	0.6290	0.4102	0.2024	0.1232	0.2934	0.2024
		1.5	0.5663	0.6111	0.3290	0.5663	0.9576	1.0409	0.5089	0.9576
	1.5	0.5	4.1178	1.0922	5.3992	4.1178	0.4489	0.2083	0.5005	0.4489
		1	0.4074	0.5020	0.0778	0.4074	0.2514	0.3335	0.0446	0.2514
		1.5	0.7876	0.7336	0.3932	0.7876	0.7876	0.7336	0.3932	0.7876
50	0.5	0.5	0.4979	0.6138	0.3151	0.4979	0.3679	1.0043	0.2483	0.3679
		1	0.0141	0.5189	0.1199	0.0141	0.0097	1.0088	0.1364	0.0097
		1.5	0.1393	0.4674	0.0579	0.1393	0.2544	1.0478	0.0992	0.2544
	1	0.5	1.5597	0.1486	0.7418	1.5597	0.4045	0.0796	0.2423	0.4045
		1	0.0146	0.2816	0.1663	0.0146	0.0099	0.2220	0.0867	0.0312
		1.5	0.5188	0.3760	0.0400	0.5188	0.6346	0.4494	0.0982	0.6346
	1.5	0.5	4.8217	0.9802	0.0896	4.8271	0.5005	0.2028	0.3224	0.5005
		1	0.0804	0.1181	0.7657	0.0804	0.0102	0.0213	0.0237	0.0102
		1.5	0.0850	0.7625	0.5981	0.8050	1.2922	1.1483	0.6945	1.2922
100	0.5	0.5	0.4234	0.6145	0.2009	0.4234	0.3198	1.0064	0.1624	0.3198
		1	0.0230	0.5203	0.0019	0.0230	0.0258	1.0132	0.0078	0.0258
		1.5	0.1962	0.4642	0.0480	0.1962	0.3446	1.0370	0.0798	0.3446
	1	0.5	1.8738	0.1355	0.8285	1.8738	0.4629	0.0701	0.2692	0.4629
		1	0.1023	0.1995	0.2629	0.1023	0.0490	0.1606	0.1066	0.0490
		1.5	0.4761	0.4151	0.1974	0.4761	0.5468	0.4478	0.1861	0.5468
	1.5	0.5	5.4201	1.2747	2.7818	5.4201	0.5257	0.2410	0.3792	0.5257
		1	0.1606	0.0098	0.3825	0.1606	0.0554	0.0098	0.1232	0.0554
		1.5	0.6737	0.6091	0.4299	0.6737	0.7439	0.6421	0.3830	0.7439

The results of the simulation study are summarized and tabulated in table 1 and 2 of the four estimators for all sample size and (α, λ) values respectively .

In each row of Table 1 and 2 we have four values of estimator that is the maximum likelihood, Least square method , Bayes method and Moment method . The best method that gives smallest value of (IMSE) and (IMAPE).

Discussion:

In table 1 and 2 ,When we compared reliability and hazard estimation for Lomax distribution by using mentioned method we find the best estimator is bayes (BYS) for small values of α and Least square method for large values and this result is true for all values of parameters and sample sizes used in the study.

Conclusion:

The estimated Reliability and Hazard function of Lomax distribution from Bayes estimation is the best compared for small values of α and Least square method for large values of α ,The maximum likelihood and moment methods gives the same values of IMSE and IMAPE for all values of α, λ and sample size .

When the number of sample size increases the Integrated Mean Square Error (IMSE) for Reliability and Hazard function decreases in all cases, But Integrated absolute mean square Error decreases in all cases except for hazard function it is increases.

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