IRAQI STATISTICIANS JOURNAL

https://isj.edu.iq/index.php/isj

ISSN: 3007-1658 (Online)

A Generalized Shrinkage-type Estimator of Population Mean in Simple Random Sampling under Conventional and Non-Conventional Measures of Auxiliary Variables

Emmanuel J. Ekpenyong[1], Loveline Chiamaka Okoro[2] and Theophilus Obijuru Nelson[3]

[1,2,3]Department of Statistics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria
[1]ej.ekpenyong@mouau.edu.ng
[2]okoro.loveline@mouau.edu.ng
[3]nelson.theophilus@mouau.edu.ng

ARTICLE INFO

Article history:

Received	24 June 2025
Revised	26 June 2025
Accepted	09 July 2025
Available online	15 July 2025

Keywords:

Efficiency
Bias
Mean Squared Error
Estimator
Shrinkage
Simple Random Sampling

ABSTRACT

In this study, a generalized shrinkage-type estimator of population mean in simple random sampling has been proposed. The proposed estimator is a combination of some of the known estimators in literature with the aim of obtaining estimators with higher efficiency. Its bias and mean squared error (MSE) have been derived using Taylor series up to the first order of approximation. The optimal MSE's of the proposed class of estimators have been obtained. Theoretical comparison of the proposed shrinkage-type estimator has been also made with other existing related ratio estimators of the population mean using auxiliary information. The conditions under which the proposed shrinkage-type estimators perform better than the other existing estimators of population mean are given. Validation of results from both simulation and real data sets application reveals that the proposed shrinkage-type estimators performed better than some existing related ratio estimators considered in this work as they are having lower mean squared errors and higher percent relative efficiencies (PREs).

1. Introduction

In sample surveys, Auxiliary information from sampling theory is employed to improve parameter estimation and boost the estimators' efficiency. The auxiliary information is obtained from auxiliary variable which is highly positively or negatively correlated with the main variable under study, (Gupta and Yadav [1]). In literature, the issue of estimating the population mean when an auxiliary variable is present has been extensively addressed. Some of estimation like the ratio, regression and product in literature. When there is a strong positive correlation between the study and auxiliary variables, the ratio technique of estimation performs quite well. On the other hand, if there is a high and negative correlation, the product

Corresponding author E-mail address: ej.ekpenyong@mouau.edu.ng
https://doi.org/10.62933/vd5wpm87

technique of estimation can be successfully adopted. Regression-type estimators are preferred if the straight line does not pass through the origin (has an intercept). In sampling theory, estimation of the population parameters is of key importance and researchers have been on the search for a more efficient estimator. Thus, the sample mean, being an unbiased estimator is the most suitable estimator for estimating population mean, but it has a reasonably large sampling variance, [1]. To reduce the problem of large sampling variance, [2] proposed the ratio estimator of the population mean which ensures better efficiency than the sample mean estimator due to the incorporation of auxiliary variables. Also the product estimator introduced by [3] is more efficient than the sample mean estimator, under a negative correlation and other conditions. For detailed study of the modified ratio type estimators, latest references can be made to [2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. However, the performance of ratio estimation could not really improve in some populations. At this point, many authors proposed estimators by using the exponential function and modified class of ratio estimation. Bahl and Tuteja [14] is the first to propose an estimator using the exponential function for the estimation of the population mean. Other authors include [10], [15], [16], [17], [18], [19], [20], [21], [22], and [23]. Their efficiencies over the regression estimator in some cases are not statistically significant, while in some cases are significant under certain conditions and data type. The continuous search for an improved estimator of population mean in terms of accuracy, efficiency and flexibility becomes imperative.

Hence, this work seeks to propose with justifications a generalized shrinkage-type estimator of population mean under simple random sampling with one auxiliary variable, which would always be more efficient than some existing estimators or compare favourably with the best of the existing estimators to be considered in this work.

2. Sampling Procedure and Notations

Let $U = \{U_{1,}\ldots,U_N\}$ be a finite population of size N and let $(yi,\ xi)$ be the value of the study variable Y and the auxiliary variable X on ith unit $Ui,i = 1,\ldots N$. Let $\bar{Y}$ and $\bar{X}$ be population means of the study variable Y and the auxiliary variable X respectively. Let a sample of size (n) be drawn by simple random sampling without replacement (SRSWOR) based on which we obtain the means $(\bar{x})$ and $(\bar{y})$ for the auxiliary variable (X) and the study variable(Y). We assume that the population mean $\bar{X}$ and the population variance S_x^2 of the auxiliary variable are known. The following notations are defined:

$C_x = S_x / \bar{X}$ **,** Coefficient of variation of the auxiliary variable

$C_y = S_y / \bar{Y}$, Coefficient of variation of the study variable

$\rho = S_{xy} / S_y S_x$, Correlation coefficient between the auxiliary and study variables

$K = \rho\left(C_y / C_x\right)$, population constant, $f = n/N$, the sampling fraction

$S_x^2 = \sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2 \Big/ N-1$, population variance of the auxiliary variable

$S_y^2 = \sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^2 \Big/ N-1$, population variance of the study variable

$S_{xy} = \sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right) \Big/ N-1$, population covariance between the auxiliary and study variables

$\bar{X} = \sum_{i=1}^{N} X_i \big/ N$, population mean of the auxiliary variable

$\bar{Y} = \sum_{i=1}^{N} Y_i \big/ N$, population mean of the study variable

$\bar{x} = \sum_{i=1}^{n} x_i \big/ n$, sample mean of the auxiliary variable

$\bar{y} = \sum_{i=1}^{n} y_i \big/ n$, sample mean of the study variable

$\beta_{1(X)} = \mu_3 \big/ S^2$, coefficient of skewness of the auxiliary variable

$\beta_{2(X)} = \mu_4 \big/ \left(S_x^2\right)^2$, coefficient of kurtosis of the auxiliary variable

$M_R = \left(X_{(1)} + X_{(N)}\right) \big/ 2$, midrange of the auxiliary variable

$T_M = \left(q_1 + 2q_2 + q_3\right) \big/ 4$, Trim mean of the auxiliary variable

$Q_D = \left(q_1 - q_3\right) \big/ 2$, Quartile deviation of the auxiliary variable

H_L = the median of the auxiliary variable

$\theta = \frac{C\bar{X}}{C\bar{X} + r}$

$\psi = {(1-f)} \big/ {n}$

3. Some related existing estimators with their mean squared errors

(i) The classical regression estimator is given as

$$t_6 = \bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right) \quad (1)$$

Its mean squared error was obtained as

$$MSE\left(t_6\right) = \psi \bar{Y}^2 C_y^2 \left(1 - \rho_{yx}^2\right) \quad (2)$$

(ii) Yunusa *et al.* [15] suggested an estimator which is given by

$$t_7 = 2^{-1} \bar{y} \left[\left(\frac{\bar{x}}{\bar{X}} \right)^{\alpha} + \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right] \right] \quad (3)$$

Where α is a suitably chosen constant

Its mean squared error is given as

$$\text{MSE}(t_7) = \bar{Y}^2 \psi \left[C_y^2 + \left(\frac{\alpha}{2} - \frac{1}{4} \right)^2 C_x^2 + 2\left(\frac{\alpha}{2} - \frac{1}{4} \right) \rho C_y C_x \right] \quad (4)$$

(iii) Yahaya *et al*. [12] suggested an estimator which is given by

$$t_8 = \bar{y} \left[k \frac{\bar{X}}{\bar{x}} + (1-k) \frac{\bar{x}}{\bar{X}} \right] \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \quad (5)$$

With a mean squared error of

$$MSE\left(t_8\right)=\psi\bar{Y}^2\left[C_y^2+\frac{\left(1-4k\right)^2}{2}C_x^2+\left(1-4k\right)\rho_{(y,x)}C_yC_x\right] \quad (6)$$

Where, $K=\rho\left(C_y/C_x\right)$

(iv) Muhammad *et al.* [7] proposed an estimator which is given by

$$t_9=\bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right)^{\gamma} \quad (7)$$

Where γ is a constant

Its mean squared error is given by

$$\text{MSE}(\text{t}_9)=\psi\bar{Y}^2\left[C_y^2+\gamma^2\delta^2C_x^2-2\gamma\delta\rho C_yC_x\right] \quad (8)$$

For optimal MSE, $\gamma^{opt}=\frac{\rho C_y}{\delta C_y}$ and

$$\text{MSE}(\hat{\bar{Y}}_m)_{\min}=\bar{Y}^2\psi C_y^2\left(1-\rho^2\right) \quad (9)$$

Where $\delta=\frac{\bar{X}}{\bar{X}+n}$

(v) Javid *et al.* [16] proposed an exponential ratio estimator which is given by

$$t_0=\left[T_1\bar{y}+T_2\right]\exp\left[\frac{C\left(\bar{X}-\bar{x}\right)}{C\left(\bar{X}+\bar{x}\right)+2r}\right] \quad (10)$$

where, T_1 and T_2 are constants and C and r are the known conventional and nonconventional measures of the auxiliary variable. Its minimum mean squared error is given by with optimal values of T_1 and T_2 that is,

$$T_1^{opt}=\frac{B_iC_i-D_iE_i+B_i}{A_iB_i+B_i-E_i^2} \qquad T_2^{opt}=\frac{\bar{Y}\left(A_iD_i-C_iE_i+D_i-E_i\right)}{A_iB_i+B_i-E_i^2}$$

$$MSE\left(t_0\right)_{\min}=\bar{Y}^2\left[1-\frac{\left(A_iD_i^2+B_iC_i^2-2C_iD_iE_i+2B_iC_i+D_i^2-2D_iE_i+B_i\right)}{\left(A_iB_i+B_i-E_i^2\right)}\right] \quad (11)$$

where, $A_i=\psi\left(C_y^2+\theta^2C_x^2-2\theta C_{yx}\right)$, $B_i=1+\psi\theta^2C_x^2$, $C_i=\psi\left(\left(\frac{3}{8}\right)\theta^2C_x^2-\left(\frac{1}{2}\right)\theta C_{yx}\right)$,

$D_i=1+\left(\frac{3}{8}\right)\psi\theta^2C_x^2$, $E_i=1+\psi\left(\theta^2C_x^2-\theta C_{yx}\right)$

$\theta=\frac{C\bar{X}}{C\bar{X}+r}$

4. The proposed generalized estimator

Following [16], a generalized shrinkage-type estimator with some modifications is proposed as

$$t_{LE}=\left[\gamma_1\bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha_1}+\gamma_2\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha_2}\right]\exp\left\{V\left[\frac{C\left(\bar{X}-\bar{x}\right)}{C\left(\bar{X}+\bar{x}\right)+2r}\right]\right\}, \qquad (12)$$

where, γ_1 and γ_2 are the minimizing constants, the values of which are to be obtained so that the resulting MSE is minimum, α_1, α_2 and V are the generalizing constants which are suitably chosen and 'C' and 'r' are conventional or non-conventional measures of auxiliary variables. The proposed estimator provides flexibility in producing members of the class which are themselves estimators with desirable characteristics.

To obtain the approximate expression for the bias and mean squared error for the proposed class of estimators, we express (11) in terms of $\ell_s^{'}$ to the first order of approximation, assuming that,

$\bar{x}=\bar{X}\left[1+\ell_x\right]$, where, $\ell_x=\dfrac{\bar{x}-\bar{X}}{\bar{X}}$

and $\bar{y}=\bar{Y}\left[1+\ell_y\right]$, where, $\ell_y=\dfrac{\bar{y}-\bar{Y}}{\bar{Y}}$

$$E(\ell_x)=E(\ell_y)=0, E(\ell_x^2)=\frac{1-f}{n}C_x^2, E(\ell_y^2)=\frac{1-f}{n}C_y^2, E(\ell_y\ell_x)=\frac{1-f}{n}\rho C_y C_x=\frac{1-f}{n}KC_x^2, K=\rho\left(C_y/C_x\right)$$

Therefore, (11) is expressed as

$$t_{LE}=\gamma_1\bar{Y}\left[1+\ell_y\right]\left[1+\ell_x\right]^{-\alpha_1}+\gamma_2\left[1+\ell_x\right]^{-\alpha_2}\exp\left\{V\left[\frac{C\left(\bar{X}-\bar{X}\left[1+\ell_x\right]\right)}{C\left(\bar{X}+\bar{X}\left[1+\ell_x\right]\right)+2r}\right]\right\}$$

$$\begin{aligned}t_{LE}=&\left[\gamma_1\bar{Y}\left[1+\ell_y\right]\left[1-\alpha_1\ell_x+\frac{\alpha_1\left(\alpha_1+1\right)\ell_x^2}{2}+\ldots\right]+\gamma_2\left[1-\alpha_2\ell_x+\frac{\alpha_2\left(\alpha_2+1\right)\ell_x^2}{2}+\ldots\right]\right]\\&\left[1-\frac{V\theta\ell_x}{2}\left[1-\frac{\theta\ell_x}{2}+\frac{\theta^2\ell_x^2}{8}+\ldots\right]+\frac{V^2\theta^2\ell_x^2}{8}\left[1-\frac{2\theta\ell_x^2}{2}+\ldots\right]+\ldots\right]\end{aligned} \qquad (13)$$

To the first order of approximations, (13) becomes

$$\begin{aligned}t_{LE}\cong&\ \gamma_1\bar{Y}-\gamma_1\left[\left(\frac{\bar{Y}V\theta}{2}+\bar{Y}\alpha_1\right)\ell_x-\bar{Y}\ell_y-\left(\frac{\bar{Y}V\theta^2}{4}+\frac{\bar{Y}V^2\theta^2}{8}+\frac{\bar{Y}\alpha_1V\theta}{2}+\frac{\bar{Y}\alpha_1\left(\alpha_1+1\right)}{2}\right)\ell_x^2\right.\\&\left.+\left(\frac{\bar{Y}V\theta}{2}+\bar{Y}\alpha_1\right)\ell_y\ell_x\right]+\gamma_2\left[1-\left(\frac{V\theta}{2}+\alpha_2\right)\ell_x+\left(\frac{V\theta^2}{4}+\frac{V^2\theta^2}{8}+\frac{\alpha_2V\theta}{2}+\frac{\alpha_2\left(\alpha_2+1\right)}{2}\right)\ell_x^2\right]\end{aligned} \qquad (14)$$

Where $\theta=\dfrac{C\bar{X}}{C\bar{X}+r}$

$$\begin{aligned}t_{LE}-\bar{Y}=&\ \bar{Y}\left(\gamma_1-1\right)-\gamma_1\left[\left(\frac{\bar{Y}V\theta}{2}+\bar{Y}\alpha_1\right)\ell_x-\bar{Y}\ell_y-\left(\frac{\bar{Y}V\theta^2}{4}+\frac{\bar{Y}V^2\theta^2}{8}+\frac{\bar{Y}\alpha_1V\theta}{2}+\frac{\bar{Y}\alpha_1\left(\alpha_1+1\right)}{2}\right)\ell_x^2+\left(\frac{\bar{Y}V\theta}{2}+\bar{Y}\alpha_1\right)\ell_y\ell_x\right]\\&+\gamma_2\left[1-\left(\frac{V\theta}{2}+\alpha_2\right)\ell_x+\left(\frac{V\theta^2}{4}+\frac{V^2\theta^2}{8}+\frac{\alpha_2V\theta}{2}+\frac{\alpha_2\left(\alpha_2+1\right)}{2}\right)\ell_x^2\right]\end{aligned} \qquad (15)$$

$$\mathrm{E}\left[t_{LE}-\bar{Y}\right]=\bar{Y}\left(\gamma_1-1\right)+\gamma_1\bar{Y}\psi\left[\left(\frac{V\theta^2}{4}+\frac{V^2\theta^2}{8}+\frac{\alpha_1V\theta}{2}+\frac{\alpha_1\left(\alpha_1+1\right)}{2}\right)C_x^2\right.$$
$$\left.-\left(\frac{V\theta}{2}+\alpha_1\right)\rho C_yC_x\right]+\gamma_2\left[1+\psi C_x^2\left(\frac{V\theta^2}{4}+\frac{V^2\theta^2}{8}+\frac{\alpha_2V\theta}{2}+\frac{\alpha_2\left(\alpha_2+1\right)}{2}\right)\right] \tag{16}$$

$$Bias(t_{LE})=\mathrm{E}\left[t_{LE}-\bar{Y}\right]=\bar{Y}\left(\gamma_1-1\right)+\gamma_1\bar{Y}Q_3+\gamma_2Q_4 \tag{17}$$

Squaring both sides of (15) and taking expectation, we derive the MSE of t_{LE} as

$$MSE(t_{LE})=E\left[t_{LE}-\bar{Y}\right]^2=\bar{Y}^2\left[\gamma_1-1\right]^2+\gamma_1^2\bar{Y}^2Q_1+\gamma_2^2Q_2-2\gamma_1\bar{Y}^2Q_3-2\gamma_2\bar{Y}Q_4+2\gamma_1\gamma_2\bar{Y}Q_5 \tag{18}$$

where,

$$Q_1=\psi\left[C_y^2+\left(\frac{V\theta^2}{2}+\frac{V^2\theta^2}{2}+2\alpha_1V\theta+\alpha_1\left(2\alpha_1+1\right)\right)C_x^2-2\left(V\theta+2\alpha_1\right)\rho C_yC_x\right]$$

$$Q_2=1+\psi2\left[\frac{V^2\theta^2}{4}+\frac{V\theta^2}{4}+\alpha_2V\theta+\alpha_2\left(\alpha_2+\frac{1}{2}\right)\right]C_x^2$$

$$Q_3=\psi\left[\frac{3}{8}\left(\frac{2V\theta^2}{3}+\frac{V^2\theta^2}{3}+\frac{4\alpha_1V\theta}{3}+\frac{4\alpha_1\left(\alpha_1+1\right)}{3}\right)C_x^2-\frac{1}{2}\left(V\theta+2\alpha_1\right)\rho C_yC_x\right]$$

$$Q_4=1+\psi\left(\frac{V\theta^2}{4}+\frac{V^2\theta^2}{8}+\frac{\alpha_2V\theta}{2}+\frac{\alpha_2\left(\alpha_2+1\right)}{2}\right)C_x^2$$

$$Q_5=1+\psi\left[\left(\frac{V\theta^2}{2}+\frac{V^2\theta^2}{2}+\alpha_1V\theta+\alpha_2V\theta+\alpha_1\alpha_2+\frac{\alpha_1\left(\alpha_1+1\right)}{2}+\frac{\alpha_2\left(\alpha_2+1\right)}{2}\right)C_x^2\right.$$
$$\left.-\left(V\theta+\alpha_1+\alpha_2\right)\rho C_yC_x\right]$$

To obtain values of γ_1 and γ_2 that optimizes the $MSE\left(t_{LE}\right)$, (18) is differentiated partially with respect to γ_1 and γ_2, and the resulting expressions equated to zero. Then, the equations are solved simultaneously to give the optimal values of γ_1 and γ_2 as

$$\gamma_1^{opt}=\frac{Q_2Q_3-Q_4Q_5+Q_2}{Q_1Q_2+Q_2-Q_5^2} \tag{19}$$

$$\gamma_2^{opt}=\frac{\bar{Y}\left(Q_1Q_4-Q_3Q_5+Q_4-Q_5\right)}{Q_1Q_2+Q_2-Q_5^2} \tag{20}$$

Substituting the optimum values of γ_1 and γ_2 into (18), we derive the minimum mean squared error of t_{LE} as

$$MSE\left(t_{LE}\right)_{\min}\cong\bar{Y}^2\left[1-\frac{\left(Q_1Q_4^2+Q_2Q_3^2-2Q_3Q_4Q_5+2Q_2Q_3+Q_4^2-2Q_4Q_5+Q_2\right)}{Q_1Q_2-Q_5^2+Q_2}\right] \tag{21}$$

Varying the values of γ_1, γ_2, α_1, α_2, V, C and r produces different members of the proposed class of estimators with desirable features. Some of these members are presented in Table 1.

Table 1 **Existing family of t_{LE} for distinct values of γ_1, γ_2, α_1, α_2, V, C and r**

t	Value of parameters								Estimators
	S/N	γ_1	γ_2	α_1	α_2	V	C	r	
	1	1	0	0	0	0	0	0	$t_y = \bar{y}$ (the sample mean estimator) $\text{MSE}(\text{t}_y) = \psi \bar{Y}^2 C_y^2$
	2	γ_1	γ_2	0	0	1	C	r	$t_0 = [\gamma_1 \bar{y} + \gamma_2] \exp\left[\frac{C(\bar{X} - \bar{x})}{C(\bar{X} + \bar{x}) + 2r}\right]$ [Javid *et al.*, 2021] $Bias(t_0) = (T_1 - 1)\bar{Y} + T_1 \bar{Y} C_i + T_2 D_i$ $MSE(t_0) = (T_1 - 1)^2 \bar{Y}^2 + T_1^2 \bar{Y}^2 A_i + T_2^2 B_i - 2T_1 \bar{Y}^2 C_i - T_2 \bar{Y} D_i + 2T_1 T_2 \bar{Y} E_i$
	3	1	0	1	0	0	0	0	$t_1 = \bar{y} \frac{\bar{X}}{\bar{x}}$ [Cochran , 1940] $Bias(\text{t}_1) = \psi \bar{Y} \left[C_x^2 - \rho C_y C_x \right]$ $\text{MSE}(\text{t}_1) = \psi \bar{Y}^2 \left[C_y^2 + C_x^2 - 2\rho C_y C_x \right]$
	4	1	0	-1	0	0	0	0	$t_2 = \bar{y} \frac{\bar{x}}{\bar{X}}$ [Murthy, 1964] $Bias(\text{t}_2) = \psi \bar{Y} \rho C_y C_x$ $MSE(t_2) = \psi \bar{Y}^2 \left(C_y^2 + C_x^2 + 2\rho C_y C_x \right)$
	5	1	0	0	0	1	1	0	$t_3 = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$ [Bahl and Tuteja, 1991] $Bias(t_3) = \bar{Y} \psi \left[\frac{3C_x^2}{8} - \frac{\rho C y C_x}{2} \right]$ $\text{MSE}(t_3) = \bar{Y}^2 \psi \left[C_y^2 + \frac{C_x^2}{4} - \rho C y C_x \right]$
	6	1	0	$-\alpha$	0	1	1	0	$t_4 = \bar{y} \left(\frac{\bar{x}}{\bar{X}} \right)^{\alpha} \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$ [kadilar, 2016] $MSE(t_4) = \psi \bar{Y}^2 \left(C_y^2 + \frac{C_x^2}{4} + 2\alpha\rho C_x C_y + \rho C_x C_y + \alpha^2 C_x^2 + \alpha C_x^2 \right)$
	7	1	0	2	0	0	0	0	$t_5 = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^2$ [Kadilar and Cingi, 2003] $\text{Bias}(t_5) = \psi \bar{Y} C_x^2 (1 - 2k)$

$$MSE(t_5) = \psi \bar{Y}^2 \left[C_y^2 + 4C_x^2 (1-k) \right]$$

Table 1 indicates some members of the proposed class of estimators that already exist. That is, some existing estimators of population mean are members of the proposed generalized class of estimators

Table 2. New members of t_{LE} for distinct values of γ_1, γ_2, α_1, α_2, V, C and r

Value of parameters								Estimators
S/N	γ_1	γ_2	α_1	α_2	V	C	r	
1	γ_1	γ_2	-2	2	1	1	0	$t_{LE_1} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-2} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right)^{2} \right] \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$
2	γ_1	γ_2	-2	-1	1	1	0	$t_{LE_2} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-2} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right)^{-1} \right] \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$
3	γ_1	γ_2	-2	0	1	1	0	$t_{LE_3} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-2} + \gamma_2 \right] \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$
4	γ_1	γ_2	2	0	1	1	0	$t_{LE_4} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{2} + \gamma_2 \right] \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$
5	γ_1	γ_2	2	-1	1	1	0	$t_{LE_5} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{2} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right)^{-1} \right] \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$
6	γ_1	γ_2	-1	1	0	1	0	$t_{LE_6} = \gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-1} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right)$
7	γ_1	γ_2	0	2	-2	1	0	$t_{LE_7} = \left[\gamma_1 \bar{y} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right)^{2} \right] \exp\left[-2 \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \right]$
8	γ_1	γ_2	-2	1	1	$B_1(x)$	1	$t_{LE_8} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-2} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right) \right] \exp\left[\frac{\beta_1(x)(\bar{X} - \bar{x})}{\beta_1(x)(\bar{X} + \bar{x}) + 2} \right]$
9	γ_1	γ_2	-2	2	1	T_m	ρ_{yx}	$t_{LE_9} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-2} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right)^{2} \right] \exp\left[\frac{T_m(\bar{X} - \bar{x})}{T_m(\bar{X} + \bar{x}) + 2\rho_{yx}} \right]$
10	γ_1	γ_2	-2	1	1	$B_2(x)$	C_y	$t_{LE_{10}} = \left[\gamma_1 \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{-2} + \gamma_2 \left(\frac{\bar{X}}{\bar{x}} \right) \right] \exp\left[\frac{\beta_2(x)(\bar{X} - \bar{x})}{\beta_2(x)(\bar{X} + \bar{x}) + 2C_y} \right]$

Table 2 shows some new members derived from the proposed class of estimators. Continuous variation of the parameters produced more members of the family of the proposed generalized class of estimators of population mean under simple random sampling strategy.

5. Efficiency Comparison

The efficiencies of the proposed estimators can be compared with other existing estimators by establishing some conditions under which they will be more efficient than the existing ones.

Let,

$$\Delta_1 = Q_1Q_4^2 + Q_2Q_3^2 - 2Q_3Q_4Q_5 + 2Q_2Q_3 + Q_4^2 - 2Q_4Q_5 + Q_2$$

$$\Delta_2 = Q_1Q_2 - Q_5^2 + Q_2$$

then $MSE\left(t_{LE}\right)_{\min} \cong \bar{Y}^2\left[1-\frac{\Delta_1}{\Delta_2}\right]$ (22)

(a) Comparison with the sample mean estimator

The proposed estimator t_{LE} is more efficient than the estimator of the sample mean if the following condition holds;

$$MSE\left(t_y\right) - MSE\left(t_{LE}\right)_{\min} > 0$$

$$\left[\psi C_y^2 - 1 + \frac{\Delta_1}{\Delta_2}\right] > 0 \qquad (23)$$

When (23) holds, the estimator t_{LE} will be more efficient than the sample mean, $\bar{y}$.

(b) Comparison with the classical ratio estimator

The condition for the proposed estimator to be more efficient than the classical ratio estimator is defined by

$$MSE\left(t_1\right) - MSE\left(t_{LE}\right)_{\min} > 0$$

$$\left[\psi\left[C_y^2 + C_x^2 - 2\rho C_y C_x\right] - 1 + \frac{\Delta_1}{\Delta_2}\right] > 0 \qquad (24)$$

(c) Comparison with the Bahl and Tuteja [14] Ratio type Exponential estimator

The proposed estimator t_{LE} is more efficient than the Bahl and Tuteja [14] Ratio type Exponential estimator if the following condition holds;

$$MSE\left(t_3\right) - MSE\left(t_{LE}\right)_{\min} > 0$$

$$\left[\psi\left[C_y^2 + \frac{C_x^2}{4} - \rho C y C_x\right] - 1 + \frac{\Delta_1}{\Delta_2}\right] > 0 \qquad (25)$$

(d) Comparison with [10] Exponential type estimator

The proposed estimator t_{LE} is more efficient than the Kadilar [10] Exponential type estimator if the following condition holds;

$$MSE\left(t_4\right) - MSE\left(t_{LE}\right)_{\min} > 0$$

$$\left[\psi\left[C_y^2 + \frac{C_x^2}{4} + 2\alpha\rho C_x C_y + \rho C_x C_y + \alpha^2 C_x^2 + \alpha C_x^2\right] - 1 + \frac{\Delta_1}{\Delta_2}\right] > 0 \qquad (26)$$

(e) Comparison with the [16] Exponential ratio estimator

The proposed estimator t_{LE} is more efficient than the Javid *et al.* [16] exponential ratio estimator if the following condition holds;

$$MSE\left(t_0\right)_{\min} - MSE\left(t_{LE}\right)_{\min} > 0$$

$$\left[\frac{\Delta_{J1}}{\Delta_{J2}} - \frac{\Delta_1}{\Delta_2}\right] > 0 \qquad (27)$$

where,

$$\Delta_{J1} = A_i D_i^2 + B_i C_i^2 - 2C_i D_i E_i + 2B_i C_i + D_i^2 - 2D_i E_i + B_i$$

$$\Delta_{J2} = A_i B_i + B_i - E_i^2$$

(f) Comparison with [7] estimator

The proposed estimator t_{LE} is more efficient than [7] estimator if the following condition holds;

$$MSE(t_9) - MSE(t_{LE})_{\min} > 0$$

$$\left[\bar{Y}^2 \psi \left[C_y^2 + \gamma^2 \delta^2 C_x^2 - 2\gamma\delta\rho C_y C_x\right] - 1 + \frac{\Delta_1}{\Delta_2}\right] > 0 \qquad (28)$$

(g) Comparison among members of the proposed class of Exponential ratio estimator

The proposed class of Exponential ratio estimator $t_{LE(i)}$ is more efficient than the proposed estimator $t_{LE(\text{j})}$ if the following conditions holds;

$$MSE(t_{LE(\text{i})}) \le MSE(t_{LE(\text{j})})$$

$$\bar{Y}^2\left[1 - \frac{\Delta_{1(i)}}{\Delta_{2(i)}}\right] \le \bar{Y}^2\left[1 - \frac{\Delta_{1(\text{j})}}{\Delta_{2(\text{j})}}\right] = \frac{\Delta_{1(\text{j})}}{\Delta_{2(\text{j})}} - \frac{\Delta_{1(i)}}{\Delta_{2(i)}} \le 0 \quad = d_j - d_i \le 0 \qquad (29)$$

where,

$$d_j = \frac{\Delta_{1(\text{j})}}{\Delta_{2(\text{j})}}, \qquad d_i = \frac{\Delta_{1(\text{i})}}{\Delta_{2(\text{i})}}$$

Thus, for any two members of the proposed class of estimator, $t_{LE(i)}$ and $t_{LE(\text{j})}$; $t_{LE(i)}$ will be more efficient than $t_{LE(\text{j})}$ if the condition given above holds.

The empirical efficiency comparison was done by obtaining the percent relative efficiency (PRE) which is evaluated as

$$PRE = \frac{Var(t_y)}{MSE(t)} \; X \; 100 \qquad (30)$$

where, $t = t_y, t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{\text{LE}_1}, t_{\text{LE}_2}, t_{\text{LE}_3}, t_{\text{LE}_4}, t_{\text{LE}_5}, t_{\text{LE}_6}, t_{\text{LE}_7}, t_{\text{LE}_8}, t_{\text{LE}_9}, t_{\text{LE}_{10}}$

A PRE that is greater than 100 shows an increase in efficiency of the proposed estimator, while the PRE that is less than 100 shows a decrease in efficiency of the proposed estimator.

6. Numerical validation

Simulation study

To validate the theoretical results of this work, simulated data were generated. In this section, a finite population of (X, Y) with size N = 1000 was generated from a bivariate normal distribution with theoretical means of μ = (5, 5) and covariance matrix given as $\Sigma = \begin{pmatrix} 9 & 1.4 \\ 1.4 & 9 \end{pmatrix}$

The steps adopted in the simulation are:

1. Select a simple random sample of sizes n = 50, 100, 200 from the population of size N = 1000 without replacement.
2. From step 1, compute the following; $\bar{x}$, $\bar{y}$, $\bar{X}$, $\bar{Y}$, S_x^2, S_y^2, S_{xy}, C_x, C_y, ρ, β1(x), β2(x), Q_D, T_M, M_R, H_L.
3. Compute the values of t.
4. Repeat the process from (3) above 10,000 times.
5. Find $t = \frac{\sum_{i=1}^{10,000} t_i}{10,000}$, Bias$(t) = \frac{\sum_{i=1}^{10,000} \left(t_i - \bar{Y}\right)}{10,000}$ and $MSE(t) = \frac{\sum_{i=1}^{10,000} \left(t_i - \bar{Y}\right)^2}{10,000}$

Table 3: Bias, MSE and PRE values under simulation study

	Population N =1000								
	n = 50			n = 100			n = 200		
Estimators	Bias	MSE	PRE	Bias	MSE	PRE	Bias	MSE	PRE
t_y	0	0.1727	100	0	0.0784	100	0	0.0331	100
t_0	-0.1458	0.1491	115.8283	-0.0230	0.0674	116.3205	0.0007	0.0287	115.3310
t_1	-0.0109	0.1879	91.9110	0.0052	0.0734	106.7711	-0.0081	0.0371	89.1609
t_2	0.0157	0.3267	52.8563	0.0093	0.1538	50.9956	-0.0071	0.0627	52.7890
t_3	-0.0104	0.1592	108.4646	0.0036	0.0671	116.8306	-0.0091	0.0309	107.1094
t_4	0.0125	0.5385	32.0710	0.0203	0.2006	39.0957	-0.0001	0.1069	30.9659
t_5	0.0538	0.6590	26.2038	0.0217	0.3005	26.0939	-0.0016	0.1261	26.2409
t_6	-0.0127	0.1600	107.9042	0.0001	0.0674	116.3336	-0.0098	0.0308	107.5441
t_7	0.0027	0.1958	88.1902	0.0065	0.0907	86.4039	-0.0081	0.0373	88.6185
t_8	-2.6673	4.3206	3.9969	-2.6325	6.9408	1.1297	-1.9471	3.8041	0.8700
t_9	-0.0079	0.1595	108.2817	0.0031	0.0730	107.4162	-0.0098	0.0307	107.9412
t_{LE_1}	-0.0123	0.1072	161.1395	-0.0004	0.0280	279.6343	-0.0054	0.0202	163.7324
t_{LE_2}	-0.0306	0.0988	174.8682	-0.0155	0.0547	143.2379	-0.0135	0.0199	166.2996
t_{LE_3}	-0.0346	0.0289	596.8681	-0.0186	0.0204	384.5054	-0.0130	0.0056	586.9739
t_{LE_4}	-0.0423	0.1113	155.1854	-0.0164	0.0508	154.4441	-0.0113	0.0228	145.1416
t_{LE_5}	-0.0383	0.0335	515.2114	-0.0175	0.0187	418.9882	-0.0118	0.0067	495.3013
t_{LE_6}	**-0.0393**	**0.0228**	**756.502**	**-0.0186**	**0.0094**	**836.2972**	**-0.0127**	**0.0044**	**757.5895 *****
t_{LE_7}	**-0.0393**	**0.0229**	**755.6094**	**-0.0186**	**0.0094**	**836.1604**	**-0.0127**	**0.0044**	**757.4161 ****
t_{LE_8}	**-0.0289**	**0.0285**	**606.8992**	**-0.0092**	**0.0112**	**702.4782**	**-0.0104**	**0.0056**	**592.3896 ***
t_{LE_9}	-0.0124	0.0833	207.1739	0.0003	0.0272	287.8873	-0.0054	0.0200	165.2757
$t_{LE_{10}}$	-0.0287	0.0286	603.1337	-0.0091	0.0112	700.9574	-0.0103	0.0056	586.6178

Table 3 shows the results of the simulation study for n = 50, 100 and 200. From the display of the results, it can be clearly seen that the proposed estimator t_{LE_6} performed with the greatest efficiency with PREs of 756.502, 836.2972 and 757.5895 followed by the proposed estimator t_{LE_7} which performed almost equally as t_{LE_6} with PREs of 755.6094, 836.1604 and 757.4161 and then the proposed estimator t_{LE_8} with PREs of 606.8992, 702.4782 and 592.3896 respectively in the three samples used for the simulation.

Real data set

To examine the performance of the proposed family of ratio estimators with some of the existing estimators discussed in literature, four (4) populations from literature have been considered as given below.

Table 4: Statistics for four natural populations

Parameter	Popln. 1	Popln. 2	Popln. 3	Popln. 4
N	34	34	34	250
n	20	10	20	89
$\bar{Y}$	856.4117	856.4117	856.4117	5073.171
$\bar{X}$	208.8823	199.4412	199.4412	29561.09
C_y	0.8561	0.8561	0.8561	1.747251
C_x	0.7205	0.7531	0.7531	2.112318
ρ_{yx}	0.4491	0.4453	0.4453	0.807536
$\beta_1(x)$	0.9782	1.1823	1.1823	4.894861
$\beta_2(x)$	0.0978	1.0445	1.0445	31.8449
Q_D	80.25	89.375	89.375	12.0
T_M	162.25	165.562	165.562	101.0
M_R	284.5	320.0	320.0	105.0
H_L	190.0	184.0	184.0	98.0

Population 1 (Source: Singh and Chaudhary [24], adapted from [16]
Y = Area under wheat crop in acres during 1974 in 34 villages.
X = Area under wheat crop in acres during 1971 in 34 villages.

Population 2 (Source: Singh and Chaudhary [24], adapted from [16]
Y = Area under wheat crop in acres during 1974 in 34 villages.
X = Area under wheat crop in acres during 1973 in 34 villages.

Population 3 (Source: Singh and Chaudhary [24], adapted from [16]
Y = Area under wheat crop in acres during 1974 in 34 villages.
X = Area under wheat crop in acres during 1973 in 34 villages.

Population 4 (Source: Ozge and Didem [25]
Y = Amount of oil olive produced (ton).
X = Number of fruits trees.

Table 5: Bias, MSE and PREs of various estimators for populations 1and 2

Estimators	Population 1			Population 2		
	MSE	**Bias**	**PRE**	**MSE**	**Bias**	**PRE**
t_y	11067.09	0.0000	100	37944.3	0.0000	100
t_0	910.041	-1.0626	1216.1080	3666.981	-4.2818	1034.7558
t_1	5342.89	1.2346	207.1370	21199.04	7.3667	178.9907
t_2	32468.98	7.9185	34.0851	113415.9	26.9195	33.4559
t_3	6245.27	-0.5268	177.2074	22230.88	-0.6024	170.6828
t_4	49049.07	1.2922	22.5633	173174	5.3425	21.9111
t_5	15296.38	-6.6839	72.3510	63180.11	-19.5528	60.0574
t_6	8834.96	0.0000	125.2647	30420.24	0.0000	124.7337
t_7	1342.93	-0.4750	824.1001	7412.827	-0.5554	511.8735
t_8	10581.52	3.588552	104.5888	34832.14	14.05395	108.9347
t_9	6556.94	-173.683	168.7842	23234.7	-173.69	163.3087
t_{LE_1}	531.56	-0.6207	2082.0153	4059.44	4.7401	934.7176
t_{LE_2}	293.39	-0.3426	3772.2033*	1006.835	-1.17564	3768.6711
t_{LE_3}	**116.37**	**-0.1359**	**9510.4729*** **	**195.3856**	**-0.22814**	**19420.2132 *** **
t_{LE_4}	665.77	-0.7774	1662.3083	2286.487	-2.66985	1659.5021
t_{LE_5}	236.57	-0.2762	4678.1816**	689.5654	-0.80518	5502.6398*
t_{LE_6}	498.88	-0.5825	2218.3681	1241.895	-1.45011	3055.3549
t_{LE_7}	498.88	-0.58253	2218.3681	1241.895	-1.45011	3055.3549
t_{LE_8}	789.134	-0.9214	1402.435	2174.632	-2.5392	1744.86
t_{LE_9}	690.924	-0.8068	1601.78	542.105	-0.633	6999.431**
$t_{LE_{10}}$	743.436	-0.8681	1488.641	1895.71	-2.2136	2001.587

Table 6: Bias, MSE and PREs of various estimators for populations 3 and 4

Estimators	Population 3			Population 4		
	MSE	**Bias**	**PRE**	**MSE**	**Bias**	**PRE**
t_y	11067.09	0.0000	100	568545.7	0.0000	100
t_0	1104.853	-1.2901	1001.6799	188388.4	-37.1343	301.7944
t_1	6183.054	2.1486	178.9907	1098714	134.1484	51.7465
t_2	33079.63	7.8515	33.4559	1700271	29.6440	33.4385
t_3	6484.006	-0.1757	170.6829	625893.3	46.6002	90.8375
t_4	50509.09	1.8235	21.9111	2889344	21.0642	19.6773

t_5	18427.53	-5.7029	60.0574	3290777	104.5044	17.2769
t_6	8872.571	0.0000	124.7337	197789.2	0.0000	287.4504
t_7	2162.074	-0.1620	511.8735	708428.3	73.7545	80.2545
t_8	10159.38	4.09907	108.9347	17595893	317.6436	3.231128
t_9	6776.788	-170.85	163.3087	867528.6	-3320.82	65.53625
t_{LE_1}	620.8999	-0.725	1782.4274	43579.85	-8.5903	1304.6068
t_{LE_2}	352.1649	-0.4112	3142.5875	64153.05	-12.6456	886.2333
t_{LE_3}	**137.2272**	**-0.1602**	**8064.7933*** **	19506.48	-3.8450	2914.6504**
t_{LE_4}	670.9437	-0.7834	1649.4812	29274.98	-5.7706	1942.0874*
t_{LE_5}	242.4661	-0.2831	4564.3865**	**7178.921**	**-1.4151**	**7919.6539*** **
t_{LE_6}	586.7311	-0.6851	1886.2286*	84036.75	-16.5649	676.5441
t_{LE_7}	586.7311	-0.6851	1886.2286*	84036.75	-16.5649	676.5441
t_{LE_8}	919.955	-1.0742	1203.003	96096.37	-18.9421	591.6412
t_{LE_9}	803.8727	-0.9387	1376.721	74838.26	-14.7518	759.6993
$t_{LE_{10}}$	867.0045	-1.0124	1276.474	103571.5	-20.4155	548.940

6. Discussion of results

In this work, a generalized shrinkage-type estimator has been proposed. The proposed estimator contains some minimizing constants γ_1 and γ_2, and some unknown constants α_1, α_2, V, C and r whose value has been chosen within the range of -2 to 2. The optimal values of the two constants γ_1 *and* γ_2 were obtained by partially differentiating the MSE and was used to obtain the minimal MSE of the proposed generalized estimator. The optimal mean squared error of this proposed generalized shrinkage-type estimator is shown in equation (21), and the optimality condition is observed to be a function of the parameters V, C and r. This generalized shrinkage-type estimator is proposed in equation (11) with appropriate choices of the unknown constants $\gamma_1, \gamma_2, \alpha_1, \alpha_2$, V, C, and r to produce the members of the estimator. Table 1 shows some existing estimators proposed by [2, 14, 26, 10, 27 and 16], which are members of the proposed generalized estimator and some other new estimators which were generated from the proposed class of estimator of population mean under simple random scheme.

Simulation study results presented in Table 3 was used for the empirical analysis. Three (3) different sample sizes of 50, 100 and 200 were selected from a population of size 1000, for the analysis. From the results, it was observed that all the proposed estimators had mean squared errors smaller than and Percent Relative Efficiencies (PREs) greater than the classical ratio estimator, the exponential ratio estimator, the classical regression estimator, Javid et al. (2021) estimator for n = 50, 100 and 200, with MSE(s) of 0.0228, 0.0094 and 0.0044 and PREs of 756.502%, 836.2972% and 757.5895% respectively. It is also observed from Table 3 that the proposed estimator t_{LE_7} performed almost equally as the estimator, t_{LE_6} with MSE(s) 0.0229, 0.0094 and 0.0044 and PREs 755.6094%, 836.1604% and 757.4161% respectively. On the other hand, the existing estimator t_8 (Yahaya *et al.*, 2020) with MSE(s) of 4.3206, 6.9408 and 3.8041 has the highest MSEs and lowest PREs of 3.9969%, 1.1297% and 0.8700% in the 3 samples used for the simulation and thus proves to be the least efficient in all the estimators considered in this work.

Four (4) real data sets as presented in Tables 4 were also used for empirical analysis. From the results as presented in Tables 5 and 6, all the proposed estimators were observed to have smaller mean squared errors than all other existing estimators considered in this work such as the classical ratio estimator, the classical regression estimator, the exponential ratio estimator, Javid *et al.* [16] and others, except the proposed estimator t_{LE_1} which has MSE larger than t_0 [16] in population 2. The proposed estimators t_{LE_3} performed with the greatest efficiency in populations 1, 2 and 3 with MSEs of 116.3674, 195.3856 and 137.2272 and PREs of 9510.4729%, 19420.2132% and 8064.7933% respectively while the proposed estimator t_{LE_5} performed with greatest efficiency in population 4 with a mean squared error of 7178.921 and PRE of 7919.6539% among all the estimators (both existing and proposed) considered in this work. On the other hand, the existing estimator t_4 has the least efficiency in population 1, 2 and 3 with mean squared errors of 49049.07, 173174 and 50509.09 and Percent Relative Efficiency (PREs) of 22.5633%, 21.9111% and 21.9111% respectively. The estimator t_8 has the least efficiency in population 4 with a mean squared error of 17595893 and PRE of 3.231128%.

7. Conclusion

From the discussions above, it can be concluded that the proposed class of estimators is superior in terms of both theoretical and empirical efficiency compared to the other existing members of the proposed class of estimators and non-members considered in this work under the optimality condition mentioned above.

The proposed class of estimator with these desirable properties is highly recommended for use in practical applications where the use of auxiliary information on a single auxiliary variable under simple random sampling without replacement and other relevant conditions are required.

Acknowledgments

The authors wish to acknowledge the reviewers of this manuscripts and various authors of leading works used in this manuscript.

Declaration of Interest:

The authors declare that there is no conflict of interest in this paper.

References

[1] Gupta, R. K. and Yadav, S. K. (2017). New efficient estimators of population mean using non-traditional measures of dispersion. *Open J. Stat.* 7(3), 394–404.

[2] Cochran, W. G., (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *Journal of Agricultural Science,* 30, 262 – 275.

[3] Robson, D. S. (1957). Application of multivariate polykays to the theory of unbiased ratio-type estimation. *Journal of American Statistical Association*, 52(282), 491-508.

[4] Subramani, J. and Kumarapandiyan, G. (2013). Estimation of population mean using deciles of an auxiliary variable. *Statistics in Transition-New Series*, 14(1), 75–88.

[5] Subzar, M., Raja, S. M., Rishu, J. and Shabeer, M. (2017). Enhancing Ratio Type Estimators for Estimating Population Mean using Auxiliary Information in Survey Sampling. *International Journal of Agricultural Statistical Sciences,* 13(1), 213-220.

[6] Yadav, S. K. and Adewara, A. A. (2013). On improved estimation of population mean using qualitative auxiliary information. *Math. Theor. Model.* 3(11), 42-50.

[7] Muhammad, I., Oyeyemi, G. M. and Kareem, A. O. (2023). Combined Exponential-Type Estimators for Finite Population Mean in Two-Phase Sampling. *Asian Journal of Probability and Statistics*, 21 (2), 44-58.

[8] Abid, M., Abbas, N., Sherwani, R. A. K. and Nazir, H. Z. (2016). Improved ratio estimators for the population mean using non-conventional measure of dispersion. *Pak. J. Stat. Oper. Res.* XII (2), 353–367.

[9] Misra, S., Gupta, R. K. and Shukla, A. K. (2012). Generalized class of estimators for estimation of finite

population variance. *International Journal of Agriculture and Statistical Science*, 8(2), 447-458.

[10] Kadilar, C. O. (2016). A new exponential type estimator for the population mean in simple random sampling. *Journal of Modern Applied Statistical Methods*, 15(2), 207– 214.

[11] Jerajuddin ,M. and Kishun, J. (2016). Modified Ratio Estimators for Population Mean Using Size of the Sample, Selected From Population. *IJSRSET*, 2(2), 10-16.

[12] Yahaya, Z., Isah, M. and Najmuddeen, M. S. (2020). Alternative Ratio-Product Type Estimator in Simple Random Sampling. *Communication in Physical Sciences*, 5(4), 418-426.

[13] Zakari, 1. Y., Muili, J. O., Tela, M. N., Danchadi, N. S. and Audu, A. (2020b). Use of Unknown Weight to Enhance Ratio-Type Estimator in Simple Random Sampling. *Lapai Journal of Applied and Natural Sciences*, 5(1), 74-81.

[14] Bahl, S. and Tuteja, R. K. (1991), Ratio and product type exponential estimators. *Journal of Information and Optimization Sciences,* 12(1), 159-164.

[15] Yunusa, M. A., Audu, A., Ishaq, O. O. and Beki, D. O. (2021). An efficient exponencial type estimator for estimating finite population mean under simple random sampling. *Annals computer science series* vol. XIX.

[16] Javid, S., Sat, G. and Ronald, O. (2021). On using the conventional and nonconventional measures of auxiliary variable for mean estimation. *Mathematical Problems in Engineering*, 2021(1), 1-13.

[17] Nitu, M. and Mandowara, V. L. (2022). Efficient family of exponential and dual estimators of finite population mean in ranked set sampling. *International Journal of Applied Mathematics & Statistical Sciences (IJAMSS)*, 9(3), 2319-3972.

[18] Enang, E. I. and Ekpenyong, E. J. (2018). On the Optimal search for efficient estimators of population mean in simple random sampling in the presence of auxiliary variable. *Hacettepe Journal of Mathematics and Statistics*, 47(4), 963-982.

[19] Ünal, C. and Kadilar, C. (2019). Improved family of estimators using exponential function for the population mean in the presence of non-response. *Communications in Statistics – Theory and Methods*, 50(1), 237-248.

[20] Ceren, U. and Kadilar, C. (2021). Improved Estimators using Exponential Function for the Population Mean in Simple and Stratified Random Samplings. *Pak. J. Stat. Oper. Res.* 17(2), 333-342.

[21] Gagan, k., Priyanka, B., Surendra, K. and Yadav, K. (2018). General exponential ratio type estimator of population mean under simple random sampling scheme. *IJCRT*, 6(1), 450-455.

[22] Ekpenyong, E. J. and Enang, E. I. (2019). Improved Exponential Ratio-In-Regression Class of Estimators of the Population Mean in Simple Random Sampling in the presence of Two Auxiliary Variables. *International Journal of Basic Science and Technology*, 5(1), 34- 48.

[23] Rajesh, S., Prabhakar, M., Ahmed, A. and Supriya, K. (2020). Exponential Type Estimator for Estimating Finite Population Mean. *Int. J. Comp. Theo. Stat.*, 7(1), 2210-1519.

[24] Singh, D. and Chaudhary, F. S. *Theory and Analysis of Sample Survey Design*, New age International Publishers, New Delhi, 1986.

[25] Ozgul, A. and Didem O. (2017). An Overview to Exponential Type Mean Estimators in Simple Random Sampling Scheme and Efficiency Comparison. *International Journal of Probability and Statistics,* 6(3), 51-64

[26] Murthy, M. N. (1964). Product method of estimation. *Sankhya,* 26, 294-307.

[27] Kadilar, C. and Cingi, H. (2003). A study on the chain ratio-type estimator, Hacettepe *Journal of Mathematics and Computation,* 32, 105 - 108.