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Numerically approximating partial differential equations

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Abstract

This research presents a new finite element technique that uses a gradient operator over generalized functions that is not well-defined. The idea of weak discrete gradients is introduced as a way to approximate weak gradients, which could be useful for numerical solutions of Partial derivatives equations . The study aims to present a comprehensive highlight for handling derivatives operators on generalized services . The approach utilizes a discrete weak gradient operator to develop numerical techniques that can solve a second-order elliptic problem. This operator is an alternative to the conventional gradient operator. The approach employed is known as weak Galerkin (WG), which is a numerical method that permits the use of entirely discontinuous functions.. The paper provides an error estimate for the weak Galerkin finite element solutions for both discrete and norms-based solutions to the second order elliptic problem. Additionally, the paper highlights the super convergence of the weak Galerkin approximation.

Keywords: *Discrete gradient; Finite element methods using Galerkin; hybrid finite element techniques; elliptic second-order issues.*

التقريب العددي للمعادلات التفاضلية الجزئية

رلى إسماعيل حمد
مديرية تربية ذي قار

خلاصة

يقدم هذا البحث تقنية جديدة للعناصر المحدودة تستخدم عامل التدرج على الوظائف المعممة غير المحددة جيدًا. تم تقديم فكرة التدرجات المنفصلة الضعيفة كوسيلة لتقريب التدرجات الضعيفة، والتي يمكن أن تكون مفيدة للحلول العددية لمعادلات المشتقات الجزئية. وتهدف الدراسة إلى تقديم نظرة شاملة للتعامل مع مشغلي المشتقات على الخدمات المعممة. يستخدم هذا النهج مشغل التدرج الضعيف المنفصل لتطوير التقنيات العددية التي يمكنها حل مشكلة إهليلجية من الدرجة الثانية. يعد هذا العامل بديلاً لمشغل التدرج التقليدي. يُعرف النهج المستخدم باسم Galerkin الضعيف(WG) ، وهو طريقة عددية تسمح باستخدام وظائف متقطعة تمامًا. توفر الورقة تقديرًا للخطأ لحلول العناصر المحدودة الضعيفة لـ Galerkin لكل من الحلول المنفصلة والقائمة على المعايير لـ مشكلة إهليلجية من الدرجة الثانية. بالإضافة إلى ذلك، يسلط البحث الضوء على التقارب الفائق لتقريب جاليركين الضعيف.

الكلمات الرئيسية: التدرج المنفصل؛ طرق العناصر المحدودة باستخدام جاليركين. تقنيات العناصر المحدودة الهجينة؛ قضايا الاهليليجيه من الدرجة الثانية.

Introduction

This paper offers an approach for numerically approximating partial differential equations that is based on a novel comprehension of approximations and differential operators. Several criteria must be satisfied to determine an unknown function in the Dirichlet problem for second-order elliptic equations.

$-\nabla.(a\nabla u) + \nabla.(bu) + cu = f \quad in\ \Omega,$ (1.1)

$u = g \quad on\ \partial\Omega,$ (1.2)

The domain Ω is either a polygon or a polyhedron.

$\mathbb{R}^d(d = 2,3), a = \left(a_{ij}(x)\right)_{d\times d} \in [L^{\infty}(\Omega]^{d^2}$ is a function with symmetric matrix values,

$b = \left(b_{\circ}(x)\right)_{d\times 1}$ The function f is a function of vector, and C is a function of scalar, defined on the domain Ω, while the matrix α meets a specified condition. there produces a constant α > 0. such that

$\alpha\xi^T \leq \xi^T a\xi, \qquad \forall\xi \in \mathbb{R}^d.$ (1.3)

We will limit our focus to two-dimensional issues simply for the sake of simplicity. It is simple to extend to higher-dimensional issues.

The standard weak form for (1.1), (1.2) seeks. $u \in \mathrm{H}^1\ (\Omega)$

Such that u = g and

$(\mathrm{a}\nabla u, \nabla u) - (bu, \nabla u) + (cu, u) = (f, u) \quad \forall u \in \mathrm{H}_0^1(\Omega),$ (1.4)

In the given equation (1.4), f and g are vector functions, and C is either a scalar or a vector function. The gradient operator is used to represent the gradient of the function. The Galerkin technique is a standard method that uses subspaces of finite dimensions in place of the trial and test spaces in the equation (1.4), and this is done correctly as described in references [1] and [2]. The solution obtained from this approach is known as a approximation of Galerkin. A key aspect of the Galerkin technique is the selection of approximation services that allow the gradient operator to be verified in the traditional sense.

Galerkin finite element methods involve using continuous piecewise polynomials for both trial and test approximation functions over a finite element partition of the domain, which is usually marked by a symbol. As a result, a significant amount of attention has been paid to ensuring that the "continuity" criterion is satisfied in traditional Galerkin finite element methods, as opposed to recent guidlines in discontinuous Galerkin methods. However, in both continuous and discontinuous Galerkin finite element techniques, the meaning of the gradient operator remains consistent with the conventional concept as described in existing literature.

There have been many numerical techniques produced for solving the problem ‘s model (1.1)-(1.2). These techniques can be divided into two

classifications: those worked on the primary variable and those worked on a variable and a flow variable (mixed formulation). The first category includes interior penalty type discontinuous Galerkin methods and standard Galerkin finite element methods. The second category includes mixed finite element and variable-based discontinuous Galerkin methods. As there are numerous publications regarding generic finite element methods, it is not feasible to outline all the noteworthy contributions that have been made by the computational mathematics research community in this article.. The main aim of the reference cited is to establish a connection between existing numerical methods and those that will be introduced in the following sections.

The weak Galerkin finite element technique, the hybridizability discontinuous Galerkin (HDG) method, and the mixed finite element approach with a hybridised illustration of Fraeijs de Veubeke are all interconnected. This relationship has been elucidated and explained in detail.. The hybridised formulation introduces the Lagrange multiplier as a new term on each element's periphery. It is known that the Lagrange multiplier approaches the original function near each element's border. When it comes to certain popular mixed finite element techniques, such as the HDG approach, the WG systems mentioned in Section 4 are actually similar (1.1). The weak Galerkin method differs from previous methods when the coefficients are variable functions that are not necessarily smooth or well-behaved. Weak gradients provide a logical framework for handling such functions, even at the interfaces between different components, in a manner that is similar to the classical approach. This concept is expected to hold true for various classes of partial derivatives equations that involve derivatives differential operators, such as divergence and curl, as well as their associated constraints.

The weak Galerkin finite element technique can be applied to solve elliptic problems in one space dimension, as well as in higher dimensions.

Our research presents a weak finite technique method to solve elliptic problems in one spatial direction . We found that this technique offers significant benefeits over the well-known weak Galerkin technique, which is commonly used for multi-dimensional problems. These advantages include improved accuracy and the ability to solve discrete equations locally, element by element. We also provide the most precise wrong estimates for the discrete H1-norm, L2-norm, and L-norm, along with few super convergence results. To support our theoretical analysis, we provide numerical examples at the end.

The numerical partial differential equation community has recently shown significant interest in the weak Galerkin finite element technique.. This method for tackling elliptic issues in a multidimensional space was first put forth by Wang and Ye. Later, a number of altered weak Galerkin approaches were researched. The

weak Galerkin finite element technique is an continuous of the traditional finite element technique that replaces classical derivatives in the variational equation with weak derivatives based on weak finite element functions. This approach allows for the usage of completely stoppage finite element services , which means that the benefit of a finite element service on an element boundary can differ from its value inside the element. Compared to the traditional discontinuous Galerkin finite element technique, the weak Galerkin method is more flexible due to this property. To learn more about this technique and how it differs from other finite element methods, readers are encouraged to refer to related articles.

Following is an outline for this essay., we present the elliptic problem using a weak finite element technique. In our study, Section 3 focuses on analyzing the fixation of the weak finite element approach, while Section four is dedicated to determining the optimal error and super convergence estimates for various norms. We also delve into the local solvability of the weak finite element system and provide experiments of numerical nature to further demonstrate our theoretical framework. Our notation for the standard Sobolev spaces on the interval I with the norm (I) is denoted as Hm throughout the study. Additionally, we use the notations (,) and to represent the inner product and norm, respectively, in the L2 space on I. To provide a positive constant that is not dependent on of the mesh size h, we use the constant C.

The process of approximating a solution to a given partial differential equation problem using the weak finite element technique.

Analyze the problem of elliptic (1.1). adding the transformation function to equation (1.1)

$p(x) = \exp\left(-\int_0^x \frac{a_1(x)}{a_2(x)}dx\right)$, we see that problem (1.1). can be transformation into the

Following from: $-(pa_2\grave{u}) + pa_0 = pf(x), \quad x \in (a,b), \; u(a) = 0, \quad \grave{u}(b) = 0.$

As a result, we will only examine elliptic problems in the following discussion.

$$\begin{cases} -(a_2(x)\grave{u}) + a_0(x)u = f(x), \quad x \in (a,b), \\ u(a) = 0, u(b)\grave{} = 0, \end{cases} \quad where\ a_2(x) \geq a_{min} > 0, a_0(x) \geq 0\ and$$

$\grave{u} = \frac{d_u}{d_x}$. *we assume that* $a_2(x) \in \mathrm{H}^1(a,b), a_0(x) \in L_\infty(a,b)$.

Error analysis

This section is dedicated to examining the error in the weak finite principle approach. The illustration indicates that the convergence rate of the weak finite element technique is either equal to or greater than that of the conventional finite

element technique. The section commences by showcasing the property of approximation for of the weak finite element space, Sh. To maintain a stability in the approximation accuracy between Sh and the space Pr(Ii) utilized for dw and rv, we will always establish the index while constructing discrete weak functions..

The local solvability and example of numbers

This section revolves around conducting an analysis of error for the weak finite element technique. Our findings reveal that the theoretical convergence rate of the weak finite element approach is either similar or higher than that of the conventional finite element method. We commence by demonstrating the approximation property of the weak finite element space, Sh. To ensure a balance in the approximation accuracy between Sh and the space Pr(Ii) utilized for dw and rv, we will always establish the index when constructing discrete weak functions.

This research focuses on employing mechanics of quarter-point tetrahedral finite elements in linear elastic fracture.

The study of body cracks has garnered significant attention in various fields, include : science of material , engineering of structure , and engineering of oil and gas reservoir , due to the prevalence of intrinsic flaws and fractures in many materials and constructions. In the meaning of linear elastic fracture mechanics (LEFM), computing stress intensity components (SIFs) with precision is vital for analyzing fractured bodies.. Due to the fact that they completely specify the stress condition close to the crack, SIFs are crucial to precisely estimating the beginning of fracture propagation. The SIFs can only be used to estimate a limited number of simple fracture configurations; for more complex crack difficulties, numerical techniques like the finite element (FE) method must be utilised. When using the FE technique to examine crack difficulties, it can be difficult to accurately replicate the fracture tip single stress field and capture the high stress gradient near to the break. This is the justification for the forty years' worth of intensive study on reliable and accurate FE methods for modelling fracture problems.

In the early 1970s, the use of the finite element (FE) technique for solving problems gained popularity. However, it was discovered that conventional components were unable to accurately capture the distinctive stress field near the fracture due to the interpolation of field variables using polynomials. This led to the development of quarter-point/singular elements by Barsoum (1976) and Henshell and Shaw (1975) by put mid-side nodes near the fracture tip or front at the quarter-point location. These elements can provide the square root stress singularity through a nonlinear mapping. Utilization of Collapsed of quarter-point hexahedra, quarter-point pentahedra, and quarter-point bricks have been to study and model three-dimensional fractures. The collapsed quarter-point hexahedron is particularly useful for modelling fracture problems due to its accurate representation of the single stress field near the crack. Simple methods, such as

removal correlation and central integral techniques, have been developed for these principles to withdraw stress intensity factors (SIFs) from the FE solution. However, using these components requires the generation of a fully structured mesh around the fracture front, which can be challenging and time-consuming, especially for non simple crack and configurations of the body.

Due to the meshing restrictions associated with using quarter-point hexahedral elements, researchers have increasingly turned to tetrahedra for addressing crack concerns. One proposed method involves using both hexahedral and tetrahedral elements, taking advantage of the latter's ability to mesh complex geometries and their effectiveness in the fracture front region when quarter-point hexahedral elements are crushed. The main drawback is the need for tie limitations or transition pyramid elements when hexahedral and tetrahedral elements meet due to the incompatibility between their interface node topologies. An alternate approach involves utilizing tetrahedra to describe the structure overall and mapping solution of the FE to a sub-model that employs hexahedra to solve for the fracture variety . However, this approach necessitates two FE model evaluations and may result in higher computational costs. More recently, it has been suggested to model the entire fractured body domain using pure tetrahedra in an arbitrary and unstructured mesh, which has been successfully utilized to study crack propagation and fragmentation. However, there has not been sufficient emphasis in the literature on the effectiveness and precision of using tetrahedral elements for modelling fracture singular fields.

overview and evaluation of quarter-point finite elements.

It is well-known that classical finite elements use polynomials to interpolate field variables in the FE domain.. The fracture tip square root single stress field can't be duplicated as a result. To gather precise field variables close to the fracture, a very fine mesh is needed without the need for any formulation for the components connected to the crack tip. Early in the 1970s, when several studies recommended using specific element formulations near the fracture tip, it was discovered that FE crack problems produced unsatisfactory results when solved with standard elements. Previous studies have primarily concentrated on developing crack tip elements (CTEs) that can effectively replicate the singular fields near the tip of fracture . These principles were employed to save the region surrounding the tip of crack , while conventional elements were utilized for the remaining domain. The early development and applications of the Cessford stress intensity factor (SIF) calculation are documented in various sources (Byskov, 1970; Tracey, 1971, 1974; Benzley, 1974; Akin, 1976).. However, the use of CTEs was limited due to several factors, including the need for transition elements to connect them to conventional elements in the remote region, algorithmic quirks in commercial FE codes, and limited support for fixed strain and stiff body motion

modes in the CTE form functions. Quarter-point elements (QPEs) were proposed by Barsoum (1976) and Henshell and Shaw (1975), which greatly improved the FE analysis of fracture problems.

Barsoum (1976) and Henshell and Shaw (1975) non dependent demonstrated that a quarter-point element with a mid-side node placed adjacent to the fracture tip can accurately select the crack tip's singularity. This modification lead to non linear pattern between natural and local coordinates, This results in a stress singularity of inverse square root nature that affects the entire element.. Using quarter-point elements allows for representation of the entire fractured body domain with a single element, eliminating the need for commercial FE codes to use CTEs. Quarter-point elements are easy to construct algorithmically and automatically meet continuity requirements for shape functions between elements because they account for rigid body motion and constant stresses during form function generation.

The characteristics of quarter-point elements sparked a phase of intense research and implementation spanning four decades. These elements have been used to analyze 2D and 3D fracture issues, Especially the quadrilateral element with four nodes and eight integration points. (Fig. 1a), which can be created by relocating nodes of mid-side near the crack tip to the quarter-point location of an isoperimetric eight-noded quadrilateral. Early research on quarter-point quadrilateral elements revealed some shortcomings, Some erroneous assumptions made about the rectangular element include the belief that it modify the square root singularity solely on the boundaries of element and an incorrect demonstration that the energy strain of this factor is unbounded. . Subsequent studies, however, revealed that stresses exhibit square root singularity at all rays originating from the fracture tip in a small region near the crack tip and propagate throughout the entire element along the element sides. Additionally, it was found that the strain energy and stiffness of these elements were limited.. To obtain correct results, these components should be deformed as little as possible from a rectangle. However, because of the big factor angle at the tip of the crack, they are not frequently used and are instead replaced with triangular shaped elements (categories ii and iii) that can better reflect the angular distribution of stress.

The formulation of tetrahedral elements in finite element analysis.

The procedure involves converting the geometry and displacement variety of a ten-node structure. Isoperimetric tetrahedral element from the local coordinate technique to the coordinate naturally the system (where $0\leq\xi,\eta,\zeta\leq1$) has been provided.

$$x(\xi,\eta,\zeta)=\sum_{i-1}^{10}N_i x_i \quad , \quad y(\xi,\eta,\zeta)=\sum_{i-1}^{10}N_i y_i \quad , \quad z(\xi,\eta,\zeta)=\sum_{i-1}^{10}N_i z_i$$

$$u(\xi,\eta,\zeta)=\sum_{i-1}^{10}N_i u_i \quad , \quad v(\xi,\eta,\zeta)=\sum_{i-1}^{10}N_i v_i \quad , \quad w(\xi,\eta,\zeta)=\sum_{i-1}^{10}N_i w_i$$

the node's displacements in the x, y, and z directions are (ui,vi,wi) and The function of shape for Ni corresponds to the node with coordinates (xi, yi, zi) in the local space., respectively (Fig. 2). A finite element with ten-noded tetrahedral shape functions is given by:

$N_1=\lambda(2\lambda-1)$, $N_2=\xi(2\xi-1)$, $N_3=\eta(2\eta-1)$, $N_4=\zeta(2\zeta-1)$, (2)

$N_5=4\lambda\xi$, $N_6=4\xi\eta$, $N_7=4\lambda\eta$,

$N_8=4\lambda\zeta$, $N_9=4\xi\zeta$, $N_{10}=\zeta(2\zeta-1)$,

Bernstein-B ezier Finite factor Modeling of Scattering of Short-Wave with Improved Conformal Perfectly Matched Layers

The Finite Element Method (FEM), which is often used for domain discretization to model wave scattering in unbounded media, requires truncation of the infinite canter and the use of perfect boundary conditions. To prevent reflections, an absorbing layer, allowing waves to exit the computational domain. These two methods are commonly used for domain truncation. The Perfectly Matched Layer (PML), proposed by B. Erenger as a material of reflectionless absorbing condition that covers the area of interest, is a popular choice for an ABC method.

. Initially developed for studying electromagnetic waves, the PML has been extended to various applications, including acoustic scattering, seismology, electrodynamics, and geophysical fluid dynamics, as evidenced by surveys and references in the literature.

ABCs fall into 2 classifications : local and non-local, : locality is known as the field's exclusive reliance on fields that are local to a certain area on the outer boundary. The advantage of local ABCs is that they maintain the FEM's computational effectiveness. The following are some examples: Bayliss, Gunzburger, and Feng. The lowest order ABCs are simple to construct, but unless the fake border is positioned sufficiently distant from the scatterer, suitable precision cannot be reached. High order ABCs make it possible to obtain good accuracy, but their use is challenging and computationally intensive. The ABCs are listed in Reference. Nonlocal ABCs includes: the truncated Dirichlet-to-Neumann (DtN) can position the artificial boundary closer to the scatterer, but this can lead to the creation of a dense sub-block in the FE global matrix next to the outer border , which increases the computational cost of solving the linear pattern, especially in 3D applications. Other methods for domain truncation and boundary condition specification include the continued-fraction ABC, double absorbing boundary technique , infinite elements, wave envelope, and boundary element methods.

However, the PML has the advantage of being able to be put near to the scatterer, similar to non-local ABCs, while still maintaining the computational efficiency of local ABCs within the FEM. For more information and discussion on these methods, readers are encouraged to consult the literature.

. However, the discrete level of the PML is not completely reflection-free, which may result in erroneous reflections of outgoing waves and potentially contaminate solution across the entire computational factor .

Hopefully, by modifying the FE discretization and PML settings, this source of inaccuracy may be reduced. The PML and its expansions to more broad geometries and applications are extensively covered in the literature. Cartesian coordinates and straight or flat artificial boundaries are used in the majority of PML designs. Deriving PMLs in different coordinate systems is important for addressing specific issues. For example, in electromagnetic wave time domain calculations, corner areas can be a significant source of reflection errors. Additionally, research has shown that a poor choice of time step can lead to long-term instabilities of Cartesian PMLs. The issues with Cartesian PML have led to its extension to cylindrical and spherical dimensions using complex coordinate stretching. The optimal PML parameters for performance were identified through theoretical research in curvilinear coordinates. The comparison of ellipsoidal PML against infinite elements demonstrated the scalability and iteration count advantages of PML. A parameter-free PML using singular absorbing services was explored, and convex-shaped geometries were addressed in the time harmonic pattern. The PML formulation involves curvilinear coordinates (ζ1, ζ2) in the layer Ωpml with x = x(ζ1, ζ2) and a well-defined orthogonal projection of x onto the interface Σ denoted by . The coordinate ζ1 represents the distance from x to p, while the connect ζ2 is a local parameterization of the interface Σ using its arc length, and n and t are the chosen unit outward normal and tangent, respectively, satisfying the Frenet formulas.

$$\frac{d_p}{d\zeta_2} = t \ \ and \quad \frac{d_n}{d\zeta_2} = \quad kt, \qquad (3.1)$$

Any point x inside the layer Ωpml can be expressed as a service of the two curvilinear coordinates ζ1 and ζ2, where ζ1 represents the distance between x and a point p, and ζ2 is a local parameterization interference of Σ using its arc of length. The curvature κ of Σ at the point p, which is also a function of ζ2, is used in the expression. The unit outward normal n and tangent t are also defined using ζ2, and satisfy the Frenet formulas.

$$x(\zeta_1, \zeta_2) = \zeta_1 n(\zeta_2) + p(\zeta_2). \qquad (3.2)$$

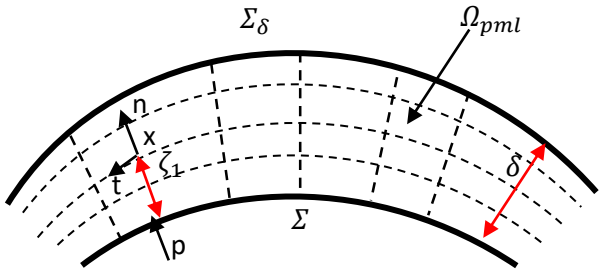

Figure 2 : orthogonal curvilinear coordinates system.

algebraic results.

The accuracy of rPML and ePML was tested through two-dimensional landmarks that utilized BBFEM with condensation of static nature, which minimizes the bandwidth and degree of freedom of high-order FE global matrices. In this technique, the inner mode DoF is removed during the assembly process of the discrete algebraic system.The inner mode solution is obtained by resolving minute linear algebraic equations at the elemental level after the solution for the element border modes. The wavenumber is k= since all of the following assumes that the medium's propagation speed is unity. The Gauss-Legendre integration method is used for evaluating element matrices of affine elements with p+1 integration points. For curved components, the number of quadrature points is determined by the desired precision and a rule of 2p+1 points is established. To evaluate the performance of the Gordon and Hall blending map approach, a benchmark test involving radiation from a Hankel source is conducted. The convergence of BBFEM coupled with rPML is then examined using a single wave scattering problem, and the discrete PML parameters' effect on accuracy is investigated. The final benchmark test involves multiple scattering and compares rPML and ePML to rBGT2 and eBGT2.

Benchmarks's Analysis

The two benchmark problems have computational domains with an annular shape, as depicted in Figure 4. The rBGT2 domain has an inner radius of "a" and an outer radius of "b". On the other hand, the rPML domain has an inner radius of "a" and an outer radius of "a+PML thickness".

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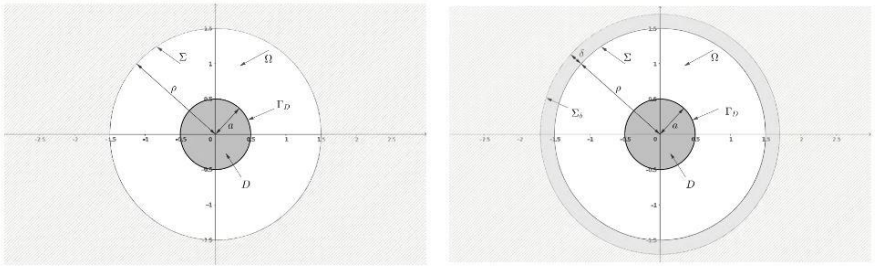

Figure 4: Computational domain for the first two benchmarks: (left) rBGT$_2$ domain; (right) rPML domain.

Both variety of quarter-point tetrahedral factors are effective in accurately modeling a square root stress singularity close to the crack front. However, there is a small area at the curved shape of the quarter-point tetrahedra connected to the fronts of curved crack where the Jacobian becomes negative. To avoid this problem, it is recommended to use quarter-point tetrahedra with straight sides to simulate the curved sides of the crack fronts. These elements can also simulate a square root displacement shapes close to the crack front, which is confirmed by numerical results on relative displacements over fracture surfaces. The displacement correlation (DC) pattern is suggested for computing precise stress intensity factor (SIF) estimates, which is a computationally inexpensive technique that can be used with unstructured models even when the crack surfaces are not aligned. The DC method can be easily incorporated into any FE code. The conclusions of using the DC approach have been proven for various fracture geometries in mode I and mixed-mode loadings, with an mean SIF computation error ranging from 1% for through-the-thickness fractures to about 4% for elongated elliptical ones. A comparison of conclusions obtained using the DC approach for standard and quarter-point elements shows that the average SIF computation error are much than doubles when quarter-point tetrahedra are used in the crack front area. The mean SIF computation error using the DC approach reaches its lowest value at an optimum distance from the fracture front, which depends on the mesh size according to a detailed parametric analysis. This study supports the use of unstructured meshes for accurate and successful investigations of fractured bodies.

Conclusion

The idea of curvilinear PMLs has been placed to frequency domain simulations of short-wave scattering using B-ezier-Bernstein based finite elements. Efficiently resolving the PML (perfectly matched layer) requires the insertion of high-order finite elements (FEs) on computational mesh grids with large elements relative to the wavelength. The Gordon and Hall blending map is effective in portraying curved geometries in these applications. Standard FEMs must choose between using an expensive PML that allows for high absorption to reduce the mirror of outgoing waves or an inexpensive PML that requires less computation to provide solution.

Numerical condition have been conducted to evaluate the effectiveness of the suggested PMLs. Studies comparing the radial PML to a second-order ABC (absorbing boundary condition) with a radial shape and those working with Hankel source radiation and wave scattering by a rigid cylinder have shown that it allows for the improvement of the exponential and algebraic convergence rates predicted for the p and h patterns of BBFEM (Blended Boundary Finite Element Method), respectively.

For scattering problems with multiple modes, the radial ABC can lead to comparable outcome if a precise result is not necessary. Numerical conclusions from a scenario involving multiple scattering by several rigid cylinders have shown that PMLs with radial and elliptical shapes can produce extremely accurate results. The research highlights the value of elliptical PMLs for reducing the computational domain. The absorption function of parabolic does not require a non-trivial optimization process since BBFEM can effectively image the rapid decay of emitted waves in the layer. Thick PMLs can be used to achieve optimal performance.

In general, PML settings do not need to be adjusted for each problem since they have been successful across a wide range of frequencies, grid resolutions, and polynomial orders. Recent studies have shown improved performance of PMLs based on unbounded absorption functions, and future research should consider a comparison study using high-order FEs in this approach.

. A technique based on curved tetrahedral Bezier-Bernstein FEs is being developed, and the use of high-order refinement methods is a potential way to enhance computing efficiency.

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