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Estimation of Power Function Distribution Parameters Using a Simulation Approach for Bivariate Type II Censored Data via Bayesian Method

Noor Al-Safi Kazem Abdul [1], Baydaa Ismeal Abdul Wahhab [2]

[1]Ministry of Higher Education and Scientific Research Baghdad, Iraq
[2]Department of Statistics, College of Administration and Economics University of Baghdad, Iraq

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ABSTRACT

In this research, we estimated a model and parameters for the power distribution function for type II doubly censored data using simulation methods and Bayesian techniques under loss functions (Linex, Entropy) by comparing them. The results showed that the Bayesian method under Lindley's approximation with the exponential loss function (Linex) was more efficient than the Bayesian method under Lindley's approximation with the Entropy loss function. Additionally, it was found that the Bayesian method using Lindley's approximation under the entropy loss function for the parameters was more efficient than the Bayesian method using Lindley's approximation under the linex loss function, evaluated by the Mean Squared Error (MSE).

1. Introduction

There are various life tests, and researchers may face difficulties in obtaining complete information from experimental units, as these units can be affected by random factors. The data we will analyze are monitored data, which include multiple types: Type I monitored data and Type II monitored data. Our study will focus on double Type II monitored data. In 2013, the researchers (Zarrin, Saxena, Kamal, Islam [2]((Estimating the distribution of the power function using the maximum likelihood and Bayesian methods. They suggested that the distribution of the power function should be considered when analyzing electronic failure data. In 2015, the researcher (Hanif)[3] He and his colleagues estimated the parameters of the power function distribution using Bayesian estimation, relying on the Weibull distribution and the generalized gamma distribution. A comparison was made between the two methods using Monte Carlo simulation, where the mean squared error was used as a statistical indicator for comparison. The results showed that the Bayesian estimator with the Weibull distribution was more efficient in estimating parameters for small samples, while the performance of the Bayesian estimator with the gamma distribution was better than the other methods in the case of large samples. The power function distribution is one of the distributions also used to assess the reliability of semiconductor devices and other products, and it is known as a flexible distribution that can provide a good fit for a specific set of failure data. It has numerous applications in various scientific fields such as finance,

* Corresponding author. E-**mail** address: Kadhimn353@gmail.com
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ecology, sociology, soil science, and economics, among others. Many researchers have applied the power function distribution to model the reliability of complex or repairable systems [4] .It is also used to describe any set of data for variables, although rare, may fit well for certain sets of failure data (breakdowns which a sample is limitedthat it is innovative, it is used in the section "Research Method" to describe the step of research and used in the section "Results and Discussion" to support the analysis of the results [3].

$$f(x,a,b) = \frac{a}{b}\left(\frac{b}{X}\right)^{1-a} \quad \text{where } 0<x<b \;;a,b>0 \quad \ldots..(1)$$

2. Type II censored data [5]

This is data that is a combination of right-censored and left-censored data and appears in many applications, especially in reliability analysis and biomedical studies. Often, several extreme values of the sample are recorded, whether they are lower or higher than a certain level, as a result of negligence by inexperienced observers or other factors. Therefore, it may make sense to exclude these observations from the original dataset. The remaining sample, from which the minimum and maximum values have been excluded, is often referred to as the Type II censored sample. When we have a size n of units under test, and we need to monitor m of the units where ، r < s < n ، m = s - r + 1 ،Thus, the censored data will be (Xr < … < Xs). In this case, the random variable cannot be determined at the same time, and therefore the test stops or ends until we obtain m units under censorship. Upon reaching s.

And the maximum likelihood function for the double data of the second type is:

$$L(a,b)=\frac{n!}{(r-1)!(n-s)!}\,(F(X_r))^{r-1}[1-F(X_r)]^{n-s}\prod_{i=r}^{s} f(X_i) \quad \ldots\ldots\ldots(2)$$

3. Bayesian Method[6][7][8]

It is a statistical method used for estimation, and the evolution of Bayesian estimation can be traced back to the eighteenth century. However, due to the inadequate computing technology and data collection capabilities of that time, Bayesian estimation was not widely used. With the continuous developments in computer technology, Bayesian estimation began to garner significant interest from scientists by the end of the twentieth century and rapidly evolved. Today, Bayesian estimation is widely used in fields such as machine learning, statistics, data mining, and others. This approach provides a systematic method that combines prior information with current data, and is considered a random variable carrying certain distributional assumptions (the prior distribution).The Bayesian method relies on current information, which is known as the likelihood function of the observations. The prior distribution function is combined with current information, represented by the maximum likelihood function, to obtain the posterior probability function. This method is useful for obtaining accurate or close-to-reality information, as the posterior distribution represents a complete description of the unknown parameter in the presence of sample information.After clarifying the steps of the Bayesian approach using tools based on the prior distribution, the maximum likelihood function, the posterior distribution, and the loss function, all these tools are used to estimate the parameters of the power function distribution. To obtain the value of the estimator, we rely on the posterior distribution, which is considered one of the foundational pillars of this method, in addition to using loss functions such as the quadratic or exponential functions, among others.

3.1. Prior Probability Density Function [10][9] -

To conduct a Bayesian analysis, it is essential to specify some prior distributions for the random variables. We assume that the variables a and b are independent random variables that follow a gamma distribution. Prior distributions play an important role in determining the posterior distribution, and these distributions are often referred to as reference distributions. The probability density function is considered vague or non-informative, as the rationale for using non-informative prior distributions is to allow the data to speak for itself. Therefore, choosing a prior distribution may increase the complexity and difficulty of the mathematical equations, while independent variables from a gamma distribution tend to be relatively simple and concise, which may reduce complex inferential and computational issues

$$g_1(a) = \frac{d^c}{\Gamma c} a^{c-1} e^{-da} \quad .....(3)$$

$$g_2(b) = \frac{p^f}{\Gamma f} b^{f-1} e^{-pb} \quad ......(4)$$

The joint prior distribution is obtained by multiplying the prior distributions above for the parameters a and b, represented by formulas (3) and (4).

$$G(a,b) = g_1(a) * g_2(b)$$

$$= \frac{d^c}{\Gamma c} a^{c-1} e^{-da} * \frac{p^f}{\Gamma f} b^{f-1} e^{-pb}$$

$\frac{1}{k} = \frac{d^c}{\Gamma c} * \frac{p^f}{\Gamma f}$ us obtain the following

Let

$$G(a,b) = \frac{1}{k} a^{c-1} b^{f-1} e^{-(da+pb)} \quad ... ... (5)$$

3-2 Posterior Probability Density Function[11][12

The posterior distribution is a fundamental concept in Bayesian statistics. As mentioned earlier, the Bayesian approach relies on the prior probability density function and the likelihood function of the observations. The posterior distribution plays a vital role in the inference process, as it represents the updated knowledge or belief about a variable or set of variables after taking prior information and observed data into account. In Bayesian analysis, the posterior distribution is obtained by multiplying the prior distribution, which reflects our initial beliefs, by the likelihood function that defines the probability of observing the data given certain parameter values. The posterior distribution provides a comprehensive summary of uncertainty, allowing us to estimate variables and make various predictions and analyses. It serves as a link between prior knowledge and observed evidence. Overall, the equation (5) representing the joint prior distribution is combined with the equation (6) representing the likelihood function of the observations, resulting in the joint posterior distribution.

And the maximum likelihood function for the power function dis.is:

$$L(a,b) = \boldsymbol{w}\,(\boldsymbol{X_r}^{\boldsymbol{a(r-1)}})[1-(\tfrac{\boldsymbol{X_s}}{\boldsymbol{b}})^{\boldsymbol{a}}]^{\boldsymbol{n-s}}$$

$$\boldsymbol{a^{s-r+1}}(\boldsymbol{b^{-as}}) \textstyle\prod_{k=1}^{n} \boldsymbol{X_i^{a-1}}) \prod(a,b) = \mathrm{G}(a,b)$$

$$L(a,b) \quad ......(6)$$

And when substituting the above in the joint posterior distribution function (7), we obtain the following:

$$= \frac{1}{k} a^{(c+s-r+1)-1} b^{(f-as)-1} e^{-(da+pb)} e^{\,a(r-1)\log xi} [1 - (\tfrac{X_s}{b})^a]^{n-s} e^{(a-1)\sum \log X_I}$$

$$= \frac{1}{k} a^{(c+s-r+1)-1} b^{(f-as)-1} e^{-(da+pb)} e^{\,a(r-1)\log xi} [1 - (\tfrac{X_s}{b})^a]^{n-s} e^{a\sum \log X_I} e^{-\sum \log X_I}$$

: Assume $y = \sum \log X_I$ we obtain the following

$$[1- (\tfrac{X_s}{b})^a]^{n-s} e^{ay} e^{-y} \frac{1}{k} a^{(c+s-r+1)-1} b^{(f-as)-1} e^{-(da+pb)} e^{\,a(r-1)\log x_i}$$

$$=[1- (\tfrac{X_s}{b})^a]^{n-s} e^{ay} e^{-y} \frac{1}{k} a^{(c+s-r+1)-1} b^{(f-as)-1} e^{-pb} e^{-da} e^{\,a(r-1)\log x_i}$$

The joint posterior distribution of the power function distribution is as follows:

$$=[1- (\tfrac{X_s}{b})^a]^{n-s} \quad e^{-y} ..(7) \quad =[1- (\tfrac{X_s}{b})^a]^{n-s} \quad e^{-y} ..(7 \quad)$$

$$\pi(a,b) = \frac{1}{k} a^{(c+s-r+1)-1} b^{(f-as)-1} e^{-pb} e^{-a(d-(r-1)logXi-y)}$$

And using some loss functions:

4- Loss functions [6]

The Bayesian method requires the use of loss functions, where the Bayesian estimator can be obtained by minimizing the expected loss function of the posterior distribution. To achieve the loss function, two conditions must be met:

$L(\hat{\Theta},\Theta) \geq 0 \;\; \forall\hat{\Theta}, \forall\Theta$-1

$L(\hat{\Theta},\Theta) = 0 \;\; \forall\hat{\Theta}, \forall\Theta$-2

4-1-Loss Function (Linex)[13]

The Linex loss function is considered one of the well-known asymmetric loss functions used by researchers. This function combines exponential and linear losses, giving greater importance to errors in one direction compared to the other, making it more realistic. This function was first used by researcher Vatian in 1975.

The Linex loss function can be obtained from the following formula:

$$Llf= e^{[h(H-\hat{H})]-h(H-\hat{H})-1} \quad ;h\neq 0 \ldots\ldots(8)$$

Where $\hat{H} =-\frac{1}{h}\log[E(e^{(-H/X)})]$

4-2-Entropy Loss Functions (Elf)[14]

Entropy loss functions are considered asymmetric functions and are a modification of the linear exponential loss function (Linex) proposed by researchers Varian in 1975 and Zellner in 1986. They were utilized by Calabria and Pulcini in 1994.

$$ELF=[\frac{\hat{H}}{H}]^{Z} - Z\,(\log(\frac{\hat{H}}{H}))-1 \;;Z\neq 0 \qquad \ldots(9)$$

The $\hat{H}$ entropy loss function can be obtained from the following formula:

$$\hat{H}=[E\,\frac{\hat{H}}{X}^{Z}]^{-\frac{1}{Z}}$$

By using the loss functions (entropy, linex), we obtain a Bayes estimator for the power distribution function as follows:

$$\hat{\theta}elf = [\frac{\int_0^\infty\int_0^\infty \theta^{-z}\prod(a,b)\,dadb}{\int_0^\infty\int_0^\infty \prod(a,b)\,dadb}]^{-\frac{1}{z}}$$

$$\hat{\theta}llf = -\frac{1}{h}\,log\left[\frac{\int_0^\infty\int_0^\infty e^{-h\theta}\prod(a,b)\,dadb}{\int_0^\infty\int_0^\infty \prod(a,b)\,dadb}\right]$$

Here, we use the Lindley approximation method to solve the complex integrals mentioned above, with the aim of obtaining Bayes estimates for the power distribution function in the context of loss functions.

5- Lindley Approximation [15]

Due to the inability to derive a closed-form Bayes estimator from the posterior distribution presented in the previous equations, we rely on the Lindley approximation to obtain approximate Bayes estimates for the parameters of the power distribution function. This aids in estimating the Bayes parameters a and b . Thus, the ratio of the model integral is:

$$I(X)=\frac{\int \Delta(a,b)\,e^{G(a,b)+L(a,b)}d(a,b)}{\int e^{G(a,b)+L(a,b)}d(a,b)}$$

We can approximate the ratio of the integral as follows:

$$\Psi_{ij} = -\frac{1}{L_{ij}}$$

$$,L_b = \frac{\partial Log(L(a,b))}{\partial b}, L_{abb} = \frac{d^{3}log(L(a,b))}{\partial a\partial b\partial b},$$

$$L_{baa} = \frac{d^{3}log(L(a,b))}{\partial b\partial a\partial a} \quad ,$$

$$L_{aba} = \frac{d^{3}log(L(a,b))}{\partial a\partial b\partial a},$$

$$L_{bb} = \frac{\partial^2 Log(L(a,b))}{\partial b^2}\;, L_{bbb} = \frac{\partial^3 Log(L(a,b))}{\partial b^3}$$

$$L_a = \frac{\partial Log(L(a,b))}{\partial a}, L_{aaa} = \frac{\partial^3 Log(L(a,b))}{\partial a^3}, L_{aa} =$$

$$\frac{\partial^2 Log(L(a,b))}{\partial a^2}, L_{bab} = \frac{d^{3}log(L(a,b))}{\partial b\partial a\partial b}$$

To obtain the Bayesian estimator b for the power function distribution under the entropy loss function as follows [16]:

$\Delta(a,b)=b^{-q}$, $\Delta_b=-\,qb^{-(q+1)}$, $\Delta_{bb}= q(q+1)b^{-(q+2)}$

$\Delta_{aa}=\Delta_{ba}=\Delta_{ab}=\Delta_a=0$

$$\hat{B}ELF= B^{-Q}+\frac{1}{2}[(Q(Q+1)B^{-(Q+2)} - 2\,QB^{-(Q+1)}\Gamma_B)\Psi_{BB} + (-2\,QB^{-(Q+1)}\Gamma_A)\Psi_{BA}] + \frac{1}{2}[(-QB^{-(Q+1)}\Psi_{BB}\Theta_1 - QB^{-(Q+1)}\Psi_{AB}\Theta_2]$$

To obtain the Bayesian estimator a for the power function distribution under the entropy loss function as follows [16]:

$\Delta(a,b)=a^{-q}$, $\Delta_a=-\,qa^{-(q+1)}$, $\Delta_{aa}= q(q+1)a^{-(q+2)}$ $\Delta_{bb}=\Delta_{ba}=\Delta_b=\Delta_{ab}=0$

$\hat{a}$elf= $a^{-q}+\frac{1}{2}[-2\, q a^{-(q+1)}\gamma_b \Psi_{ab}+(q(q+1)a^{-(q+2)}-2\, q a^{-(q+1)}\gamma_a)\Psi_{aa}]+\frac{1}{2}[-q a^{-(q+1)}\Psi_{ba}\Theta_1 - q a^{-(q+1)}\Psi_{aa}\Theta_2]$

To obtain the Bayesian estimator b for the power function distribution under the linex loss function as follows [16] :

$\Delta(a,b)=e^{-qb}$, Δ_b=- qe^{-qb} , $\Delta_{bb}= q^2 e^{-qb}$

Δ_{aa}=Δ_{ba}=Δ_{ab}= Δ_a=0

$\hat{b}$LLF=

$e^{-qb}+\frac{1}{2}[(q^2 e^{-qb}-2\, Qe^{-qb}\,\Gamma_b)\Psi_{bb}+(-2\, Qe^{-qb}\,\gamma_a)\Psi_{ba}]+\frac{1}{2}[(-Qe^{-qb}\Psi_{bb}\Theta_1 - Qe^{-qb}\,\Psi_{ab}\Theta_2]$

They all aim to achieve a single goal. Simulation is defined as a mathematical approach used to address and implement problems on a computer. It is considered the language of the age, assisting researchers in their studies by providing descriptive models rather than ideal models. Its purpose is to provide answers to a variety of questions of interest to many researchers, reflecting what happens in real systems.Therefore, simulation is used to save time and effort and to achieve accurate results in addressing the problems researchers face.At this stage, the methods are compared. used in this research will be made according to the following steps:

1. Determining Sample Size n Six sample sizes were chosen(n = 10, 15, 20, 50, 100, 150) . For the purposes of conducting Type II double monitoring on the samples, values of r were defined as (r = 4, 10, 20) based on the size of each sample. Additionally, values of (s) were set as (s = 8, 10, 13, 17, 40, 60, 80, 140), remaining constant in a single experiment but varying from one experiment to another. Table 1 illustrates what has been mentioned above.

as follows:

X= b $u^{\frac{1}{a}}$

The data generation formula above follows a power distribution with parameters (a, b). Based on this, dual type II censoring observation data is applied according to the values of (r, s) and the sample sizes.

4- Consequently, a comparison is made to select the best method from those used in the research, based on the application of real data through the use of the statistical measure of the Mean Squared Error (MSE) for the model and the parameters of the power distribution as follows:

i- The Mean Squared Error for the parameters of the power distribution according to the following formula:

$$MSE(\hat{\theta}\,)=\frac{\sum_{i=1}^{r}(\hat{\theta}-\theta)^2}{R}$$

ii- The Mean Squared Error for the power distribution model according to the following formula:

$$MSE(\hat{y})=\frac{\sum_{i=1}^{n}(\hat{y}-y)^2/n}{R}$$

Table 1: shows the sample sizes and the values of (r, s).

n	10	15	20	50	100	150
r	4			10,20		
s	8,10,13,17			40,60,80,140		

- Choose hypothetical values for the shape parameter (a) and the scale parameter (b) for the power distribution function, and assume the constant value (2=q) as the loss function constant used in the Bayesian method, as shown in the following Table (2):

Table 2: shows the hypothetical values for the parameters for the proposed experiments.

exp	1	2	3	4	5	6	7	8	9	10
a	1	1	3	1	6	8	7	4	2	5
b	2.5	2.3	4	0.5	5	2.6	0.6	0.75	1.5	2.5

The simulation experiments are repeated (R=1000) for each trial.

3- Then, we generate data following a power distribution using the inverse of the cumulative function as follows

Table 3: shows the Mean Squared Error (MSE) for the parameters of the power distribution for each method when (

Table 4: presents the Mean Squared Error (MSE) for the parameters of the power distribution for all methods when (a = 2) and (b = 3).

			Bias				**The best**	
			Bias				**The best**	
N	**r**	**s**	**Lindley**					
n	**r**	**s**	**Lindley**					
			Mse a^elf)(	**Mse b^elf)(**	**Mse a llf)(**	**Mse b llf)(**	**a^**	**b^**
50	**20**	**40**	**5.05E-05**	**0.000167**	**7.84E-05**	**0.00018**	**elf**	**llf**
			Mse a^elf)(	**Mse b^elf)(**	**Mse a llf)(**	**Mse b llf)(**	**a^**	**b^**
		40	**2.54E-05**	**8.35E-05**	**3.93E-05**	**8.99E-05**	**elf**	**elf**
10	**4**	**8**	**0.000283**	**0.000835**	**3.90E-04**	**0.000899**	**llf**	**elf**
100	**20**	**60**	**2.49E-05**	**8.35E-05**	**3.93E-05**	**8.99E-05**	**elf**	**elf**
		8	**0.000192**	**0.000558**	**2.61E-04**	**0.0006**	**llf**	**elf**
		80	**2.48E-05**	**8.35E-05**	**3.93E-05**	**8.99E-05**	**elf**	**elf**
15	**4**	**10**	**0.000183**	**0.000557**	**2.61E-04**	**0.000599**	**llf**	**elf**
		40	**1.70E-05**	**5.57E-05**	**2.62E-05**	**5.99E-05**	**elf**	**elf**
		13	**0.000177**	**0.000557**	**2.61E-04**	**0.000599**	**llf**	**elf**
150	**20**	**60**	**1.67E-05**	**5.57E-05**	**2.62E-05**	**5.99E-05**	**elf**	**elf**
		8	**0.000146**	**0.000419**	**0.000197**	**0.00045**	**elf**	**elf**
		80	**1.65E-05**	**5.56E-05**	**2.62E-05**	**5.99E-05**	**elf**	**elf**
20	**4**	**10**	**0.000139**	**0.000418**	**0.000196**	**0.00045**	**elf**	**elf**
		140	**1.64E-05**	**5.56E-05**	**2.62E-05**	**5.99E-05**	**elf**	**elf**
		13	**0.000133**	**0.000418**	**0.000196**	**0.00045**	**elf**	**elf**
		17	**0.00013**	**0.000417**	**0.000196**	**0.000449**	**elf**	**elf**

Table (4) shows that the Bayesian method with lindley approximation, in the case of the entropy loss function (ELF), is the best for sample sizes (50, 100, 150) because it has the lowest MSE

Table 5: presents the Mean Squared Error (MSE) for the parameters of the power distribution for all methods when (a = 3) and (b = 4).

n	**r**	**s**	**Bias**				**The best**	
			Lindley					
			Mse a^elf)(	**Mse b^elf)(**	**Mse a llf)(**	**Mse b llf)(**	**a^**	**b^**
10	**4**	**8**	**0.000838**	**0.001551**	**0.000895**	**0.0016**	**elf**	**elf**
15	**4**	**8**	**0.00056**	**0.001034**	**0.000597**	**0.001067**	**elf**	**elf**

		10	**0.000555**	**0.001034**	**0.000598**	**0.001067**	**elf**	**elf**
		13	**0.000551**	**0.001034**	**0.000598**	**0.001067**	**elf**	**elf**
20	**4**	**8**	**0.000421**	**0.000776**	**0.000448**	**0.0008**	**elf**	**elf**
		10	**0.000417**	**0.000775**	**0.000448**	**0.0008**	**elf**	**elf**
		13	**0.000413**	**0.000775**	**0.000449**	**0.0008**	**elf**	**elf**
		17	**0.000412**	**0.000775**	**0.000449**	**0.0008**	**elf**	**elf**

Table (5) shows that the Bayes method using the linidley approximation under the entropy loss function (elf) is the best for sample sizes (10, 15, 20) because it has the lowest Mean Squared Error (MSE).

Table 6: shows the Mean Squared Error (MSE) of the parameters of the power distribution for all methods when (a = 3) and (b = 4).

N	**r**	**s**	**Bias**				**The best**	
			Lindley					
			Mse $\hat{a}$(elf)	**Mse $\hat{b}$(elf)**	**Mse $\hat{a}$(llf)**	**Mse $\hat{b}$(llf)**	**$\hat{a}$**	**$\hat{b}$**
50	**10**	**40**	**0.000164**	**0.00031**	**0.00018**	**0.00032**	**elf**	**elf**
100	**10**	**40**	**8.18E-05**	**0.000155**	**8.98E-05**	**0.00016**	**elf**	**elf**
		60	**8.16E-05**	**0.000155**	**8.98E-05**	**0.00016**	**elf**	**elf**
		80	**8.15E-05**	**0.000155**	**8.98E-05**	**0.00016**	**elf**	**elf**
150	**10**	**40**	**5.46E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**
		60	**5.44E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**
		80	**5.44E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**
		140	**5.43E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**

Table (6) shows that the Bayesian method using the lindley approximation, in the case of the entropy loss function (elf), is the best with sample sizes (50, 100, 150) because it has the lowest Mean Squared Error (MSE).

Table 7: shows the Mean Squared Error (MSE) for the parameters of the power distribution function for all methods when (a = 3) and (b = 4).

N	r	S	Bias Lindley				The best	
			Mse $\hat{a}$elf)(	Mse $\hat{b}$elf)(	Mse $\hat{a}$llf)(	Mse $\hat{b}$llf)(	$\hat{a}$	$\hat{b}$
50	**20**	**40**	**0.000164**	**0.00031**	**0.00018**	**0.00032**	**elf**	**elf**
100	**20**	**40**	**8.20E-05**	**0.000155**	**8.98E-05**	**0.00016**	**elf**	**elf**
		60	**8.17E-05**	**0.000155**	**8.98E-05**	**0.00016**	**elf**	**elf**
		80	**8.15E-05**	**0.000155**	**8.98E-05**	**0.00016**	**elf**	**elf**
150	**20**	**40**	**5.47E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**
		60	**5.45E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**
		80	**5.44E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**
		140	**5.43E-05**	**0.000103**	**5.99E-05**	**0.000107**	**elf**	**elf**

Table (7) shows that the Bayesian method using the lindley approximation in the case of the entropy loss function (elf) is the best for sample sizes (50, 100, 150) because it has the lowest Mean Squared Error (MSE

Table 8 : presents the Mean Squared Error (MSE) for the model and for each method when (a = 3) and (b = 4).

N	r	S	mse(Llf)	mse(elf)	The Best
10	**4**	**8**	**0.666348**	**0.853248**	**Llf**
15	**4**	**8**	**0.638702**	**0.812896**	**Llf**
		10	**0.6083**	**0.856578**	**Llf**
		13	**0.590535**	**0.890274**	**Llf**
20	**4**	**8**	**0.623919**	**0.789623**	**Llf**
		10	**0.594909**	**0.835978**	**Llf**
		13	**0.57883**	**0.871717**	**Llf**
		17	**0.56901**	**0.895847**	**Llf**
50	**10**	**40**	**0.53918**	**0.902442**	**llf**
100	**10**	**40**	**0.532929**	**0.890812**	**llf**
		60	**0.530056**	**0.902833**	**llf**
		80	**0.528699**	**0.907992**	**llf**
150	**10**	**40**	**0.530046**	**0.885436**	**llf**
		60	**0.527845**	**0.898767**	**llf**
		80	**0.526703**	**0.904374**	**llf**
		140	**0.525212**	**0.910698**	**llf**
50	**20**	**40**	**0.544883**	**0.895627**	**llf**

100	**20**	**40**	**0.537385**	**0.881613**	**llf**
		60	**0.531678**	**0.900479**	**llf**
		80	**0.529533**	**0.906978**	**llf**
150	**20**	**40**	**0.533524**	**0.874566**	**llf**
		60	**0.529255**	**0.89598**	**llf**
		80	**0.527457**	**0.903191**	**llf**
		140	**0.525443**	**0.910436**	**llf**

Table (8) shows that the Bayesian method using the lindley approximation in the case of the exponential loss function (LLF) is the best among the sample sizes (10, 15, 20, 50, 100, 150) because it has the lowest mean squared error (MSE).

4. Conclusions

Based on the findings of the research, the following conclusions have been reached:
1. The Bayesian method with Lindley's approximation under the Entropy Loss Function (ELF) demonstrated its efficiency in estimating the parameters of the power distribution compared to the Bayesian method with Lindley's approximation under the linex Loss Function (LLF) when calculating the Mean Squared Error (MSE).
2. The Bayesian method with Lindley's approximation under the linex Loss Function (LLF) proved its efficiency in estimating the model of the power distribution compared to the Bayesian method with Lindley's approximation under the Entropy Loss Function (ELF) when calculating the Mean Squared Error (MSE).
3. The values of the Mean Squared Error (MSE) for the parameters of the power distribution and the model decrease as the sample size increases. is the translation of the provided ref

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