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## S–Domination Number in Graphs


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RESEARCH ARTICLE

# $\mathbb{S}$–Domination Number in Graphs

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## ABSTRACT

Let $\mathbb{G}$ ($\mathbb{V}$, $\mathbb{E}$) be a graph, the set $D \subseteq \mathbb{V}$ is dominating set if each vertex $v \in \mathbb{V}$ is either in D or is adjacent to a vertex in D and if there is no dominating subset of D, D will minimal dominating of a graph $\mathbb{G}$. The domination number $\gamma(\mathbb{G})$ is the minimum cardinality of all members of a minimal dominating set of a graph $\mathbb{G}$. If $\mathbb{V} - D$ includes dominating set $D'$ of vertices of a graph $\mathbb{G}$ then $D'$ is called an inverse dominating set to D such that $\gamma^{-1}(\mathbb{G})$ is the minimum cardinality of every member of minimal inverse dominating set of $\mathbb{G}$. Throughout this paper, two new parameters of domination number which are called the $\mathbb{S}$-domination number $\gamma_s(\mathbb{G})$ and the inverse $\mathbb{S}$-domination number ${\gamma_s}^{-1}(\mathbb{G})$ are introduced such that $\mathbb{S}$-dominating set and inverse $\mathbb{S}$-dominating set are proper sets. Theoretical parts and sides of these definitions are discussed. The results and properties of this definition are tackled, especially the definitions are studied on special graphs for instance cycle, path, complete, complete bipartite, wheel and complement of these graphs additionally for another graph helm, lollipop and Dutch windmill have been tackled.

**Keywords:** Certain graphs, Complement of certain graphs, Invers $\mathbb{S}$-dominating set, $\mathbb{S}$-dominating set, $\gamma_s$–set

## Introduction

Graph theory one of the prosperous branches of modern mathematics and computer applications. Domination number is an important number of graph theory. where this number finds solutions to several life problems. Many important applications of domination number in various fields like biological sciences, engineering, social and physical. Domination in mathematics appeared in several fields including algebraic graph [1,2] topological graph [3,4] labeled graph [5] fuzzy graph [6–8] and others.

The initial of domination number appearance in. [9] $D \subseteq \mathbb{V}$ is dominating set if every vertex $v \in \mathbb{V}$ is either an element of $D$ or is adjacent to an element of D and it is minimal dominating set refers to the sets of all minimal dominating of a graph $\mathbb{G}$. The domination number $\gamma(\mathbb{G})$ is the minimum cardinality of every members of minimal dominating set of $\mathbb{G}$. Further if $D'$ is dominating set in $\mathbb{V} - D$ of $\mathbb{G}$ then $D'$ is called an inverse dominating set to D such that $\gamma^{-1}(\mathbb{G})$ is the minimum cardinality of every inverse dominating set of $\mathbb{G}$, after that different definitions appeared for domination number [10–12] and the invers of domination number. [13,14]

In this paper, a new parameters of dominating set is inserted which is called the $\mathbb{S}$ – dominating set and this definition depends on conditions placed on the outside of the dominating set ($\mathbb{V} - D$) that is in the neighborhood of the vertices outside the dominating set where D is the proper set dominates by numbers of vertices of a graph $\mathbb{G}$ and $\gamma_s(\mathbb{G})$ denote to the $\mathbb{S}$-domination number. Also Throughout this paper, the invers of $\mathbb{S}$-dominating set of the graph $\mathbb{G}$ is introduce called inverse $\mathbb{S}$-dominating set and ${\gamma_s}^{-1}(\mathbb{G})$ represent the inverse $\mathbb{S}$-domination number.

Finally, some properties and results for the new concepts of domination numbers are discussed and calculated for certain graphs including cycle, path, complete, complete bipartite, wheel and for the complement of the same certain graph additionally this numbers determined by varies graph as helm, lollipop and Dutch windmill. For more details about each concept, see. [15–17]

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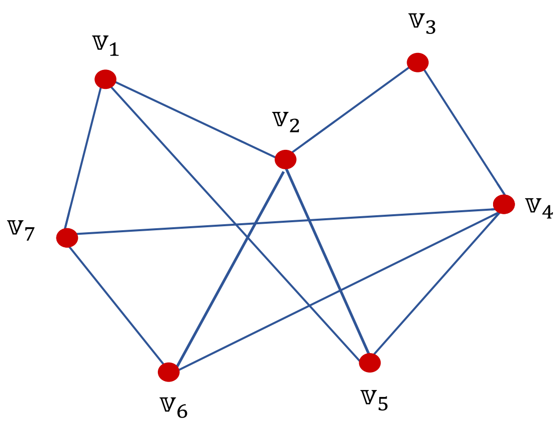


**Fig. 1.** The M$\mathbb{S}$DS of the graph.

## The main results of $\mathbb{S}$–domination number

Some results and propositions of $\mathbb{S}$-domination number are discussed in this section additionally some certain graphs and other extra graphs are calculated.

**Definition 1:** For a graph $\mathbb{G}$, a proper set $D \subset \mathbb{V}(\mathbb{G})$ is called a $\mathbb{S}$-dominatind set ($\mathbb{S}$DS) if $|N(\mathbb{v}) \cap D| =$ even$\forall \mathbb{v} \in \mathbb{V} - D$.

**Definition 2:** Let $\mathbb{G}$ be a graph and D be an $\mathbb{S}$DS then D is said to be minimal $\mathbb{S}$-dominatind set if has no proper subset $D' \subseteq D$ is a $\mathbb{S}$DS, M$\mathbb{S}$DS denote all minimal $\mathbb{S}$DS of $\mathbb{G}$.

**Definition 3:** The minimum cardinality of all M$\mathbb{S}$DS is said to be a $\mathbb{S}$-domination number and is denoted by $\gamma_s(\mathbb{G})$.

**Definition 4:** A proper $\mathbb{S}$-dominating set D with cardinality $\gamma_s(\mathbb{G})$ in a graph $\mathbb{G}$ is called $\gamma_s$−set

**Example 1:** *Consider $\mathbb{G}$ is a graph of seven vertices which is given in Fig. 1. the* M$\mathbb{S}$DS *of* $\mathbb{G}$ *are* $D_1 = \{\mathbb{v}_1, \mathbb{v}_2, \mathbb{v}_4, \mathbb{v}_5\}$, $D_2 = \{\mathbb{v}_1, \mathbb{v}_2, \mathbb{v}_3, \mathbb{v}_7\}$ *and* $D_3 = \{\mathbb{v}_2, \mathbb{v}_4, \mathbb{v}_6, \mathbb{v}_7\}$ *therefore* $\gamma_s(\mathbb{G}) = 4$.

## Results and discussion

**Proposition 1:** Consider $\mathbb{G}$ is a graph and D is a $\mathbb{S}$DS, then

(1) Every isolated and pendant vertex belongs to $\mathbb{S}$DS.
(2) Every vertex $\mathbb{v}$ of odd degree and $N(\mathbb{v}) \subseteq D$ then $\mathbb{v} \in D$.
(3) $n \geq 3$.

**Proof:**

(1) By definition (1), the result is obtained.
(2) Let $\mathbb{v} \notin D$ and $\deg(\mathbb{v}) = m$, m is odd. Now, let $N(\mathbb{v}) = \{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_m\}$ and $\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_m \in D$ then by definition $|N(\mathbb{v}) \cap D| =$ m but m is odd and $\mathbb{v} \notin D$ this is contradiction; hence it must be $\mathbb{v} \in D$.
(3) Let $n = 1, 2$ then $D = \mathbb{V}$ by (1), this contradiction with D is proper set. Thus $n \geq 3$.

**Proposition 2:** Let $\mathbb{G}$ is a connected graph of order $n \geq 3$ and has at least two vertices of degree $n - 1$ then $\gamma_s(\mathbb{G}) = 2$.

**Proof:** Let $\mathbb{G}$ be a graph of order $n \geq 3$, consider $\{\mathbb{v}_i, \mathbb{v}_j\}$ be to vertices of degree $n - 1$ on $\mathbb{G}$, since these vertices are adjacent to each vertex in $\mathbb{G}$, then $D = \{\mathbb{v}_i, \mathbb{v}_j\}$ is M$\mathbb{S}$DS with minimum cardinality thus, $\gamma_s(\mathbb{G}) = 2$.

**Proposition 3:** Let $P_n$ be a path graph of order n then $\gamma_s(P_n) = 2 + \lfloor \frac{n-2}{2} \rfloor$.

**Proof:** Let $\{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$ be the vertices on the path, $n \geq 3$, since $\mathbb{v}_1$ and $\mathbb{v}_n$ have one neighborhood. So, by Proposition 1 Case 1 they must belong to every $\mathbb{S}$DS. Hence the reman vertices are $n - 2$. Now, let $D = \{\mathbb{v}_{2+2i}, i = 0, 1, \ldots, \lfloor \frac{n-1}{2} \rfloor - 1\} \cup \{\mathbb{v}_1, \mathbb{v}_n\}$, it is clear that D is M$\mathbb{S}$DS, since each vertex in $\mathbb{V} - D$ has two vertices in D which are adjacent to it. To prove it is minimum $\mathbb{S}$DS, assume that $K = D - \{\mathbb{v}\}$ is $\mathbb{S}$DS, it is obvious that $\mathbb{v} \neq \mathbb{v}_1, \mathbb{v}_n$ by Proposition 1 case 1. So, $\mathbb{v} \in D - \{\mathbb{v}_1, \mathbb{v}_n\}$ then there is a vertex in $\mathbb{V} - K \cup \{\mathbb{v}_1, \mathbb{v}_n\}$ is dominated by lonely vertex. Therefore, K is not $\mathbb{S}$DS. Thus, D is M$\mathbb{S}$DS with $\gamma_s(P_n) = 2 + \lfloor \frac{n-2}{2} \rfloor$.

**Proposition 4:**

(1) $\gamma_s(C_n) = \lceil \frac{n}{2} \rceil$, where $C_n$ is a cycle graph.
(2) $\gamma_s(K_n) = 2$, where $K_n$ is a complete graph.

**Proof:**

(1) Let $D = \{\mathbb{v}_{1+2i}, i = 0, 1, \ldots, \lceil \frac{n-2}{2} \rceil\}$ it is prominent that D is $\mathbb{S}$DS of all vertices from $\mathbb{v}_1$ to $\mathbb{v}_n$, since each vertex in $\mathbb{V} - D$ has two vertices in D which are adjacent to it. Again, in the same manner of the Proposition 3 proves D is M$\mathbb{S}$DS with $\gamma_s(C_n) = \lceil \frac{n}{2} \rceil$.
(2) By Proposition 2, the result is obtained.

**Proposition 5:** Let $K_{m,n}$ be a complete bipartite graph then $\gamma_s(K_{m,n}) = \begin{cases} n, & \text{if } m = 1 \text{ and n is even} \\ 2, & \text{if } m = 2 \text{ and } n \geq 1 \\ 4, & \text{if } m,\ n > 2. \end{cases}$

**Proof:** From definition of a complete bipartite graph $K_{m,n}$ suppose that $\mathbb{V}_m$ and $\mathbb{U}_n$ are the partition sets of it where $\mathbb{V}_m = \{v_1, v_2, \ldots, v_m\}$ and $\mathbb{U}_n = \{u_1, u_2, \ldots, u_n\}$. So, there are three cases disputed below

Case 1: let n is even and $m = 1$ say $v_1$ since every vertex in the set $\mathbb{U}_n$ has one neighborhood $v_1$ so, by Proposition 1 Case 1 they must be belong to every $\mathbb{S}$DS. Now, let $D = \mathbb{U}_n$, it is obvious that D is $\mathbb{S}$DS. But $v_1$ is adjacent to every vertex in D therefore $N(v_1) = \mathbb{U}_n$ and $|\mathbb{U}_n|$ is even hence it mustn't belong to D thus $\Upsilon_s(K_{1,n}) = n$. If n is odd and $m = 1$ say $v_1$ then every vertex in the set $\mathbb{U}_n$ by Proposition 1 Case 1 must be belong to every $\mathbb{S}$DS. But $v_1$ has an odd degree (since n is odd) and every neighborhood of it is belongs to every $\mathbb{S}$DS, so by Proposition 1 Case 2 it must belong to every $\mathbb{S}$DS, hence each vertex in the graph $K_{1,n}$ where n is odd and $m = 1$ belong to $\mathbb{S}$DS this contradiction with D is the proper $\mathbb{S}$DS. Hence if n is odd and $m = 1$ $K_{1,n}$ has no $\mathbb{S}$DS.

Case 2: suppose $m = 2$ and $n \geq 1$, consider $D = \{v_1, v_2\}$ since each vertex in $\mathbb{V} - D$ is adjacent to only $v_1$ and $v_2$ hence D is $\mathbb{S}$DS but $|D| < 2$ if every vertex is remoted from D then it becomes no $\mathbb{S}$DS therefor D is $\gamma_s$−set.

Case 3: let $m, n > 2$, let $D = \{v_1, v_2, u_1, u_2\}$ since each vertex in $\mathbb{V}_m - D$ is adjacent to $u_1$ and $u_2$ also every vertex in $\mathbb{U}_n - D$ is adjacent to $v_1$ and $v_2$ hence D is $\mathbb{S}$DS. Assume that $K = D - \{v\}$ is $\mathbb{S}$DS, then there is a vertex in $\mathbb{V} - K$ is dominated by odd vertices therefore K is not SDS. Thus D is $\gamma_s$−set.

**Proposition 6:** If $W_n$ is a wheel graph, then $\gamma_s(W_n) = \begin{cases} \lfloor \frac{n}{3} \rfloor + 2, & n \equiv 0, 2 \bmod 3 \text{ and } n \neq 9, 5 \\ \lfloor \frac{n}{3} \rfloor + 1, & n \equiv 1 \bmod 3 \text{ and } n = 9, 5 \end{cases}$

**Proof:** Let $W_n = \{v_1, v_2, \ldots, v_n\}$, $v_n$ be the center vertex of the wheel graph, there are two cases disputed below.

Case 1: suppose that $n = 9, 5$, since $W_n = C_{n-1} + K_1$ and $D = \{v_{1+2i}, i = 0, 1, \ldots, \lceil \frac{n-2}{2} \rceil\}$ is $\mathbb{S}$DS of $C_{n-1}$ by proof of Proposition 4 Case 1, it is clear that $|D|$ is even. So, the vertex $v_n$ is not belong to D. Hence D is $\gamma_S$−set.

Case 2: let $D_1 = \{v_{2+3i}, i = 0, 1, \ldots, \lfloor \frac{n-2}{3} \rfloor\}$, then three subcases are classification as below.

Subcase 1: if $n \equiv 0 \bmod 3$, then $D = D_1 \cup \{v_1, v_n\}$,

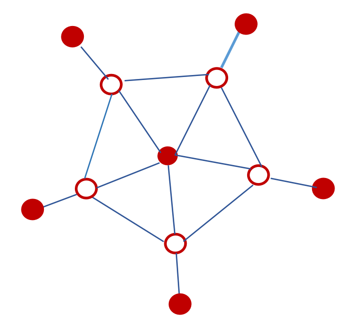

**Fig. 2.** The M$\mathbb{S}$DS of graph $\mathbf{H}_5$.

Subcase 2: if $n \equiv 1 \bmod 3$, then $D = D_1 \cup \{v_n\}$,

Subcase 3: if $n \equiv 2 \bmod 3$, then $D = D_1 \cup \{v_1\}$,

In each of the preceding two Subcases 1 and 3, it is clear that every vertex in $\mathbb{V} - D_1 \cup \{v_n\}$ has two vertices that are adjacent to it except $\{v_1\}$ has three vertices which are adjacent to it in $D_1 \cup \{v_n\}$ therefor it must belong to every S-dominating by Proposition 1 Case 2. Hence $D = D_1 \cup \{v_1, v_n\}$ is $\mathbb{S}$DS. Assume there is an $\mathbb{S}$DS $K = D - \{v\}$. Then there is at least one vertex in $\mathbb{V} - K$ that is dominated by one vertex in K since every vertex in $\mathbb{V} - D$ has two vertices that are adjacent to it in D. Therefore K is not a SDS. Hence D is $\gamma_s$−set.

But in subcase (2), since every vertex in $\mathbb{V} - D$ has two vertices that are adjacent to it. Hence D is $\mathbb{S}$DS. Again, by the same manner in above (1) and (3) D is $\gamma_s$−set.

**Proposition 7:** Consider $H_n$ IS a helm graph then $\gamma_s(H_n) = n + 1$.

**Proof:** The helm graph $H_n$ is gotten from wheel graph $W_m = C_{m-1} + K_1$ and creating a new vertex for every vertex of cycle $C_{m-1}$ such that $n = m - 1$ as given in Fig. 2. Since this graph has n pendant vertices by Proposition 1 Case 1 they must belong to every $\mathbb{S}$DS and let $v$ be the center vertex of the wheel graph belong to $\mathbb{S}$DS. Thus, every vertex in $\mathbb{V} - D$ has two vertices in D which are adjacent to it therefore removing any vertex from D then it becomes no $\mathbb{S}$DS therefor $\gamma_s(G) = n + 1$.

**Proposition 8:** Consider $L_{m,n}$ is a lollipop graph then $\gamma_s(L_{m,n}) = \lceil \frac{n}{2} \rceil + 2$.

**Proof:** The lollipop graph gotten from communicating complete graph $K_m$ with path graph $P_n$ by bridge. The set of vertices of $P_n$ are $\{v_1, v_2, \ldots, v_n\}$ and the set of vertices of $K_m$ are $\{u_1, u_2, \ldots, u_m\}$, $v_n$ is the end vertex of $P_n$ and $u_1$ is the vertex of degree m, from Proposition 1 Case 1 $v_n$ belong to every $\mathbb{S}$DS. Let $D = D_1 \cup \{v_n, u_1, u_2\}$ where $D_1 = \{v_{2+2i}, i = 0, 1, \ldots, \lfloor \frac{n-1}{2} \rfloor - 1\}$. It is clear that D is

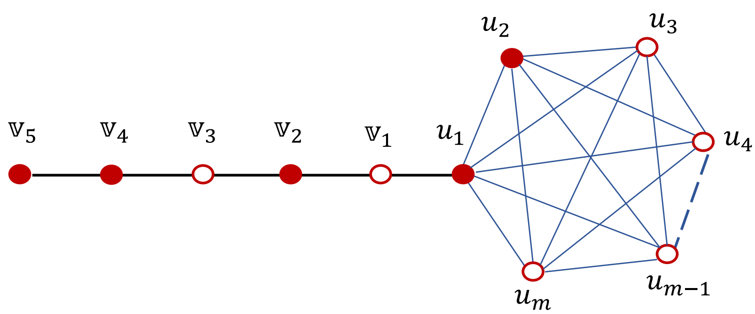


**Fig. 3.** The M$\mathbb{S}$DS of graph $\mathbf{L_{m,5}}$.

$\mathbb{S}$DS since each vertex in $\mathbb{V} - D$ has two vertices in D which are adjacent to it such that $D_1 \cup \{\mathbb{v}_{n,} u_1\}$ is $\mathbb{S}$DS of every vertex on $P_n$ where $u_1$ dominate the vertex $\mathbb{v}_1$. also $\{u_1, u_2\}$ are dominate $m-1$ vertices from $K_n$ thus by proof of Propositions 3 and 4 Case 2 removing any vertex from D then it becomes no $\mathbb{S}$DS therefor D is M$\mathbb{S}$DS with $\gamma_s(L_{m,n}) = \lceil \frac{n}{2} \rceil + 2$. (For an example see Fig. 3).

**Proposition 9:** Consider $D_n^m$ is a Dutch windmill graph then $\gamma_s(D_n^m) = m\lfloor \frac{n-1}{2} \rfloor + 1$.

**Proof:** The Dutch windmill graph gotten from sharing m copies of cycle graph $C_n$ in one vertex. Let $\{\mathbb{v}_n, \mathbb{v}_i^j, i = 1, \ldots, n-1\} \forall j = 1, \ldots, m$ be the set of vertices on the Dutch windmill graph $D_n^m$ such that $\mathbb{v}_n$ is the common vertex, consider $D = \{\mathbb{v}_n, \mathbb{v}_{2+2i}^j, i = 0, \ldots, \lfloor \frac{n-1}{2} \rfloor - 1\}, \; j = 1, \ldots, m$ it is clear that D is $\mathbb{S}$DS of $D_n^m$ since every vertex in $\mathbb{V} - D$ has two vertices in D which is adjacent to it. To prove D is M$\mathbb{S}$DS, assume that $K = D - \{\mathbb{v}\}$ is $\mathbb{S}-$domination then there are two cases depending on the place of omitted vertex.

Case 1: if $\{\mathbb{v}\} = \{\mathbb{v}_n\}$ then the vertices $\{\mathbb{v}_1^j, \mathbb{v}_{n-1}^j\} \forall j = 1, \ldots, m$ is dominated by one vertex. Therefore K is not $\mathbb{S}$DS. Thus, D is M$\mathbb{S}$DS with $\gamma_s(D_n^m) = m\lceil \frac{n-1}{2} \rceil + 1$.

Case 2: if $\{\mathbb{v}\}$ is every vertex in $D - \{\mathbb{v}_n\}$ then there is a vertex in $\mathbb{V} - K$ is dominated by one vertex. Therefore, K is not $\mathbb{S}$DS. Thus, D is M$\mathbb{S}$DS with $\gamma_s(D_n^m) = m\lceil \frac{n-1}{2} \rceil + 1$. (For an example see Fig. 4).

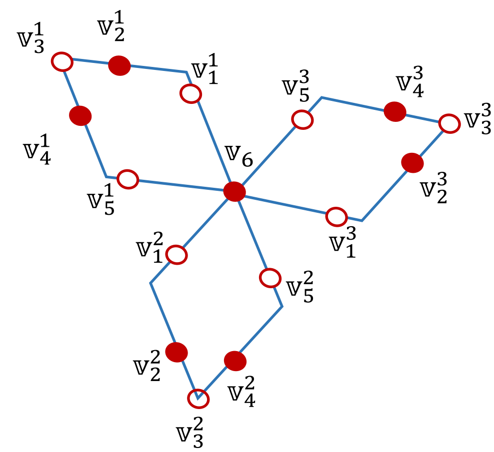


**Fig. 4.** The M$\mathbb{S}$DS of graph $\mathbf{D_6^3}$.

## *More $\mathbb{S}$ –domination number in the complement of the certain graph*

For this section, the concept $\mathbb{S}$ –domination number is discussed for complement of some certain graph

**Proposition 10:** If $P_n$ is a path graph with order $n \geq 4$, then $\gamma_s(\overline{P_n}) = \begin{cases} \lceil \frac{n}{3} \rceil, & n \equiv 1,\ 2,\ 3 \bmod 6 \\ \lceil \frac{n}{3} \rceil + 1, & n \equiv 0,\ 4,\ 5 \bmod 6 \end{cases}$

**Proof:** Let $\{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$ be the vertices on the $(\overline{P_n})$ and let $D_1 = \{\mathbb{v}_{2+3i}, i = 0, 1, \ldots, \lfloor \frac{n-2}{3} \rfloor\}$, there are two cases disputed as below:

Case 1: $D = \begin{cases} D_1 \cup \{\mathbb{v}_{n-1}\}, & \text{If } n \equiv 1 \bmod 6 \\ D_1 & \text{If } n \equiv 2, 3 \bmod 6 \end{cases}$

For this case, it is obvious every vertex in $\mathbb{V} - D$ is adjacent to only one vertex in $P_n$, this vertex is adjacent to $\lceil \frac{n}{3} \rceil - 1$ in $\overline{P_n}$ (for an example see Fig. 5(a)), but this is even. Thus, D is $\gamma_s-$set.

Case 2: $D = \begin{cases} D_1 \cup \{\mathbb{v}_n\}, & \text{if } n \equiv 0 \bmod 6 \\ D_1 \cup \{\mathbb{v}_1, \mathbb{v}_n\} & \text{if } n \equiv 4 \bmod 6 \\ D_1 \cup \{\mathbb{v}_1\} & \text{if } n \equiv 5 \bmod 6 \end{cases}$

For this case, it is obvious each vertex in $\mathbb{V} - D$ is adjacent to only one vertex in $P_n$, this vertex is adjacent to $\lceil \frac{n}{3} \rceil$ in $\overline{P_n}$ (for an example see Fig. 5(b)), but this is even. Thus, D is $\gamma_s-$set.

**Proposition 11:** If $C_n$ is cycle graph with $n \geq 5$, then

$$\gamma_s(\overline{C_n}) = \begin{cases} \frac{n}{2}, & n \equiv 0 \bmod 4 \\ \frac{n}{2} + 1, & n \equiv 2 \bmod 4 \\ \lceil \frac{n}{3} \rceil, & n \equiv 1, 3 \bmod 4 \text{ except } n \equiv 5 \bmod 6 \\ \lceil \frac{n}{3} \rceil + 1, & n \equiv 5 \bmod 6 \end{cases}$$

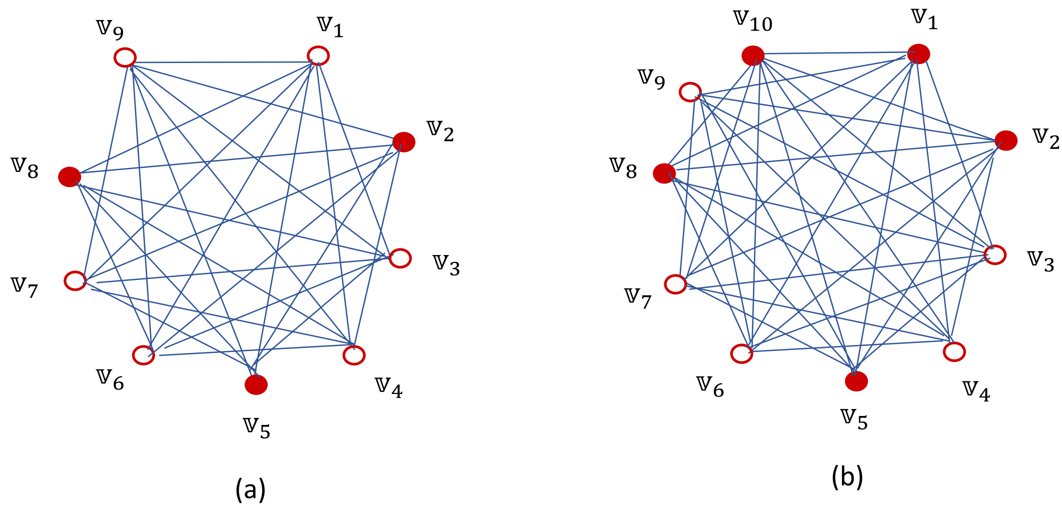


**Fig. 5.** The M$\mathbb{S}$DS of graphs $\overline{P_9}$ and $\overline{P_{10}}$.

**Proof:** Let $\{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$ be the vertices on the $(\overline{C_n})$, there are two cases disputed below.

Case 1: let $D_1 = \{\mathbb{v}_{2+2i}, i = 0, 1, \ldots, \lceil \frac{n}{3} \rceil\}$, then there are two subcases.

Subcase 1: If $n \equiv 0 \bmod 4$, then $D = D_1$, it is obvious each vertex in $\mathbb{V} - D$ is adjacent to only two vertices in $C_n$, and this vertex is adjacent to $\frac{n}{2} - 2$ in $\overline{C_n}$ (for an example see Fig. 6(a)), but this is even.

Subcase 2: If $n \equiv 2 \bmod 4$, then $D = D_1 \cup \{\mathbb{v}_1\}$, it is obvious each vertex in $\mathbb{V} - D$ is adjacent to only two vertices in $C_n$, and this vertex is adjacent to $\frac{n}{2} - 1$ in $\overline{C_n}$, but this is even.

Thus, in each of the preceding two subcases D is $\gamma_s-$set.

Case 2: let $D_1 = \{\mathbb{v}_{1+3i}, i = 0, 1, \ldots, \lfloor \frac{n-1}{3} \rfloor\}$, then

$$D = \begin{cases} D_1 \cup \{\mathbb{v}_n\}, & \text{If } n \equiv 5 \bmod 6 \\ D_1, & \text{If } n \equiv 1, 3 \bmod 4 \text{ except } n \equiv 5 \bmod 6 \end{cases}$$

For this case, it is prominent that each vertex in $\mathbb{V} - D$ is adjacent to only one vertex in $C_n$. Therefore by the same manner in the case 1 from proof of Proposition 10 proves that D is $\gamma_S-$set. (For an example see Fig. 6(b)).

**Proposition 12:** If $K_{m,n}$ is a complete bipartite graph, then $\gamma_s(\overline{K_{m,n}}) = \begin{cases} 3, & \text{if } m = 1,\ n \geq 3 \\ 4, & \text{if } m \geq 2,\ n \geq 3 \end{cases}$

**Proof:** There are two cases disputed below.

Case 1: let $m = 1$ say $\mathbb{v}_1$. Since $\mathbb{v}_1$ is isolated vertex then it is belong to every $\mathbb{S}$DS by Proposition 1 Case 1 so, $\gamma_s(\overline{K_{m,n}}) = 1 + \gamma_s(K_n)$ where $\gamma_s(K_n)$ is found by Proposition 4 Case 2. Hence the required is obtained.

Case 2: it is true the graph $\overline{K_{m,n}}$ is isomorphic to the union of two components $K_m$ and $K_n$ so, $\gamma_s(\overline{K_{m,n}}) = \gamma_s(K_m) + \gamma_s(K_n)$, now by Proposition 4 Case 2 is found $\gamma_s(K_m)$ and $\gamma_s(K_n)$ which is required.

**Proposition 13:** If $W_n$ is a wheel graph with $n \geq 6$, then

$$\gamma_s\left(\overline{W_n}\right) = \begin{cases} \lceil \frac{n+1}{3} \rceil + 1, & n \equiv 0, 2 \bmod 4 \text{ except } n \equiv 4 \bmod 6 \\ \lceil \frac{n}{3} \rceil & n \equiv 4 \bmod 6 \\ \lceil \frac{n}{2} \rceil, & n \equiv 1 \bmod 4 \\ \lceil \frac{n}{2} \rceil + 1, & n \equiv 3 \bmod 4 \end{cases}$$

**Proof:** The graph $\overline{W_n}$ is isomorphic to the union of two components $\overline{C_n}$ and $K_1$ such that $K_1 = 1$ say $\mathbb{v}_1$. Since $\mathbb{v}_1$ is an isolated vertex then it belongs to every $\mathbb{S}$DS by Proposition 1 Case 1. Hence $\gamma_s(\overline{W_n}) = \gamma_s(\overline{C_{n-1}}) + 1$, now by using Proposition 4 Case 1 to obtained $\gamma_s(\overline{C_{n-1}})$ which is required.

## *Inverse $\mathbb{S}$-domination number*

Some results and propositions of invers $\mathbb{S}$-domination number have been discussed in this section additionally for some certain graphs and other extra graphs calculated.

**Definition 5:** Let $\mathbb{G}$ be a graph and the proper set D be an $\gamma_s-$set. If $\mathbb{V} - D$ contains proper $\mathbb{S}$-dominating set $K \subseteq \mathbb{V}$ of vertices in a graph $\mathbb{G}$ then K is said to be an inverse $\mathbb{S}$-dominating set to D and denoted by MI$\mathbb{S}$DS.

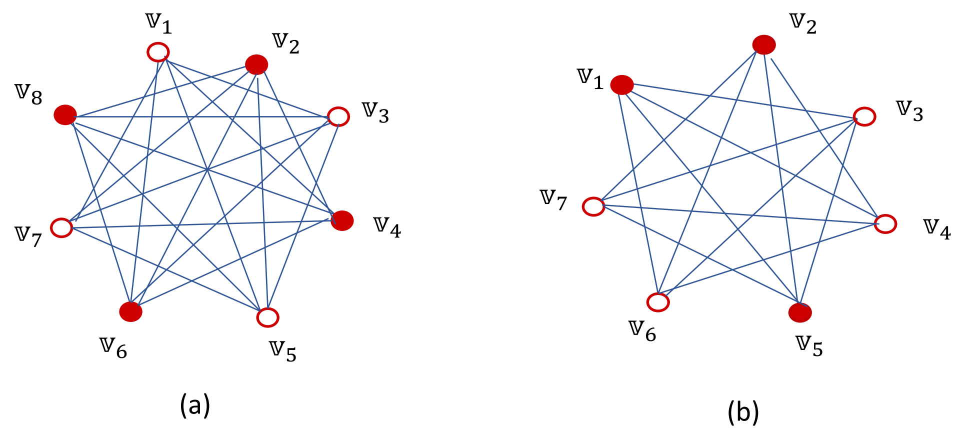


**Fig. 6.** The M$\mathbb{S}$DS of graphs $\overline{C_8}$ and $\overline{C_7}$.

**Definition 6:** Let $\mathbb{G}$ be a graph and K be an inverse $\mathbb{S}$-dominating set then K is said to be minimal inverse $\mathbb{S}$-dominating set if has no proper subset $F \subseteq K$ is an inverse $\mathbb{S}$DS, and $\gamma_s{}^{-1}(\mathbb{G})$ dented to all number of MI$\mathbb{S}$DS.

**Proposition 14:** Consider $\mathbb{G}$ is a graph and D be an $\mathbb{S}$ – dominating set, then

(1) If $\mathbb{G}$ has isolated and pendant vertex then $\mathbb{G}$ has no invers $\mathbb{S}$-dominating set.,
(2) If has vertex $\mathbb{v}$ of odd degree and $N[\mathbb{v}] \subseteq D$ then $\mathbb{G}$ has no invers $\mathbb{S}$-dominating set.
(3) $\gamma_s(\mathbb{G}) \leq \gamma_s{}^{-1}(\mathbb{G})$

**Proof:** (1) and (2) the result is obtained by Proposition 1.

**Proposition 15:** Consider $\mathbb{G}$ is a graph then,

(1) For $\mathbb{G} \cong P_n$, $\mathbb{G}$ has no invers.
(2) For $\mathbb{G} \cong C_n$,

$$\gamma_s{}^{-1}(\mathbb{G}) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ \text{has no inverse}, & \text{if n is odd} \end{cases}$$

(3) For $\mathbb{G} \cong K_n$, $\gamma_s{}^{-1}(\mathbb{G}) = 2$, where $n \geq 4$

**Proof:**

(1) Since the graph $P_n$ has two pendant vertices. Hence by Proposition 14 Case 1 $P_n$ has no invers.
(2) Let $C_n = \{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$, then there are two cases as follows.

Case 1: If n is even, then $D = \{\mathbb{V}_{1+2i}, i = 0, 1, \ldots, \frac{n-2}{2}\}$, it is prominent that D is $\gamma_S$–set by proof of Proposition 4 Case 1. Now let $D^{-1} = \{\mathbb{v}_{2+2i}, i = 0, 1, \ldots, \frac{n}{2}\}$, it is true $D^{-1}$ is $\mathbb{S}$DS and $D \cap D^{-1} = \emptyset$. Then $D^{-1}$ is an I$\mathbb{S}$DS in $C_n$. Since $|D^{-1}| = |D|$ thus $D^{-1}$ is MI$\mathbb{S}$DS with $\gamma_s{}^{-1}(C_n) = \frac{n}{2}$.

Case 2: if n is odd, from Proposition 4 Case 1 $|D| > |\mathbb{V} - D|$. Assume that there is an I$\mathbb{S}$DS $D^{-1}$ such that $|D^{-1}| \geq |D|$. But $|D^{-1}| \leq |\mathbb{V} - D| < |D|$ therefore $\gamma_s(C_n) \geq \gamma_s{}^{-1}(C_n)$ this contradiction with Proposition 14 Case 3. Thus, the graph $C_n$ has no I$\mathbb{S}$DS in this case.

(3) Let $n \geq 4$ and $D = \{\mathbb{v}_1, \mathbb{v}_2\}$ be $\gamma_s$–set by proof of Proposition 4 Case 2. So, can take different vertices from this to dominate all vertices for instant $D^{-1} = \{\mathbb{v}_3, \mathbb{v}_4\}$. it is prominent that $D \cap D^{-1} = \emptyset$, then $D^{-1}$ is an I$\mathbb{S}$DS. Since $|D^{-1}| = |D|$ thus $D^{-1}$ is MI$\mathbb{S}$DS and $\gamma_s{}^{-1}(K_n) = 2$.

**Proposition 16:** Let $W_n$ be the wheel graph with n vertices, then $\gamma_s{}^{-1}(W_n) =$

$$\begin{cases} \text{has no inverse}, & \text{if } n \equiv 0, 2 \bmod 3,\ n \neq 5, 9 \\ \lceil \frac{n-1}{2} \rceil, & \text{if } n = 5, 9 \\ n - \lceil \frac{n}{3} \rceil, & \text{if } n \equiv 1 \bmod 3 \end{cases}$$

**Proof:** Let $W_n = \{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$ be the vertex set of the wheel graph and $\mathbb{v}_n$ be the vertex center of the wheel graph, there are three cases disputed below.

Case 1: if $n = 9, 5$, since $W_n = C_{n-1} + K_1$ and $D = \{\mathbb{v}_{1+2i}, i = 0, 1, \ldots, \lceil \frac{n-2}{2} \rceil\}$ is $\gamma_S$–set of $C_{n-1}$ and $W_n$ by proof of Proposition 4 Case 1 and Proposition 6. Now let $D^{-1} = \{\mathbb{v}_{1+2i}, i = 0, 1, \ldots, \frac{n}{2}\}$, it is true $D^{-1}$ is $\mathbb{S}$DS of $C_{n-1}$ and $D \cap D^{-1} = \emptyset$. Then $D^{-1}$ is I$\mathbb{S}$DS in $W_n$, Since $|D^{-1}| = |D|$ thus $D^{-1}$ is MI$\mathbb{S}$DS and $\gamma_s{}^{-1}(W_n) = \lceil \frac{n-1}{2} \rceil$.

Case 2: if $n \equiv 0, 2 \bmod 3,\ n \neq 5, 9$, since every vertex in the wheel graph of odd 3-degree except $\mathbb{v}_n$ where $\mathbb{v}_n \in D$. But $N[\mathbb{v}_1] \in D$ by proof of Proposition 6. Hence by Proposition 14 Case 2 $W_n$ has no I$\mathbb{S}$DS in this case.

Case 3: if $n \equiv 1 \bmod 3$, let $D = \{\mathbb{v}_{2+3i}, i = 0, 1, \ldots, \lfloor \frac{n-2}{3} \rfloor\} \cup \{\mathbb{v}_n\}$, it is prominent that D is

$\gamma_s-$set by proof of Proposition 6. We can take different vertices from this to dominate all vertices. Now let $D^{-1} = \mathbb{V} - D$, it is true $D^{-1}$ is $\mathbb{S}$DS and $D \cap D^{-1} = \emptyset$. Then $D^{-1}$ is an I$\mathbb{S}$DS in $W_n$. Now assume there is an I$\mathbb{S}$DS $K = D^{-1} - \{\mathbb{v}\}$ then there is at least one vertex in $\mathbb{V} - K$ is dominated by one vertex in K. Therefore, K is not $\mathbb{S}$DS. Thus $D^{-1}$ is MI$\mathbb{S}$DS and ${\gamma_s}^{-1}(W_n) = n - \lceil \frac{n}{3} \rceil$.

**Proposition 17:** If $K_{m,n}$ is a bipartite graph with n, m vertices, then

$$ {\gamma_s}^{-1}(K_{m,n}) = \begin{cases} \text{has no inverse,} & \text{if } m = 1 \text{ and } n \geq 1, \\ & \text{if } m = 2, 3 \text{ and n is odd} \\ n, & \text{if } m = 2, 3 \text{ and n is even} \\ 4, & \text{if } m, n \geq 4 \end{cases} $$

**Proof:** Suppose that $\mathbb{V}_m$, $\mathbb{U}_n$ are the partition sets of $K_{m,n}$ it where $\mathbb{V}_m = \{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_m\}$ and $\mathbb{U}_n = \{u_1, u_2, \ldots, u_n\}$, then there are three cases disputed below.:

Case 1: If $m = 1(2, 3)$ and $n \geq 1$(odd) the rustle is true by Proposition 14 Case 2.

Case 2: Let $m = 2, 3$ and n is even since $D = \{\mathbb{v}_1, \mathbb{v}_2\}$ and each vertex in $\mathbb{V} - D$ are adjacent to $\mathbb{v}_1$ and $\mathbb{v}_2$ hence ${\gamma_s}^{-1}(K_{m,n}) = n$

Case 3: Let $m, n \geq 4$ and $D = \{\mathbb{v}_1, \mathbb{v}_2, u_1, u_2\}$ be $\gamma_s-$set by proof of Proposition 5. So, can take different vertices from this to dominate all vertices for instant $D^{-1} = \{\mathbb{v}_3, \mathbb{v}_4, u_3, u_4\}$ and by the same manner in case (1) of proof Proposition 15 Case 2 prove that $D^{-1}$ is MI$\mathbb{S}$DS and ${\gamma_s}^{-1}(K_{m,n}) = 4$.

**Proposition 18:**

(1) Consider $H_n$ is a helm graph then $H_n$ has no invers.
(2) Consider $L_{m,n}$ is a lollipop graph then $L_{m,n}$ has no invers.

**Proof:**

(1) Since the graph $H_n$ has n pendant vertices. Hence by Proposition 14 Case 1 $H_n$ has no I$\mathbb{S}$DS.
(2) Since the graph $L_{m,n}$ has one pendant vertex from the path $P_n$. Hence by Proposition 14 Case 1 $L_{m,n}$ has no I$\mathbb{S}$DS.

**Proposition 19:** Consider $D_n^m$ be the Dutch wind-mill graph then

$$ {\gamma_s}^{-1}(D_n^m) = \begin{cases} m(\frac{n}{2}), & \text{if n is even} \\ \text{has no invers,} & \text{if n is odd} \end{cases} $$

**Proof:**

(1) Let $\mathbb{V}(D_n^m) = \{\mathbb{v}_n, \mathbb{v}_i^j, i = 1, \ldots, n-1\} \forall j = 1, \ldots, m$, then there are two cases as follows.

Case 1: If n is even, then $D = \{\mathbb{v}_n, \mathbb{v}_{2+2i}^j, i = 1, \ldots, \lfloor \frac{n-1}{2} \rfloor - 1\}, j = 1, \ldots, m$, it is prominent that D is $\gamma_s-$set by proof of Proposition 9. We can take different vertices from this to dominate all vertices. Now let $D^{-1} = \mathbb{V} - D$ such that $D^{-1} = \{\mathbb{v}_{1+2i}^j, i = 0, \ldots, \lfloor \frac{n-1}{2} \rfloor\}, j = 1, \ldots, m$ it is true $D^{-1}$ is $\mathbb{S}$DS and $D \cap D^{-1} = \emptyset$. Then $D^{-1}$ is an I$\mathbb{S}$DS in $D_n^m$. By applied proof of Proposition 15 Case 2 for m copies cycle of $D_n^m$ obtained $D^{-1}$ is MI$\mathbb{S}$DS and ${\gamma_s}^{-1}(D_n^m) = m(\frac{n}{2})$.

Case 2: If n is odd, from Proposition 4 Case 1 $|D| > |\mathbb{V} - D|$. By the same manner in Case 2 of proof of Proposition 15 Case 2 prove that the graph $D_n^m$ has no I$\mathbb{S}$DS.

### *Inverse $\mathbb{S}$ –domination number in the complement of the certain graph*

For this section, the concept inverse $\mathbb{S}$-domination number has been discussed for complement of some certain graph

**Proposition 20:** Let $\mathbb{G}$ be a graph then:

(1) For $\mathbb{G} \cong \overline{P_n}$, ${\gamma_s}^{-1}(\mathbb{G}) =$
$$ \begin{cases} \lceil \frac{n}{3} \rceil, & \text{if } n \equiv 1, 2, 3 \bmod 6 \\ \text{has no invers,} & \text{if } n \equiv 0, 4, 5, \bmod 6 \end{cases} $$
(2) For $\mathbb{G} \cong \overline{C_n}$, ${\gamma_s}^{-1}(\mathbb{G}) =$
$$ \begin{cases} \frac{n}{2}, & \text{if } n \equiv 0 \bmod 4 \\ \lceil \frac{n}{3} \rceil, & \text{if } n \equiv 1, 3 \bmod 4 \text{ except } n \equiv 5 \bmod 6 \\ \text{has no invers,} & \text{if } n \equiv 2 \bmod 4 \text{ and } n \equiv 5 \bmod 6 \end{cases} $$
(3) For $\mathbb{G} \cong \overline{W_n}$, $\mathbb{G}$ has no I$\mathbb{S}$DS.

**Proof:**

(1) Let $\overline{P_n} = \{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$, then there are two cases disputed below.

Case 1: if $n \equiv 1, 2, 3 \bmod 6$, then $D = \{\mathbb{v}_{2+3i}, i = 0, 1, \ldots, \lfloor \frac{n-2}{3} \rfloor\}$, it is obvious that D is $\gamma_s-$set by proof of Proposition 10. Now let $D^{-1} = \{\mathbb{v}_{1+3i}, i = 0, 1, \ldots, \lfloor \frac{n}{3} \rfloor\}$, it is clear that $D \cap D^{-1} = \emptyset$ and $D^{-1}$ is an $\mathbb{S}$DS. Since $|D^{-1}| = |D|$ thus $D^{-1}$ is MI$\mathbb{S}$DS and ${\gamma_s}^{-1}(\overline{P_n}) = \lceil \frac{n}{3} \rceil$.

Case 2: if $n \equiv 0, 4, 5 \bmod 6$, then $D = \{\mathbb{v}_{2+3i}, i = 0, 1, \ldots, \lfloor \frac{n-2}{3} \rfloor\} \cup \{\mathbb{v}_n\}$, such that D is $\gamma_s-$set by proof of Proposition 10. Since there is no two vertices $\{\mathbb{v}_i, \mathbb{v}_j\}$ such that $d(\mathbb{v}_i, \mathbb{v}_j) = 3$ (d is the distance between $\mathbb{v}_i$ and $\mathbb{v}_j$) belong to $\mathbb{S}$DS to dominate all vertices in $\mathbb{V} - D$, thus $\overline{P_n}$ has no I$\mathbb{S}$DS.

(2) Let $\{\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_n\}$ be the vertices on the graph $(\overline{C_n})$ then there are three cases as follows.

Case 1: if $n \equiv 0 \bmod 4$ then $D = \{\mathbb{v}_{2+2i}, i = 0, 1, \ldots, \lceil \frac{n}{3} \rceil\}$, such that D is $\gamma_s$−set by proof of Proposition 11. Now let $D^{-1} = \{\mathbb{v}_{1+2i}, i = 0, 1, \ldots, \lceil \frac{n}{3} \rceil\}$, it is clear that $D \cap D^{-1} = \emptyset$, then $D^{-1}$ is an I$\mathbb{S}$DS. Since $|D^{-1}| = |D|$ thus $D^{-1}$ is MI$\mathbb{S}$DS and ${\gamma_s}^{-1}(\overline{C_n}) = \frac{n}{2}$.

Case 2: if $n \equiv 1, 3 \bmod 4$ except $n \equiv 5 \bmod 6$, then $D = \{\mathbb{v}_{1+3i}, i = 0, 1, \ldots, \lfloor \frac{n-1}{3} \rfloor\}$, it is clear that D is $\gamma_s$−set by proof of Proposition 11. Now let $D^{-1} = \{\mathbb{v}_{2+3i}, i = 0, 1, \ldots, \lfloor \frac{n-2}{3} \rfloor\} \cup \{\mathbb{v}_{n-1}\}$, it is clear that $D \cap D^{-1} = \emptyset$, then $D^{-1}$ is an I$\mathbb{S}$DS. Since $|D^{-1}| = |D|$ thus $D^{-1}$ is MI$\mathbb{S}$DS and $\gamma_s(\overline{C_n}) = \lceil \frac{n}{3} \rceil$.

Case 3: If $n \equiv 2 \bmod 4$ and $n \equiv 5 \bmod 6$. From proof of Proposition 11 $|D| > |\mathbb{V} - D|$. By the same manner in Case 2 of proof of Proposition 15 Case 2 prove that the graph $\overline{C_n}$ has no invers.

(3) The graph $(\overline{W_n})$ has isolated vertex, hence by Proposition 11 Case 1 it has no I$\mathbb{S}$DS.

**Proposition 21:** Let $K_{m,n}$ be complete bipartite graph, then ${\gamma_s}^{-1}(\overline{K_{m,n}})$

$$= \begin{cases} \text{has no invers}, & \text{if } m = 1, 2, 3 \text{ and } n \geq 1 \\ 4, & \text{if } m, n \geq 4 \end{cases}$$

**Proof:** There are two cases disputed below.

Case 1: Let $n \geq 1$, since there is isolated vertex if $m = 1$ and there are two pendant vertices where $m = 2, 3$, thus $K_{1,n}$, $K_{2,n}$ and $K_{3,n}$ has no inverse by Proposition 14 Case 1.

Case 2: Let $m, n \geq 4$, it is clear that the graph $\overline{K_{m,n}}$ is isomorphic to the union of two components $K_m$ and $K_n$ so ${\gamma_s}^{-1}(\overline{K_{m,n}}) = {\gamma_s}^{-1}(K_m) + {\gamma_s}^{-1}(K_n)$, now by Proposition 15 Case 3 ${\gamma_s}^{-1}(K_m)$ and ${\gamma_s}^{-1}(K_n)$ is found. Which is required.

## Conclusion

Through the results new two parameter have gotten of domination number in graph, namely, $\mathbb{S}$-domination number and inverse $\mathbb{S}$-domination number. Many properties are found for these numbers especially determined to some certain graphs and the complement of these graphs. In addition, these concepts will be tackled on two operations in some graphs and in digraph. Also in other fields of mathematics for instances algebraic graph, topological graph, labeled graph, fuzzy graph and other.

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- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.
- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Babylon, Iraq.

## References

1. Goswami J, Sarmah M. On domination in the total torsion element graph of a module. Proyecciones (Antofagasta). 2023;42(3):795–814. https://doi.org/10.22199/issn.0717-6279-4904.
2. Patwari D, Saikia HK, Goswami J. Some results on domination in the generalized total graph of a commutative ring. J Algebra Rel Top. 2022;10(1):119–128. https://doi.org/10.22124/jart.2021.19238.1265.
3. Devi RN, Muthumari G. View on neutrosophic over topologized domination graphs. Neutrosophic Sets Syst. 2021;47:520–532. http://dx.doi.org/10.5281/zenodo.5775174.
4. Jwair ZN, Abdlhusein MA. Some dominating results of the topological graph. Int J Nonlinear Anal Appl. 2023;14(2):133–140. https://doi.org/10.22075/ijnaa.2022.6404.
5. Al-harere MN, Omran AAA. Binary operation graphs. AIP Conf Proc. 2019;2086(1):030008–3. https://doi.org/10.1063/1.5095093.
6. Meenakshi A, Kannan A, Mahdal M, Karthik K, Guras R. A comparative study of fuzzy domination and fuzzy coloring in an optimal approach. Mathematics. 2023;11(18):4019. https://doi.org/10.3390/math11184019.
7. Manjusha OT. Global domination in fuzzy graphs using strong arcs. J Fuzzy Ext Appl. 2023;4(1):8–17. https://doi.org/10.22105/jfea.2022.354342.1227.
8. Ramya S, Lavanya S. Contraction and domination in fuzzy graphs. 2023; TWMS J App Eng Math. 2023;13(1):133–142.
9. Berge C. Theory of graphs and its applications, Methuen Co. London: 1962. 257 P.
10. Omran AA, Oda HH. Hn-domination in graphs. Baghdad Sci J. 2019;16(1):242–247. https://doi.org/10.21123/bsj.2019.16.1(Suppl.).0242.
11. Abed SS, Al-Harere MN. Rings domination in graphs. Int J Nonlinear Anal Appl. 2022;13(2):1833–1839. https://doi.org/10.22075/ijnaa.2022.6544.

12. Hassan JA, Salim JI. J-domination in graphs. Eur J Pure Appl. 2023;16(4):2082–2095. https://doi.org/10.29020/nybg.ejpam.v16i4.4883.
13. Kahat SSH, Al-harere MN. Inverse equality co-neighborhood domination in graphs. J Phys Conf Ser. 2021;1879(3): 032036. http://dx.doi.org/10.1088/1742-6596/1879/3/032036.
14. Jayasree TG, Iyer RR. New parameters on inverse domination. IJMET. 2019;10(3):1111–1116.
15. Haynes TW, Hedetniemi S, Slater P. Fundamentals of domination in graphs. 1st ed. CRC press: 2013. 464 P. https://doi.org/10.1201/9781482246582.
16. Mary AA, Anjaline W. Minimum neighborhood domination of split graph of graphs. Baghdad Sci J. 2023; 20(1): 0273–0273 I. https://doi.org/10.21123/bsj.2023.8404.
17. Mohammed MA, Al-Mayyahi SYA, Virk AR, Rehman HM. Irregularity indices for line graph of Dutch windmill graph. Proyecciones (Antofagasta). 2020;39(4):903–918. http://dx.doi.org/10.22199/issn.0717-6279-2020-04-0056.

# رقم الهيمنة $\mathbb{S}$ في البيانات

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## الخلاصة

لنفرض $\mathbb{G}\ (\mathbb{V},\mathbb{E})$ هو رسم بياني متكون من مجموعة من الرؤس $\mathbb{V}$ ومجموعة اضلاع $\mathbb{E}$ ,المجموعة الجزئية $D$ من مجموعة الرؤوس $\mathbb{V}$ تكون مجموعة هيمنه اذا كل رأس$\mathbb{v}$ ينتمي الى مجموعة الرؤس $\mathbb{V}$ هو اما عنصر في مجموعة الهيمنه $D$ او يجاور عنصر في هذه المجموعه و اذا لا توجد مجموعه هيمنه محتواة في $D$ فان $D$ تكون مجموعة هيمنه صغرى في الرسم البياني $\mathbb{G}$ . $\gamma(\mathbb{G})$ هو رمز لأصغر رقم هيمنه بالنسبه لأرقام جميع المجاميع المهيمنه الصغرى للرسم البياني $\mathbb{G}$. اذا مجموعة الرؤس $\mathbb{V}$ من دون مجموعة رؤس مجموعة الهيمنه $D$ تحتوي مجموعة همينه $\mathrm{D}'$ من الرؤس في الرسم البياني فان هذه المجموعه هي معكوس المجموعة المهينه للرسم البياني $\mathbb{G}$ حيث $\gamma^{-1}(\mathbb{G})$ هو رمز لأصغر رقم معكوس مجموعة هيمنه بالنسبه لأرقام جميع مجاميع معكوس الهيمنه الصفرى للرسم الباني $\mathbb{G}$. خلال هذا البحث يتم تقديم تعريف جديد لرقم الهيمنه يسمى رقم الهيمنه $\mathbb{S}$ يرمز له $\gamma_{\mathbb{S}}(\mathbb{G})$ و ايضا تم تعريف معكوس رقم الهيمنه $\mathbb{S}$ يرمز له $\gamma_{\mathbb{S}}{}^{-1}(\mathbb{G})$ و درست خصائص هذا الرقم و معكوسه علما ان مجموعة الهينه$\mathbb{S}$ و معكوسها هي مجموعة فعليه بالاضافة الى ذالك نوقش هذا اللرقم و معكوسه بالنسبه لانواع خاصة في الرسم البياني مثلا المسار, الحلقة, السرسم البياني التام, ثنائي التجزئة التام ,العجلة و المتممات لهذه الرسوم البيانيه بالاضافة الى ذالك درست هذه الارقام بالنسبه الى رسوم بيانيه مختلفة كالرسم البياني للخوذة , المصاصة و طاحونة الهواء الهولندية.

**الكلمات المفتاحية:** الرسوم البيانية الرئيسيه، المتممات للرسوم البيانية الرئيسيه، معكوس المجموعة المهيمنه $\mathbb{S}$، المجموعة المهيمنه $\mathbb{S}$، رقم المجموعة المهيمنه الصغرى $\mathbb{S}$.