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Comparison of the LEO Estimator Method and the Two-Parameter LEO Method in Estimating the Parameters of the Conway–Maxwell–Poisson Regression Model in the Presence of the Multicollinearity Problem

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ABSTRACT

The aim of the research is to compare the Leo method and the two-parameter Leo method in estimating the parameters of the Conway-Maxwell-Poisson regression model in the presence of the Multicollinearity problem, While Poisson regression serves as a standard tool for modeling the association between a count response variable and explanatory variables, it is well documented that this approach is limited by Poisson's assumption of equal dispersion of the data, The Conway–Maxwell–Poisson (COMP) regression model has established itself as a viable alternative for real count data that Accounting for over- or under-dispersion, COM-Poisson regression can flexibly model associations that include the discrete count response variable and covariates, Using the simulation method (Mont-Carlo) to generate data tracking the Conway-Maxwell-Poisson regression model, and these data suffer from the problem of linear multiplicity according to the influencing and variable factors, including (sample size, degree of correlation, different values of the dispersion parameter, number of explanatory variables) and the average squares of error were relied upon as a criterion for comparing the methods of estimating the parameters of the model, Through the results of the simulation, the superiority of the new modified Leo estimator with two parameters over the estimator Leo**,** In the future study, the Generalized Mutual Verification Standard (GCV) can be used to select the bias parameters of the new modified two-parameter Leo estimator (CPNMTPL) for greater efficiency, The results indicate that the number of publications is growing, and the management and business area is the one that contributes the most, with the countries that produce in co-authorship also providing the most publications.

1. Introduction

Regression models are one of the most important models used in modern studies, especially research and health studies because of their important results, Poisson regression is a common tool for modeling counting data and is applied in medical sciences, engineering, etc. However, real data is often too much or little dispersed, and we cannot apply Poisson regression, to overcome this problem, we consider the regression model based on the Conway Maxwell Poisson distribution (COMP) to be the best distribution for this problem, in general Estimator of the maximum probability of estimating unknown parameters of the regression model (Conway-Maxwell-Poisson (COMP) However, in the presence of the problem of Multicollinearity, the estimates become unstable due to their large variance and standard error, to solve this problem, a

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new Liu estimator was proposed for the Conway-Maxwell-Poisson regression model with excessive and even dispersion and lack of dispersion, the number of data modeling is improving in many areas of research, Regression models of counting data are used with data that suffer from excessive or under-dispersion, counting data regression models include the Poisson model, the negative binomial (NB) model, the bell model, and the Conway-Maxwell-Poisson model, in many areas of research, the commonly used model is the Poisson model. However, the Poisson model assumes that the mean and variance of the response variable are equal, in most cases, the response variable data can be excessive and dispersed, in these cases, The binomial regression model is used. negative (NB) because it is more flexible than the Poisson regression model in absorbing excessive dispersion However, the Conway-Maxwell-Poisson model is more flexible than the negative binomial (NB) model because it can be used in both cases of excess or minus dispersion, the Conway-Maxwell-Poisson (Com-Poisson) distribution was proposed by Conway and Maxwell, this distribution applies to real counting data that express dispersion data, plus or minus, So the COM-Poisson regression is a flexible model for linking a discrete enumeration response variable with common (explanatory) variables, a COM-Poisson distribution is flexible enough to deal with dispersion in counting data (whether excess or minus) with an additional dispersion parameter denoted by kama (γ), which is a two-parameter generalization of the Poisson distribution.

2.CONWAY–MAXWELL–POISSON REGRESSION

As a generalization of the Poisson distribution and a common alternative to other discrete distributions, the Conway–Maxwell–Poisson (CMP) distribution has the flexibility to characterize redundant or dispersed data clearly (Zhan & Young, 2024).
The distribution of Conway-Maxwell-Poisson according to (Daly & Gaunt, 2016) is defined as

$$P(Y_i = y_i | x_i , z_i) = \frac{1}{Z(\lambda_i , \gamma_i)} \frac{\lambda_i^{y_i}}{(y_i!)^{\gamma_i}} \quad , \; y_i = 0,1,2,\ldots \qquad (1)$$

Where $Z(\lambda_i, \gamma_i)$ It is known as the normalization factor and as in the following formula

$$Z(\lambda_i , \gamma_i) = \sum_{n=0}^{\infty} \frac{\lambda_i^n}{(n!)^{\gamma_i}} \qquad (2)$$

n : Indicates the sample size.
γ_i : Indicates the dispersion parameter of the Conway–Maxwell–Poisson distribution.
And that

$$\lambda_i = exp(x_i' \beta) \qquad (3)$$

$$\gamma_i = exp(-g_i' \delta) \qquad (4)$$

β: indicates a vector whose dimensions (k+1)×1 vector parameters ($\beta_0, \beta_1, \beta_2, \ldots, \beta_k$).
δ : indicates a vector of dimensions (m+1)×1 vector parameters ($\delta_o, \delta_1, \delta_2, \ldots, \delta_k$).
Common variables are represented by vectors x_i and g_i.

3.The problem of Multicollinearity

The term multiple linear relationship refers to the situation in which two or more illustrative variables are closely related to each other.

4.Methods for estimating Conway-Maxwell-Poisson regression parameters

In this paragraph, the methods for estimating the regression parameters of Conway Maxwell Poisson will be explained (COMP).

4.1 COM–Poisson–Liu estimator

Several studies on regression models have shown that the Liu type estimator is a good alternative to the character regression estimator (Tanış & Asar, 2024).
(Akram et al., 2022) and (Rasheed et al., 2022), presented a Liu estimator for the COMP model and named the CPL estimator, as follows:

$$\hat{\beta}_d = (Z + I)^{-1} (Z + dI) \hat{\beta}_{ML}; 0 < d < 1 \qquad (5)$$

$$MSE(\hat{\beta}_d) = tr\left(MMSE(\hat{\beta}_d)\right) = \hat{\gamma} \sum_{j=1}^{p} \frac{(\lambda_j + d)^2}{\lambda_j (\lambda_j + 1)^2} + \sum_{j=1}^{p} \frac{\alpha_j^2 (d-1)^2}{(\lambda_j + 1)^2} \qquad (6)$$

4.2 Proposed COM–Poisson–new modified two-parameter Liu estimator

After (Sami, Amin, Akram, et al., 2022) (Sami, Amin, & Butt, 2022) and (Abonazel, 2023), the new modified Liu estimator for the Conway–Maxwell–Poisson

(COMP) regression model was proposed Based on the two parameters, our proposed estimator is obtained by increasing

$$-(k+d_o)\hat{\beta}_k = \beta + \epsilon$$

$$\hat{\beta}_{k,do} = (Z+I)^{-1}(Z-(k+d_o)I)\hat{\beta}_k \; ; k > 0 \, , 0 < d_o < 1 \quad (7)$$

$$\text{MSE}(\hat{\beta}_{\text{k,do}}) = \text{tr}\left(\text{MMSE}(\hat{\beta}_{\text{k,do}})\right) = \hat{\gamma}\sum_{j=1}^{p}\frac{\lambda_j(\lambda_j-k-do)^2}{(\lambda_j+1)^2(\lambda_j+k)^2} + \sum_{j=1}^{p}\frac{\alpha_j^2(k(2\lambda_j+1)+\lambda_j(1+d_o))^2}{(\lambda_j+1)^2(\lambda_j+k)^2} \quad (8)$$

5. Comparison between the Liu estimator (CPL) and the two - parameter Liuestimator (CPNMTPL)

A two-parameter Leo estimator (CPNMTPL) is better than a CPL estimator if condition is met

$$\hat{\gamma}(e_j^2c_j^2-\lambda_j^2b_j^2) > \alpha_j^2\lambda_j[\left(k(2\lambda_j+1)+\lambda_j(1+d_o)\right)^2 - c_j^2(d-1)^2]; \forall j = 1,\dots,p \;\; ; k>0 \;\; ; 0 < d, d_o < 1$$

$$b_j = (\lambda_j - k - d_o) \;\; \& \;\; c_j = (\lambda_j + k) \;\; , \;\; e_j = (\lambda_j + d)$$

The difference between the Mean Mean Least Squares of Error (MMSE) matrix for Leo estimators (CPL) and the two-parameter Leo estimators (CPNMTPL) is as follows:

$$MMSE(\hat{\beta}_d) - MMSE(\hat{\beta}_{k,do}) = \hat{\gamma}\,\psi\,\Lambda_I^{-1}\Lambda_d\,\Lambda^{-1}\,\Lambda_d\Lambda_I^{-1}\psi' + (d-1)^2\psi\Lambda_I^{-1}\alpha\,\alpha'\,\Lambda_I^{-1}\psi' - \hat{\gamma}\,\psi\,\Lambda_I^{-1}\Lambda_{k,do}\Lambda_k^{-1}\Lambda\Lambda^{-1}\Lambda\Lambda_k^{-1}\Lambda_{k,do}\Lambda_I^{-1}\psi' - \psi(\Lambda_I^{-1}\Lambda_{k,do}\Lambda_k^{-1}\Lambda - I)\alpha\alpha'(\Lambda_I^{-1}\Lambda_{k,do}\Lambda_k^{-1}\Lambda - I)\psi'$$

So we can rewrite the previous equation as follows:

$$MSE(\hat{\beta}_d) - MSE(\hat{\beta}_{k,do}) = \sum_{j=1}^{p}\left(\frac{\hat{\gamma}\,e_j^2 + (d-1)^2\lambda_j\,\alpha_j^2}{\lambda_j a_j^2} - \frac{\hat{\gamma}\,\lambda_j b_j^2 + \alpha_j^2(k(2\lambda_j+1)+\lambda_j(1+d_o))^2}{a_j^2c_j^2}\right)$$

$$e_j = (\lambda_j + d)$$

$$MSE(\hat{\beta}_d) - MSE(\hat{\beta}_{k,do}) > 0$$

$$\hat{\gamma}e_j^2c_j^2 + \lambda_j\alpha_j^2c_j^2(d-1)^2 - \hat{\gamma}\lambda_j^2b_j^2 - \alpha_j^2\lambda_j[\left(k(2\lambda_j+1)+\lambda_j(1+d_o)\right)^2 > 0$$

$$\rightarrow \hat{\gamma}(e_j^2c_j^2-\lambda_j^2b_j^2) - \alpha_j^2\lambda_j\left[\left(k(2\lambda_j+1)+\lambda_j(1+d_o)\right)^2 - c_j^2(d-1)^2\right] > 0$$

$$\rightarrow \hat{\gamma}(e_j^2c_j^2-\lambda_j^2b_j^2) > \alpha_j^2\lambda_j[\left(k(2\lambda_j+1)+\lambda_j(1+d_o)\right)^2 - c_j^2(d-1)^2]$$

$$MSE(\hat{\beta}_d) > MSE(\hat{\beta}_{k,do})$$

$$\hat{\gamma}(e_j^2c_j^2-\lambda_j^2b_j^2) > \alpha_j^2\lambda_j\left[\left(k(2\lambda_j+1)+\lambda_j(1+d_o)\right)^2 - c_j^2(d-1)^2\right] \; ; \; \forall j = 1,\dots,p$$

A two-parameter Leo estimator (CPNMTPL) is better than a CPL estimator.

6.Estimation of bias parameters

We can use the following estimators for parameter d in the Leo estimator (CPL):

$$\hat{d}_1 = \frac{1}{p}\sum_{j=1}^{p}\left(\frac{\hat{\alpha}_{ML(j)}^2}{\left(\frac{\hat{\gamma}}{\lambda_j}+\hat{\alpha}_{ML(j)}^2\right)}\right) \quad (9)$$

$$\hat{d}_2 = max_j\left(0, min_j\left(\frac{\hat{\alpha}_{ML(j)}^2-\hat{\gamma}}{max_j\left(\frac{\hat{\gamma}}{\lambda_j}\right)+max_j\left(\hat{\alpha}_{ML(j)}^2\right)}\right)\right) \quad (10)$$

We can use the following estimator for parameter k in the character regression magnitude (CPR):

$$\hat{k} = min_j\left(\frac{\hat{\gamma}}{\hat{\alpha}_{ML(j)}^2}\right) \quad (11)$$

$$\hat{k}_1 = min_j\left(\frac{\hat{\gamma}}{2\hat{\alpha}_{ML(j)}^2+\left(\frac{\hat{\gamma}}{\lambda_j}\right)}\right) \quad (12)$$

$$\hat{k}_2 = \min_j\frac{p\hat{\gamma}}{\sum_{j=1}^{p}(2\hat{\alpha}_{ML(j)}^2+\left(\frac{\hat{\gamma}}{\lambda_j}\right))} \quad (13)$$

$$\hat{k}_3 = \frac{\min_j(\lambda_j)(\hat{\gamma}-\min_j\hat{\alpha}_{ML(j)}^2)}{\hat{\gamma}+\min_j(\lambda_j)\min_j(\hat{\alpha}_{ML(j)}^2)} - \hat{d}_1 \quad (14)$$

(SABRI, 2013) presented the following formula:

$$\hat{d}_3 = \max_j\left(0, \frac{1}{p}\sum_{j=1}^{p}\left(\frac{\hat{\alpha}_{ML(j)}^2-1}{\frac{1}{\lambda_j}+\hat{\alpha}_{ML(j)}^2}\right)\right) \quad (15)$$

7. Simulation

Simulations are the imitation of a process or system in the real world over time, simulations require the use of models; the model represents the main characteristics or behaviors of the chosen system or process, while simulation represents the evolution of the model over time, although simulation can still be performed "manually", nowadays it always implies the use of a computer to create an artificial history of a system to draw conclusions about its characteristics and actions.

Simulation experiments were conducted using some different levels of (n), (γ), (ρ), (p), (σ^2) and as follows:

Different sample sizes were used, n = 50, 100, 150, 250.

Different values were used for the dispersion parameter, $\gamma = 0.8, 1, 1.5$.

Different degrees of correlation were used between the illustrative variables, $\rho =$ 0.90, 0.95, 0.99.

The number of explanatory variables adjusted p=3, 5.

Cases have been set for different discrepancy values (Hamood, 2019), σ^2 =0.3, 1.5, 4.

$$x_{ij} = m_{ij}\sqrt{1-\rho^2} + \rho m_{ip} \quad , i = 1, \dots \dots, n;\ j = 1, \dots \dots, p \qquad (16)$$

$$y_i = \mu_i = \exp(\beta_1 x_{i1} + \cdots + \beta_p x_{ip}) \quad , \ i = 1, \dots , n \qquad (17)$$

Where as $\sum_{j=1}^{p} \beta_j^2 = 1$; $\beta_1 = \cdots = \beta_p$ As in (Imoto, 2014) and (Abonazel, Awwad, et al., 2023) (Abonazel, Saber, et al., 2023).

Output data is duplicated (x_{ij}, y_i) ، L = 1000 Time to calculate the MSE standard simulation as follows:

$$MSE(\hat{\beta}) = \frac{1}{L}\sum_{l=1}^{L}(\hat{\beta}_l - \beta)'(\hat{\beta}_l - \beta) \qquad (18)$$

8. Simulation results

Tables (1), (2) and (3) show the mean squares of error (MSE) values for the two estimation methods when the number of illustrative variables (P=3) and sample size (n=50, 100, 150, 250) respectively show that the two-parameter Leo estimator is better than the Leo estimator in the Conway–Maxwell–Poisson regression model in all cases.

At the sample size (50) and the number of illustrative variables and the values of the correlation coefficients (0.90, 0.95, 0.99) it was found that the method of the two-parameter Leo estimator (CPNMTPL) in its estimated form, in which it depends on two shrinkage parameters (K1, d3) shown in Tables (3-1), (3-2) and (3-3), has the lowest mean squares of error (MSE) compared to the other method.

At the sample size (250) and for all ρ values, we note that the method of the two-parameter Leo estimator (CPNMTPL) in its estimated formula, in which it depends on two shrinkage parameters (K1, d3), has the lowest mean squares of error (MSE).

Tables (4), (5) and (6) show the mean squares of error (MSE) values for the two estimation methods when the number of independent variables (p=5) and sample size (n=50, 100, 150, 250) respectively show that the two-parameter Leo estimator is better than the Leo estimator in the Conway–Maxwell–Poisson regression model in all cases.

At the sample size (50) and the values of the correlation coefficients (0.90, 0.95, 0.99), we note that the method of the two-parameter Leo estimator (CPNMTPL) in its estimated formula, in which it depends on two contraction parameters (K1, d3), has the lowest mean squares of error (MSE) of its value respectively according to the values of the correlation coefficients (0.0059621, 0.0047924, 0.0042797) in Table (3-4) and its value (0.0088887, 0.0071987, 0.0064513) in Table (3-5) and its value (0.0181250, 0.0149066, 0.0134559) in Table (3-6).

At the sample size (250), we notice that the method of the two-parameter Leo estimator (CPNMTPL) in its estimated formula, in which it depends on two contraction parameters (K1, d3) has the lowest mean squares of error (MSE) and for all ρ values.

Where it can be easily observed the decline in the values of the mean squares of error (MSE) as the sample size increases, as it shows one of the new characteristics when the estimator approaches the real value of the parameter by increasing the sample size and the constant factor (number of explanatory variables, correlation coefficient) This indicates that the greater the sample size, the lower the value of the mean squares of error (MSE).

Table (1) Values of the mean of error squares for all estimation methods when the number of explanatory variables p = 3 & $\gamma = 0.80$ and $\sigma^2 = 0.3$.

N		**50**			**100**			**150**			**250**		
P		**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**
$\hat{\beta}_d$	D1	0.0044858	0.0050338	0.0062898	0.0026260	0.0029464	0.0036925	0.0018883	0.0021201	0.0026521	0.0014536	0.0016294	0.0020316
	D2	0.0044727	0.0050174	0.0062643	0.0026254	0.0029457	0.0036828	0.0018860	0.0021172	0.0026476	0.0014522	0.0016275	0.0020288
$\hat{\beta}_{k,d}$	**K1, d3**	0.0044214	0.0049530	0.0061642	0.0002779	0.0002812	0.0036063	0.0018783	0.0021074	0.0026323	0.0014450	0.0016186	0.0020149
	K2, d3	0.0048930	0.0055411	0.0070626	0.0010782	0.0011628	0.0040713	0.0019604	0.0022106	0.0027924	0.0015037	0.0016920	0.0021281
	K3, d3	0.0044482	0.0049863	0.0062148	0.0002852	0.0002878	0.0036592	0.0018821	0.0021122	0.0026398	0.0014495	0.0016242	0.0020236

Table (2) Values of the mean of error squares for all estimation methods when the number of explanatory variables p = 3 & $\gamma = 1$ and $\sigma^2 = 1.5$.

N		**50**			**100**			**150**			**250**		
P		**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**
$\hat{\beta}_d$	D1	0.0066279	0.0074159	0.0092134	0.0039641	0.0044244	0.0055183	0.0028136	0.0031498	0.0039189	0.0020121	0.0022530	0.0028041
	D2	0.0066030	0.0073850	0.0091662	0.0039549	0.0044235	0.0055008	0.0028093	0.0031442	0.0039102	0.0020097	0.0022500	0.0027997
$\hat{\beta}_{k,d}$	**K1, d3**	0.0063808	0.0071081	0.0087437	0.0038593	0.0002904	0.0053176	0.0027882	0.0031016	0.0038446	0.0019808	0.0022139	0.0027694
	K2, d3	0.0076056	0.0086118	0.0109651	0.0043763	0.0015449	0.0062754	0.0029601	0.0033734	0.0042581	0.0021357	0.0024065	0.0029741
	K3, d3	0.0065301	0.0072906	0.0090104	0.0039293	0.0002951	0.0054460	0.0028012	0.0031317	0.0038902	0.0020036	0.0022421	0.0027904

Table (3) Values of the mean of error squares for all estimation methods when the number of explanatory variables p = 3 & $\gamma = 1.5$ and $\sigma^2 = 4$.

N		**50**			**100**			**150**			**250**		
P		**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**	**0.9**	**0.95**	**0.99**
$\hat{\beta}_d$	D1	0.0135405	0.0150794	0.0185648	0.0083564	0.0092752	0.0114221	0.0058267	0.0064940	0.0080078	0.0037369	0.0041783	0.0051863
	D2	0.0134723	0.0149954	0.0184393	0.0083292	0.0092739	0.0114206	0.0058137	0.0064779	0.0079830	0.0037314	0.0041714	0.0051758
$\hat{\beta}_{k,d}$	**K1, d3**	0.0128229	0.0141954	0.0172451	0.0078746	0.0003007	0.0003025	0.0057024	0.0063401	0.0076748	0.0036687	0.0040933	0.0050561
	K2, d3	0.0161396	0.0182042	0.0229836	0.0098220	0.0027713	0.0033104	0.0063614	0.0071497	0.0091755	0.0040073	0.0045126	0.0056885
	K3, d3	0.0132358	0.0146912	0.0179443	0.0081904	0.0003030	0.0003045	0.0057818	0.0064374	0.0078934	0.0037171	0.0041530	0.0051458

Table (4) Values of the mean of error squares for all estimation methods when the number of explanatory variables p = 5 & γ = 0.80 and σ^2=0.3.

N		50			100			150			250		
P		0.9	0.95	0.99	0.9	0.95	0.99	0.9	0.95	0.99	0.9	0.95	0.99
$\hat{\beta}_d$	D1	0.0043300	0.0048555	0.0060600	0.0025480	0.0028580	0.0035691	0.0018336	0.0020575	0.0025717	0.0012492	0.0014004	0.0017475
	D2	0.0043229	0.0048466	0.0060462	0.0025453	0.0028546	0.0035641	0.0018323	0.0020559	0.0025692	0.0012485	0.0013995	0.0017461
$\hat{\beta}_{k,d}$	**K1, d3**	0.0042797	0.0047924	0.0059621	0.0021613	0.0023803	0.0028540	0.0018257	0.0020476	0.0025562	0.0011788	0.0013126	0.0016128
	K2, d3	0.0057461	0.0066121	0.0087128	0.0053392	0.0060603	0.0076606	0.0020888	0.0023771	0.0030647	0.0020770	0.0024001	0.0031714
	K3, d3	0.0043080	0.0048276	0.0060156	0.0023650	0.0026163	0.0031612	0.0018300	0.0020529	0.0025645	0.0012330	0.0013785	0.0017081

Table (5) Values of the mean of error squares for all estimation methods when the number of explanatory variables p = 5 & γ = 1 and σ^2=1.5.

N		50			100			150			250		
P		0.9	0.95	0.99	0.9	0.95	0.99	0.9	0.95	0.99	0.9	0.95	0.99
$\hat{\beta}_d$	D1	0.0065962	0.0073795	0.0091655	0.0038908	0.0043551	0.0054156	0.0027897	0.0031232	0.0038853	0.0019319	0.0021631	0.0026914
	D2	0.0065822	0.0073621	0.0091389	0.0038863	0.0043495	0.0054070	0.0027873	0.0031201	0.0038806	0.0019306	0.0021614	0.0026890
$\hat{\beta}_{k,d}$	**K1, d3**	0.0064513	0.0071987	0.0088887	0.0038632	0.0043206	0.0053624	0.0027694	0.0030977	0.0038460	0.0016881	0.0019589	0.0022408
	K2, d3	0.0097036	0.0111874	0.0147674	0.0047723	0.0054499	0.0070747	0.0033313	0.0037966	0.0049088	0.0039107	0.0041241	0.0056910
	K3, d3	0.0065449	0.0073140	0.0090600	0.0038776	0.0043385	0.0053897	0.0027823	0.0031138	0.0038705	0.0018319	0.0020946	0.0024649

Table (6) Values of the mean of error squares for all estimation methods when the number of explanatory variables p = 5 & γ = 1.5 and σ^2=4.

N		50			100			150			250		
P		0.9	0.95	0.99	0.9	0.95	0.99	0.9	0.95	0.99	0.9	0.95	0.99
$\hat{\beta}_d$	D1	0.0141653	0.0157816	0.0194332	0.0083693	0.0093295	0.0115023	0.0059819	0.0066701	0.0082302	0.0042643	0.0047633	0.0058967
	D2	0.0141210	0.0157270	0.0193516	0.0083577	0.0093157	0.0114831	0.0059738	0.0066601	0.0082150	0.0042603	0.0047585	0.0058896
$\hat{\beta}_{k,d}$	**K1, d3**	0.0134559	0.0149066	0.0181250	0.0052459	0.0056079	0.0063268	0.0058634	0.0065232	0.0080078	0.0035307	0.0038661	0.0045809
	K2, d3	0.0250820	0.0288132	0.0375679	0.0161222	0.0174429	0.0201160	0.0082468	0.0094388	0.0122883	0.0090435	0.0101718	0.0126251
	K3, d3	0.0139159	0.0154599	0.0189069	0.0058805	0.0062769	0.0070494	0.0059500	0.0066295	0.0081649	0.0038883	0.0042746	0.0050980

9. Application

Conway-Maxwell-Poisson regression was applied in the presence of the Multicollinearity problem, and Table (7) shows the estimates of the regression coefficients of the Leo estimator with the best parameters in the experimental aspect of the Conway-Maxwell-Poisson regression model in the presence of the problem of Multicollinearity multiplication and its Mean Squares of Error (MSE**).**

The results of estimating the parameters of the Conway-Maxwell-Poisson regression model in the presence of the problem of Multicollinearity and mean squares of error (MSE) for the two-parameter Leo estimator can be expressed by writing a number of programs in MATLAB R2013a.

The data that have been adopted in this aspect are related to the analysis of the impact of environmental factors on the level of air pollution in Iraq, which was obtained from the Statistics and Information Systems Authority at the Ministry of Planning if the values of the variables were recorded during the years 2021 and 2022, and the Conway-Maxwell-Poisson regression model includes: The dependent variable (The dependent variable): Y: the level of air pollution. Explanatory variables include: X1: traffic level. X2: The quantity of industries in the region. X3: the amount of precipitation.

Sample size (n=50).

Table (7) Estimation of the parameters of the Conway-Maxwell-Poisson regression model for the two-parameter Leo estimator.

Estimator		β_1	β_2	β_3	MSE
$\hat{\beta}_{k,d}$	K1, d3	1.0874939	0.1018377	0.0839914	$1.39e^{-14}$

Through Table (7) applying the Conway-Maxwell-Poisson regression in the presence of the problem of linear multiplicity of the two-parameter Leo estimator on real data, we note that the value of the mean squares of error of the two-parameter Leo estimator was ($1.39e^{-14}$).

It was found through the table that the value of the parameter β_1 has reached its value (β_1=1.0874939) and this means that the variable X1 has a direct relationship with the dependent variable (air pollution level) The higher the level of traffic, the higher the level of air pollution, while the value of the parameter β_2 has reached its value (β_2=0.1018377) and this means that the variable X2 has a direct relationship with the dependent variable (air pollution level) The more the amount of industries in the region, the greater the level of High air pollution, as for the value of the parameter β_3, its value was (β_3 = 0.0839914), and this means that the X3 variable has a direct relationship with the dependent variable (air pollution level), the greater the amount of rain, the greater the level of decrease in air pollution.

10.Conclusion

1. Study the possibility of developing new, more efficient algorithms to solve parameter estimation problems in the Conwy-Maxwell-Poisson regression model.
2. Conway Maxwell Poisson's regression the CMPRE model has proven effective in modeling counting data with a wide range of dispersion.
3. In the future study, the Generalized Mutual Verification Standard (GCV) can be used to select the bias parameters of the two-parameter Leo estimator (CPNMTPL) for greater efficiency.
4. We conclude that the D2 estimator is better than the D1 estimator in the Leo method with one parameter when P=3 and P=5.
5. The new modified two-parameter Leo estimator method (CPNMTPL) can be relied upon because it performs better than the Leo estimator method

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