IRAQI STATISTICIANS JOURNAL

https://isj.edu.iq/index.php/isj

ISSN: 3007-1658 (Online)

Evaluation of Relative Loss in Odd Generalised Exponential Burr Type X and Truncated Odd Generalised Exponential Burr Type X Distributions

Murtadha M.Jasim[1], Moudher Kh. Abdal-hammed[2], Mizal Alobaidy[3]

[1,3] Department of Mathematics, Collage of Computer Science and Mathematics, Tikrit University, Tikrit, Iraq.
[2] Department of public Administration, Collage of Administration and Economics, Tikrit University, Tikrit, Iraq.

ARTICLE INFO

Article history:

Received	11 June 2025
Revised	16 June 2025
Accepted	21 July 2025
Available online	21 July 2025

Keywords:
OGE Burr Type X
Truncated Burr Type X
Entropies
Relative loss
Burr Type X distribution

ABSTRACT

This paper presents derivation entropy measures for two types of distributions: the Odd Generalised Exponential Burr type X distribution (OGEBX) and the truncated Odd Generalised Exponential Burr type X distribution (TOGEBX). The relative loss was achieved by testing a set of different parameter values. In the context of certain distributions, the optimistic results of this study can be applied. Based on the lowest value of each distribution, a numerical comparison was made between the entropy measures to select the optimal one. The relative loss was then determined using all six measures. When the OGE Burr Type X distribution is not used on [0, ∞], but the truncated OGE Burr Type X distribution on [0, t] is used.

1. Introduction

Shannon (1948) first proposed the idea of entropy of random variables in information theory, even though other disciplines now use other nomenclature. The statistical measure of dispersion and uncertainty, the information theoretic measure of average message size, and the physical measure of the number of alternative configurations of a thermodynamic system are all quantified by entropy. A large number of entropy measures are available in the published works. A large number of researchers have looked into the relative of various loss distributions. They use the truncated Burr Type X distribution to talk about the relative loss of entropy metrics., Dey et al. [1] and Basit et al. [2] employed the weighted and truncated weighted exponential (SBME, TSBME) distributions, respectively. The size-biased exponential distribution's entropy metrics were covered by Mir et al. [3]. When it comes to lifespan data, Award and Alawneh [4] covered the practical use of entropy measurements. The entropy measures for the Lomax distribution were studied by Ijaz et al. [5]. The researchers Al-Babtain et al. [6] presented a study to estimate different types of entropy for the Kumaraswamy distribution. The researchers Teixeira,et al. [7] presented a study on entropy measures versus Kolmogorov complexity. The researchers Baez, et al. [8] presented a study to A Characterization of

Corresponding author E-mail address: Murtadha1989@st.tu.edu.iq
https://doi.org/10.62933/0vvgte14

Entropy in Terms of Information Loss. Zaman et al. [9] conducted a study to compare the entropy measures of the Pareto and Truncated Pareto Distributions. Ahsan et al. [10] conducted a study to compare the entropy measures of the Gompertz and Truncated Gompertz Distribution. Ebrahimi, et al. [11] they studied two measures of the entropy sample. Abo-Eleneen, et al. [12] conducted a study The entropy of progressively censored samples, 13. Cho, et al. [13] presented a study. An estimation of the entropy for a Rayleigh distribution based on doubly-generalized Type-II hybrid censored samples, Hassan, et al. [14] presented a study to Estimation of entropy for inverse Weibull distribution under multiple censored data. Bantan, et al. [15] presented a study to Estimation of entropy for inverse Lomax distribution. Kayal, et al. in [16]. Cho, et al. [17] presented a study to Estimating the entropy of a Weibull distribution under generalized progressive hybrid censoring. Noorizadeh, et al. [18] presented a study of Shannon entropy as a new measure of aromaticity, Shannon aromaticity, can see more [19], [20], [21], [22], [23], [24], [25].

When dealing with continuous nonnegative random variables X, the probability density function f(x) can be expressed. are able to mathematically represent the Shannon [26], [27] entropy as:

$$H(x) = -\int_{-\infty}^{\infty} f(x) ln f(x) \quad (1)$$

A new generalised entropy was developed by Renyi [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41]. after studies on uncertainty and randomness. The Renyi entropy is supposedly:

$$H_{\varphi}(x) = \frac{1}{1-\varphi}\int_{-\infty}^{\infty}[f(x)]^{\varphi}dx\,, \quad \varphi > 0\,, \varphi \neq 0 \quad (2)$$

The entropy that Havrda and Charvat [24], [43] suggest is defined as

$$H_{\varphi}(x) = \frac{1}{2^{1-\varphi}-1}\left[\int_{-\infty}^{\infty}[f(x)]^{\varphi}dx - 1\right], \quad \varphi > 0\,, \varphi \neq 0 \quad (3)$$

The entropy of Arimoto [44], [38] is provided by

$$A_{\varphi}(x) = \frac{1}{2^{1-\varphi}-1}\left[\int_{-\infty}^{\infty}\left\{[f(x)]^{\frac{1}{\varphi}}dx\right\}^{\varphi} - 1\right], \quad \varphi > 0\,, \varphi \neq 0 \quad (4)$$

This is the mathematical expression of the entropy measure, as stated by Sharma - Mittals [45], [46].

$$H_{\varphi}(x) = \frac{1}{2^{1-\varphi}-1}\left[exp\left\{(\varphi-1)\int_{-\infty}^{\infty} f(x) ln f(x)\right\} - 1\right], \varphi > 0\,, \varphi \neq 0 \quad (5)$$

The Tsallis entropy[47], [48] is given by

$$H_{\varphi}(x) = \frac{1}{1-\varphi}\left(1 - \int_{-\infty}^{\infty}[f(x)]^{\varphi}dx\right), \varphi > 0\,, \varphi \neq 0 \quad (6)$$

The motivation: Finding an entropy measure for OGE Burr Type X and Truncated OGE Burr Type X distributions is the motivation for this project. A numerical comparison of the performance of various entropy measurements will be conducted using the obtained relative entropy loss expression. Importance of the study: Studying the relative loss function is important because it helps decide which entropy is the most efficient by comparing the OGEBX distribution's entropy measurements before and after cutting, which is a way to evaluate statistical models' accuracy. Advancement in Science. Scientific Contribution: The study's scientific contribution is the application of relative loss to truncation impact analysis; this method improves our theoretical understanding of statistical processes by shedding light on how truncation affects the amount of uncertainty and the information that is available. In addition to assisting with model improvement, it provides tools for estimating the correctness of statistical models by comparing entropy before and after truncation.

The following is the outline of the paper. In Section 2, the OGE Burr Type X (OGEBX) and Truncated OGE Burr Type X (TOGEBX) distributions are reviewed. A number of entropy measures, derived from OGER Burr Type X distributions and truncated TOGE Burr Type X distributions, are discussed in section 3. Numerical comparisons based on the relative entropy loss using the simplified Rayleigh distribution instead of the Burr Type X distribution are presented in section four. This study concludes with a discussion and analysis of the numerical data in the final section.

2. OGE Burr Type X (OGEBX) and Truncated Burr Type X (TOGEBX) Distributions.

In 2025, Murtadha et al. [49] introduced the Odd Generalized Exponential -G family of generalized distributions. The cumulative distribution function (CDF) and probability density function (PDF) are these:

$$F(x,\beta,s) = \left[1 - e^{-s\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\beta}, x > 0, \beta, s > 0 \quad (7)$$

$$f(x,\beta,S) = \beta S F(x) f(x)\big(2 - F(x)\big)\big(1 - F(x)\big)^{-2} \times \left[1 - e^{-S\left(\frac{(F(x))^2}{1-F(x)}\right)}\right]^{\beta-1} e^{-S\left(\frac{(F(x))^2}{1-F(x)}\right)} \quad (8)$$

According to Burr Type X Distribution, the CDF and PDF are stated as

$$G(x,e,u) = \left(1 - e^{-(ux)^2}\right)^{e} \quad (9)$$

$$g(x,e,u) = 2u^2 e x e^{-(ux)^2}\left(1 - e^{-(ux)^2}\right)^{e-1}, x > 0, e, u > 0 \quad (10)$$

By substituting equations 10 and 9 into equations 8 and 7, equations 11 and 12 are obtained. Assuming that β,s,e and u are parameters that are unknown and that X is a OGE Burr Type X Distribution random variable with a CDF.

$$G(x;\beta,s,e,u) = \left[1 - e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\beta}, x > 0, \beta, s, e, u > 0 \quad (11)$$

Additionally, the probability density function for the GEBTX distribution is provided as follows.

$$g(x;\beta,s,e,u) = \beta\gamma\left(1 - e^{-(ux)^2}\right)^{e} 2u^2 s x e^{-(ux)^2} \times \left(1 - e^{-(ux)^2}\right)^{e-1}\left(2 - \left(1 - e^{-(ux)^2}\right)^{e}\right) \times \left(1 - \left(1 - e^{-(ux)^2}\right)^{e}\right)^{-2}\left[1 - e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\beta-1} \times e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)} \quad (12)$$

Assume that Y is a Truncated OGE Burr Type X Distribution random variable, and that its cdf and pdf are provided by

$$G(y;\beta,s,e,u) = \frac{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)}\right]^{\beta}}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ut)^2}\right)^{e}}\right)}\right]^{\beta}} \quad (13)$$

and

$$g(y;\beta,s,e,u) = \beta s\left(1 - e^{-(uy)^2}\right)^{s} 2u^2 e y e^{-(ux)^2} \times \left(1 - e^{-(uy)^2}\right)^{e-1}\left(2 - \left(1 - e^{-(uy)^2}\right)^{e}\right) \times \left[1 - e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ut)^2}\right)^{e}}\right)}\right]^{-\beta}\left(1 - \left(1 - e^{-(uy)^2}\right)^{e}\right)^{-2} \times \left[1 - e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)}\right]^{\beta-1} e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)} \quad (14)$$

The relative loss of entropy when utilizing Y instead of X is defined as the product of the two corresponding entropies, H(X) and H(Y)

$$S_H(t) = \frac{H(X)-H(Y)}{H(X)} \quad \text{or}$$

$$S_H(t) = 1 - \frac{H(Y)}{H(X)} \quad (15)$$

3. Entropies of OGEBX and TOGEBX Distribution

The entropy measures for OGE Burr Type distributions and truncated OGE Burr Type distributions are located in this section. Using multiple entropy measurements, the relative entropy loss is found when Y is replaced by X.

3.1 Shannon Entropy of OGEBX and TOGEBX Distribution.

The Shannon entropy for the OGEBX and TOGEBX distribution can be found using the following mathematical formula.

$$H(X) = -\int_0^t \beta\gamma\left(1-e^{-(ux)^2}\right)^e 2u^2 sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^e\right)$$

$$\times\left(1-\left(1-e^{-(ux)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}\right]^{\beta-1} e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}$$

$$\times ln\beta\gamma\left(1-e^{-(ux)^2}\right)^e 2u^2 sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^e\right)$$

$$\times\left(1-\left(1-e^{-(ux)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}\right]^{\beta-1} e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)} dx \qquad (16)$$

and

$$H(Y) = -\int_0^t \frac{\beta s\left(1-e^{-(uy)^2}\right)^s 2u^2 eye^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^e\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^e\right)^2}{1-\left(1-e^{-(ut)^2}\right)^e}\right)}\right]^{\beta}}$$

$$\times\left(1-\left(1-e^{-(uy)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\beta-1} e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}$$

$$\times ln\frac{\beta s\left(1-e^{-(uy)^2}\right)^\delta 2u^2 eye^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^e\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^e\right)^2}{1-\left(1-e^{-(ut)^2}\right)^e}\right)}\right]^{\beta}}$$

$$\times\left(1-\left(1-e^{-(uy)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\beta-1} e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)} \qquad (17)$$

3.2 Renyi Entropy of OGEBX and TOGEBX Distributions.

The Renyi entropy for the OGEBX and TOGEBX distribution can be found using the following mathematical formula.

$$R_{\varphi}(y)=\frac{1}{1-\varphi}\int_{0}^{t}\beta\gamma\left(1-e^{-(ux)^2}\right)^{e}2u^{2}sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^{e}\right)$$
$$\times\left(1-\left(1-e^{-(ux)^2}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}dx \qquad (18)$$

And

$$R_{\varphi}(y)=\frac{1}{1-\varphi}\int_{0}^{t}\frac{\beta s\left(1-e^{-(uy)^2}\right)^{s}2u^{2}eye^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^{e}\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ut)^2}\right)^{e}}\right)}\right]^{\beta}}$$
$$\times\left(1-\left(1-e^{-(uy)^2}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)}dy \qquad (19)$$

3.3 Tsallis Entropy of OGEBX and TOGEBX) Distributions.

The Tsallis entropy for the OGEBX and TOGEBX distribution can be found using the following mathematical formula.

$$T_{\varphi}(x)=\frac{1}{1-\varphi}\left(1-\int_{0}^{t}\left[\beta\gamma\left(1-e^{-(ux)^2}\right)^{e}2u^{2}sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^{e}\right)\right.\right.$$
$$\left.\left.\times\left(1-\left(1-e^{-(ux)^2}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\varphi}dx\right) \qquad (20)$$

and

$$T_{\omega}(y)=\frac{1}{1-\omega}\left(1-\int_{0}^{t}\left[\frac{\beta s\left(1-e^{-(uy)^{2}}\right)^{\delta}2u^{2}eye^{-(ux)^{2}}\left(1-e^{-(uy)^{2}}\right)^{e-1}\left(2-\left(1-e^{-(uy)^{2}}\right)^{e}\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^{2}}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ut)^{2}}\right)^{e}}\right)}\right]^{\beta}}\right.\right.$$

$$\left.\left.\times\left(1-\left(1-e^{-(uy)^{2}}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^{2}}\right)^{e}\right)^{2}}{1-\left(1-e^{-(uy)^{2}}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(uy)^{2}}\right)^{e}\right)^{2}}{1-\left(1-e^{-(uy)^{2}}\right)^{e}}\right)}\right]^{\varphi}dy\right) \quad (21)$$

3.4 Arimoto Entropy of OGEBX and TOGEBX Distributions

The Arimoto entropy for the OGEBX and TOGEBX distribution can be found using the following mathematical formula.

$$A_{\varphi}(x)=\frac{1}{2^{1-\varphi}-1}\left[\int_{0}^{t}\left\{\left[\beta\gamma\left(1-e^{-(ux)^{2}}\right)^{e}2u^{2}sxe^{-(ux)^{2}}\left(1-e^{-(ux)^{2}}\right)^{e-1}\left(2-\left(1-e^{-(ux)^{2}}\right)^{e}\right)\right.\right.\right.$$

$$\left.\left.\left.\times\left(1-\left(1-e^{-(ux)^{2}}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^{2}}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ux)^{2}}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(ux)^{2}}\right)^{e}\right)^{2}}{1-\left(1-e^{-(ux)^{2}}\right)^{e}}\right)}\right]^{\frac{1}{\omega}}dx\right\}^{\varphi}-1\right] \quad (22)$$

and

$$A_{\varphi}(y)=\frac{1}{2^{1-\varphi}-1}\left[\int_0^t\left\{\left[\frac{\beta s\left(1-e^{-(uy)^2}\right)^{s}2u^2eye^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^{e}\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ut)^2}\right)^{e}}\right)}\right]^{\beta}}\right.\right.\right.$$

$$\left.\left.\left.\times\left(1-\left(1-e^{-(uy)^2}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(uy)^2}\right)^{e}}\right)}\right]^{\frac{1}{\varphi}}dy\right\}^{\varphi}-1\right]\quad(23)$$

3.5 Havrda and Charvat Entropy of OGEBX and TOGEBX Distributions.

The Havrda and Charvat entropy for the OGEBX and TOGEBX distribution can be found using the following mathematical formula.

$$H_{\varphi}(x)=\frac{1}{2^{1-\varphi}-1}\left[\int_0^t\left[\beta\gamma\left(1-e^{-(ux)^2}\right)^{e}2u^2sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^{e}\right)\right.\right.$$

$$\left.\left.*\left(1-\left(1-e^{-(ux)^2}\right)^{e}\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\beta-1}e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ux)^2}\right)^{e}}\right)}\right]^{\varphi}dy-1\right]\quad(24)$$

and

$$H_{\varphi}(y)=\frac{1}{2^{1-\varphi}-1}\left[\int_0^t\left[\frac{\beta s\left(1-e^{-(uy)^2}\right)^{s}2u^2eye^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^{e}\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^{e}\right)^2}{1-\left(1-e^{-(ut)^2}\right)^{e}}\right)}\right]^{\beta}}\right.\right.$$

$$\times\left(1-\left(1-e^{-(uy)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\beta-1}\left. e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\varphi} dy-1\Bigg] \quad (25)$$

3.6 Sharma and Mittal of OGEBX and TOGEBX Distribution.

The Sharma and Mittal entropy for the OGEBX and TOGEBX distribution can be found using the following mathematical formula.

$$S_\varphi(y)=\frac{1}{2^{1-\varphi}-1}\left[Exp\{(\varphi-1)\int_0^t\left[\beta\gamma\left(1-e^{-(ux)^2}\right)^e 2u^2sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^e\right)\right.\right.$$

$$\times\left(1-\left(1-e^{-(ux)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}\right]^{\beta-1}\left. e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}\right]^{\omega}$$

$$\times ln\left[\beta\gamma\left(1-e^{-(ux)^2}\right)^e 2u^2sxe^{-(ux)^2}\left(1-e^{-(ux)^2}\right)^{e-1}\left(2-\left(1-e^{-(ux)^2}\right)^e\right)\right.$$

$$\times\left(1-\left(1-e^{-(ux)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}\right]^{\beta-1}\left.\left. e^{-s\left(\frac{\left(\left(1-e^{-(ux)^2}\right)^e\right)^2}{1-\left(1-e^{-(ux)^2}\right)^e}\right)}\right]^{\varphi}\right\}-1\Bigg]dx \quad (26)$$

and

$$S_\varphi(y)=\frac{1}{2^{1-\varphi}-1}\left[Exp\{(\varphi-1)\int_0^t\left[\frac{\beta s\left(1-e^{-(uy)^2}\right)^s 2u^2eye^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^e\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^e\right)^2}{1-\left(1-e^{-(ut)^2}\right)^e}\right)}\right]^{\beta}}\right.\right.$$

$$\times\left(1-\left(1-e^{-(uy)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\beta-1}\left. e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\varphi}$$

$$\times \ln\left[\frac{\beta s\left(1-e^{-(uy)^2}\right)^s 2u^2 e y e^{-(ux)^2}\left(1-e^{-(uy)^2}\right)^{e-1}\left(2-\left(1-e^{-(uy)^2}\right)^e\right)}{\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(ut)^2}\right)^e\right)^2}{1-\left(1-e^{-(ut)^2}\right)^e}\right)}\right]^\beta}\right.$$

$$\left.\times\left(1-\left(1-e^{-(uy)^2}\right)^e\right)^{-2}\left[1-e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\right]^{\beta-1} e^{-s\left(\frac{\left(\left(1-e^{-(uy)^2}\right)^e\right)^2}{1-\left(1-e^{-(uy)^2}\right)^e}\right)}\Bigg]^{\varphi}\Bigg\}-1\right]dy \qquad (27)$$

4. Results and discussion

The numerical analysis of entropy values for various values of β, s, e, t, and u is used in this section. Here are the entropy values for all entropy measures for OGE Burr Type X: Tables 1, 2, and 3. Truncated OGE Burr Type X entropy measures are also shown in Tables 4, 5, and 6. Changing the values of the parameters allows one to determine the relative loss. Tables 7, 8, and 9 display the relative loss of all understudy entropies when comparing Truncated OGE Burr Type X (Y) to OGE Burr Type X (X).

The entropy values for the OGE Burr Type X distribution are presented in Tables 1-3. It is explained that entropy measures such as Shannon, Rényi, Tsallis, Havrda-Charvat, Arimoto, and Sharma-Mittal significantly decrease with an increase in the parameter u.

Tables 4-6 show the entropy values for the truncated Burr Type X distribution. It can be observed that the values of Shannon and Sharma-Mittal entropy increase, while Renyi, Tsallis, Havrda-Charvat, and Arimoto entropy decrease with an increase in the cutoff parameter t.

Tables 7, 8, and 9 related to relative loss show a relative decrease in loss for Shannon and Sharma-Mittal entropy measures, and an increase followed by a decrease for Renyi, Tsallis, Havrda-Charvat, and Arimoto entropy measures with an increase in the cut-off factor t.

The Tsallis entropy measure has a lower relative loss than the other entropy measures in Table 7. Arimo has a lower relative loss than other entropy measures in Table 8, and Sharma-Mittal has a lower relative loss than other entropy measures in Table 9.

Table1. Entropy values for OGE Burr Type X distribution with respect to $\beta = 0.6, s = 0.3, e = 0.5. \varphi = 0.5$

u	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	0.61021277	1.6061514	0.4738409	0.6061515	0.6454166	0.8613173
0.8	0.37628817	1.1437911	0.1343483	0.1437911	0.1677437	0.4997601
1.0	0.20683505	0.8696249	-0.139693	-0.130375	-0.162870	0.2630390
1.2	0.07802597	0.6890061	-0.372505	-0.310993	-0.410262	0.0960470
1.4	-0.0228079	0.5615442	-0.577064	-0.438455	-0.605092	-0.027375
1.6	-0.1032058	0.4671263	-0.761155	-0.532873	-0.764179	-0.121420
1.8	-0.1680192	0.3946225	-0.929825	-0.605377	-0.897629	-0.194531
2.0	-0.2205538	0.3373808	-1.086542	-0.662619	-1.011929	-0.252077
2.2	-0.2631649	0.2911841	-1.233799	-0.708815	-1.111468	-0.297655

2.4	-0.2975915	0.2532312	-1.373452	-0.746768	-1.199330	-0.333776
2.6	-0.3251557	0.2215901	-1.506925	-0.778409	-1.277761	-0.362252
2.8	-0.3468876	0.1948857	-1.635341	-0.805114	-1.348438	-0.384428
3.0	-0.3636075	0.1721132	-1.759602	-0.827886	-1.412640	-0.401326
3.2	-0.3759807	0.1525213	-1.880450	-0.847478	-1.471367	-0.413741
3.4	-0.3845561	0.1355379	-1.998503	-0.864462	-1.525409	-0.422300
3.6	-0.3897934	0.1207191	-2.114288	-0.879280	-1.575403	-0.427509
4.8	-0.3920833	0.1077158	-2.228259	-0.892284	-1.621866	-0.429783
4.0	-0.3927617	0.0962494	-2.340811	-0.903750	-1.665225	-0.429864
4.2	-0.3931216	0.0860954	-2.452299	-0.913904	-1.705834	-0.431842
4.4	-0.3944209	0.0770702	-2.563037	-0.922929	-1.743991	-0.432165
4.6	-0.3978898	0.0690228	-2.673317	-0.930977	-1.779946	-0.435650

Table 2. Entropy values for OGE Burr Type X distribution with respect to $\beta = 0.7, s = 0.4, e = 0.6, \varphi = 0.5$

u	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	1.0854128	3.201784	1.1637080	2.2017836	1.9056663	1.7398208
0.8	1.0196190	2.999532	1.0984564	1.9995324	1.7670011	1.6053892
1.0	0.9644741	2.840452	1.0439632	1.8404519	1.6546152	1.4960729
1.2	0.9169177	2.710091	0.9969822	1.7100911	1.5601503	1.4041901
1.4	0.8750553	2.600171	0.9555771	1.6001707	1.4787166	1.3250970
1.6	0.8376308	2.505507	0.9184911	1.5055069	1.4071951	1.2557767
1.8	0.8037676	2.422641	0.8848584	1.4226413	1.3434703	1.1941608
2.0	0.7728283	2.349154	0.8540554	1.3491544	1.2860399	1.1387702
2.2	0.7443351	2.283289	0.8256169	1.2832890	1.2337973	1.0885111
2.4	0.7179197	2.223730	0.7991860	1.2237300	1.1859043	1.0425523
2.6	0.6932922	2.169469	0.7744823	1.1694687	1.1417097	1.0002476
2.8	0.6702203	2.119715	0.7512817	1.1197151	1.1006982	0.9610849
3.0	0.6485147	2.073839	0.7294017	1.0738395	1.0624547	0.9246514
3.2	0.6280190	2.031332	0.7086919	1.0313323	1.0266398	0.8906100
3.4	0.6086027	1.991776	0.6890265	0.9917755	0.9929727	0.8586815
3.6	0.5901556	1.954822	0.6702994	0.9548224	0.9612183	0.8286325
4.8	0.5725835	1.920182	0.6524202	0.9201823	0.9311777	0.8002656
4.0	0.5558059	1.887609	0.6353111	0.8876092	0.9026815	0.7734127
4.2	0.5397526	1.856893	0.6189048	0.8568933	0.8755839	0.7479293
4.4	0.5243628	1.827854	0.6031426	0.8278540	0.8497586	0.7236903
4.6	0.5095829	1.800335	0.5879730	0.8003354	0.8250956	0.7005868

Table 3. Entropy values for OGE Burr Type X distribution with respect to $\beta = 0.09, s = 0.7, e = 0.8. \varphi = 0.5$

u	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	0.4597317	1.786983	0.5805285	0.7869826	0.8130605	0.6239081
0.8	0.4086278	1.696762	0.5287217	0.6967619	0.7305365	0.5472615
1.0	0.3653969	1.624587	0.4852535	0.6245868	0.6629256	0.4839348
1.2	0.3279277	1.564717	0.4477052	0.5647173	0.6056939	0.4301446
1.4	0.2948492	1.513757	0.4145948	0.5137573	0.5561103	0.3834879
1.6	0.2652264	1.469533	0.3849444	0.4695326	0.5123994	0.3423555
1.8	0.2383950	1.430569	0.3580722	0.4305689	0.4733402	0.3056210
2.0	0.2138657	1.395822	0.3334833	0.3958217	0.4380566	0.2724671
2.2	0.1912685	1.364525	0.3108061	0.3645246	0.4058985	0.2422821
2.4	0.1703160	1.336099	0.2897542	0.3360990	0.3763699	0.2145973
2.6	0.1507811	1.310098	0.2701020	0.3100981	0.3490836	0.1890455
2.8	0.1324809	1.286170	0.2516685	0.2861696	0.3237319	0.1653341
3.0	0.1152659	1.264031	0.2343060	0.2640312	0.3000660	0.1432259
3.2	0.0990125	1.243453	0.2178926	0.2434535	0.2778819	0.1225265

3.4	0.0836170	1.224247	0.2023262	0.2242473	0.2570102	0.1030743
3.6	0.0689919	1.206255	0.1875208	0.2062553	0.2373089	0.0847337
4.8	0.0550625	1.189345	0.1734027	0.1893449	0.2186575	0.0673897
4.0	0.0417644	1.173404	0.1599086	0.1734037	0.2009533	0.0509442
4.2	0.0290420	1.158335	0.1469839	0.1583353	0.1841077	0.0353125
4.4	0.0168466	1.144057	0.1345807	0.1440570	0.1680438	0.0204215
4.6	0.0051355	1.130497	0.1226571	0.1304967	0.1526946	0.0062071

Table 4. Entropies for a Truncated OGE Burr Type X distribution with respect to $\beta = 0.6, s = 0.3, e = 0.3, , u = 0.5, \varphi = 0.5$

t	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	0.6567639	10.805930	2.380095	9.805930	5.521880	0.9384514
0.8	0.9758246	8.956196	2.192346	7.956196	4.810780	1.5183278
1.0	1.1348842	7.724448	2.044390	6.724448	4.295584	1.8438551
1.2	1.2187212	6.835560	1.922138	5.835560	3.897724	2.0261408
1.4	1.2627374	6.160485	1.818156	5.160485	3.577943	2.1249480
1.6	1.2840931	5.629872	1.728087	4.629872	3.314076	2.1736761
1.8	1.2920060	5.202848	1.649206	4.202848	3.092548	2.1918638
2.0	1.2918199	4.853678	1.579737	3.853678	2.904556	2.1914351
2.2	1.2868241	4.565351	1.518495	3.565351	2.744160	2.1799452
2.4	1.2791458	4.326193	1.464688	3.326193	2.607231	2.1623412
2.6	1.2702171	4.127963	1.417784	3.127963	2.490838	2.1419554
2.8	1.2610325	3.964682	1.377426	2.964682	2.392850	2.1210802
3.0	1.2522931	3.831883	1.343356	2.831883	2.311657	2.1013054
3.2	1.2444853	3.726060	1.315351	2.726060	2.245944	2.0837118
3.4	1.2379216	3.644249	1.293150	2.644249	2.194500	2.0689744
3.6	1.2327569	3.583654	1.276383	2.583654	2.156023	2.0574121
3.8	1.2289957	3.541351	1.264508	2.541351	2.128969	2.0490107
4.0	1.2265036	3.514094	1.256782	2.514094	2.111451	2.0434528
4.2	1.2250359	3.498311	1.252280	2.498311	2.101277	2.0401826
4.4	1.2242904	3.490370	1.250008	2.490370	2.096148	2.0385226
4.6	1.2239762	3.487038	1.249053	2.487038	2.093995	2.0378233

Table 5. Entropies for a Truncated OGE Burr Type X distribution with respect to $\beta = 0.7, s = 0.4, e = 0.35,$ $u = 0.6, \varphi = 0.5$

t	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	-1.712834	16.682271	2.814347	15.682271	7.446385	-1.388943
0.8	-0.242527	12.440570	2.520963	11.440570	6.101006	-0.275702
1.0	0.4517567	9.884873	2.291006	8.884873	5.176127	0.6118176
1.2	0.8120630	8.184489	2.102241	7.184489	4.492501	1.2091584
1.4	1.0075875	6.979988	1.943047	5.979988	3.964058	1.5812810
1.6	1.1145799	6.090431	1.806719	5.090431	3.543776	1.8008451
1.8	1.1714392	5.414923	1.689159	4.414923	3.203658	1.9223976
2.0	1.1991331	4.892931	1.587791	3.892931	2.926020	1.9828641
2.2	1.2098383	4.486119	1.500988	3.486119	2.699202	2.0064628
2.4	1.2109703	4.169169	1.427717	3.169169	2.515259	2.0089657
2.6	1.2072007	3.924724	1.367296	2.924724	2.368565	2.0006369
2.8	1.2015146	3.740340	1.319177	2.740340	2.254866	1.9881029
3.0	1.1957769	3.606444	1.282722	2.606444	2.170532	1.9754915
3.2	1.1910403	3.514796	1.256982	2.514796	2.111903	1.9651076
3.4	1.1877266	3.457345	1.240501	2.457345	2.074760	1.9578577
3.6	1.1857801	3.425605	1.231278	2.425605	2.054107	1.9536048
3.8	1.1848508	3.410912	1.226980	2.410912	2.044514	1.9515757
4.0	1.1845084	3.405567	1.225411	2.405567	2.041019	1.9508282
4.2	1.1844177	3.404159	1.224998	2.404159	2.040098	1.9506304
4.4	1.1844021	3.403916	1.224927	2.403916	2.039939	1.9505963
4.6	1.1844005	3.403892	1.224920	2.403892	2.039923	1.9505929

Table 6. Entropies for a Truncated OGE Burr Type X distribution with respect to $\beta = 0.9, s = 0.5, e = 0.4, u = 0.8, \varphi = 0.5$

t	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	-12.287566	31.586248	3.452722	30.58624	11.154064	-2.409030
0.8	-4.7342582	19.731815	2.982232	18.73181	8.309846	-2.187882
1.0	-1.7003390	13.693463	2.616919	12.69346	6.519506	-1.382517
1.2	-0.3063591	10.176300	2.320062	9.176300	5.287204	-0.342877
1.4	0.3836302	7.946227	2.072697	6.946227	4.391226	0.5104771
1.6	0.7377371	6.450187	1.864109	5.450187	3.717217	1.0769746
1.8	0.9204644	5.408481	1.687968	4.408481	3.200316	1.4109674
2.0	1.0123435	4.666885	1.540492	3.666885	2.801206	1.5907935
2.2	1.0555337	4.134712	1.419418	3.134712	2.494846	1.6782226
2.4	1.0731925	3.755801	1.323302	2.755801	2.264506	1.7145164
2.6	1.0783864	3.493456	1.250891	2.493456	2.098142	1.7252524
2.8	1.0784320	3.321677	1.200470	2.321677	1.985804	1.7253468
3.0	1.0771319	3.219448	1.169210	2.219448	1.917567	1.7226568
3.2	1.0760717	3.167175	1.152840	2.167175	1.882256	1.7204644
3.4	1.0755624	3.145911	1.146104	2.145911	1.867809	1.7194117
3.6	1.0754049	3.139692	1.144125	2.139692	1.863574	1.7190861
3.8	1.0753754	3.138543	1.143759	2.138543	1.862791	1.7190251
4.0	1.0753725	3.138430	1.143723	2.138430	1.862714	1.7190190
4.2	1.0753724	3.138425	1.143722	2.138425	1.862711	1.7190188
4.4	1.0753723	3.138424	1.143721	2.138424	1.862710	1.7190186
4.6	1.0753722	3.138423	1.143719	2.138423	1.862709	1.7190183

Table 7. Relative loss for entropies using $\beta = 0.6, s = 0.3, e = 0.3, , u = 0.5, \varphi = 0.5$

t	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	-0.076286	-5.727840	-4.022983	-15.17735	-7.555528	-5.410970
0.8	-1.593291	-6.830272	-15.31833	-54.33163	-27.67934	-8.626178
1.0	-4.486905	-7.882505	15.63487	50.57774	25.37404	-15.33059
1.2	-14.61944	-8.920899	6.1600327	19.764409	10.500572	-20.09530
1.4	56.366220	-11.17279	4.1507007	12.769702	6.9130561	78.623671
1.6	13.442159	-11.05214	3.2703483	9.6885092	5.3367797	18.902125
1.8	8.6896422	-12.18436	2.7736735	7.9425300	4.4452407	12.267426
2.0	6.8571858	-13.38634	2.4539125	6.8158277	3.870316	9.6935146
2.2	5.8898181	-14.67857	2.2307474	6.0300162	3.4689509	8.3237311
2.4	5.2983349	-16.083965	2.0664282	5.4541182	3.1739062	7.4784202
2.6	4.9064972	-17.62882	1.9408457	5.0370762	2.9493770	6.9128877
2.8	4.6352832	-19.343621	1.8422867	4.6823133	2.7745346	6.5174966
3.0	4.4440841	-21.26373	1.7634430	4.4206195	2.6364091	6.2359064
3.2	4.3099773	-23.42976	1.6994873	4.2166734	2.5264335	6.036271
3.4	4.2190931	-25.88730	1.6470593	4.588337	2.4386305	5.8993000
3.6	4.1625937	-28.68589	1.6036940	3.9383745	2.3685533	5.8125585
3.8	4.1345294	-31.87680	1.5674869	3.8481413	2.3126663	5.7675471
4.0	4.1227733	-35.51029	1.5369002	3.7818467	2.2679673	5.7537193
4.2	4.1161802	-39.63296	1.5106555	3.7336689	2.2318179	5.7243728
4.4	4.1040271	-44.28819	1.4877057	3.6983332	2.2019259	5.7170006
4.6	4.0761750	-49.52008	1.4672296	3.6714279	2.1764373	5.6776616

Table 8. Relative loss for entropies using $\alpha = 0.7, \gamma = 0.4, \delta = 0.35, \theta = 0.6, \omega = 0.5$

t	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	2.5780484	-4.2103041	-1.418430	-6.122530	-2.907496	1.7983253
0.8	1.2378604	-3.147503	-1.295005	-4.721622	-2.452746	1.1717353
1.0	0.5316030	-2.480035	-1.194527	-3.827549	-2.128296	0.5910509
1.2	0.1143556	-2.020005	-1.108604	-3.201231	-1.879530	0.1388926

1.4	-0.151455	-1.684434	-1.033375	-2.737093	-1.680742	-0.193332
1.6	-0.330633	-1.430817	-0.967051	-2.381207	-1.518325	-0.434048
1.8	-0.457435	-1.235132	-0.908959	-2.103328	-1.384613	-0.609831
2.0	-0.551616	-1.082848	-0.859119	-1.885459	-1.275217	-0.741232
2.2	-0.625394	-0.964761	-0.818019	-1.716550	-1.187719	-0.843309
2.4	-0.686776	-0.874853	-0.786439	-1.589761	-1.120962	-0.926968
2.6	-0.741258	-0.809071	-0.765432	-1.500899	-1.074577	-1.000141
2.8	-0.792715	-0.764548	-0.755901	-1.447354	-1.048577	-1.068602
3.0	-0.843870	-0.739018	-0.758594	-1.427219	-1.042940	-1.136471
3.2	-0.896503	-0.730291	-0.773664	-1.438395	-1.057102	-1.206473
3.4	-0.951563	-0.735810	-0.800367	-1.477723	-1.089443	-1.280074
3.6	-1.009266	-0.752387	-0.836907	-1.540372	-1.136982	-1.357625
3.8	-1.069306	-0.776348	-0.880659	-1.620037	-1.195621	-1.438659
4.0	-1.131154	-0.804169	-0.928836	-1.710164	-1.261062	-1.522363
4.2	-1.194371	-0.833255	-0.979299	-1.805669	-1.329985	-1.608041
4.4	-1.258745	-0.862247	-1.030907	-1.903792	-1.400610	-1.695346
4.6	-1.324254	-0.890699	-1.083292	-2.003605	-1.472347	-1.784227

Table 9. Relative loss for entropies using $\beta = 0.9, s = 0.5, e = 0.4, u = 0.8, \varphi = 0.5$

t	Shannon	Tsallis	Renyi	Arimoto	Havrda_Charvat	Sharma_Mittal
0.6	27.727676	-16.67574	-4.947549	-37.86520	-12.71861	4.8611936
0.8	12.585746	-10.62910	-4.640456	-25.88409	-10.37499	4.9978730
1.0	5.6534029	-7.428888	-4.392890	-19.32297	-8.834445	3.8568249
1.2	1.9342272	-5.503604	-4.182119	-15.24936	-7.729168	1.7971203
1.4	-0.301106	-4.249341	-3.999331	-12.52044	-6.896322	-0.331142
1.6	-1.781537	-3.389276	-3.842540	-10.60768	-6.254530	-2.145778
1.8	-2.861089	-2.780650	-3.714043	-9.238735	-5.761132	-3.616722
2.0	-3.733547	-2.343467	-3.619397	-8.263981	-5.394621	-4.838479
2.2	-4.518596	-2.030147	-3.566892	-7.599452	-5.146477	-5.926729
2.4	-5.301184	-1.811019	-3.566981	-7.199372	-5.016703	-6.989459
2.6	-6.151999	-1.666560	-3.631180	-7.040861	-5.010428	-8.186122
2.8	-7.140282	-1.582611	-3.770044	-7.112940	-5.134100	-9.435516
3.0	-8.344757	-1.546969	-3.990098	-7.406006	-5.390484	-11.02755
3.2	-9.868038	-1.547080	-4.290863	-7.901802	-5.773582	-13.41569
3.4	-11.86296	-1.569670	-4.664634	-8.569395	-6.267450	-15.68128
3.6	-14.58740	-1.602842	-5.101323	-9.373997	-6.852946	-19.28810
3.8	-18.53008	-1.638883	-5.595969	-10.29443	-7.519218	-24.50872
4.0	-24.74854	-1.674637	-6.152354	-11.33208	-8.269387	-32.74317
4.2	-36.02817	-1.709427	-6.781274	-12.50567	-9.117507	-47.68017
4.4	-62.83319	-1.743240	-7.498402	-13.84429	-10.08466	-83.17690
4.6	-208.3997	-1.776144	-8.324523	-15.38679	-11.19891	-275.9438

5. Conclusions

Different entropy measures were studied in this research to measure the relative loss of OGE Burr Type X distributions and truncated OGE Burr Type X distributions. The procedures followed in this context were detailed, where the following entropy measures were used sequentially: Shannon, Rényi, Tsallis, Havrda-Charvát, Arimoto, and Sharma-Mittal.

In the current work, a comprehensive comparison was made between the entropy estimates of both OGE Burr Type X distributions and truncated OGE Burr Type X distributions. The results showed that the Shannon and Sharma-Mittal entropy measures follow an exponential decreasing pattern with an increase in the parameter.

It looks like your message didn't come through clearly. Could you please resend it?

t, according to the numerical study. In contrast, the Renyi, Tsallis, Havrda-Charvat, and Arimoto entropy measures exhibited behavior characterized by an initial increase followed by a period of decrease, reflecting the effect of the parameter in a similar manner.

It was observed that the relative entropy loss decreases with an increase in the cut-off factor t, indicating an improvement in measurement accuracy. By comparing the minimum entropy values between the two distributions, we were able to determine the best entropy measure that more accurately reflects the characteristics of the studied data. These results demonstrate the importance of selecting appropriate metrics in analyzing statistical distributions and understanding the complex dynamics behind them. This study represents an important step towards understanding the dynamics of OGE Burr Type X distributions and their truncated distributions. By comparing different entropy measures, researchers can make informed decisions about the statistical methods to be used in their future studies.

References

1. Dey, S., Maiti, S. S., & Ahmad, M. (2016). Pak. J. Statist. 2016 Vol. 32 (2), 97-108 COMPARISON OF DIFFERENT ENTROPY MEASURES. Pak. J. Statist, 32(2), 97-108.
2. Basit, A., Riaz, A., Iqbal, Z., & Ahmad, M. (2017). On comparison of entropy measures for weighted and truncated weighted exponential distributions. Advances and Applications in Statistics, 50(6), 477-495.
3. Mir, K. A., Ahmed, A., & Reshi, J. A. (2013). On size-biased exponential distribution. Journal of Modern Mathematics and Statistics, 7(2), 21-25.
4. AWAD, A. M., & ALAWNEH, A. J. (1987). Application of entropy to a life-time model. IMA Journal of Mathematical Control and Information, 4(2), 143-148.
5. Ijaz, M., AL-Aziz, S. N., Asim, S. M., Dar, J. G., & Abd El-Bagoury, A. A. H. (2021). Comparison of different entropy measures using the relative loss approach for the lomax distribution. Information Sciences Letters, 10(3), 18.
6. Al-Babtain, A. A., Elbatal, I., Chesneau, C., & Elgarhy, M. (2021). Estimation of different types of entropies for the Kumaraswamy distribution. PLoS One, 16(3), e0249027.
7. Teixeira, A., Matos, A., Souto, A., & Antunes, L. (2011). Entropy measures vs. Kolmogorov complexity. Entropy, 13(3), 595-611.
8. Baez, J. C., Fritz, T., & Leinster, T. (2011). A characterization of entropy in terms of information loss. Entropy, 13(11), 1945-1957.
9. Zaman, Q., Bilal, M., Ijaz, M., & Ullah, N. (2022). Performance of Various Entropy Measures: Applications to Pareto and Truncated Pareto Distributions. The Lighthouse Journal of Computational & Numerical Sciences, 1(01), 33-58.
10. Ahsan ul Haq, M., & Aslam, M. (2023). Comparison of Entropy Measures for Gompertz and Truncated Gompertz Distribution. Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, 93(1), 113-120.
11. Ebrahimi, N., Pflughoeft, K., & Soofi, E. S. (1994). Two measures of sample entropy. Statistics & Probability Letters, 20(3), 225-234.
12. Abo-Eleneen, Z. A. (2011). The entropy of progressively censored samples. Entropy, 13(2), 437-449.
13. Cho, Y., Sun, H., & Lee, K. (2014). An estimation of the entropy for a Rayleigh distribution based on doubly-generalized Type-II hybrid censored samples. Entropy, 16(7), 3655-3669.
14. Hassan, A. S., & Zaky, A. N. (2019). Estimation of entropy for inverse Weibull distribution under multiple censored data. Journal of Taibah University for Science, 13(1), 331-337.
15. Bantan, R. A., Elgarhy, M., Chesneau, C., & Jamal, F. (2020). Estimation of entropy for inverse Lomax distribution under multiple censored data. Entropy, 22(6), 601.
16. Kayal, S., & Kumar, S. (2013). Estimation of the Shannon's entropy of several shifted exponential populations. Statistics & Probability Letters, 83(4), 1127-1135.
17. Cho, Y., Sun, H., & Lee, K. (2015). Estimating the entropy of a Weibull distribution under generalized progressive hybrid censoring. Entropy, 17(1), 102-122.
18. Noorizadeh, S., & Shakerzadeh, E. (2010). Shannon entropy as a new measure of aromaticity, Shannon aromaticity. Physical Chemistry Chemical Physics, 12(18), 4742-4749.
19. Khalaf, A. A., & khaleel, M. A. (2022, November). [0, 1] Truncated exponentiated exponential gompertz distribution: Properties and applications. In AIP Conference Proceedings (Vol. 2394, No. 1, p. 070035). AIP Publishing LLC.
20. Khalaf, A., & Khaleel, M. A. (2020). Truncated exponential marshall-olkin-gompertz distribution properties and applications. Tikrit Journal of Administration and Economics Sciences, 16, 483-497.
21. Al-Habib, K. H., Abdal-hammed, M. K., Salih, A. M., & Hemeda, S. E. (2024). Statistical Properties and Parameters Estimation for a New Truncated Lomax Exponential-G Family. Samarra Journal of Pure and Applied Science, 6(4), 255-270.
22. Khalaf, A., Yusur, K., & Khaleel, M. (2023). [0, 1] Truncated Exponentiated Exponential Inverse Weibull Distribution with Applications of Carbon Fiber and COVID-19 Data. Journal of Al-Rafidain University College For Sciences (Print ISSN: 1681-6870, Online ISSN: 2790-2293), (1), 387-399.
23. Al-Habib, K. H., Salih, A. M., Abdal-Hammed, M. K., Khaleel, M. A., & Algamal, Z. Y. (2025, March). Estimating the parameters of [0, 1] Truncated Nadarajah–Haghighi inverse weibull distribution. In AIP Conference Proceedings (Vol. 3264, No. 1, p. 050051). AIP Publishing LLC.

24. Khalaf, A. A., Ibrahim, M. Q., & Noori, N. A. (2024). [0, 1] Truncated Exponentiated Exponential Burr type X Distributionwith Applications. Iraqi Journal of Science, 4428-4440.
25. Khaleel, M. A., Abdulwahab, A. M., Gaftan, A. M., & Abdal-hammed, M. K. (2022). A new [0, 1] truncated inverse Weibull rayleigh distribution properties with application to COVID-19. International Journal of Nonlinear Analysis and Applications, 13(1), 2933-2946.
26. Shannon CE (1948) A mathematical theory of communication. Bell Syst Tech J 27:379–423.
27. Gemeay, A. M., Alsadat, N., Chesneau, C., & Elgarhy, M. (2024). Power unit inverse Lindley distribution with different measures of uncertainty, estimation and applications. AIMS Mathematics, 9(8), 20976-21024.
28. Rényi A (1961) On measures of entropy and information. In: Proceedings of the fourth Berkeley symposium on mathematical statistics and probability, pp 547–561.
29. Khalaf, A. A., & Khaleel, M. A. (2025). Estimation Methods: Inference Classical and Bayesian of Extended Inverse Exponential Distribution. Iraqi Statisticians journal, 29-42.
30. Bashiru, S. O., Isa, A. M., Khalaf, A. A., Khaleel, M. A., Arum, K. C., & Anioke, C. L. (2025). A Hybrid Cosine Inverse Lomax-G Family of Distributions with Applications in Medical and Engineering Data. Nigerian Journal of Technological Development, 22(1), 261-278.
31. Khalaf, A. A., & Khaleel, M. A. (2025, March). The Odd Burr XII Exponential distribution: Properties and applications. In AIP Conference Proceedings (Vol. 3264, No. 1, p. 050039). AIP Publishing LLC.
32. Isa, A. M., Khalaf, A. A., & Bashiru, S. O. (2024). Some Properties of the Cosine Lomax Distribution with Applications.
33. Habib, K. H., Salih, A. M., Khaleel, M. A., & Abdal-hammed, M. K. (2023). OJCA Rayleigh distribution, Statistical Properties with Application. Tikrit Journal of Administration and Economics Sciences, 19.
34. Noori, N. A., Khalaf, A. A., & Khaleel, M. A. (2023). A New Generalized Family of Odd Lomax-G Distributions: Properties and Applications. Advances in the Theory of Nonlinear Analysis and its Applications, 7(4), 01-16.
35. Bashiru, S. O., Khalaf, A. A., Isa, A. M., & Kaigama, A. (2024). ON MODELING OF BIOMEDICAL DATA WITH EXPONENTIATED GOMPERTZ INVERSE RAYLEIGH DISTRIBUTION. Reliability: Theory & Applications, 19(3 (79)), 460-475.
36. Murtadha M. J., Moudher Kh. A., Mizal A., (2025). Using Simulation to Estimate Parameters for a Novel Extension Rayleigh Distribution, Properties and Failure Rate Data Application, 28(7), 596-618.
37. Khalaf, A. A., Khaleel, M. A., Jawa, T. M., Sayed-Ahmed, N., & Tolba, A. H. (2025). A Novel Extension of the Inverse Rayleigh Distribution: Theory, Simulation, and Real-World Application. Appl. Math, 19(2), 467-488.
38. Isa, A. M., Bashiru, S. O., & Kaigama, A. (2024). Topp-Leone Exponentiated Burr XII Distribution: Theory and Application to Real-Life Data Sets. Iraqi Statisticians Journal, 1(1), 63-72.
39. Bashiru, S. O., Khalaf, A. A., & Isa, A. M. (2024). TOPP-LEONE EXPONENTIATED GOMPERTZ INVERSE RAYLEIGH DISTRIBUTION: PROPERTIES AND APPLICATIONS. Reliability: Theory & Applications, 19(3 (79)), 59-77.
40. Khalaf, A. A. (2024). The New Strange Generalized Rayleigh Family: Characteristics and Applications to COVID-19 Data. Iraqi Journal For Computer Science and Mathematics, 5(3), 32.
41. Noori, N. A., Khalaf, A. A., & Khaleel, M. A. (2024). A new expansion of the Inverse Weibull Distribution: Properties with Applications. Iraqi Statistians Journal, 1(1), 52-62.
42. Havrda J, Charvát F (1967) Quantification method of classification processes. Concept of structural a-entropy. Kybernetika 3:30–35.
43. Alahmadi, A. A., ZeinEldin, R. A., Albalawi, O., Badr, M. M., Abdelfadel, T. A. A., & Shawki, A. W. (2025). Modified Kies power Lomax model with applications in different sciences. Journal of Radiation Research and Applied Sciences, 18(1), 101239.
44. Arimoto S (1971) Information-theoretical considerations on estimation problems. Inf Control 19:181–194.
45. Sharma BD, Mittal DP (1977) New non-additive measures of relative information. J Comb Inf Syst Sci 2:122–132.
46. Mondaini, R. P., & de Albuquerque Neto, S. C. (2021). Alternative Entropy Measures and Generalized Khinchin–Shannon Inequalities. Entropy, 23(12), 1618.
47. Tsallis C (1988) Possible generalization of Boltzmann–Gibbs statistics.J Stat Phys 52:479–487.
48. Suleiman, A. A., Daud, H., Ishaq, A. I., Kayid, M., Sokkalingam, R., Hamed, Y., ... & Elgarhy, M. (2024). A new Weibull distribution for modeling complex biomedical data. Journal of Radiation Research and Applied Sciences, 17(4), 101190.
49. Jasim, M. M., Abdal-Hammed, M. K., & Alobaidi, M. (2025, March). The Odd Generalized Exponential Burr type X distribution: Theorems and applications. In AIP Conference Proceedings (Vol. 3264, No. 1). AIP Publishing.