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Application of the linear programming model to solve comprehensive planning plans in case of fuzzy data
تطبيق نموذج البرمجة الخطية لحل خطط التخطيط الشاملة في حالة البيانات الضبابية

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Abstract:
In the era of modern technology, the decision-making process has become highly significant within companies and industrial institutions. With the advancement in technology, there has been an increased focus on applying quantitative methods in decision-making to seek the most appropriate and optimal decision, especially in production companies . This study examines the method of employing the linear programming model using Iskander's method as an aid in solving aggregate production planning (APP) problems in a manner that aligns with the existence of information and data that may be inherently vague and uncertain. The study illustrates the efficiency of this method through its practical application. The study determined the production quantity that the company should produce in a way that enables it to achieve the maximum possible profit given the lowest available possibilities .

Keywords: Production planning, linear programming, fuzzy logic, belonging functions

المستخلص : المستخلص في عصر التكنولوجيا الحديثة اصبحت عملية اتخاذ القرار(making decision) لها أهمية كبيرة داخل الشركات والمؤسسات الصناعية ومع زيادة التطور التكنلوجي أزداد الاهتمام بتطبيق الأساليب الكمية في عملية اتخاذ القرار بحثا عن القرار الانسب والأمثل خصوصاً في الشركات الإنتاجية ، اذا تناولت هذه الدراسة طريقة توظيف نموذج البرمجة الخطية (Liner programming) من خلال الاعتماد على طريقة (Iskander's) كوسيلة مساعدة في حل مشاكل التخطيط الاجمالي للانتاج (Aggregate production planning) (APP) بما يتلائم مع وجود المعلومات والبيانات التي قد تكون في طبيعتها غامضة وغير مؤكدة (Uncertainty) مع توضيح مدى كفاءة تلك الطريقة من خلال الجانب التطبيقي للدراسة ،حيث تم التوصل الى تحديد كمية الانتاج الواجب على الشركة انتاجها بالشكل الذي يؤهل الشركة الى تحقيق اقصى ربح ممكن في حالة توفر ادنى الامكانيات (The lowest possibilities)المتاحة لديها .

الكلمات المفتاحية: تخطيط الانتاج ، البرمجة الخطية ، المنطق الضبابي ، دوال الانتماء

Introduction:

Companies and institutions strive to find the best methods and means to achieve their goals, whether it is maximizing profits or minimizing costs to the lowest possible level. This pursuit has led them to explore various programs and models to use in their different activities, whether related to planning or implementing plans and making decisions that align with their objectives. Production planning is a crucial aspect for institutions in general because it provides a forward-looking perspective on the entire production process. Through it, the required production facilities are determined and arranged, involving the process of manufacturing products using all available resources such as human resources, raw materials, and working hours, to provide services to consumers and clients. This aims to achieve pre-set goals such as maximizing profits, improving performance, and reducing costs while maintaining the necessary level of quality. This approach offers suggestions and solutions that help increase production and thereby achieve the highest possible profits. One of the most important methods used in this field is linear programming. This method is based on a mathematical principle where various problems faced by companies and institutions are represented in the form of a mathematical model. It is formulated as an objective function, which may either be to maximize profits or minimize costs, subject to a set of constraints that limit this function. [Al-Mushahdani, Azraa Kamel 2015], [Bakheet, Abdul Jabbar Khudair, Rasha Radhawi Kamel 2016]. However, the application of linear programming has seen significant changes in recent times. It used to rely on certainty, meaning the presence of real and confirmed data. However, there are many situations that are inherently uncertain or unclear, especially with the rapid technological advancements in the current era. Therefore, fuzzy linear programming is considered the most practical approach to handle such scenarios. This shift led researchers, notably Zimmermann in 1978, to modify the model by incorporating fuzzy logic, an AI tool introduced by Lotfi Zadeh in 1965. Zadeh made adjustments to mathematical laws by creating fuzzy numbers. Consequently, linear programming began to be applied under uncertain or ambiguous conditions, with all parameters and variables in the model being subject to probability and fuzziness. This approach is now referred to as fuzzy linear programming. To address this ambiguous model, numerous contributions have been made by various researchers, including Iskander, which will be detailed in the practical section. [Hamza Ibrahim Hamza, et al., 2021], [Israa H. Hasan, Iden H. Al Kanani, 2024].

Research Methodology

1-1-1 Research Problem

The research problem is identified as the ambiguity or uncertainty that attracts researchers' attention, motivating them to find suitable solutions by examining the reality of the operations and activities of companies or institutions, particularly service companies in Iraq, and experiencing the challenges they face.

1-1-2 Research Objective

The primary goal of this research is to clarify how to use the fuzzy linear programming model by relying on Iskander's method for total production planning in an uncertain environment. The aim is to determine the optimal quantity in a straightforward and unbiased manner, given the available resources within the institution, including human resources and economic capabilities.

1-2 Previous Studies

In 2019, researchers Marwan Abd al-Hamid Ashour and Rana Nidal Ibrahim conducted a study titled "Total Production Planning Using Linear Programming with Practical Application." This study aimed to develop a mathematical model for total production planning at Baghdad Soft Drinks Company by relying on total planning strategies (work hour control strategy, inventory level control strategy). Among the key findings of the study was the importance of utilizing the available production capacity during periods of lower demand to invest in subsequent months when demand exceeds available capacity. Additionally, the study highlighted the need to minimize the use of overtime due to its relatively high costs.

In 2020, researchers Al-Dulaimi, Sami Ahmed Abbas, and Ahmed Ali Salman conducted a study titled "Optimizing the Application of the Material Requirements Planning (MRP) System in Modeling Production Planning Decisions: An Empirical Study at Al-Rafidain Dairy Factory - Abu Ghraib." The results of this research were summarized in controlling production quantities, capacities, and inventory in its various forms by carefully balancing the main production materials and the required quantities of raw materials. This approach significantly reduced overall costs. The study also recommended enhancing the product sample's profile due to its competitive features against imported products.

In the same year (2020), researcher Zouheira A'arab conducted a study titled "Improving the Performance of Economic Institutions Using Numerical Linear Programming - A Case Study of Emb Metal Packaging." This study aimed to apply the traditional linear programming model to improve the performance of economic institutions, specifically at Emb Metal Packaging. The study concluded that it was possible to determine the optimal production mix that maximizes the institution's profit. It was also found that the results provided by the linear programming model were more effective than the actual production achieved, and sensitivity analysis identified the range within which the linear programming values remain optimal**.**
In 2021, researchers Hamdan Zainab and Mallel Rabi'a conducted a study titled "The Role of Linear Programming in Decision-Making: A Case Study of the Abrasive Materials Production Company in Saida Province." This study aimed to highlight the role of applying linear programming in decision-making within the abrasive materials company, using the POM QM V5 software. The study concluded that the optimal quantities to achieve the highest level of profit were identified. The institution should follow a production plan involving the production of X1 (ceramic products) valued at 282.89 and X2 (bakelite products) valued at 23,055.16, while avoiding the production of X3 (pressure discs), thus maximizing its turnover to a value of 16,879,550.

Theoretical Aspect

2-1 Some Basic Concepts in Fuzzy Sets:

Fuzzy Logic : Fuzzy logic is a mathematical approach used to model issues of uncertainty. It provides a general framework for solving problems related to representing approximate or imprecise information and offers the necessary mechanisms for utilizing such information [Deepak, G.,at all , 2021],[Deb, M., and De, P.K. ,2015]. The concept of fuzzy logic was introduced in 1965 by the scientist Lotfi Zadeh, who was the first to propose fuzzy logic, also known as "qualitative logic" or "fuzzy logic" [Zadeh, L. A., 1965]. Fuzzy logic is considered an effective model for simulating human cognitive thinking and is an extension of multi-valued logic based on expert systems that involve uncertain data in the decision-making process [Bezdek, J. C., 1993]. Fuzzy Logic: Fuzzy logic is a mathematical approach used to model issues of uncertainty. It provides a general framework for solving problems related to representing approximate or imprecise information and offers the necessary mechanisms for utilizing such information. The concept of fuzzy logic was introduced in 1965 by the scientist Lotfi Zadeh, who was the first to propose fuzzy logic, also known as "qualitative logic" or "fuzzy logic" [Zadeh, L. A., 1965]. Fuzzy logic is considered an effective model for simulating human cognitive thinking and is an extension of multi-valued logic based on expert systems that involve uncertain data in the decision-making process [Bezdek, J. C., 1993]. Fuzzy logic is based on fuzzy sets, which are used when precise mathematical formulas are not available. The fuzzy set theory was developed to accommodate the properties of vagueness and ambiguity and to reduce the need for precise quantitative inputs in decision-making and analysis. Many data suffer from uncertainty, which means they can be represented by multiple values rather than a single value, thus falling into the realm of fuzzy sets instead of classical sets. The fuzziness is handled using membership functions, which determine the degree of membership of elements in a fuzzy set, ranging between 0 and 1. The general form of a membership function for a fuzzy set is represented as $\mu\_(A)(x): x \rightarrow [0,1]$. When the membership degree of an element is 1, it means the element fully belongs to the fuzzy set. Conversely, when the membership degree is 0, it means the element does not belong to the fuzzy set at all. Degrees between 0 and 1 indicate varying levels of membership. For instance, if the membership degree is close to 0.5, the element is partially (50%) in the fuzzy set and partially (50%) out of it, known as the "Equilibrium Point." Membership degrees approaching 0.9 mean the element is 90% in the fuzzy set and 10% out, indicating a higher degree of membership. Thus, fuzzy sets are an extension of classical set theory, with classical sets being a special case of fuzzy sets [Al-Mashhadani, Athraa Kamel, 2015], [Iraq Tariq Abbas, 2012].

2-2 Membership function

are of significant importance in defining fuzzy set theory. They are used to calculate the degree of membership of an element in a fuzzy set. Membership functions are typically represented by a graph where the vertical axis (y-axis) represents the membership degrees of the fuzzy set, and the horizontal axis (x-axis) represents the ordinary values of the fuzzy variable. The essential condition for a membership function is [Klir, G., & Yuan, B., 1997]:

$$M_{\tilde{V}_{(u)}} \to [0,1] \quad \ldots\ldots\ldots (1)$$

And also

$$\sum_{i=1}^{n} M_{\tilde{V}_{(u)}} = 1 \quad \ldots\ldots\ldots (2)$$

There are several methods to express membership functions. They can be expressed using a numerical approach or a functional approach. Examples include the trapezoidal function, the triangular function, and other fuzzy functions [Osama H. Mohammed, 2012]. In this research, we will focus on a general case of the triangular membership function.

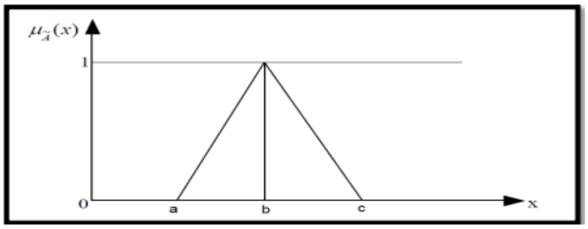
2-3 Triangular Membership function

Assuming that A tilde $\tilde{A} = (a\,,b\,,c)$ represents a triangular fuzzy number, where, the membership function of the element A can be represented as follows [Bo Yuan and George J. Klir, 1995], [M. G. Iskander, 2002]:

$$M_{\tilde{A}(x)} = \begin{cases} 0 & if\ x \le a \\ \frac{x-a}{b-a} & if\ a \le x \le b \\ \frac{c-x}{c-b} & if\ b \le x \le c \\ 1 & if\ x \ge c \end{cases} \quad \ldots\ldots\ldots\ldots(3)$$

Where (a,b,c) can be described as the values along the range of the random variable X in the fuzzy set. Here, a represents the minimum limit of the random variable X , c represents the maximum limit of the random variable X, and b represents the middle or the median value of the random variable X . The fuzzy membership function for the random variable X can be illustrated as follows [Iden Hassan, Nabeel H. Saeed, 2011], [Abdul Razak, Hamadan, 2004].

Figure(2-1) Illustration of a Triangular Membership Function



2-4 General Formulation of the Fuzzy Linear Programming Model According to Iskander's Method

This formulation assumes that all parameters, including coefficients of decision variables in the objective function and constraints, as well as the right-hand side values (available quantities), take on fuzzy values that are not precisely defined, which is common in practical scenarios. For instance, in many cases, selling prices are not precise, leading to a fuzzy or imprecise profit margin. Additionally, raw material prices may be unclear due to fluctuations in currency exchange rates, making costs also ambiguous or uncertain. The general formulation for these cases is as follows:

$maximization\ or\ minimization\ Z = \tilde{C}_1 x_1 + \tilde{C}_2 x_2 + \tilde{C}_3 x_3 \pm \cdots + \tilde{C}_n x_n$

Subject to :

$\tilde{a}_{11} x_1 + \tilde{a}_{12} x_1 + \tilde{a}_{13} x_1 + \cdots + \tilde{a}_{1n} x_n (\leq = \geq) \tilde{b}_1$

$\tilde{a}_{21} x_1 + \tilde{a}_{22} x_1 + \tilde{a}_{23} x_1 + \cdots + \tilde{a}_{2n} x_n (\leq = \geq) \tilde{b}_2$

. . . .

. . . .

. . . .

$\tilde{a}_{m1} x_1 + \tilde{a}_{m2} x_1 + \tilde{a}_{m3} x_1 + \cdots + \tilde{a}_{mn} x_n (\leq = \geq) \tilde{b}_m$

$i = 1, \dots, m \quad , j = 1, \dots, n \quad , \quad X_{ij \geq 0}$

……………..…………… (4)

Where:

X_{ij} : Represents the decision variables.

$\tilde{C}_j$:- Represents the fuzzy coefficients associated with the decision variables.

$\tilde{a}_{ij}$: Represents the fuzzy parameters associated with the decision variables in the constraint functions.

$\tilde{b}_j$: Represents the fuzzy values for the right-hand side of the constraints (available quantities).

The above model shows that all coefficients and parameters take on a fuzzy, ambiguous form, which could be represented by a triangular membership function or a trapezoidal membership function. If the membership function is triangular, its values are defined as follows:

$\tilde{C}_j = (C_{jL} = C_j - \sqrt{1-\alpha}\,, C_j\,, C_{jU} = C_j + \sqrt{1-\alpha}$ …… (5)

$\tilde{a}_{ij} = (a_L = a_{ijL} - \sqrt{1-\alpha}\,, a_{ij}\,, a_{ijU} = a_{ij} + \sqrt{1-\alpha}$ …… (6)

$\tilde{b}_i = (b_{iL} = b_i - \sqrt{1-\alpha}\ ,\ b_i\ ,\ b_{iU} = b_i + \sqrt{1-\alpha}$ …… (7)

Where α is defined as the minimum degree of membership that any element in the fuzzy set can have. The value of α\alphaα lies within the closed interval [0,1] and is predetermined by the decision-maker. It represents the level of confidence or the minimum probability required (minimum possible or required). Values denoted by L represent the lower bounds, while values denoted by U represent the upper bounds.

2-5 formulation a fuzzy linear programming model to its traditional form

The general formulation of the fuzzy linear programming model will be converted into its traditional form based on equations (5), (6), and (7) using the Iskander methodology in the following model:

$\max \text{ or } \min Z = \left[(1-\alpha)\left(C_1 + \sqrt{1-\alpha}\right) + \alpha C_1\right] X_1 + \left[(1-\alpha)\left(C_2 + \sqrt{1-\alpha}\right) + \alpha C_2\right] X_2 + \cdots + \left[(1-\alpha)\left(C_n + \sqrt{1-\alpha}\right) + \alpha C_n\right] X_n$

subject to:

$\left[(1-\alpha)\left(a_{11} - \sqrt{1-\alpha}\right) + \alpha a_{11}\right] X_1 + \left[(1-\alpha)\left(a_{12} - \sqrt{1-\alpha}\right) + \alpha\, a_{12}\right] X_2 + \cdots + \left[(1-\alpha)\left(a_{1n} - \sqrt{1-\alpha}\right) + \alpha a_{1n}\right] X_n\ (\leq = \geq)\left[(1-\alpha)\left(b_1 + \sqrt{1-\alpha}\right) + \alpha b_1\right]$

……..(8)

$\left[(1-\alpha)\left(a_{21} - \sqrt{1-\alpha}\right) + \alpha\, a_{21}\right] X_1 + \left[(1-\alpha)\left(a_{22} - \sqrt{1-\alpha}\right) + \alpha\, a_{22}\right] X_2 + \cdots + \left[(1-\alpha)\left(a_{2n} - \sqrt{1-\alpha}\right) + \alpha\, a_{2n}\right] X_n\ (\leq = \geq)\left[(1-\alpha)\left(b_2 + \sqrt{1-\alpha}\right) + \alpha b_2\right]$

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$$[(1-\alpha)(a_{m1}-\sqrt{1-\alpha})+\alpha\, a_{m1}]\, X_1 + [(1-\alpha)(a_{m2}-\sqrt{1-\alpha})+\alpha\, a_{m2}]\, X_2 + \cdots + [(1-\alpha)(a_{mn}-\sqrt{1-\alpha})+\alpha\, a_{mn}]\, X_n \ (\leq = \geq) [(1-\alpha)(b_m+\sqrt{1-\alpha})+\alpha b_m]$$

$$i = 1,2,\dots,m \qquad j = 1,2,\dots,n$$

$$0 \leq \alpha \leq 1 \quad , \quad X_{ij} \geq 0$$

The Practical Aspect

To illustrate the effectiveness of the proposed model using Iskander's method, we will consider a company that develops a weekly production plan to achieve the maximum profit by producing three products constrained by several limitations. The data is outlined in the following table. Table (3-1) shows the fuzzy data about the variables and coefficients related to the objective function and the resource constraints for the manufactured products.

Table (3-1)

Product	X1	X2	X3	Fuzzy Constraints
Machine Capacity	270	310	290	150,000
Raw Material 1	50	10	20	120,000
Raw Material 2	40	55	65	200,000
Working Hours	1.5	2	3	300
Transportation Cost	0.55	0.78	0.90	50,000
Demand	1	1	1	60,000
Profit	20	26	33	

3-2 Building the model according to the traditional formula

Based on the table provided, the production process data can be formulated into a fuzzy linear programming model. Here’s how you can structure the data and constraints in the model:

Max $Z = 20X_1 + 26X_2 + 33X_3$

s.to

$270X_1 + 310X_2 + 290X_3 \leq 150000$

$50X_1 + 10X_2 + 20X_3 \leq 120000$

$40X_1 + 55X_2 + 65X_3 \leq 200000$

$1.5X_1 + 2X_2 + 3X_3 \leq 300$

$0.55X_1 + 0.78X_2 + 0.90X_3 \leq 50000$

$X_1 + X_2 + X_3 \leq 60000$

$X_1, X_2, X_3 \geq 0$

3-3 Converting the linear programming model from fuzzy to classical formulation

Max $Z = [\,(1-\alpha)(20+\sqrt{1-\alpha}\,)+\alpha 20\,]X_1 + [\,(1-\alpha)(26+\sqrt{1-\alpha}\,)+\alpha 26\,]X_2 + [\,(1-\alpha)(33+\sqrt{1-\alpha}\,)+\alpha 33\,]X_3$

s.to

$[\,(1-\alpha)(270+\sqrt{1-\alpha}\,)+\alpha 270\,]X_1 + [\,(1-\alpha)(310+\sqrt{1-\alpha}\,)+\alpha 310\,]X_2 + [\,(1-\alpha)(290+\sqrt{1-\alpha}\,)+\alpha 290\,]X_3 \leq [\,(1-\alpha)(150000+\sqrt{1-\alpha}\,)+\alpha 150000\,]$

$[\,(1-\alpha)(50+\sqrt{1-\alpha}\,)+\alpha 50\,]X_1 + [\,(1-\alpha)(10+\sqrt{1-\alpha}\,)+\alpha 10\,]X_2 + [\,(1-\alpha)(20+\sqrt{1-\alpha}\,)+\alpha 20\,]X_3 \leq [\,(1-\alpha)(120000+\sqrt{1-\alpha}\,)+\alpha 120000\,]$

$[\,(1-\alpha)(40+\sqrt{1-\alpha}\,)+\alpha 40\,]X_1 + [\,(1-\alpha)(55+\sqrt{1-\alpha}\,)+\alpha 55\,]X_2 + [\,(1-\alpha)(65+\sqrt{1-\alpha}\,)+\alpha 65\,]X_3 \leq [\,(1-\alpha)(200000+\sqrt{1-\alpha}\,)+\alpha 200000\,]$

$[(1 - \alpha)(1.5 + \sqrt{1 - \alpha}) + \alpha 1.5]X_1 + [(1 - \alpha)(2 + \sqrt{1 - \alpha}) + \alpha 2]X_2 + [(1 - \alpha)(3 + \sqrt{1 - \alpha}) + \alpha 3]X_3 \leq [(1 - \alpha)(300 + \sqrt{1 - \alpha}) + \alpha 300]$

$[(1 - \alpha)(0.55 + \sqrt{1 - \alpha}) + \alpha 0.55]X_1 + [(1 - \alpha)(0.78 + \sqrt{1 - \alpha}) + \alpha 0.78]X_2 + [(1 - \alpha)(0.90 + \sqrt{1 - \alpha}) + \alpha 0.90]X_3 \leq [(1 - \alpha)(50000 + \sqrt{1 - \alpha}) + \alpha 50000]$

$[(1 - \alpha)(1 + \sqrt{1 - \alpha}) + \alpha]X_1 + [(1 - \alpha)(1 + \sqrt{1 - \alpha}) + \alpha]X_2 +[(1 - \alpha)(1 + \sqrt{1 - \alpha}) + \alpha 1] X_3 \leq [(1 - \alpha)(60000 + \sqrt{1 - \alpha}) + \alpha 60000]$

$X_1, X_2, X_3 \geq 0$

3-4 Solve the model according to the change in the value of α.

We will solve the model according to the value α, which expresses the level of confidence and satisfaction of the decision maker, where its value is between (0,1) using the Lingo program. We will reach the following results shown in the table below: Table (3-2) shows the simulation results of the model solution according to the change in the value of α

Table (3-2)

Product	α=0.1	α=0.25	α=0.50	α=0.75	α=1
X1	243.99	576.52	0	0	364
X2	335.43	0	279.101	0	321
X3	0	0	266.002	489.24	0
(Maximum Profit)	33832.331	25477.809	31000.654	23114.021	28000

Results

Based on the obtained results, the following conclusions can be drawn:

1. When the value is α=0.1 , Must be produced 243.99 units from X1 ,and 335.43 units from X2 , And not producing any unit of the product X3 To make a profit of 33832.331
2. When the value is α=0.25 , Must be produced 576.52 units from X1 ,and not producing any unit of each the product X2 , X3 To make a profit of 25477.809 .
3. When the value is α=0.50 , Must be produced 279.101 units from X2 ,and 266.002 units from X2 , And not producing any unit of the product X1 To make a profit of 31000.654.
4. When the value is α=0.75 , Must be produced 489.24 units from X3 , and not producing any unit of each the product X1 , X2 To make a profit of 23114.021.
5. When the value is α=1 , Must be produced 364 units from X1 ,and 321units from X2 , And not producing any unit of the product X3 To make a profit of 28000

Conclusion

Based on the previous results, the production company should choose the first scenario when the value of α equals 0.1 to achieve the maximum profit of 33,832.331 monetary units compared to the other scenarios. In this case, 243.99 units of product X1 and 335.43 units of product X2 should be produced, while production of product X3 should be halted entirely.

Recommendations

 Focusing on developing the linear programming model by utilizing modern methods and techniques due to their ability to solve many problems through creating specific programs for operations research problems.

 Emphasizing the importance of incorporating the multi-objective linear programming model under the researcher Iskandar's method as an auxiliary tool in future research.

 Working on employing modern scientific methods by using the linear programming model in production planning for economic companies, with the necessity of establishing an information system within companies or institutions, especially economic ones, to facilitate the collection of necessary data for future planning. Additionally, working on linking economic institutions with universities to obtain solutions to the problems they face.

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