Iraqi Statisticians Journal / Vol. 2 (2025): Special issue for ICSA2025: 351-357

IRAQI STATISTICIANS JOURNAL

https://isj.edu.iq/index.php/isj

ISSN: 3007-1658 (Online)

Comparison of the Quasi Maximum Likelihood and Transformation Methods in Estimating the Spatial Autoregressive Model for Panel Data Using Simulation

Ahmed Karim Awdah[1], Ahmed Abd Ali Akar[2]

[1,2] Department of Statistics, College of Administration and Economics, Al-Mustansiriyah University.

ARTICLE INFO

Article history:

Received	21/11/2024
Revised	26/11/2024
Accepted	14/1/2025
Available online	15/5/2025

Keywords:

Rook standard
Panel data
Quasi Maximum Likelihood method
The Transformation method
Comparison Criterion Mean Aboslute Percentage Error (MAPE).

ABSTRACT

This research deals with the study of the spatial autoregressive model for panel data, where the model parameters were estimated using two estimation methods (the Quasi maximum likelihood method, the transformation method) in the presence of the modified spatial weight matrix according to the Rook adjacency criterion. The two methods were compared using the comparison criterion of the average absolute relative error with the aim of reaching the best estimation method. Simulation experiments were also used on two values of the cross-sections (n=20,60) and two different time periods (T=5,20) and four different sample sizes (nt=100, 300, 400,1200). Through the comparison criterion (MAPE), the best method for estimating the model was reached, which is the transformation method (TTA).

1. Introduction

Interest in spatial regression analysis began in the nineteen century, when theories and concepts emerged, such as the distance law, which determines that the similarity between phenomena decreases with increasing distance between them. These ideas were the cornerstone of understanding spatial effects. In the middle of the twentieth century, with the development of devices, it became possible to develop analytical models that depend on spatial data, which led to the emergence of spatial regression analysis, which is considered one of the statistical methods that were created for the relationship between the dependent variable and the explanatory variables in the presence of spatial dependence. Recently, this analysis has attracted great interest in research by researchers in various sciences, due to the possibility of using it in many applied fields such as international economics, environmental economics, agricultural economics, etc. Ignoring spatial dependence in the data may lead to failure to achieve the analytical hypotheses, which in turn leads to inefficient and biased estimates. Therefore, it was necessary to adopt estimation methods that take into account spatial dependence to obtain efficient and unbiased estimates. Recently, the importance of spatial econometrics has emerged, which deals with the effects of interaction between spatial units, and researchers have realized the importance of introducing panel data models to benefit from the advantages they provide[5,8]. The research aims to make a comparison between the two estimation methods represented by the quasi-maximum likelihood method and the transformation method to estimate the spatial

* Corresponding author. E-**mail** address: ahmed.awdah@uomustansiriyah.edu.iq
https://doi.org/10.62933/nwnkt518

autoregressive model for panel data (SARPD) under the modified spatial weight matrix according to the rook adjacency criterion, and by using the comparison criterion of the mean absolute percentage error (MAPE) we obtain the best estimation method.

2. Spatial Dependence

The presence of spatial dependence between a set of sample data means that the observation in area (A) depends on the observation in area (B) when A≠B according to the following formula[6]:-

$$Y_A = f(Y_B) \qquad , \quad A = 1,2,3,\ldots L \qquad (1)$$

It is noted from the above formula that the dependence can be between several observations since area (A) can take any value from (L), and there are two reasons for this to happen[6].

- Measurement errors of observations in adjacent spatial units.
- The spatial dimension of socio-demographic or economic activity, which may be the most important aspect of the modeling problem.

3. Spatial Weight Matrix

It is also called (spatial correlation matrix) as this matrix is used in analyzing the spatial relationships between observations in order to understand and analyze the spatial effects between phenomena and is symbolized by the symbol (M) and it is a positive, non-random, non-symmetric matrix with dimensions n×n and it is built based on adjacency, so it gives the value (1) to adjacent spatial units while it gives (0) to non-adjacent spatial units[2,12].

4.Modified Spatial Adjacency Matrix

This matrix is called "modified" because it is an extension of the spatial adjacency matrix after making the appropriate modifications to it so that the row sum in it is equal to one. It is constructed according to the following formula[1,8]:

$$M_{AB}^{adj} = \frac{M_{AB}}{\sum M_{AB}} \qquad , 0 < M_{AB}^{adj} < 1 \qquad (2)$$

M_{AB}: is a measure used to compare the degree of closeness and to know the relationship between the two regions.

5. Contiguity Criterion Rook

This criterion is considered one of the most used criteria due to its simplicity and realism. This criterion is built when a cell shares a common side with another adjacent cell, and it is given a value of (1) when the two adjacent cells share a common side, and it is given a value of (0) in other cases. The following formula can explain this[1,11]:

$$M_{Rook} = \begin{cases} M_{AB} = 1 & \text{Ifthe two cells have a common border} \\ M_{AB} = 0 & \text{Other than that} \end{cases} \qquad (3)$$

6. Panel Data

It is the data whose observations are recorded for (n) cross-sections over limited time periods, where these sections represent (regions, states, countries...etc.), and panel data has advantages in that it provides better efficiency and an increase in degrees of freedom as well as linear multiplicity between variables. The following table shows the form of panel data[3].

Table 1. shows the form of panel data

n	T	Y_{nt}	X_1	...	X_K
1 2 : n	1 : : 1	Y_{11} Y_{21} : Y_{n1}	$X_{1(11)}$ $X_{1(21)}$: $X_{1(n1)}$	... ... : ...	$X_{k(11)}$ $X_{k(21)}$: $X_{k(n1)}$
1 2 : n	2 : : 2	Y_{12} Y_{22} : Y_{n1}	$X_{1(12)}$ $X_{1(22)}$: $X_{1(n2)}$	... ... : ...	$X_{k(12)}$ $X_{k(22)}$: $X_{k(n2)}$
1 2 : n	t : : t	Y_{1t} Y_{2t} : Y_{nt}	$X_{1(1t)}$ $X_{1(2t)}$: $X_{1(nt)}$	... ... : ...	$X_{k(1t)}$ $X_{k(2t)}$: $X_{k(nt)}$

7. Spatial Auto Regressive Model Panel Data

The spatial autoregressive model for panel data with fixed effects is expressed mathematically by the following formula[4]:

$$Y_{nt} = \varphi_t M_{nt} Y_{nt} + X_{nt}\beta_t + \epsilon_{nt} + C_n \quad (4)$$

Y_{nt}: represents the value of the response variable in observation (n) at time period (t), Its dimensions are (n×1).
C_n: represents the value of the intersection point in observation(A), Its dimensions are (n×1).
β_t: represents the value of the slope of the regression line, Its dimensions are (k×1).
φ_t: the spatial dependence parameter.
Mnt: the spatial adjacency matrix, Its dimensions are (n×n).
X_{nt}: represents the value of the explanatory variable in observation(n) at time period (t), Its dimensions are (n×k).
ϵ_{nt}: represents the value of the error of observation (n) at time period (t), Its dimensions are (n×1).

8 . Estimation Methods

8.1 Quasi Maximum Likelihood Method (QMLE)

This method is used to estimate the model parameters consistently, especially when the number of cross-sections (n) is specific and close to (T), and thus the problem of cross-sectional parameters appears. Equation (4) can be written in terms of errors and reformulated as follows[4,7]:

$$\epsilon_{nt} = S_n Y_{nt} - X_{nt}\beta_t - C_n \quad (5)$$

$$S_n = (I_n - \varphi_t M_{nt}) \;, \quad \theta = (\varphi, \beta, \sigma^2)$$

$$l_{n,T}(\theta, C_n) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^{nT} e^{\frac{-\epsilon'_{nt}(\theta)\epsilon_{nt}(\theta)}{2\sigma^2}} \quad (6)$$

$$\ln \; l_{n,T}(\theta, C_n) = \frac{-nT}{2}\ln(2\pi\sigma^2) + T\ln|S_n| - \frac{1}{2\sigma^2}\sum_{t=1}^{T}\epsilon'_{nt}(\theta)\epsilon_{nt}(\theta) \quad (7)$$

$$\frac{dlnl}{dC_n} = \frac{1}{\sigma^2}\sum_{t=1}^{T}(S_n Y_{nt} - X_{nt}\beta_t - C_n) \quad (8)$$

$$C_n = \frac{1}{\mathrm{T}}\sum_{t=1}^{T}(S_n Y_{nt} - X_{nt}\beta_t) \quad (9)$$

$$ln \;\; l_{n,T}(\theta, C_n) = \frac{-nT}{2}ln(2\pi\sigma^2) + Tln|S_n| - \frac{1}{2\sigma^2}\sum_{t=1}^{T}[S_n Y_{nt} - X_{nt}\beta_t - \frac{1}{T}\sum_{t=1}^{T}(S_n Y_{nt} - X_{nt}\beta_t)]^2 \quad (\mathbf{10})$$

From the equation above, we take the last term and simplify. To get rid of the fixed spatial effects, we take C_n. By taking the differences to the original values, the effects will not appear in the equation. We can reformulate it after taking the deviations as follows:

$$\tilde{Y}_{nt} = S_n^{-1}\tilde{X}_{nt}\beta_t + S_n^{-1}\tilde{\epsilon}_{nt} \quad (11)$$

$$M_{nt}\tilde{Y}_{nt} = G_n\,\tilde{X}_{nt}\beta_t + G_n\tilde{\epsilon}_{nt} \quad (12)$$

$$G_n = M_{nt}S_n^{-1}$$

Based on the formula, we find the first derivative to obtain the estimated formulas for the model parameters, which are as follows[7]:

$$\hat{\beta}_{nT} = [\sum_{t=1}^{T}\tilde{X}'_{nt}\tilde{X}_{nt}]^{-1}\left[\sum_{t=1}^{T}\tilde{X}'_{nt}\,S_n\tilde{Y}_{nt}\right] \quad (13)$$

$$\hat{\sigma}^2_{nT} = \frac{\sum_{t=1}^{T}\tilde{\epsilon}'_{nt}\tilde{\epsilon}_{nt}}{nT} \quad (14)$$

As for the parameter φ_t, it is found numerically.

8.2 The Transformation Method (TTA)

This method depends on performing a transformation to exclude spatial effects by reducing the number of observations by one observation for each sample, meaning (from (N×T) to (N×(T-1)) in the case of spatial effects in the model. This is done by multiplying the variables by the normal orthogonality matrix of the characteristic vectors, whose elements consist of $F_{T,T-1}, \frac{1}{T}l_T$ Which represent the characteristic vectors of the matrix (J_T) which are $J_T = I_T - \frac{1}{T}l_T l'_T$ since: I_T: the identity matrix has dimensions (T×T), l_T: a vertical vector with dimensions (T×1) consisting of one integer,

$F_{T,T-1}$: a matrix with dimensions (T×T) consisting of the characteristic vectors of the J_T matrix.The natural orthogonality criterion $F_{T,T-1}$ has several conditions that must be taken into consideration and which are relied upon to achieve orthogonality. The conditions are as follows[9]:

$$\left.\begin{array}{l} F'_{T,T-1}F_{T,T-1} = I_{T-1} \\ l'_T F_{T,T-1} = \underline{0} \\ F'_{T,T-1} l_T = \underline{0} \\ F_{T,T-1}F'_{T,T-1} = J_T \\ J_T F_{T,T-1} = F_{T,T-1} \\ F'_{T,T-1}F_{T,T-1} + \frac{1}{T} l_T l'_T = I_T \\ \left[F_{T,T-1}, \frac{1}{\sqrt{T}} l_T \right]' \left[F_{T,T-1}, \frac{1}{\sqrt{T}} l_T \right] = I_T \end{array}\right] \quad (15)$$

The model variables are multiplied by the amount $F_{T,T-1}$ as follows [7,9]:

$$\left.\begin{array}{l} [Y^*_{n1}, Y^*_{n2}, \dots Y^*_{n,T-1}] = [Y^*_{n1}, Y^*_{n2}, \dots Y^*_{nT}] F_{T,T-1} \\ [X^*_{n1,k}, X^*_{n2,k}, \dots X^*_{n,T-1,k}] = [X^*_{n1,k}, X^*_{n2,k}, \dots X^*_{nT,k}] F_{T,T-1} \end{array}\right] \quad (16)$$

$$[\epsilon^{*'}_{n1}, \epsilon^{*'}_{n2}, \dots, \epsilon^{*'}_{n,T-1}]' = \left(F'_{T,T-1} \otimes I_n\right) [\epsilon^{*'}_{n1}, \epsilon^{*'}_{n2}, \dots, \epsilon^{*'}_{n,T}]' \quad (17)$$

$$E(\epsilon^{*'}_{n1}, \epsilon^{*'}_{n2}, \dots, \epsilon^{*'}_{n,T-1})' \left(\epsilon^{*'}_{n1}, \epsilon^{*'}_{n2}, \dots, \epsilon^{*'}_{n,T-1}\right) = \sigma^2 \left(F'_{T,T-1} \otimes I_n\right)\left(F_{T,T-1} \otimes I_n\right) = \sigma^2 I_{n(T-1)} \quad (18)$$

Through the conditions in equation (15), we get rid of the spatial effects parameter (C_n) by multiplying it by($F_{T,T-1}$)

$$[C_n, C_n, \dots, C_n]' F_{T,T-1} = 0 \quad (19)$$

After applying the transformation method, the model becomes as follows[9]:

$$Y^*_{nt} = \varphi_t M_{nt} Y^*_{nt} + X^*_{nt}\beta_t + \epsilon^*_{nt} \qquad t = 1,2,\dots,T-1 \quad (20)$$

$$M_{nt}Y^*_{nt} = G_n X^*_{nt}\beta_t + G_n \epsilon^*_{nt} \quad (21)$$

$$G_n = M_{nt}(I_n - \varphi_t M_{nt})^{-1}$$

$$lnl = \frac{-n(T-1)}{2}\ln(2\pi\sigma^2) + (T-1)ln|I_n - \varphi_t M_{nt}| - \frac{1}{2\sigma^2}\sum_{t=1}^{T-1} \epsilon^{*'}_{nt}\epsilon^*_{nt} \quad (22)$$

Based on the formula, we find the first derivative to obtain the estimated formulas for the model parameters, which are as follows[7]:

$$\hat{\beta}_{nT} = \left[\sum_{t=1}^{T-1} X^{*'}_{nt} X^*_{nt}\right]^{-1} \left[\sum_{t=1}^{T-1} X^{*'}_{nt} S_n Y^*_{nt}\right] \quad (23)$$

$$\hat{\sigma}^2_{nT} = \frac{\sum_{t=1}^{T-1} \epsilon^{*'}_{nt}\epsilon^*_{nt}}{n(T-1)} \quad (24)$$

As for the parameter φ_t, it is found numerically.

9.Comparison Criterion Mean Aboslute Percentage Error (MAPE)

Through this criterion, the best method is chosen from among the methods of estimating the model. The lower the value of the criterion, the better the method is in estimating the model. It can be calculated according to the following formula[10]:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} \left|\frac{\rho}{Y_t}\right| \times 100\% \quad (25)$$

$$\rho = Y_t - \hat{Y}_t$$

ρ: The difference between the real value and the estimated value when observing t

Y_t: Real values at observation t

n: Sample size.

10.Description Of Simulation Experiments

By using a statistical program (Matlab), a number of simulation experiments were conducted, which included the following:

First: - Using two values for the cross-sections (n=20, 60) and two different time periods (T=5,20) and thus we have three different sample sizes which are (nt=100,300,400,1200) and also using two values for the variance (σ^2= 0.2, 0.9) and using two values for the spatial dependence which are (φ_t=0.2, 0.6) and this experiment is repeated (1000) times.

Second: Generating random variables: In this paragraph, the random variables included in the spatial autoregressive model for panel data with fixed effects will be generated, which are as follows:

a) Generation of the independent variable: One explanatory variable is generated according to the regular distribution X~U(0,1).

b) Generation of random errors: They are generated according to the normal distribution $\epsilon_{nt} \sim N(0,\sigma^2)$

c) Generation of the dependent variable: The dependent variable (Y) is generated to fit the reality of the problem under study

d) Determination of the modified spatial weight matrix: In this paragraph, the modified spatial weight matrix is found according to the Rook contiguity criterion.

11.Results and discussion

After implementing simulation experiments and with the aim of comparing the two methods of Quasi maximum likelihood estimation (QMLE) and Transformations (TTA) and in light of the modified spatial weight matrix and using the comparison criterion of the mean absolute relative error (MAPE), the comparison was made and the results were as in the tables below.

Table (2) shows the results of estimating the parameters of the spatial autoregressive model for panel data when $\sigma^2 = 0.2$

		n	t	nt	Methods	mean($\widehat{\beta_t}$)	MAPE(β_t)	$\hat{\varphi}_t$	MAPE of Model	Best Method
$\sigma^2 = 0.2$	(β_t= 1 , φ_t = 0.2)	20	5	100	QMLE	0.8020	0.2110	0.2170	0.2812	TTA
					TTA	1.0511	0.1806	0.2770	0.2542	
		20	20	400	QMLE	0.8112	0.2000	0.0660	0.3234	QMLE
					TTA	0.8926	0.4320	0.0640	0.3580	
		60	5	300	QMLE	0.8149	0.1851	0.0967	0.3070	TTA
					TTA	1.0962	0.1328	0.1693	0.2147	
		60	20	1200	QMLE	0.8160	0.1843	0.0265	0.3338	QMLE
					TTA	1.0885	0.2310	0.0381	0.3579	

From Table (2), we note that the lowest value of the comparison criterion (MAPE) was at a sample size of (300), which means that the best method is transformations because it achieved the lowest value of the comparison criterion.

Table (3) shows the results of estimating the parameters of the spatial autoregressive model for panel data when $\sigma^2 = 0.2$

		n	t	nt	Methods	mean($\widehat{\beta_t}$))	MAPE(β_t)	$\hat{\varphi}_t$	MAPE of Model	Best Method
$\sigma^2 = 0.2$	(β_t= 0.3 , φ_t = 0.6)	20	5	100	QMLE	0.0820	0.2180	0.0129	0.5693	TTA
					TTA	0.3371	0.1719	0.0215	0.4572	
		20	20	400	QMLE	0.0770	0.2230	0.0031	0.5674	TTA
					TTA	0.3254	0.2662	0.0043	0.4730	
		60	5	300	QMLE	0.0769	0.2231	0.0061	0.5683	TTA
					TTA	0.3441	0.1310	0.0076	0.4514	
		60	20	1200	QMLE	0.0771	0.2229	0.0015	0.5687	TTA
					TTA	0.4067	0.2036	0.0014	0.4343	

From Table (3), we note that the lowest value of the comparison criterion (MAPE) was at a sample size of (1200), which means that the best method is transformations because it achieved the lowest value of the comparison criterion.

Table (4) shows the results of estimating the parameters of the spatial autoregressive model for panel data when $\sigma^2 = 0.9$

		n	t	nt	Methods	mean($\widehat{\beta_t}$)	MAPE(β_t)	$\widehat{\varphi}_t$	MAPE of Model	Best Method
$\sigma^2 = 0.9$	(β_t= 1 , φ_t = 0.2)	20	5	100	QMLE	0.8451	0.4985	0.5804	0.8936	QMLE
					TTA	0.9038	0.4957	0.5250	0.9438	
		20	20	400	QMLE	0.8390	0.4895	0.2452	0.8855	QMLE
					TTA	0.8740	0.8150	0.1294	1.0131	
		60	5	300	QMLE	0.8120	0.2958	0.4171	0.8677	QMLE
					TTA	0.9933	0.2816	0.3782	0.8836	
		60	20	1200	QMLE	0.7916	0.3235	0.1497	0.8874	QMLE
					TTA	0.9917	0.5140	0.0895	0.9251	

From Table (4), we note that the lowest value of the comparison criterion (MAPE) was at a sample size of (300), which means that the best method is Quasi maximum likelihood because it achieved the lowest value of the comparison criterion.

Table (5) shows the results of estimating the parameters of the spatial autoregressive model for panel data when $\sigma^2 = 0.9$

		n	t	nt	Methods	mean($\widehat{\beta_t}$)	MAPE(β_t)	$\widehat{\varphi}_t$	MAPE of Model	Best Method
$\sigma^2 = 0.9$	(β_t= 0.3 , φ_t = 0.6)	20	5	100	QMLE	0.0960	0.2671	0.1759	1.1689	TTA
					TTA	0.2883	0.4100	0.1209	1.1668	
		20	20	400	QMLE	0.0781	0.2655	0.0511	1.1809	TTA
					TTA	0.2568	0.4814	0.0236	0.7008	
		60	5	300	QMLE	0.0766	0.2270	0.0930	1.2096	TTA
					TTA	0.2990	0.2644	0.0544	0.8864	
		60	20	1200	QMLE	0.0773	0.2261	0.0252	1.2154	TTA
					TTA	0.2825	0.3097	0.0095	1.2021	

From Table (5), we note that the lowest value of the comparison criterion (MAPE) was at a sample size of (400), which means that the best method is transformations because it achieved the lowest value of the comparison criterion.

11.1summary Of Final Tables For Estimation Methods

Table (6) shows a summary of the MAPE values across all tables.

n	t	nt	$\sigma^2 = 0.2$		$\sigma^2 = 0.9$		Repetition (QMLE)	Repetition (TTA)	Total repetition
			β_t =1, φ_t=0.2	β_t =0.3, φ_t=0.6	β_t =1, φ_t=0.2	β_t =0.3, φ_t=0.6			
20	5	100	TTA	TTA	QMLE	TTA	1	3	4
	20	400	QMLE	TTA	QMLE	TTA	2	2	4
60	5	300	TTA	TTA	QMLE	TTA	1	3	4
	20	1200	QMLE	TTA	QMLE	TTA	2	2	4
							6	10	16

From the table above, we notice that the best estimation method is the transformation method (TTA), and its sequence is the first, as its preference is repeated (10) times out of a total of (16), while the Quasi maximum likelihood method (QMLE) has its preference repeated (6) times out of a total of (16). This indicates that the Quasi maximum likelihood method is less efficient than the transformation method.

12. Conclusions

The results, using the Mean Absolute Relative Error (MAPE) comparison criterion, showed that the best method is the Transformational Approach (TTA) because it achieved the lowest MAPE value at a sample size of (300). We also note that the best MAPE results are achieved when the number of cross-sections is (n=60). The Quasi-Maximum

Likelihood (QMLE) values also appeared in some cases as high values, indicating that this method is less efficient than the TTA method.

References

[1] Ahmed.A. A.,(2021)."Estimation The Spatial Durban Regression Model For Anemia Pattients Sample In Some Region Of Al-Karth/ Baghdad ". Al-Mustiansiryah University.

[2] Anwar, S. A. & Ahmed, A. A. (2023)." Estimation of the Durbin spatial semi-paramteric regression model using the sub-segment regression method". Statistics Department, Collage Of Management And Economic, Al-Mustiansiryah University,Iraq.

[3] Daniel, X. & Sock H. L. (2007)"Introduction to panel Data Analysis". Miller/ Handbook of Researth Methods in Public Administration AU5384-C032, p.572

[4] Juncog,G. & Xi,Q. (2020) "fixed effects spatial panel data model with time-varying spatial dependence ’’ . Economics Letters , Vol ; (196).

[5] Elhorst, J. P.(2014),"Spatial Econometics From Cross-Sectional Data to Spatial Panels", Faculty of Economics and Business, Univeristy of Groningen, Groningen, The Netherlands.

[6] LeSge, J. P.(1999)."The Theory and Practice of Spatail Econometrics " Department of Economics, University of Toledo:pp.(3-7).

[7] Lee,L.F. & Yu,J.(2010)’’Estimation of spatial autoregressive Panel data models with fixed effects’’ Journal of Econometrics , Vol ; (154) , pp.(165-185).

[8] Luc, A.(1988),"Spatial Econometric: Methods and Models", Department of Geography and Economics University of California, Santa Barbara.

[9] Shaheed,S.A. & AL-Saffar, R. S. (2020) "Measuring the Impact of Environmental Sustainablity on Tuberculosis Rates Using the Two-Stage Least Squares Mathod in the Polled Model’’ international Journal on Advanced Science Engineering Information Technology, Vol ; (10) .No(6).

[10] Khair,U. & H. Fahmi, & S. Al Hakim, & R. Rahim,(2017)."Forecasting Error Calculation with Mean Absolute Deviation and Mean Absolute Percentage Error". International Conference on Information and Communication Technology (IconICT) IOP conf, Series : Journalof Physics:conf, Series.Vol. (930),012002

[11] Wadhah, S. I. & Ghiath, H. M. & wafaa, J. H. (2021),"Comparison and Estimation of a spatial Autoregressive (SAR)Model for cancer in Baghdad Regions", Interational Journal of Agricultural and statistical Sciences.

[12] Wadhah, S. I. & Nawras, S. M. (2022), "Estimation of the general spatial regression model (SAC) by maximam Likelihood method". Int. J. Nonlinear Anal Appl. (13), (2947-2957).