IRAQI STATISTICIANS JOURNAL

https://isj.edu.iq/index.php/isj

ISSN: 3007-1658 (Online)

Safe Bayesian Quantile Regression

Mohammed Sadeq Shniat[1], and Ahmad Naeem Flaih[2]

[1,2]Department of Statistics , College of Administration and Economics, University of Al-Qadisiyah,, 58001 Al-Qadisiyah, Iraq

ARTICLE INFO

Article history:
Received 20/11/2024
Revised 26/11/2024
Accepted 14/1/2025
Available online 15/5/2025

Keywords:
Bayesian estimation
learning rate
safe Bayes
posterior distribution
Simulation

ABSTRACT

In Bayesian estimation of the quantile regression parameters become more accurate estimate if the likelihood function equipped with learning rate parameter (safe Bayesian). Learning rate parameter can be se to solve the problem of overfitting of the estimated model. The amount of the data (likelihood) can be controlled by the learning rate parameter which reflects on the good conclusion that drawn. Bayesian estimation under the likelihood with leaning rate parameter results the so called learning rate generalized posterior. Choosing the appropriate learning rate parameter is the key idea of this paper, simulation study has conducted based on suggesting that the learning rate parameter follows the multinomial distribution. New Gibbs sampler algorithm has developed beside the quantile regression. Real data analysis has done with the response variable that represents the creatinine in the blood along with some predictor variables. Based on the results of simulation study and real data we have conclude that the proposed model is perform well along with other different regression models.

1. Introduction

Quantile regression model (QR) analysis is concerned in estimating of the quantiles of the conditional distribution (Y|X) and is the most popular robust regression that has applied in many scientific data fields. The median regression is a special case of quantite regression when the loss function is of absolute form, which is a robust estimated regression model. [1] introduced the quantile regression which is become increasable popular regression model, the QR in general gives an overall guess of the predictor variables effects with different quantiles levels (0<q<1). From a Bayesian perspective, [2] introduced the Bayesian quantile regression, for more information about on quanlite regression, see [3]. [4] proposed new technique to estimates the unknown parameters of the quantile regression from the Bayesian point of view. They developed new Gibbs Sampler algorithm based on new hierarchical prior distribution model. They used the skewed Laplace distribution as the distribution of likelihood of response variable. [5] Introduced employing of safe Bayesian parameter with the likelihood function to address the misspecification model of the response variable (likelihood) in classical Bayesian regression. The proposed method has applied to the multiple linear regression with the variable selection method, lasso. [6] Introduced the safe Bayesian estimation method along with penalized lasso variable selection procedure, in this paper the hierarchical model have developed the model that proposed by Park and Casella in 2008. Variable selection procedure under adaptive lasso quantile regression model has been intensively studied in [7]. Also, the variable selection procedure in quantile regression for improving the prediction accuracy and to improving the interpretability of the estimated quantile regression model has been investigated [8]. Tobit quantile regression studied to investigate the parameter estimation via

* Corresponding author.E-**mail** address: ahmad.flaih@qu.edu.iq
https://doi.org/10.62933/8ang6p97

Bayesian technique by utilizing g-prior distribution [9]. Bayesian estimation of the Tobit quantile regression model form a variable selection procedure with elastic met penalized method has studied [10]. The previous literature review articles assumed that the response variable (likelihood) follows normal distribution in Bayesian estimation of quantile regression model. The misspecification of likelihood distribution leads to poor prediction. To solve the misspecification problem, the safe Bayesian is the solution. In safe Bayesian the learning rate control the weight of the likelihood function to adjusted the misspecification problem of normal distribution. We assumed that the learning rate parameter of safe Bayesian has uniform (0,1) random variable and then new hierarchical model and new posterior distribution have developed to implement the Gibbs sampler algorithm.

2. Safe Bayesian Quantile Regression

Quantile regression model (QRM) was proposed by [1]. QRM provides different models of the response variable based on qth-quantiles which is defined as $Q(q) = F^{-1}(q)$ = infmum $\{x: F(x) \geq q\}$ where $F(x) = p_r(X \leq x)$ is the CDF of random Variable X, and $0<q<1$, [11]. The QRM estimators are found by minimizing the weighted sum of the absolute residuals. Now suppose that QRM is defined as,

$$y_i = x_i^T \beta(q) + e_i(q), \quad (1)$$

where $p_r(-\infty < e_i(q) < 0) = q.$ The following optimization problem can be used to estimate $\beta(q)$,

$$\hat{\beta} = argmin[\sum_{i:y_i \geq x_i^T\beta}^{\beta} q|y_i - x_i^T\beta(q)| + \sum_{i:y_i < x_i^T\beta}^{\beta} (1-q)|y_i - x_i^T\beta(q)|)$$
$$= argmin \sum_{i=1}^{n} \rho_q(y_i - x_i^T\beta(q))$$
$$= argmin \sum_{i=1}^{n} \rho_q(e_{i(q)}) \quad (2)$$

. Making inference about the quantile regression parameters can be followed by parametric approach if the error distribution $f_q(e_i)$ is specified. The skewed Laplace distribution is the commonly choice of error distribution; see [2], and [12] for more details. The (SLD) have the following a probability density function

$$f_q(e_i) = \frac{q(1-q)}{\sigma_q} \exp[-\frac{|e_i| + (2q-1)e_i}{2\sigma_q} \quad (3)$$

[4] introduced a proposition that allows the SLD to be viewed as scale mixture of normal distribution, mixing with exponential distribution,

$$y_i = x_i^T\beta(q) + a\tau_i + b\sqrt{\sigma^2\tau_i}z_i \quad (4)$$

For employing the safe Bayesian procedure with quantile regression, we write down the generic posterior distribution,

$$\pi(\gamma|y^n, x, \alpha) = \frac{\left(f(y^n|x^n,\gamma)\right)^{\alpha}\pi(\gamma)}{\int\left(f(y^n|x^n,\gamma)\right)^{\alpha}\pi(\gamma)d\gamma} \quad (5)$$

where $\alpha \in [\,0\,,1\,]$ is learning parameter, $\pi(\gamma|.)$ is the posterior probability, $f(y^n|.)$ is the likelihood function, and $\pi(\gamma)$ is the prior probability. The goal of this paper is to study the safe Bayesian quantile regression with lasso and adaptive lasso methods, therefore,

$$\beta^{\wedge}{}_{lasso} = argmin_{\beta} \sum_{i=1}^{n} \rho_q\left(y_i - x_i^T\beta(q)\right) + \lambda||\beta||_1 \quad \textbf{(6)}$$

where $\lambda > 0$ is the shrinkage parameters and $\|\beta\|_1$ is the L_1-norm penalty function. See [13] for more information on lasso quantile regression. [14] stated that in Bayesian framework one can assume that β_j can have Laplace distribution.

Also, the Bayesian adaptive lasso quantile regression can be defined by,

$$\hat{\beta}_{ad.lasso} = argmin_{\beta} \sum_{i=1}^{n} \rho_q(y_i - x_i^T\beta(q)) + \sum_{i=1}^{k} \lambda_j|\beta_j|.). \quad \textbf{(7)}$$

Based on model (1) and posterior distribution (2), the following is the hierarchal model,

$$\begin{aligned} &y_i = x_i{}^T(\text{q}) + a\tau_i + b\sqrt{\sigma^2\tau_i}z_i, \\ &y|x,\beta,\sigma^2,\tau_i,z_i,\alpha \sim [N(x_i{}^T\beta(\text{q}) + a\tau_i, b^2\sigma^2\tau_i]^{\alpha}, \\ &z_i \sim \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}z_i{}^2), \end{aligned} \quad (8)$$

$$\tau_i|\sigma^2 \sim \prod_{i=1}^{n} \frac{1}{\sigma^2} exp(\frac{\tau_i}{\sigma^2}), \quad (9)$$
$$\alpha \sim uniform(0, 1),$$
$$\sigma^2 \sim IG(a_{\sigma^2}, b_{\sigma^2}),$$
$$\beta|s, \lambda \sim \prod_{j=1}^{p} \frac{\lambda^2}{2\sigma^2} exp\{-\frac{\lambda^2|\beta_j|}{\sigma^2}\},$$
$$\lambda^2 \sim Gamma(c, d).$$

the conditional distribution of $(y|.)$ in lasso quantile regression model is

$$f(y|x, \tau_i, \beta(q), z_i, \sigma^2) = \left[\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi b^2\sigma^2\tau_i}} exp\{-\frac{(y_i - x_i'\beta(q) - a\tau_i)^2}{2b^2\sigma^2\tau_i}\right]^{\alpha}$$

$$= \left(\frac{1}{(b^2\sigma^2\tau_i)^{\frac{n-1}{2}}}\right)^{\alpha} exp\left\{-\frac{\sum_{i=1}^{n-1}\alpha(y_i - x_i^T\beta(q) - a\tau_i)^2}{2b^2\sigma^2\tau_i}\right\},$$

$$\propto \frac{1}{(b^2\sigma^2\tau_i)^{\frac{\alpha(n-1)}{2}}} exp\left\{\frac{-\sum_{i=1}^{n-1}\alpha(y_i - x_i^T\beta(q) - a\tau_i)^2}{2\alpha^{-1}b^2\sigma^2\tau_i}\right\} \quad (10)$$

it is obvious that the last expression is a normal distribution with mean ($x_i'\beta(q) + a\tau_i$) and variance ($\alpha^{-1}b^2\sigma^2\tau_i$).

and the posterior distributions are as follows, the full conditional posterior distribution of τ_i is,

$$\pi(\tau_i|\beta(q), y, \sigma^2, \lambda^2, s) \propto (\tau_i)^{\left(\frac{2-\alpha}{2}\right)-1} exp\{-\frac{1}{2}\left[\frac{(y_i - x'\beta)^2}{\alpha^{-1}b^2\sigma^2}.\tau_i^{-1} + (\frac{a^2 + 2\alpha^{-1}b^2}{\alpha^{-1}b^2\sigma^2})\tau_i\right]\}$$

which is follow generalized inverse Gaussian (GIG) distribution with

$$x = \tau_i, \ p = \frac{2-\alpha}{2}, a_1 = (\frac{a^2+2\alpha^{-1}b^2}{\alpha^{-1}b^2\sigma^2}), \ and \ b_1 = \frac{\sum(y_i - x'\beta(a))^2}{\alpha^{-1}b^2\sigma^2} \quad (11)$$

the full conditional posterior distribution of **s** is

$$\pi(s_j|x, y, \beta, \sigma^2, \lambda^2) \propto \pi(\beta_j(q)|s_j).\pi(s_j|\lambda^2) \propto (S_j)^{-\frac{1}{2}}$$

again , recall the GIG distribution, can be viewed as GIG distribution.

The full conditional posterior distribution of β is ,

$$\pi(\beta_j|x, y, \sigma^2, \lambda^2, \tau_i, S) \propto f(y|x, \tau_i, S, \sigma^2, \lambda^2) \times \pi(\beta_j|S_j)$$

$$\propto \{-\frac{1}{2}\left[\left(\sum_{i=1}^{n}\frac{\alpha x_{ij}^2}{b^2\sigma^2\tau_i} + \frac{1}{S_j}\right)B_j^2 - 2\sum_{i=1}^{n}\frac{\tilde{y}_i x_{ij}}{\alpha^{-1}b^2\sigma^2\tau_i}\beta_j\right]$$

where $x_i = (x_{i1}, \ldots, x_{ik}), and \ \tilde{y}_i = a\tau_i - y_i$.

let, $\Psi_1^{-1}\left[\sum_{i=1}^{n}\frac{\alpha x_{ij}^2}{b^2\sigma^2\tau_i} + \frac{1}{S_j}\right]$

and

$$\Psi_2 = \Psi_1 \alpha b^{-2}(\sigma^2)^{-1}\sum_{i=1}^{n}\tilde{y}_i x_{ij}\tau_i^{-1} \quad (12)$$

Then, we can say that Ψ_2 is the mean of β_j and Ψ_1 is the variance of β_j , so β_j has the normal distribution with mean Ψ_2 and variance Ψ_1.

The full conditional distribution of σ^2 is

$$\pi(\sigma^2|x, y, \tau_i, \beta, S, \lambda^2) \propto \pi(y|x, \tau_i, \beta, \sigma^2)\ \pi(\tau_i|\sigma^2).\pi(\sigma^2).\pi(\beta|\sigma^2)$$

$$\propto (\sigma^2)^{-\left(\frac{3n\alpha+2a_\sigma+2p}{2}\right)-1} exp\left\{-\frac{1}{\sigma^2}\left[\sum_{i=1}^{n}\frac{\alpha(y_i - x_i'\beta(q) - a\tau_i)^2}{b^2\tau_i} + \sum_{i=1}^{n}\tau_i + b_\sigma + \frac{\lambda^2}{2}\sum_{j=1}^{p}S_j\right]\right\}, \quad (13)$$

then, we can conclude that the distribution of σ^2 is inverse-Gamma with shape parameter $(\frac{3n\alpha+2a_\sigma+2p}{2})$ and scale parameter ,

$$\left[\sum_{i=1}^{n}\frac{\alpha(y_i - x_i'\beta(q) - a\tau_i)^2}{b^2\tau_i} + \sum_{i=1}^{n}\tau_i + b_\sigma + \frac{\lambda^2}{2}\sum_{j=1}^{p}S_j\right].$$

The full conditional posterior of learning rate parameter α is given by

$$\pi(\alpha|x, y, \beta, \tau_i, \sigma^2) \propto \pi(y|x, \beta, \sigma^2, \alpha, \tau_i) \times \pi(\alpha)$$

$$\propto \prod_{i=1}^{n-1}\left[\left(\frac{1}{\sqrt{2\pi b^2\sigma^2\tau_i}}\right) exp\left\{\frac{(y_i - x_i'\beta(q) - a\tau_i)^2}{2b^2\sigma^2\tau_i}\right\}\right]^{\alpha}$$

with $p = (\frac{1}{\sqrt{2\pi b^2\sigma^2\tau_i}}) exp\left\{\frac{(y_i - x_i'\beta(q) - a\tau_i)^2}{2b^2\sigma^2\tau_i}\right\}$

and $\sum_{i=1}^{n-1} p_i = 1$, $\sum_{i=1}^{n-1}\alpha_i = n - 1$.

then it can be defined as probability mass function of multinomial distribution ,

$$\boldsymbol{\alpha} \sim \boldsymbol{M}_{\boldsymbol{n-1}}(\boldsymbol{n-1}; \boldsymbol{p}_1, \ldots, \boldsymbol{p}_{\boldsymbol{n-1}}).$$

Finally, we can write the full condition posterior distribution of $\boldsymbol{\lambda^2}$ as fellows,

$$\boldsymbol{\pi}(\boldsymbol{\lambda^2}|\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\tau_i}, \boldsymbol{\beta}, \boldsymbol{S}, \boldsymbol{\sigma^2}) \propto \boldsymbol{\pi}(\boldsymbol{s}|\boldsymbol{\lambda^2}).\boldsymbol{\pi}(\boldsymbol{\lambda^2})$$
$$\propto (\boldsymbol{\lambda^2})^{\boldsymbol{p+c-1}} \boldsymbol{exp}\left\{-\boldsymbol{\lambda^2}\left[\frac{\sum_{i=1}^{n} \boldsymbol{S_j}}{\boldsymbol{2\sigma^2}} + \boldsymbol{d}\right]\right\}$$

then, $\boldsymbol{\lambda^2}$ has a Gamma distribution with shape parameter (p+c) and scale parameter $\left(\left[\frac{\sum_{i=1}^{n} \boldsymbol{S_j}}{\boldsymbol{2\sigma^2}} + \boldsymbol{d}\right]\right)$.

It is worth to noting that the previous hierarchical model and the posterior distribution are applicable or both the lasso and adaptive lasso with easy assumption about the parameter (λ in lasso) to be (λ_j in adaptive lasso), See [15] for more details for learning rate parameter and safe Bayesian.

3. Simulation and Real Data Analysis

This section we discuss the performance of the proposed model (SBALQ) and investigate the implementing of the Gibbs sampling algorithm in generating the samples of the interested parameters from the proposed posterior distributions. The obtained results are compared with quantile regression (QR) model, Bayesian quantile regression (BQR) model, and Bayesian adaptive lasso quantile regression (BALQR) model. In simulation study we discussed two examples under two sample sizes (n=50) and three quantile levels (0.25, 0.5, 0.75), also we set 12000 iteration and burn-in the first 2000 iterations for stability purposes. Furthermore, We

ran 200 replications to measure the quality of performance of the proposed model and other methods, The Bias, mean square error (MSE), and mean absolute error (MAE) have used as quality criterions for coefficient estimates,

$$\boldsymbol{Bias} = \hat{\beta}_j - \beta_j,$$

and

$$\boldsymbol{MSE} = \frac{1}{n}\sum_{i=1}^{n}({y_i}^{actual} - {y_i}^{estimated})^2$$
$$= (\hat{\beta} - \beta)^t \textstyle\sum(\hat{\beta} - \beta), \quad (14)$$

and

$$\boldsymbol{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|{y_i}^{actual} - {y_i}^{estimated}\right|, \quad (15)$$

where ${y_i}^{actual} = X\beta^{actual}$, ${y_i}^{estimated} = X\hat{\beta}$, and $\sum$ is the variance- covariance matrix of X.

3.1 Simulation example 1

This example assumed the true vector of parameters estimates as follows،

β = (3, 1.5, 0, 0, 2, 0, 0, 0)

and the process of generating data are as follow،

$$Y = X'\beta + N(0, \sigma^2) \quad (16)$$

So, we have eight explanatory variables with assumed that the *i-th* and *j-th* explanatory variables have pairwise Correlation equal to $\sum_{ij} = \rho^{|i-j|}$. We also set $\rho = 0.5$ and $\sigma = 3$. For computational purposes we standardized the covariates values and centered the response variable values. This example have discussed by [14]. Also, we use the posterior distribution of the learning rate parameter δ to estimate the mean value and we obtained that ($\alpha = 0.7747028$) for (n=50) and ($\alpha = 0.783529$) for (n=150).

the following are the results of simulation example one for (n=50) listed in some tables and figures.

Table 1: Coefficient estimates with three quantile levels (0.25,0.5,0.75) for simulation example 1 (n=50)

True parameters		3	1.5	0	0	2	0	0	0
Quantiles	Metods	B1	B2	B3	B4	B5	B6	B7	B8
0.25	QR	2.68469	1.79879	0.23676	-0.09336	2.32150	-0.67356	-0.17293	-0.23385
	BQR	2.76381	1.68071	-0.03631	0.01092	2.42359	-0.14793	0.11575	-0.03346
	BALQR	3.17801	1.59240	0.00357	-0.08165	2.17809	-0.26892	0.15751	-0.02704
	SBALQ	3.10837	1.51302	0.00278	0.00267	2.06093	-0.03371	0.13587	0.00134
0.5	QR	2.79558	1.66124	-0.12127	-0.02138	2.19287	-0.10568	0.12302	-0.01766
	BQR	2.85902	1.68141	-0.03797	0.00146	2.81306	-0.15681	0.12383	-0.03727
	BALQR	2.89899	1.63731	0.02249	-0.00762	2.22607	-0.15008	0.13853	-0.00728
	SBALQ	3.03667	1.56381	0.00565	0.00080	2.13395	-0.00765	0.08522	0.00013
0.75	QR	2.83549	1.71965	-0.16288	0.10580	1.91538	-0.07855	0.13692	0.01528
	BQR	2.85902	1.68141	-0.03797	0.00146	2.13131	-0.15681	0.12383	-0.03727
	BALQR	2.78192	1.65303	-0.17532	0.04015	1.87931	-0.09141	0.05420	0.00997
	SBALQ	2.97201	1.56484	0.01915	-0.00051	1.93058	-0.19779	0.00950	0.00454

Table 2: Bias values with three quantile levels (0.25,0.5,0.75) for simulation example 1 (n=50)

Quantiles	Metods	B1	B2	B3	B4	B5	B6	B7	B8
0.25	QR	0.31531	0.29879	0.23676	0.09336	0.32150	0.67356	0.17293	0.23385
	BQR	0.23619	0.18071	0.03631	0.01092	0.42359	0.14793	0.11575	0.03346
	BALQR	0.17801	0.09240	0.00357	0.08165	0.17809	0.26892	0.15751	0.02704
	SBALQ	0.10837	0.01302	0.00278	0.00267	0.06093	0.03371	0.13587	0.00134
0.5	QR	0.20442	0.16124	0.12127	0.02138	0.19287	0.10568	0.12302	0.01766
	BQR	0.14098	0.18141	0.03797	0.00146	0.81306	0.15681	0.12383	0.03727
	BALQR	0.10101	0.13731	0.02249	0.00762	0.22607	0.15008	0.13853	0.00728
	SBALQ	0.03667	0.06381	0.00565	0.00080	0.13395	0.00765	0.08522	0.00013
0.75	QR	0.16451	0.21965	0.16288	0.10580	0.08462	0.07855	0.13692	0.01528
	BQR	0.14098	0.18141	0.03797	0.00146	0.13131	0.15681	0.12383	0.03727
	BALQR	0.21808	0.15303	0.17532	0.04015	0.12069	0.09141	0.05420	0.00997
	SBALQ	0.02799	0.06484	0.01915	0.00051	0.06942	0.19779	0.00950	0.00454

Table 3: MES and MAE values with three quantile levels (0.25,0.5,0.75) for simulation example 1 (n=50)

Quantiles	Methods	MSE	MAE
0.25	QR	1.14450	0.90623
	BQR	0.96664	0.68844
	BALQR	1.16697	0.84866
	SBALQ	0.89856	0.56029
0.5	QR	1.01219	0.70341
	BQR	0.96317	0.68849
	BALQR	0.91329	0.67290
	SBALQ	0.73764	0.55007

0.75	QR	1.21445	0.75564
	BQR	0.96317	0.68849
	BALQR	1.25521	0.79354
	SBALQ	0.81077	0.64128

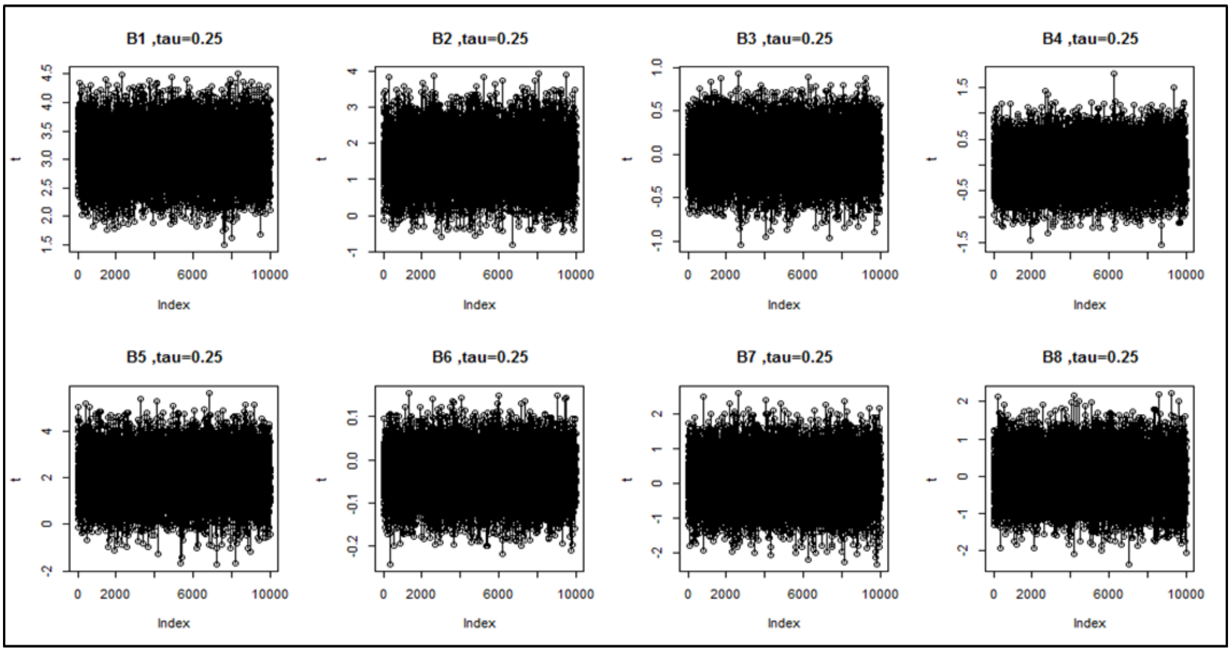

Figure 1. Trace plots of estimators values under quantile level (0.25) for simulation example 1 with n=50

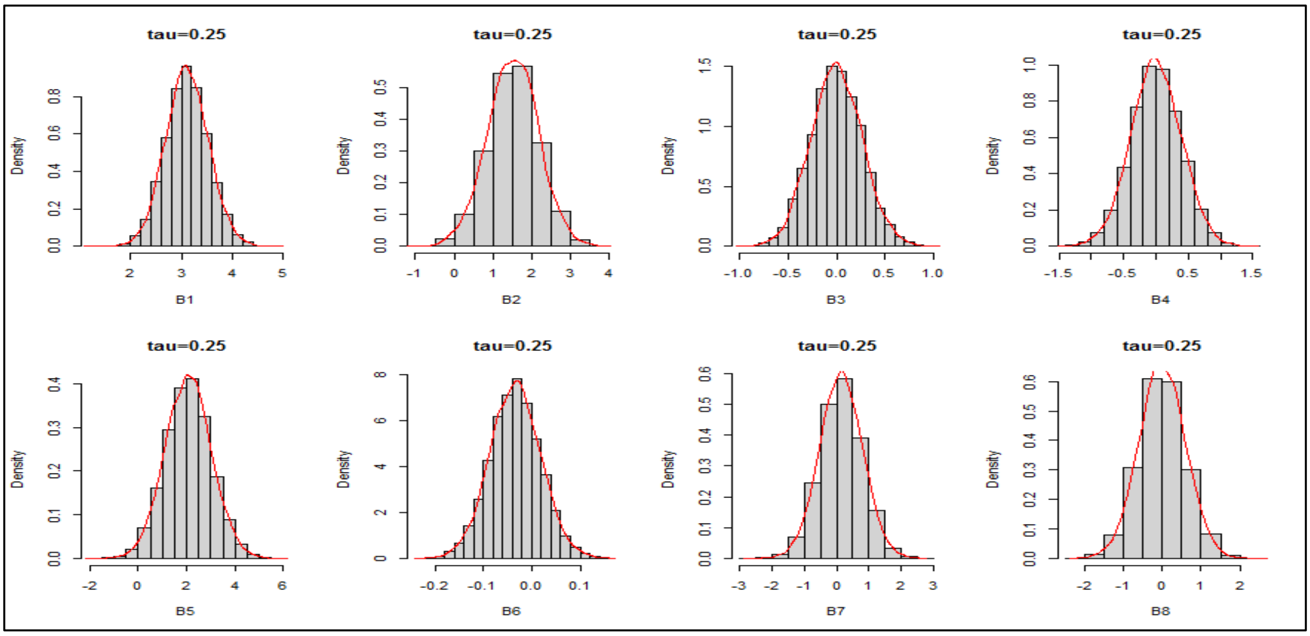

Figure 2. Histograms of estimators values under quantile level (0.25) for simulation example 1 with n=50

From tables 1-3, we can see that the proposed method (SBALQ) produced fairly closer estimates to the true vector over all quantile levels. Also, we can see that the proposed model performed well as competitive model comparing with the result of other models. Also, From figures 1-2, we can see that Gibbs sampling algorithm have good convergence and well mixing and that obviously from the trace plots and the histograms shows that the generated samples o the regression parameters estimates follows the normal distribution under all the quantile levels.

3.2 Real Data Analysis

In this subsection, the researcher analyzed real data represented by the level of creatinine in the blood (response variable) and its relationship with a group of explanatory variables. Data were collected from the Ministry of Health in Iraq, Al-Diwaniyah general hospital, where the data recorded in one of the center of kidney diseases and transplantation. This data represents (120) people with kidney failure of both sexes for the period from 20/5/2022 until 20/12/2022. The researcher will employ the proposed method and the methods referred to in the experimental side on this real data with the aim of identifying the most important factors affecting the level of creatinine in the blood and the cause of kidney failure in patients. The model that we try to fit includes the following variables:

1- The dependent variable (Y) is Serum Creatinine, which represents the level of creatinine in the blood.
2- X_1, the kidney failure (failure=0, not failure=1)
3- X_2 , which represents red wolf syndrome.
4- X_3 whose symbol on the analysis sheet is (R.B Sugar), represents the percentage of random sugar in the blood.
5- X_4 whose symbol on the analysis sheet is (B. Urea), represents the percentage of urea in the blood.
6- X_5 its symbol on the analysis sheet (L.D.L), which represents low-density lipoprotein analysis.
7- X_6 and its symbol on the analysis sheet (H.D.L), which represents high-density lipoprotein analysis.
8- X_7, whose symbol on the analysis sheet is (Ca), analysis of the level of calcium in the blood.
9- X_8, whose symbol on the analysis sheet is (HTC), represents the analysis of the size of packed red blood cells.
10- X_9 and its symbol on the analysis sheet (HP) represents the analysis of the hemoglobin test in the blood.
11- X_{10}, whose symbol on the analysis paper is (PVC), represents the amount and percentage of red blood cells in whole blood.
12- X_{11} and its symbol on the analysis sheet (WBC), which represents the test for the number of white blood cells in the blood.
13- X_{12}, whose symbol on the analysis sheet is (cholesterol), represents the percentage of cholesterol in the blood.
14- X_{13}, the blood group
15- X_{14}, whose symbol on the analysis sheet is (PTC), represents the analysis of the procalcitonin test.
16- X_{15}, whose symbol on the analysis paper is (MPV), represents the average platelet volume test.

After we run the R code that implemented in simulation study, we employed the same code to analyze the real data and the results are summarized in the following tables and figures. The estimated value of learning rate parameter is (α= 0.4858727).

Table 4: Coefficient estimates with three quantile levels (0.25,0.5,0.75)

Quantiles	0.25				0.5				0.75			
Methods	QR	BQR	BALQR	SBALQ	QR	BQR	BALQR	SBALQ	QR	BQR	BALQR	SBALQ
B1	-0.0104	-0.0144	-0.0019	-0.2358	-0.0189	-0.0144	-0.0018	-0.1154	0.0131	-0.0144	-0.0015	-0.0765
B2	0.0030	-0.0017	-0.0039	-0.0007	-0.0098	-0.0017	-0.0036	-0.0986	-0.0096	-0.0017	-0.0039	-0.0007
B3	0.0666	0.0388	0.0305	0.1948	0.0312	0.0388	0.0302	0.0964	0.0564	0.0388	0.0299	0.0635
B4	-0.0069	-0.0134	-0.0034	-0.1112	-0.0161	-0.0134	-0.0035	-0.0546	-0.0155	-0.0134	-0.0038	-0.0370
B5	-0.0277	-0.0002	0.0019	-0.0007	0.0150	-0.0002	0.0018	-0.0002	-0.0129	-0.0002	0.0018	-0.0008
B6	0.0122	0.0180	0.0442	0.0488	0.0043	0.0180	0.0439	0.0268	0.0480	0.0180	0.0444	0.0174
B7	0.7121	0.7252	0.7223	0.8348	0.7490	0.7252	0.7217	0.6632	0.7404	0.7252	0.7223	0.7416
B8	-0.0038	0.0001	0.0103	-0.0002	-0.0223	0.0001	0.0095	-0.0009	0.0214	0.0001	0.0097	-0.0209
B9	0.0112	-0.0125	-0.0213	-0.2363	-0.0133	-0.0125	-0.0231	-0.1171	-0.0110	-0.0125	-0.0232	-0.0770
B10	0.0120	0.0263	0.0329	-0.2963	0.0206	0.0263	0.0328	-0.1477	0.0416	0.0263	0.0335	-0.0972
B11	-0.0259	-0.0382	-0.0195	-0.0196	-0.0324	-0.0382	-0.0192	-0.0083	-0.0416	0.0382	-0.0190	-0.0002
B12	-0.0082	-0.0014	0.0023	-0.0244	0.0039	-0.0014	0.0018	-0.0005	-0.0157	-0.0014	0.0027	-0.0006
B13	-0.0259	-0.0566	-0.0565	-0.3707	-0.0534	-0.0566	-0.0554	-0.1825	-0.0854	-0.0566	-0.0563	-0.1211
B14	-0.0420	-0.0268	-0.0034	0.0000	-0.0273	-0.0268	-0.0049	0.0001	-0.0084	-0.0268	-0.0046	0.0002
B15	-0.0388	-0.0131	-0.0068	0.0009	-0.0167	-0.0131	-0.0075	0.0006	-0.0113	-0.0131	-0.0078	0.0467

Table 5: MSE and MAE values with three quantile levels (0.25,0.5,0.75)

Quantiles	Methods	MSE	MAE
0.25	QR	0.717836	0.445268
	BQR	0.662855	0.371944
	BALQR	0.619001	0.328681
	SBALQ	0.524813	0.237693
0.5	QR	0.817087	0.532218
	BQR	0.712855	0.431944
	BALQR	0.618630	0.328678
	SBALQ	0.532901	0.271013
0.75	QR	0.922883	0.624321
	BQR	0.742855	0.531944
	BALQR	0.638984	0.428577
	SBALQ	0.555001	0.308962

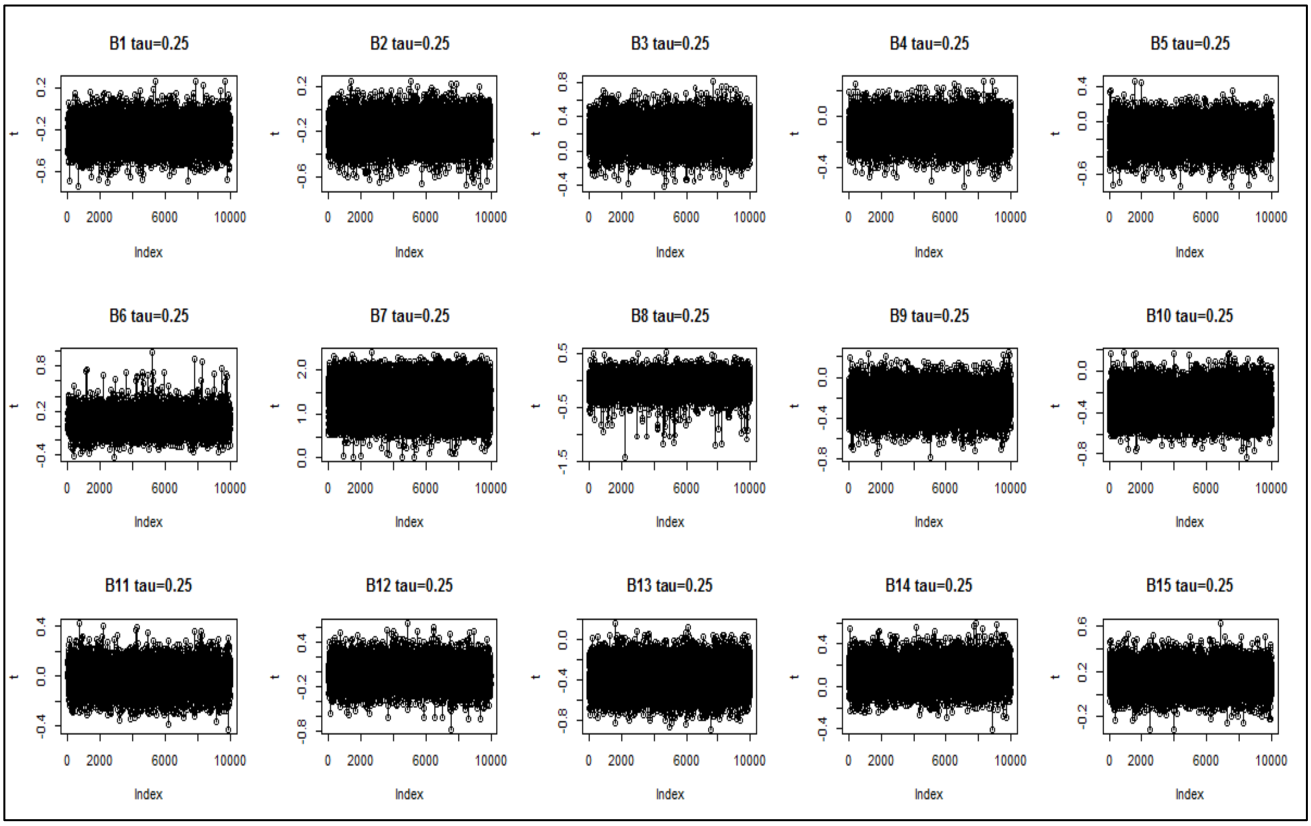

Figure 3: Trace plots of estimator's values under quantile level (0.25)

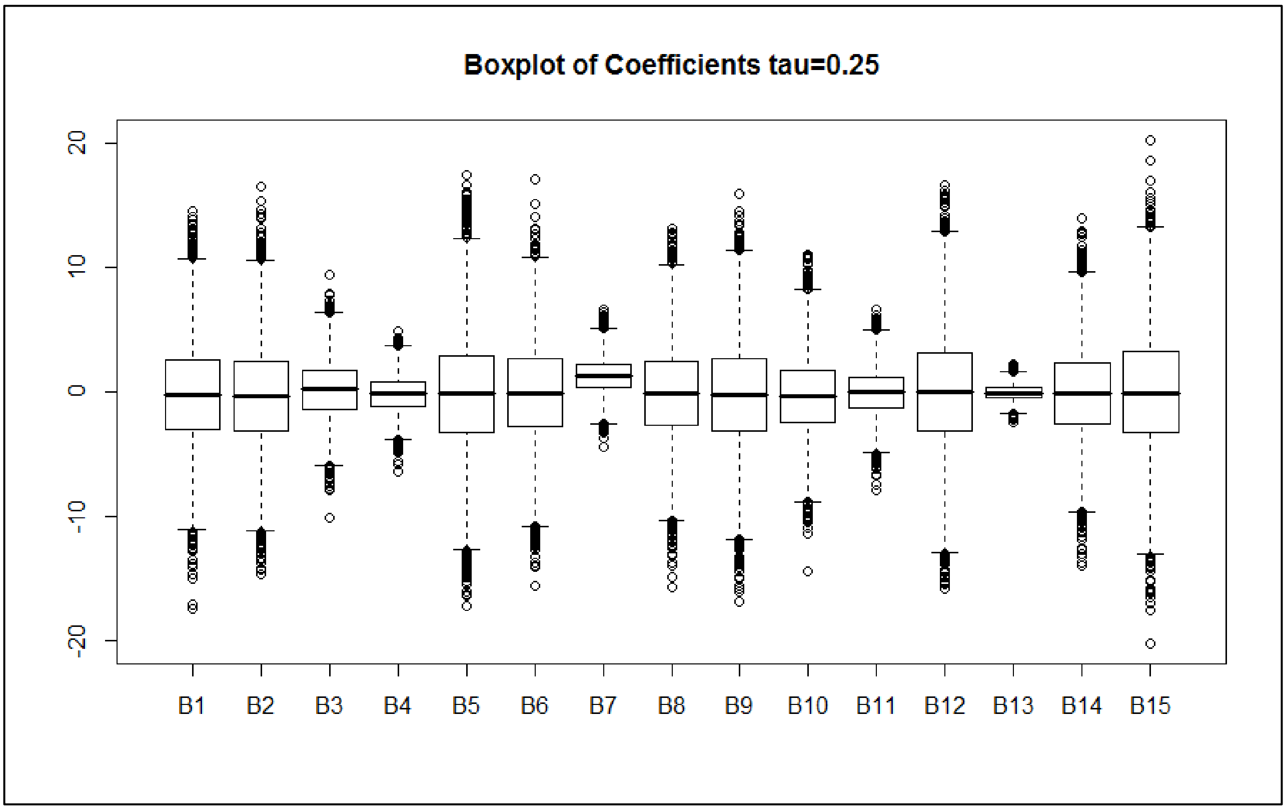

Figure 4: Box plots of estimator's values under quantile level (0.25)

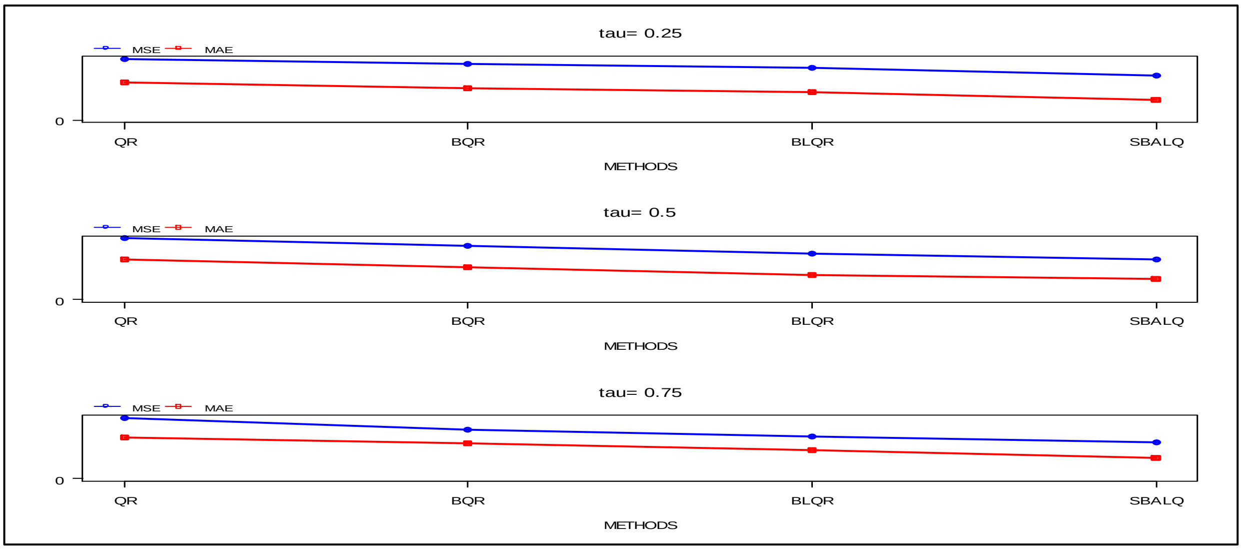

Figure 5: Line plots of Trace plots of MSE and MAE values under quantile levels (0.25, 0.50, 0.75)

From table 4, obviously some of the parameter estimates of the proposed model (SBALQ) are very close to zero at quantile level (0.25) and that indicates this methods behave as variable selection method (X2= kidney failure, X5= the percentage of urea in the blood, X8= the level of calcium in the blood., X14= procalcitonin test, X15= the average platelet volume test.) and that means these variable are irrelevant predictor variables to the response variable (level of creatinine in the blood). Also, the proposed methods behave as variable selection method on the quantile level (050, and 0.75). The (BALQR) method follows in terms of variable selection procedure our proposed method. Therefore, we can say that the proposed method is a comparable in its performance for the other existing methods. From table 5, the proposed method (SBALQ) obtained less values for MSE and MAE over all the quantile levels and that indicates this method performed well.

Moreover, Figures 3- 4 show that indicate the Gibbs sampling algorithm convergence is efficiently and do well mixing for the generating chains of samples. Also, the histograms indicate that the distribution parameter estimates is normal.

4 Conclusions

the safe Bayesian technique used for addressing the problem of likelihood distribution misspecification distribution in quantile regression model. The safe Bayesian method was studied under the Lasso and adaptive lasso quantile regression models based on the scale mixture of normal mixing with exponential density of Laplace prior distribution that proposed by proposed by Park and Casella (2008). The learning rate parameter was raised to the likelihood function to works as related weight for the misspecified prior distribution model. New hierarchical model of prior distributions was developed by assuming the learning rate has uniform prior distribution (0, 1). The multinomial distribution was derived as posterior distribution for the learning rate parameter. Gibbs sampling algorithm used to generate samples from the proposed posterior distributions. Simulation and real data results demonstrated that the proposed model have well performance comparing with other methods based one (MSE, MAE, Bias) and under different sample sizes and different quantile levels.

References

[1] Koenker, R. and G. J. Bassett (1978). Regression quantiles. Econometrica 46, 33- 50.

[2] Yu, K. and Moyeed, R. (2001). Bayesian quantile regression. Statistics & Probability Letters, 54:437-447.

[3] Yu, K., Z. Lu, and J. Stander. (2003). Quantile regression: Applications and current research areas. The Statistician 52, 331-350.

[4] Kozumi, H. and G. Kobayashi (2011). Gibbs sampling methods for Bayesian quantile regression. Journal of Statistical Computation and Simulation 81, 1565-1578.

[5] Grünwald, P. D. and Ommen, T.V. (2014). Inconsistency of Bayesian Inference for Misspecified Linear Models, and a Proposal for Repairing It. eprint arXiv: 1412.3730.

[6] Heide, R. de. (2016). the safe-bayesian lasso, master thesis. mathematical institute. Universiteit Leiden.

[7] Alhamzawi, R. (2016). Bayesian elastic net Tobit quantile regression. Communications in Statistics-Simulation and Computation, 45(7), 2409-2427.

[8] Alkenani A, and Masllam, B.S. (2019). Journal of probability and Statistics, Vol. 2019.Group identification and variably. Selection in quantile regression.

[9] Alhamzawi, R. and K. Yu (2013). Bayesian tobit quantile regression using g- prior distribution with ridge parameter. Computational Statistics & Data Analysis, revised manuscript.

[10] Alhamzawi, R. (2013). Tobit quantile regression with the adaptive lasso penalty. In The Fourth International Arab Conference of Statistics, 450 ISSN (1681 - 6870).

[11] Hao, L. & Naiman, D. (2007) Quantile Regression (07-149). Sage Publications.

[12] Yu, K. and J. Zhang (2005). A three-parameter asymmetric Laplace distribution and its extension. Communications in Statistics Theory and Methods 34, 1867-1879.

[13] Li, Y. and J. Zhu (2008). 11-norm quantile regressions. Journal of Computational and Graphical Statistics 17, 163-185.

[14] Tibshirani, R. (1996). Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society. Series B (Methodological), 267-288.

[15] Mzedawee, A. N. H. (2023). Safe Bayesian Inference of Lasso Censored Regression Based on Multinomial Distribution. AL-Qadisiyah Journal for Administrative and Economic sciences. Vol.25. Issue.4.