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Solutions of Types of Differential Equations: Analytical Method

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Abstract:

Differential equations are fundamental in mathematical modeling across various fields such as science, engineering, medicine, economics, astronomy, and environmental science. The theory of differential equations has evolved alongside the sciences where these equations appear, and their solutions are applied. Despite their significance, only a limited number of differential equations have analytical solutions, and even in cases where solutions exist, deriving them can be a complex process. Consequently, numerical methods play a crucial role in solving these equations, as they offer an alternative when analytical solutions are not feasible. This paper discusses the analytical methods for solving various types of differential equations, exploring their applications in diverse real-world problems such as Newton's second law of dynamics, radioactive decay, wave equations, and electrical circuits.

Keywords: Differential Equations,Analytical Solutions,Numerical Methods, Integral Equations

1.1 Introduction

The differential equation is the most important tool used for mathematical modeling in various fields of science, engineering, medicine, economics, astronomy, environmental science, and many other fields. The theory of differential equations developed in harmony with the sciences in which equations appear and results are applied.

Differential equations are crucial and commonly employed tools in mathematical modeling. However, only a limited number of these equations have analytical solutions, and even when they do, finding them can be quite challenging. Therefore, numerical methods play a vital role in solving differential equations and should not be overlooked.

The most famous differential equations are Newton's second law of dynamics (Mechanics), radioactive decay in nuclear physics, wave equation, Kepler's problem, simple pendulum problem, electrical circuits, chemical kinetic problems" etc. An equation is said to be a differential equation "if it contains a dependent variable and its derivatives of one or more independent variables" [1, 2].

1.2 Types of differential equations

A differential equation can be classified into the following types

1.3 Ordinary Differential Equation

"A differential equation in which an unknown function is a function of a single independent variable" is called an ordinary differential equation (ODE) [1].

1.4 Partial differential equation

"The differential equation in which the unknown function is a function of several independent variables and their partial derivatives" is called the partial differential equation (PDE) [2].

1.5 Stochastic differential equation

"A differential equation involving one or more terms is a random process, called a stochastic differential equation (SDE) and the solution of these equations is itself a random process" [3].

1.6 Differential Equation of Delay

"A differential equation involving the derivative of the unknown function at a given time in terms of function values in previous times is called the differential lag equation (DDE). In the ODE, unknown functions and their derivatives are evaluated at the same time, i.e. there are no historical functions. But when the rate of change of a time-dependent process is judged not only by its current state, but also by some previous state in its mathematical formulation, a DDE appears. [4,5].

The general first-order DDE is represented as follows

$$\left.\begin{aligned} w'(v) &= g(v, w(v), w(\tau(v))), v_0 \le v \le v_g, \\ w(v) &= \phi(v), v \le v_0, \end{aligned}\right\}$$

where $g : [v0, vg] \times$ way $\times$ way $\longrightarrow$.

Since for some $v \geq v0$, we can see that $\tau(v) < v0$, the *initial function* $\varphi(v)$ is needed for the integrity of the problem rather than a simple prime value $v0$, as happens for ODEs.

The differential equation of relative delay The relative first-order DDE is represented as follows

$$\left.\begin{aligned} w'(v) &= g(v, w(v), w(pv)), 0 < v \le T, \\ w(v) &= w_0, \end{aligned}\right\}$$

where $pp \in (0.1)$.

The multiple pantograph equation is a special type of relative DDE that can be used in control systems, electrodynamics, quantum mechanics, and number theory.

The differential equation of constant delay "The first-order constant DDE is represented as follows"

$$\left.\begin{aligned} w'(v) &= g(v, w(v), w(v-\tau)), v_0 \le v \le v_g, \\ w(v) &= \phi(v), v \le v_0, \end{aligned}\right\}$$

where $\tau > 0$ is a real constant.

The differential equation of the delay of the time variable "The DDE "ime of the first order is represented as followsvariable at the t

$$\left.\begin{aligned} w'(v) &= g(v, w(v), w(v-\tau(v))), v_0 \le v \le v_g, \\ w(v) &= \phi(v), v \le v_0, \end{aligned}\right\}$$

where $\tau(v)$ is some function of v for $v > 0$.

Differential equation of neutral delay

« DDE in which the highest derivatives of an unknown function occur with or without delay, called neutral DDE ».

The class neutral DDE is represented as follows

$$\left.\begin{aligned} w'(v) &= g(v, w(v), w'(v-\tau)), v_0 \le v \le v_g, \\ w(v) &= \phi(v), v \le v_0, \end{aligned}\right\}$$

where $\tau > 0$ is a real constant [4, 5].

1.7 Differential Algebraic Equation

"A differential equation that is a combination of a differential equation and an algebraic equation, given in implicit form" is called a differential algebraic equation (DAE). The most important class of AEDs is a semi-explicit AED, which is represented as[8].

$$\left.\begin{aligned} w' &= f(v, w, z), \\ 0 &= g(v, w, z), \end{aligned}\right\}$$

where W is the dependent variable, v is the independent variable, and z is the algebraic variable. The AED system (1.2.6) is not well defined in the mathematical sense, which will lead to the failure of any direct discrimination method. Hessenberg forms are more important categories than the AED mentioned below.

Heisenberg-1 index[12].

$$\left.\begin{aligned} w' &= f(v, w, z), \\ 0 &= g(v, w, z), \end{aligned}\right\}$$

Supposing that jacobite $\frac{\partial g}{\partial z}$ is not singular for all v. Semi-Explicit Index -1 DAE

System.

Heisenberg-2 index

$$\left.\begin{aligned} w' &= f(v,w,z), \\ 0 &= g(v,w), \end{aligned}\right\}$$

where $\frac{\partial g}{\partial w}\frac{\partial f}{\partial z}$ it is not singular for any *V*. Here, there is no algebraic variable *z* in The second equation is therefore a pure indicator -2 DAE. Heisenberg-3 index

□ ‹(v, largeur, y, z) f=′ w
□□□
′

‹(V, W, P) g= and

0 = *h*(*v*,*y*), □□□

where the product of everything $\frac{\partial h}{\partial y}\frac{\partial g}{\partial w}\frac{\partial f}{\partial z}$ is non-singular *for V* [6].

1.8 Integral differential equation

A differential equation that contains the unknown function on one side as an ordinary derivative and also has its presence on the other side under the integral sign is called the differential equation (IDE). [9].

The general form of the integral equation is represented in *w* (*v*) as follows

$$w'(v) = f(v) + \int_{\alpha(v)}^{\beta(v)} K(v,t)\,w(t)\,dt,$$

where *K(*v, t*)* is called the kernel of the integral equation.

Integral equations are mainly classified into two categories, the Fredholm and Volterra integral equations.

Fredholm integral equation

"The Fredholm integral equation is represented as follows"

$$\phi(v)\,w(v) = f(v) + \lambda \int_a^b K(v,t)\,w(t)\,dt, v,t \in [a,b]$$
,

When the limit of the integral *a* and *b are* constants, *K(*v, t*)* is called the kernel of the integral equation, *the function f* (*v*) receives a function, and *λ* is a parameter.

Volterra Integral Equation The Volterra Integral Equation is represented as follows

$$\phi(v)\,w(v) = f(v) + \lambda \int_a^v K(v,t)\,w(t)\,dt,$$

When the limit of the integral is functions in *v*, K*(*v, t*)* is called the kernel of the integral equation, the function *f* (*v*) receives a function, and *λ* is a parameter [7].

2.1 Methods and techniques for solving a differential equation

2.2 Edomian decomposition method

The Adomian Decomposition Method (ADM) is a sequential solution technique for solving differential equations. ADM can provide a solution to a differential equation in sequential form, which is determined by a recursive relation using Adomian polynomials. An effective implementation of the method can lead to an accurate and numerical solution of a broad and general class of dynamical systems representing physical problems [8, 9].

2.3. Symmetric Disorder Method

The symmetric disorder method (SMP) is a sequential solution method for obtaining a solution of different types of differential equations. HPM is built on the idea of small parameters. These small parameters are so sensitive that even a slight change in one of them will change the results. A careful selection of modest parameters leads to the best results. However, a poor selection of small parameters has negative effects. [10].

2.4 Variable Iteration Method

The variable recurrence method (VIM) is also a sequential solution method for solving linear and nonlinear differential equations. The implementation of VIM involves the determination of the Lagrange multiplier. In VIM, first create the debug function, then select Lagrange Multiples, and then select the initial iteration [11].

2.5 Contrast Conversion Method

The differential transformation method (DTM) is one of the important series solution methods that is examined in linear and nonlinear differential equations. The solution obtained in this way is in the form of polynomials and is an approximation of the exact solution. Because this method uses an iterative procedure to obtain a higher-order serial solution. It avoids the symbolic computation of derivatives and thus produces a sequential term in an easy and efficient way [12, 13].

2.6 Plume Removal Method

The Banach Contraction Method (BCM) is based on the concept of an iterative function, which uses previous iterative solutions to create a new, more precise solution. [13].The iterative method is repeated until convergence is reached [14].

2.7 Integral equation

The differential equation is changed to an integral equation, i.e. an equation in which the unknown is inside the integral.

2.8 Elementary Value Problem and Limit

The problem of a boundary value consists of a differential equation and all the necessary elementary or boundary conditions. The solution of a differential equation will satisfy the boundary conditions as well as the differential equation anywhere within the boundary. Different types of differential equations can be subject to limit value problems. The Dirichlet problem, which consists of finding harmonic functions, is one of the first limit value problems to be explored. The Sturm-Liouville problems constitute a large category of important limit value problems. The autonomous functions of the differential operator are used to study these problems [15].

Types of limit value issues

Limit value issues are categorized as below

- ***Dirichlet or first duration condition***

The Dirichlet boundary condition is the condition in which the value of the function itself is determined. If the solution takes a value of zero along the boundary, then the case is called homogeneous Dirichlet otherwise it is heterogeneous. [16].

- ***Neumann Secondary Limits or Condition***

"Newman's boundary condition provides the value of the ordinary derivative of the function."

Here, the derivative of the solution along the boundary is determined. [17].

- ***Third boundary condition or mixed boundary condition***

"The mixed limit clause determines the value of the solution and its derivative is described along the limit" [16, 17].

In this thesis, we used the following methods

1. Differential Transfer Method
2. Fa di Bruno formula and Bell polynomials
3. Step Method
4. Panach Withdrawal Method

3.1 Differential Conversion Method

DTM was introduced by Pukhov [18] and Zhou [19] as "The "Taylor transformation" was utilized in the analysis of electrical circuits. DTM is closely associated with Taylor's "expansion of genuine analytical functions." It

can be applied to address a wide range of issues across all types of differential equations, including normal, partial, late, fractional, fuzzy, and more.).
"DTM is an iterative method for determining the sequential solution of linear and nonlinear differential equations. Compared to the traditional chain method, which requires symbolic computation, DTM converts differential equations into algebraic equations that can be solved repeatedly.
Suppose that *w*(*v*) is a real analytic function in the domain of Ω and *that v = v0* is a random point in Ω. Then, w(v*) can* be extended in the Taylor series to the vicinity of the point *v = v0*.
the derivative k of the The differential transformation of" [96] Definition 1.6.1 *:s followsis defined a v0 to (v) function w*

$$W(k)[v_0] = \frac{1}{k!}\left[\frac{d^k w(v)}{dv^k}\right]_{v=v_0},$$

where W(*k*)[*v0*] represents *the differential transformation from w*(*v*) *to v = v0".*
The inverse differential transfer is given by" [96] Definition 1.6.2

$$w(v) = \sum_{k=0}^{\infty} W(k)[v_0](v - v_0)^k .”$$

Using definitions 1.6.1 and 1.6.2, the function w can be represented as a :inTaylor cha

$$w(v) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{d^k w(v)}{dv^k}\right]_{v=v_0}(v - v_0)^k .$$

(1 + *N*) Thus, the solution is approximated by the finite terminology practically
N

in (*v*) = ^{XW}(*k*)[*v0*](*v* − *V0*)^k.

k = 0
The formulas that will be used in the next ch are presented and collected in Theorem
"Suppose that W (*k*)[*v0*] *is the differential transformation of the function w* (*v*) *at v = v0".*
(A) *If* $w(v) = w_1'(v)$, *then for any v0*
In(*k*)[*v0*] = (*k* + 1)In_1(*k* + 1)[*v0*] *for k* = 0,1,2,....
(B) *If* $w(v) = w_1^{(n)}(v)$, *then for any v0*
W(*k*)[*v0*] = (*k* +1)(*k* +2)...(*k* +n)*W1*(*k* +n))[*v0*] *pour k = 0,1,2,....*
(C) *Si w*(*v*) = *w1*(*v*) · *W2*(*V*), *Alors pour tout V0*
k

In(*k*)[*v0*] = XW1(*l*)[v_0]*In*$_2$(*k* − *l*)[*v0*] *for k* = 0,1,2,....
l=0

(D) *Si w*(*v*) = *w1*(*v*) · *w2*(*v*)... · *wn*(*v*), *alors pour n'importe quel v0*

k k−*l1k*−*l1*−*l2*−...−ln−2

In(*k*)[*v0*] =XX X
... *W1*(l_1)[*v0*]*W2*(*l2*)[*v0*] *l1=0* *l2=0*
ln−*1=0*
... *Wn*−*1*(*ln*−1)[*v0*]*Wn*(*k* − *l1* − *l2* − ... − *ln*−*1*)[*v0*]

pour k = 0,1,2,....

(E) *If w*(*v*) = *eαv then for all v0* ∈ R

$$W(k)[v_0] = \frac{e^{\alpha v_0}\alpha^k}{k!}$$ *for k* = 0,1,2,....

(F) *If w*(*v*) = *vr*, *r* ∈ R *then for any v such that* |v − *v0*| < |v0|

$$W(k)[v_0] = \binom{r}{k} v_0^{r-k}$$ *for k* = 0,1,2,...,

"where $\binom{r}{k} = \frac{r(r-1)\ldots(r-k+1)}{k!} = \frac{(r)_k}{k!}$, *and* ($r$)$_k$ *represents the Pochhammer symbol".*

(G) *If w*(*v*) = *vn*, *n* ∈ N0, *then for v0* = 0
W(*k*)[*v0*] = *δ*(*k* − *n*) *pour k = 0,1,2,...,*
"where δ(*k* − *n*) = *δkn is the Kroniker delta".*

Guidance: The evidence for the formulas will be derived directly from definitions 1.6.1 and 1.6.2 respectively.

(A) Using equation (1.6.14), we have

$$\begin{aligned} w_1'(v) &= \frac{d}{dv}\sum_{k=0}^{\infty} W_1(k)[v_0](v-v_0)^k \\ &= \sum_{k=1}^{\infty} kW_1(k)[v_0](v-v_0)^{k-1} \\ &= \sum_{k=0}^{\infty} (k+1)\, W_1(k+1)[v_0](v-v_0)^k [90] \end{aligned}$$

(B) We will use induction to continue. The first step is to show the formula for n = 1, specifically the formula (1.6.17) that has already been established. Next, we assume that the formula holds true for n., et nous le prouverons pour $n+1$. Nous supposons que si $w(v)=w_1^{(n)}(v)$, alors pour tout v_0 nous avons $W(k)[v_0] = (k+1)(k+2)\ldots(k+n)W_1(k+n))[v_0]$ pour $k = 0,1,2,\ldots$. Cela signifie que

$$\begin{aligned} w_1^{(n)}(v) &= w(v) \\ &= \sum_{k=0}^{\infty} W(k)[v_0](v-v_0)^k \\ &= \sum_{k=0}^{\infty} (k+1)(k+2)\ldots(k+n)W_1(k+n))[v_0](v-v_0)^k \end{aligned}.$$

Then, for $n+1$, we have

$$\begin{aligned} w_1^{(n+1)}(v) = w'(v) &= \frac{d}{dv}\sum_{k=0}^{\infty} W(k)[v_0](v-v_0)^k = \sum_{k=1}^{\infty} kW(k)[v_0](v-v_0)^{k-1} \\ &= \sum_{k=1}^{\infty} k(k+1)(k+2)\ldots(k+n)W_1(k+n))[v_0](v-v_0)^{k-1} \end{aligned}.$$

By changing the limits of the last sum, we obtain

$$w_1^{(n+1)}(v) = \sum_{k=0}^{\infty}(k+1)(k+2)\ldots(k+n)(k+n+1)W_1(k+n+1))[v_0](v-v_0)^k.$$

This implies that if $w(v) = w_1^{(n+1)}(v)$, then for any v_0 , we have "$W(k)[v_0] = (k+1)(k+2)\ldots(k+n)(k+n+1)W_1(k+n+1))[v_0]$ pour $k = 0,1,2,\ldots$», ce qui prouve la formule.

(C) Supposons que $w_1(v) = \sum_{l=0}^{\infty} W_1(l)[v_0](v-v_0)^l$ et $w_2(v) = \sum_{m=0}^{\infty} W_2(m)[v_0](v-v_0)^m$.

Then

$$\begin{aligned} w(v) &= w_1(v)\cdot w_2(v) = \sum_{l=0}^{\infty} W_1(l)[v_0](v-v_0)^l \cdot \sum_{m=0}^{\infty} W_2(m)[v_0](v-v_0)^m \\ &= \sum_{l=0}^{\infty}\sum_{m=0}^{\infty} W_1(l)[v_0]W_2(m)[v_0](v-v_0)^{l+m}. \end{aligned}$$

Si l'on substitue $k = l + m$, on obtient

$$w(v) = \sum_{l=0}^{\infty}\sum_{k-l=0}^{\infty} W_1(l)[v_0]W_2(k-l)[v_0](v-v_0)^k$$
$$= \sum_{l=0}^{\infty}\sum_{k=l}^{\infty} W_1(l)[v_0]W_2(k-l)[v_0](v-v_0)^k.$$

By changing the order of summation in the last sum, we get

$$w(v) = \sum_{k=0}^{\infty}\sum_{l=0}^{k} W_1(l)[v_0]W_2(k-l)[v0](v-v0)^k,$$

ce qui implique que $W(k)[v0] = \sum_{l=0}^{k} W1(l)[v_0]W_2(k-l)[v0]$ pour $k = 0,1,2,....$

(D) (D) We will use induction to proceed. The initial step is to demonstrate the formula for n = 2, specifically the formula (1.6.19) that has already been established. Following that, we will verify the validity of the formula for n + 1 by assuming it holds for the product of the function at n. In other words, we will assume that if u(v) = w2(v) · w3(v) · ... · wN+1(v), then for any v0. [20].

$$In(k)[v0] = \sum_{l2=0}^{k}\sum_{l3=0}^{k-l2}\cdots\sum_{ln=0}^{k-l2-l3-...-ln-1} W2(l_2)[v0]W3(l3)[v0]$$

$$\dots W_n(ln)[v_0]Wn_{+1}(k-l2-l3-\dots-ln)[v0]$$

pour k = 0,1,2,.... Mettre $w(v) = w1(v) \cdot u(v)$. Ensuite, selon (1.6.19), nous avons

$$W(k)[v0] = \sum_{l1=0}^{k} W1(l_1)[v0]U(k-l1)[v0]$$ pour k = 0,1,2,... C'est

$$In(k)[v0] = \sum_{l1=0}^{k} W1(l_1)[v0]\sum_{l2=0}^{k-l1}\sum_{l3=0}^{k-l1-l2}\cdots\sum_{ln=0}^{k-l1-l2-l3-...-ln-1} W2(l_2)[v0]W3(l3)[v0]\dots$$

$$\dots W_n(ln)[v_0]Wn_{+1}(k-l1-l2-l3-\dots-ln)[v0]$$

for k = 0,1,2,..., which proves the formula.

(E) For any $v0 \in$ R, we can extend the function $w(v) = e\alpha v$ in the Taylor series as

$$w(v) = e^{\alpha v} = \sum_{k=0}^{\infty} \frac{\alpha^k e^{\alpha v_0}}{k!}(v-v_0)^k.$$

Comparison with the returns in definition 1.6.2

$$W(k)[v_0] = \frac{e^{\alpha v_0}\alpha^k}{k!}$$, for k = 0,1,2,....

(F) "If x and y are real numbers such that |x|> |y|, and r is any complex number, then
Newton's generalization of the binomial is "

$$(x+y)^r = \sum_{k=0}^{\infty} \binom{r}{k} x^{r-k} y^k$$,

"where $\binom{r}{k} = \frac{r(r-1)\dots(r-k+1)}{k!}$ ".

Let's rewrite *virtual reality* as
$vr = (v - v0 + v_0)^r = (v0 + (v - v0))^r$.
The application of the returns of the equation

$$v^r = \sum_{k=0}^{\infty} \binom{r}{k} v_0^{r-k} (v - v_0)^k \ [101]$$

(G) For $v0$ = 0, we can extend the function $w(v) = vn$, where $n \in$ N0, in Taylor series as

$$w(v) = v^n = \sum_{k=0}^{\infty} \delta(k-n) v^k$$,

where $\delta(k - n) = \delta kn$ is the Kronecker delta. We compare it to definition 1.6.2 and conclude that:
$W(k)[0] = \delta(k - n)$
for k = 0,1,2,..., which proves the formula.

Some of the results of the differential transformation, used in this case, are listed in Table 1.1 and can be proved using definitions

Table 1.1: NCM Results.

	Original Function	Transformed function
	$\frac{d^n w(v)}{dv^n}$	$(k+1)(k+2)(k+3)\dots(k+n)W(k+n)$
	(	s i k
1	1,	=
2	$U\delta(k - n) =$	n

		$\frac{\alpha^k}{k!}$	0, sikꟷn
		$\sum_{i=0}^{k} W_1(i)\, W_2(k-i)$	
3	$e\alpha v$	$\frac{\alpha^k}{k!}\sin\left(\frac{k\pi}{2}+\beta\right)$	
4	$w1(v)w2(v)$	$\frac{\alpha^k}{k!}\cos\left(\frac{k\pi}{2}+\beta\right)$	
5	$\sin(\alpha v+\beta)$		
6	$\cos(\alpha v+\beta)$		

4.1 Two-Dimensional Differential Transformation Method

The basic definitions and operations of two-dimensional DTM are presented here.

Definition *"The differential transformation of the function w (x,t) is represented in the following form*

$$W(k,h)=\frac{1}{k!h!}\left[\frac{\partial^{k+h}w(x,t)}{\partial x^k\partial t^h}\right]_{x=x_0,t=t_0},$$

where w (x,t) is an analytic function and continuously differentiable with respect to time t in a specified domain".

Definition *"The inverse differential transform of W (k,h) can be determined as*

$$in\ (x,t) = \sum_{k=0}^{\infty}\sum_{h=0}^{\infty} W(k,h)(x-x0)^k(t-t0)^h".$$

Combining the equation, we get

$$“w(x,t)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}w(x,t)}{\partial x^k\partial t^h}\right]_{x=x_0,t=t_0}(x-x_0)^k(t-t_0)^h.”$$

It is clear from the above definitions that the idea of two-dimensional differential transform is based on the two-dimensional expansion of the Taylor series. Some of the two-dimensional differential transformation results used in this document are presented in Table 1.2. Evidence of the results can be seen in [21].

Table 1.2: NCM results for the EDP.

	Original Function	Transformed function
1	« $u(x,t)v(x,t)$ »	« $\sum^{k}\sum^{h} U(r,h-s)V(k-r,s)$ »

		$r=0\ s=0$
2	$\frac{\partial u(x,t)}{\partial x}$	$(k+1)U(k+1,h)$
3	$\frac{\partial u(x,t)}{\partial t}$	$(h+1)U(k,h+1)$
4	“$\frac{\partial^{r+s} u(x,t)}{\partial x^r \partial t^s}$”	"$(k+1)(k+2)\ldots(k+r)(h+1)(h+2)\ldots(h+s)U(k+r,h+s)$"

4.2 Faa di Bruno formula and Bell polynomials

Eric Temple Bell invented Bell's polynomials, which were first used to examine ensemble scores. Bell polynomials appear in a variety of applications, including combinatorics, analysis, statistics, and more. Exponential partial Bell polynomials are polynomials with an infinite number of variables $x_1, x_2, \ldots$, and it is generally known that Bell polynomials can be used to obtain several specific combinatorial sequences, such as Stirling numbers, Lah numbers, and idempotent numbers [22].

In the existing literature, it has been noted that differential transformation is not applied directly to

nonlinear terms such as $\sqrt[n]{w}$, $n \geq 2$ or $\ln(w)$. However, the differential transformation of nonlinear terms can be determined using the Faa di Bruno formula in a more efficient manner. Some of the necessary notations and definitions of Bell polynomials defined as:

Definition *"Partial exponential Bell polynomials are polynomials $B_{k,l}(x_1,\ldots,x_{k-l+1})$ in an infinite number of variables $x_1, x_2 \ldots$ defined by the expansion of the series*

$$\sum_{k\geq l} B_{k,l}\left(x_1,\ldots,x_{k-l+1}\right)\frac{t^k}{k!} = \frac{1}{l!}\left(\sum_{m\geq 1} x_m \frac{t^m}{m!}\right)^l, l=0,1,2,\ldots$$
”

Definition *"Partial ordinary Bell polynomials are polynomials* $\widehat{B}_{k,l}(x_1,\ldots,x_{k-l+1})$ *in an infinite number of variables* $\hat{x}_1,\ldots,\hat{x}_{k-l+1}$ *defined by the expansion of the series"*

$$\sum_{k\geq l} \hat{B}_{k,l}\left(\hat{x}_1,\ldots,\hat{x}_{k-l+1}\right)t^k = \left(\sum_{m\geq 1} \hat{x}_m t^m\right)^l, l=0,1,2,\ldots$$

"The relation between the partial exponential Bell polynomials *Bk,l* and the partial ordinary Bell polynomials *Bˆk,l* is[23].

$$B_{k,l}(x_1,\ldots,x_{k-l+1}) = \frac{k!}{l!}\hat{B}_{k,l}\left(\frac{x_1}{1!},\frac{x_2}{2!},\ldots,\frac{x_{k-l+1}}{(k-l+1)!}\right).”$$

Lemma *([98]) "The partial ordinary Bell polynomials Bˆk,l* (*xˆ1,...,xˆk−l+1*), *l* = 0,1,2,...,k ≥ *l satisfy the recurrence relation*

$$\hat{B}_{k,l}(\hat{x}_1,\ldots,\hat{x}_{k-l+1}) = \sum_{l=1}^{k-l+1}\frac{il}{k}\hat{x}_i\hat{B}_{k-i,l-1}(\hat{x}_1,\ldots,\hat{x}_{k-i-l+2}),$$

where $\hat{B}_{0,0}$ *= 1 and* $\hat{B}_{k,0}$ *= 0 for k* ≥ 1".

Theorem 1.8.4 *[98] "Let g and f be analytic real functions near v0 and g*(*v0*) *respectively, and let h be the composition h*(*v*) = *f* (*g*(*v*))*. The differential transformation of the functions g, f, and h is represented by G*(*k*)*, F*(*k*)*, and H*(*k*) *respectively. Then H*(*k*) *satisfies the relations*

$$H(0) = F(0),$$

$$H(k) = \sum_{l=1}^{k} F(l).\hat{B}_{k,l}(G(1),\ldots,G(k-l+1)) \text{ for } k \geq 1.”$$

Some initial terms of "ordinary parital Bell polynomials *Bˆk,l* (*xˆ1,...,xˆk−l+1*)" using Lemma 1.8.3 are generated as

k = 1	*xˆ1*			
k = 2	*xˆ2*			$\hat{x}_1^2$
k = 3	*xˆ3*		2xˆ1xˆ2	$\hat{x}_1^3$
k = 4	*xˆ4*	$2\hat{x}_1\hat{x}_3+\hat{x}_2^2$	$3\hat{x}_1^2\hat{x}_2$	$\hat{x}_1^4$
k = 5		...		

and so on.

4.3 Step Approach

The step method is examined to solve the DDEs by converting them into proportionality DDEs on successive intervals $[\lambda_k,\lambda_{k+1}]$. Rewrite equation (1.2.1), we have

$$in'(v) = g[V, W(v) in(v-T)],\ c. > 0,$$

where the initial function $\psi(v)$ defined in $v \in [-\tau,0]$ and the initial condition is $w(0) = w0$ then

Successive iterations are defined as the first iteration

$$w_1'(v) = g[v, w_1(v), \psi(v-\tau)],\ 0 \leq v \leq \lambda,$$

Second iteration

$$w2'(v) = g[v, w_2(v), w1(v-\tau)],\ \lambda \leq v \leq 2\lambda,$$

...
Nth iteration

$$w_n'(v) = g\left[v, w_n(v), w_{n-1}(v-\tau)\right], \ (n-1)\lambda \le v \le n\lambda,$$

where $n \in Z+$, it is clear that each interval provides a solution of the equation [24].

4.4 Banach Contraction Method

BCM is an iterative method that is based on the concept of a recursive function, which uses previous iterative solutions to generate a new, more precise solution. The iterative method is repeated until convergence is reached [25].

In this thesis, we will study limit value problems at three singular points, differential algebraic equations, delay differential equations, ordinary differential equations, partial differential equations, and integrative-differential equations using DTM, Bell polynomials and BCM.

Computer code has been developed using Mathematica software (version 11.1.1) and MATLAB software (version 9.7.0), to obtain numerical results for different types of differential equations that are discussed numerically/analytically and presented through tables and graphs.

Results:

The study reveals that analytical solutions to differential equations, while important, are not always attainable or easy to compute. Numerical methods, such as finite difference methods and Runge-Kutta methods, are vital for obtaining solutions when analytical approaches are insufficient or infeasible. The analysis of various applications, such as mechanics, nuclear physics, and electrical circuits, demonstrates the practical importance of both analytical and numerical techniques in solving real-world problems. Furthermore, the research emphasizes the necessity of combining these methods to enhance the accuracy and efficiency of solutions, especially in complex systems.

Conclusion

DTM has been found to solve integral and integrator-differential equations in series solution rather than using linearization or discretization, resulting in significant time and computational resource savings. Compared to other methods, DTM is simple and easy to apply.

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وقائع المؤتمر العلمي البحثي الدوري الثامن للباحثين من حملة الشهادات العليا
شعبة البحوث والدراسات التربوية/ قسم الاعداد والتدريب وبالتعاون مع مركز
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((الاستدامة ودورها في تنمية القطاع التربوي))
للمدة 2025/2/12

حلول أنواع المعادلات التفاضلية: الطريقة التحليلية

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مستخلص البحث:
تعد المعادلات التفاضلية أساسية في النمذجة الرياضية عبر العديد من المجالات مثل العلوم والهندسة والطب والاقتصاد والفلك وعلوم البيئة. تطورت نظرية المعادلات التفاضلية جنبًا إلى جنب مع العلوم التي تظهر فيها هذه المعادلات وتُطبق حلولها. وعلى الرغم من أهميتها، فإن عددًا محدودًا فقط من المعادلات التفاضلية لديه حلول تحليلية، وحتى في الحالات التي تتوفر فيها الحلول، يمكن أن يكون اشتقاقها عملية معقدة. ومن ثم، تلعب الطرق العددية دورًا حيويًا في حل هذه المعادلات، حيث تقدم بديلاً عندما تكون الحلول التحليلية غير ممكنة. يناقش هذا البحث الطرق التحليلية لحل أنواع مختلفة من المعادلات التفاضلية، مستعرضًا تطبيقاتها في مشكلات واقعية متنوعة مثل قانون نيوتن الثاني في الديناميات، التحلل الإشعاعي، معادلات الموجة، والدوائر الكهربائية.
الكلمات المفتاحية :المعادلات التفاضلية، الحلول التحليلية، الطرق العددية، المعادلات التكاملية.