Baghdad Science Journal

Volume 22 | Issue 7 Article 16

7-18-2025

Experimental Formula of Single Particle Level Density in Pre-Equilibrium Reaction

Sarah Shakir Ali
Physics Department college of Science For women, University of Baghdad, Baghdad, Iraq,
sara.ali2204m@csw.uobaghdad.edu.iq

Ali Dawoud Salloum
Physics Department college of Science For women, University of Baghdad, Baghdad, Iraq,
alisaloum39@yahoo.com

How to Cite this Article

Ali, Sarah Shakir and Salloum, Ali Dawoud (2025) "Experimental Formula of Single Particle Level Density in Pre-Equilibrium Reaction," *Baghdad Science Journal*: Vol. 22: Iss. 7, Article 16.
DOI: https://doi.org/10.21123/2411-7986.4996

BAGHDAD SCIENCE JOURNAL 2025;22(7):2300–2306

RESEARCH ARTICLE

Experimental Formula of Single Particle Level Density in Pre-Equilibrium Reaction

Sarah Shakir Ali *, Ali Dawoud Salloum

Physics Department, College of Science for Women, University of Baghdad, Baghdad, Iraq

ABSTRACT

In this study, a new formula for single particle level density was found from experimental values of single particle level density using the curve fitting. This new formula was substituted in Ericson's formula which is used to calculate the partial level density in pre-equilibrium nuclear reactions and in the corrected formulas of William's formula, spin corrected formula and surface formula. The results for each formula were compared with the results for the same formula using single particle level density from the equidistant spacing model ESM. It was noticed that there is good agreement between the results of single particle level density from ESM and single particle level density from curve fitting especially at energies less than 20MeV. With increasing energy, the results of single particle level density from curve fitting become higher than the results of single particle level density from ESM. In one-component Ericson's formula the difference is clearer than in other formulas, in a spin correction the difference becomes less than one-component Ericson's and in case of two-components Ericson's correction, surface correction and Williams's correction the difference is so little that it is difficult to notice. The increase in level density of single particle level density from curve fitting is greater than the level density of single particle level density from ESM, reflecting the increase in level density with increasing energy, therefore, as it is shown the single particle level density from curve fitting is more accurate than single particle level density from equidistant spacing model ESM.

Keywords: Nuclear level density, Nuclear models, Nuclear reactions, Pre-compound reactions, Pre-equilibrium region, Statistical models of nucleus

Introduction

To study the energy levels of any nucleus theoretically, at energies above 2Mev, the nuclear levels cannot be treated as separate levels because the spacing between the levels decreases with increased energy since The number of the levels increases with the energy leading to an overlap between the levels, therefore, dealing with each level separately becomes impossible, thus a new concept called Nuclear Level density NLD was introduced which describes a huge number of level.[1–4]

Nuclear Level density NLD has great importance in the study of nuclear reactions in the construction and design of nuclear reactors, as well as in the study of reactions within stars and it also has applications in the medical fields.[5] Also the cross section of compound nucleus reactions and the pre-compound reaction depend mainly On the NLD.[6,7]

In the case of pre-compound nucleus; not all nucleons are present because the distribution of the incident particle energy is not complete, therefore, the NLD in pre-compound is called partial level Density PLD.[8]

The density of partial states depends on the parameter called single particle level density(g). In this research, the formula for the individual particle g is obtained from using curve fitting to experimental data by taking g from the reference and substituting it in Ericson's equation, which describes the partial

Received 27 November 2023; revised 13 April 2024; accepted 15 April 2024.
Available online 18 July 2025

* Corresponding author.
E-mail addresses: sara.ali2204m@csw.uobaghdad.edu.iq (S. S. Ali), alisaloum39@yahoo.com (A. D. Salloum).

https://doi.org/10.21123/2411-7986.4996

level density of states and is also substituted in the corrected formulae. And a comparison was made with the results of the same formula when single particle level density (g) is taken from equidistant spacing model ESM. The aim of this study is to get a new formula of g from the experimental values and the agreement with the ESM results gives good credit to ESM model results.

Materials and methods

Theory

The pre-equilibrium region is a stage of the nuclear reaction that determines the completeness of the distribution of energy among all the nucleons in the nucleus.

This region was suggested by the researcher J.J Griffin[8] when a part of the emission cross section cannot be interpreted theoretically neither by the compound nucleus model nor by the direct reaction model.

The pre-equilibrium cross -section depends on the PLD and the first description of the PLD was by Griffin using Ericson' s formula.[8,9]

$$\omega_1(n,E) = \frac{g^n E^{n-1}}{p!h!(n-1)!} \tag{1}$$

The symbols p, h and n are particle number, hole number, and exciton number respectively
E is the excitation energy, and g is the single particle level density, the exciton number is given by

$$n = P + h \tag{2}$$

This formula is called the one component Ericson's formula because it considers all nucleons, protons and neutrons as a same type of particles.

But if the protons and the neutrons are considered as different types of particles Ericson's formula is written in the following form and called two-component Ericson's formula.[9]

$$\omega_2(n,E) = \frac{(g_\pi)^{n_\pi}\ (g_\upsilon)^{n_\upsilon} E^{n-1}}{p_\pi!h_\pi!p_\upsilon!h_\upsilon!(n-1)!} \tag{3}$$

g_π and g_υ are the single particle level density formula of protons and neutrons respectively, they are given by[9]

$$g_{\pi=\frac{z}{A}g} \tag{4}$$

$$g_{\nu=\frac{N}{A}g} \tag{5}$$

Where Z and N are the number of protons and neutrons respectively

And the formula from fitting of the experimental value is

$$g = 0,58A + 1,4$$

n=$p_\pi + h_\pi + p_\upsilon + h_\upsilon$ is the total exciton number. It can be written as
$n = n_\pi + n_\upsilon$ where n_π is the exciton number of proton and n_υ is the exciton number of neutrons.

Ericson's formula is a crude formula, but many corrections where added to it, in this paper some of these corrections are taken like:

1-William's correction, in this correction the effect of the Pauli exclusion principle was added to the PLD formula and the one- component formula becomes[9]:

$$\omega_1(n,E) = \frac{g^n\left(E - A_{p,h}\right)^{n-1}}{p!h!(n-1)!} \tag{6}$$

The effect of Pauli's principle is represented by a factor $A_{P,h}$ which is called pauli blocking factor

This factor subtracts from excitation energy and is given by[9]

$$A_{p,h} = \frac{P(P+1) + h(h-3)}{4g} \tag{7}$$

In case of two - components the PLD formula becomes[9]

$$\omega_2(n,E) = \frac{g_\pi^{n_\pi} g_\nu^{n_\nu}\left(E - A_{p_\pi,h_\pi,p_\nu h_\nu}\right)^{n-1}}{p_\pi!h_\pi!p_\nu!h_\nu!(n-1)!} \tag{8}$$

$$A_{p_\pi,h_\pi,p_\nu h_\nu} = \frac{p_\pi(p_\pi+1) + h_\pi(h_\pi-3)}{4g_\pi} + \frac{p_\nu(p_\nu+1) + h_\nu(h_\nu-3)}{4g_\nu} \tag{9}$$

2-Spin correction

In this correction the PLD is multiplied by the factor$R(J)$ that represents the Gaussian distribution of the angular momentum.[9]

$$R(J) = \frac{2J+1}{2\sqrt{2\pi}\sigma^3} exp\left[-\frac{\left(J+\frac{1}{2}\right)^2}{2\sigma^2}\right] \tag{10}$$

The parameter (J) represent the total angular momentum of the target nucleus and σ is the spin cut off parameter.

Then the PLD become[9]

$$\rho(E,J) = \omega_1(n,E)R(J) \tag{11}$$

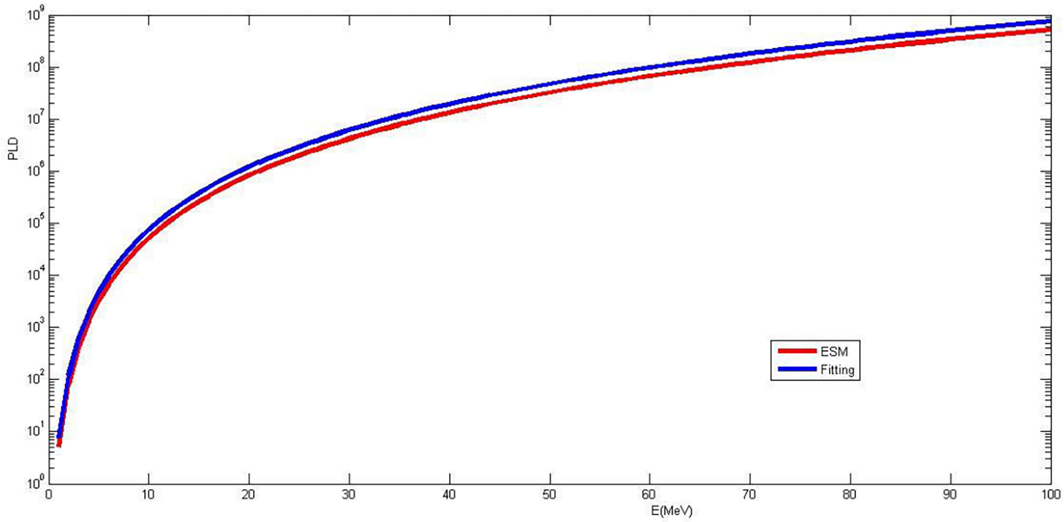

Fig. 1. Comparison of one component Ericson from ESM and fitting.

3-Surface correction

Since the nucleon at the surface of the nucleus is not surrounded by other nucleons like the ones inside the nucleus, therefore, the nuclear potential on the surface is shallower than inside the nucleus. This difference between the potential value is added as a correction called surface correction and it is added to PLD as in equation[9]

$$\omega_1(n, E, V) = \omega_1(n, E, \infty) \times f_1(n, E, V) \tag{12}$$

The function $F_1(n, \text{E}, \text{v})$ is the correction due to surface effect in the ESM it is given by[9]

$$F_1(n, E, v) = \sum_{j-0}^{h} (-1)^j C_j^h \left[\frac{E - jv(\boldsymbol{h})}{E}\right]^{\boldsymbol{n}-1} \times \theta\left(E - \boldsymbol{jv}(\boldsymbol{h})\right) \tag{13}$$

This is done in the initial particle -nucleus interaction by replacing the nuclear potential depth V_0 by $\overline{V_1}$

This means instead of putting $= V_0$, we put $V = \overline{V_1}$.

This new potential depth $\overline{V_1}$, is defined as the "average effective well depth

$V = V_0 = 38\ MeV \qquad h > 1$

$V = \overline{V_1} \qquad h \leq 0$

Results and discussion

In this section, a comparison will be made between the results of the Ericson's equation and its corrections when g is used from ESM and the same equations but when g is obtained from curve fitting, the results are drown using Mat. Lab program.

Fig. 1 shows a comparison between the results of PLD from the one component Ericson formula with single particle level density g from ESM and PLD from one-component Ericson when single particle level density g from curve fitting for the experimental values are taken from reference. It is noticed that the two curves are identical starting from more than 10Mev, after that the fitting curve becomes slightly higher than the curve from ESM. This can be interpreted as; since the energy levels increase with increasing the energy the PLD must increase and since ESM considers the spacing between the levels equal, therefore, it cannot describe the increase in PLD while it appears for g from fitting, hence this leads us to think that g from fitting is better than single particle level density g from ESM. The curve fitting results have good agreement with the results in published papers.[10]

Fig. 2 shows a comparison between the Ericsson curve for the cases of two components when g is from ESM and the Ericsson curve when g is from fitting. It is noticed that both curves start from energies higher than 0Mev and increase with increasing energy and there is a good agreement at all energy values. The curve fitting results have good agreement with the results in published papers.[10]

Fig. 3 shows a comparison of the PLD curve of William correction when g is taken from the Ericson formula and the curve of the William correction when g is taken from the fitting. From this figure,

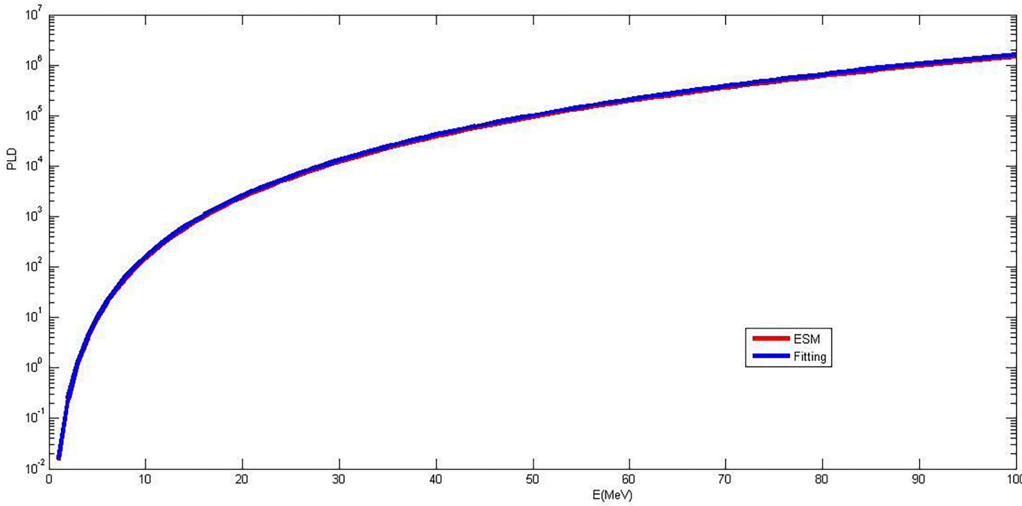

Fig. 2. Comparison of two -component Ericson from ESM and fitting.

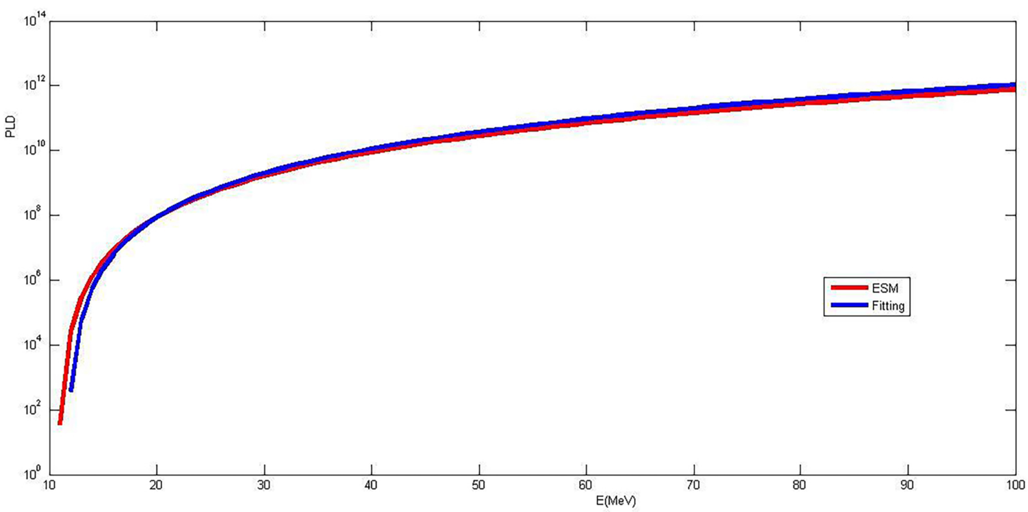

Fig. 3. A comparison between the results of one-component Williams's formula when g *from ESM and* g from fitting.

it is observed that both Williams's curves start after 10Mev since the factor A_{Ph} substructs from E, then the curve when E $<$ A_{ph} is decreased, therefore, when neglecting these regions one finds there is a good agreement between both Williams's curses from ESM and fitting. The two curves increase with increasing the excitation energy.

The curve fitting results have good agreement with the results in published papers.[10]

Fig. 4 shows a comparison between the corrected PLD curve for the total angular momentum when g is taken from Ericsson, and the curve of angular momentum when g is taken from curve fitting. It is noted that the two curves start at 1Mev and increase with increasing the energy. There is a good agreement between both curves till up to 30Mev then the curve of PLD from fitting rises slightly above the curve from ESM this may be interpreted as the g from fitting being better than g from ESM to describe the increase in levels with increasing the energy. The curve fitting results have good agreement with the results in published papers.[10]

Fig. 5 explains the comparison between the results of PLD from the surface corrected formula when g is from ESM and *when* g is from curve fitting. It is noticed that both curves start from 10Mev because all values of PLD when $jv > E$ are neglected, therefore, the curve does not start from 1Mev. Both curves increase e with increasing the energy and there is a good agreement between the two curves .It is noticed in all figures that the agreement is at low energies and with increasing the energy the curve from fitting becomes higher than the curve from ESM but the magnitude of the difference changes

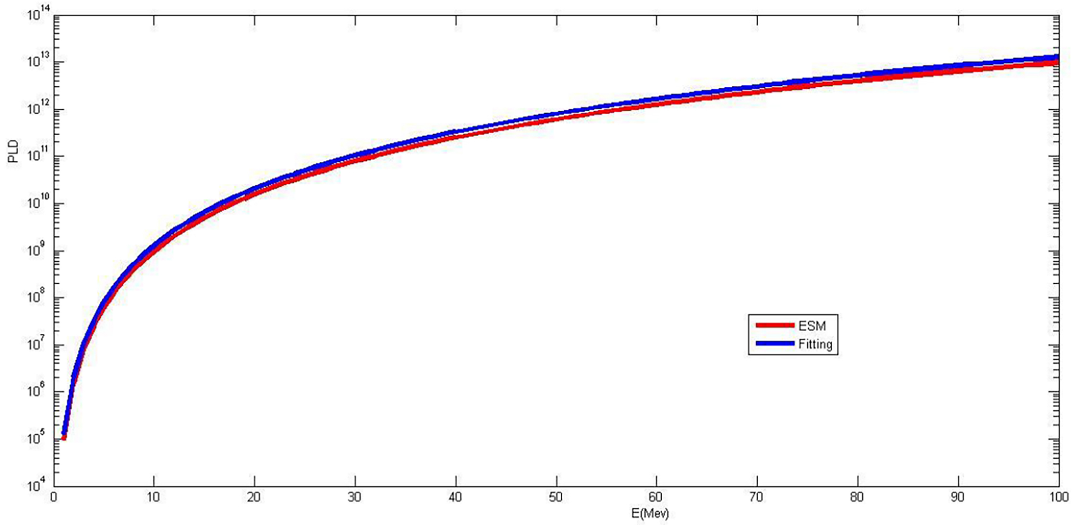

Fig. 4. Comparison between the results of PLD from spin corrected formula when g *From ESM and* g from fitting.

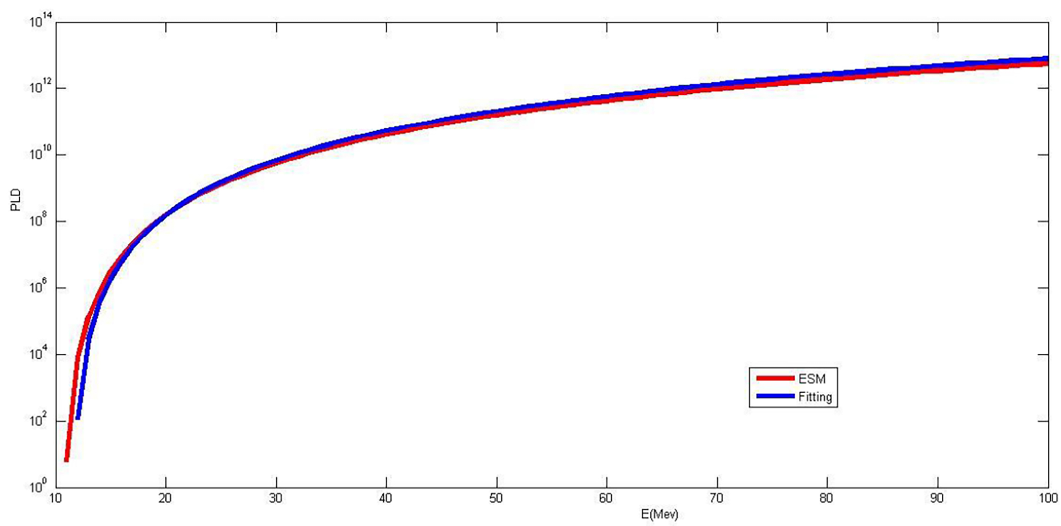

Fig. 5. A comparison of PLD from surface corrected formula when g *from ESM and* g from fitting.

from one formula to another. The curve fitting results have good agreement with the results in published papers. [10] PLD energy better than ESM,therefore, one can say that g from fitting is more realistic than that from ESM.

Conclusion

The results of all PLD formulae increase with increasing the excitation energy E .In general most of PLD curves from fitting g and those from ESM show good agreement and with increasing the energy the PLD curve from fitting becomes slightly above that from ESM, the increase depends on the formula this mean's the g from fitting can describe the increase of

Authors' declaration

- Conflicts of Interest: None.
- We hereby confirm that all the Figures in the manuscript are ours. Furthermore, any Figures and images, that are not ours, have been included with the necessary permission for re-publication, which is attached to the manuscript.
- No animal studies are present in the manuscript.

- No human studies are present in the manuscript.
- Ethical Clearance: The project was approved by the local ethical committee at University of Baghdad.

Authors' contribution statement

A. D. S proposed the topic of the research, guided and supervised the student as well as reviewed and proofread the research S.S. A wrote the paper and made the calculations then the discussion and the results.

References

1. RahmatineJad A, Bezbakh AN. Levelenlargethispage5pt density parameters of heaviest nuclei. Acta Phys pol B. 2020;13(3):491–498. https://doi.org/10.5506/APhysPolBSupp.13.491.
2. Magner AG, Sanzhur AI, Fedotkin SN, Levon AI, Shlomo S. Level density within a micro-macroscopic approach. Nucl Phys A. 2022;1021:122423. https://doi.org/10.1016/j.nuclphysa.2022.122423.
3. Pahlavani MR, Dinan MM. Thermal properties off 172Yb and 162Dy isotopes in the back -shifted Fermi gas model with temperature-dependent pairing energy. J Phys. 2019;93(37):1–10. https://doi.org/10.1007/s12043-019-1799-y.
4. Mohanto G, Rout PC, Ramachandran K, Mirgule ET, Srinivasan B, Kundu A. Probing collective enhancement in nuclear level density with evaporation α particle spectra. Phys Rev. 2022;105(3):1–11. https://doi.org/10.1103/PhysRevC.103.0343092.
5. Goriely S, Larsen A, Mucher D. Comprehensive test of nuclear level density models. Phys Rev C. 2022;106(044315):1–11. https://doi.org/10.48550/arXiv.2301.11013.
6. Bucurescu D, Egidy TV. Systematic of nuclear level density parameters. Phys J phys G. 2005;31(10):44311. https://doi.org/10.1088/0954-3899/31/10/052.
7. Rahmatinejad A, Bezbakh A N, Shneidman T M, Adamian G, Antonenko N V, Jachimowicz P, *et al.* Level-density parameters in super heavy nuclei. Phys Rev C 103. 2021;103(3):1–10. https://doi.org/10.1103/PhysRevC.103.034309.
8. Abdulla AM, Salloum AD. A comparison between the theoretical cross section based on the partial level density formulae calculated by the exciton model with the experimental data for ${}_{79}{}^{197}Au$ nucleus. Baghdad Sci J. 2021;18(1):139. https://doi.org/10.21123/bsj.2021.18.1.0139.
9. Sukovoj AM., Khitrov VA. Partial level density of the n-quasiparticle excitation in the nuclei of the $39 < A < 201$ region. Phys Part Nucl. 2006;37(6):899–922. https://doi.org/10.1134/S1063779606060049.
10. Alwan TA, Hamed BS. Study the nuclear structure of some even-even Ca Isotopes using the microscopic theory. Baghdad Sci J. 2023;20(1):235–244. http://dx.doi.org/10.21123/bsj.2022.6924.

صيغة تجريبية لكثافة الحالات الجزئية في تفاعلات قبل التوازن

سارة شاكرعلي ، علي داود سلوم

قسم الفيزياء،كلية العلوم للبنات، جامعة بغداد، بغداد، العراق

الخلاصة

في هذه الدراسة تم ايجاد صيغة جديدة لكثافة الحالات الجزئية g من القيم العملية لg باستخدام توفيق المنحنيات. وهذه الصيغة الجديدة عُوضت في صيغة اركسون المستخدمة لحساب كثافة الحالات الجزئية في تفاعلات لما قبل توازن التفاعل النووي وفي الصيغ المصححة وهم صيغة وليم والصيغة المصححة للزخم الزاوي الكلي والصيغة المصححة لتأثير السطح. النتائج لكل صيغة قورنت مع النتائج لنفس الصيغة باستخدام g من نموذج الفسح المتساوية ESM ، حيث لوحظ أن هناك اتفاق جيد بين النتائج عندما g من ESM و g من توفيق المنحنيات وخصوصاً عند الطاقات اقل من 20Mev. ومع زيادة الطاقة فأن النتائج عندما g من توفيق المنحنيات تكون اكثر منها عندما g من ال ESM لبعض منهم. في حالة صيغة اركسون للمركبة الواحدة يكون الفرق أكثر وضوحا منه بالصيغ الأخرى في الصيغة المصححة للزخم الزاوي الكلي يكون الفرق اقل منه في صيغة اركسون للمركبة الواحدة. وفي حالة صيغة اركسون للمركبتين وصيغة وليم وصيغة السطح يكون الفرق صغير جدا لذلك تكون ملاحظته بصعوبة والزيادة في كثافة الحالات عندما g من توفيق المنحنيات اكثر من كثافة الحالات عندما g من ESM يعكس الزيادة في كثافة الحالات مع زيادة الطاقة ،لذلك وكما هو واضح أن g من توفيق المنحنيات تكون أكثر دقة من g من ESM.

الكلمات المفتاحية: كثافة الحالات النووية، الموديلات النووية ، التفاعلات النووية ،تفاعلات النواة قبل المركبة ،منطقه قبل التوازن،الموديلات الأحصائية للنواة.