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ℷ –commuting operator equation

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Abstract

In this work, we lead a study on new popularization for hyponormal operation, that is (h, M). hyponormal operator. Too, we've given various_characteristics for this concept. As well as, we solve the commuting operator equation for (h, M)-hypononmal operation.

Keywords: Hilbert space, Bounded linear 0perators, Hyponormal operators, M-hyponormal operator,(M, h)-hyponormal operators.

معادلة عامل النقل – ℷ

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خلاصة

في هذا العمل، قمنا بإجراء دراسة حول تعميم جديد للعملية الناقصية، وهي (.h,M)عامل غير طبيعي. لقد قدمنا أيضًا خصائص مختلفة لهذا المفهوم. بالإضافة إلى ذلك، قمنا بحل معادلة مشغل التنقل للعملية الناقصية (.h,M)

الكلمات المفتاحية: فضاء هيلبرت، المؤثرون الخطيون المحدودون، عوامل النقص الطبيعي، عوامل النقص الطبيعيM ، M ، - hعوامل النقص الطبيعي.

1. Introduction

In the fifties of the last country Halmos P.R. [5] presented the concept of hyponormal operator, after which many studies and were presented on this subject and many studies and many generalizations were made for this concept. In (1961) Berberain S.K. [1] was given selected properties of hyponormal operators. In (1962) Stampfli J.G. [11]proved selected properties of hyponormal operators, one of them is proved that the hyponormal operators T is normal if $T^n$is normal, where n is positive integer number. In (1966) Coburn L.A. [3] was proved that Weyls theorem holds for any hyponormal operator.

In (1967) Istratescu V. [6], introduced the class of paranormal operators on Banach space. In (1972) Devi S. [4] well-defined the course of quasi-

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hyponormal operators and generalizing the notion of hyponormal in other way. In (1974) Patel S.M. [8] introduce the *-paranormal operators, and in the same year Stampfli J.G. (unpublished) has initials the study of M-hyponormal operators and Wadhwa B.L. [13] investigated some spectral theoretic aspects of M-hyponormal operators in his doctoral dissertation submitted to the Indians University. In (1975) Shah N.C. and Sheth I.H. [9], were studied some properties of *-paranormal operators.

Now this paper, we lead a new popularization for hyponormal operators, that is (h, M)-hyponormal operator then we discuss certain_characteristics that_for present concept hold, in addition, we give some examples for the properties that this concept dose not achieve. Then we will give the condition that must be met to hold these properties.

Definition 1.1.

Let $T: \mathcal{H} \longrightarrow \mathcal{H}$ stay_a restricted lined factor on a Hilbert planetary $\mathcal{H}$, then T is called hyponormal operator if $T^*T \geq TT^*$, that is $(T^* T_x, x) \geq (TT^*_x, x)$, for every $x \epsilon \mathcal{H}$.

Definition 1.2.

Hire agreement $T: \mathcal{H} \longrightarrow \mathcal{H}$ stay a restricted lined on a Hilbert planetary $\mathcal{H}$, now T is believed to be $\mathcal{M}$-hyponrmal operator if there exists a positive real number $\mathcal{M} \geq 1$, such that $MT^*T \geq TT^*$.

Definition 1.3.

Let $T: \mathcal{H} \longrightarrow \mathcal{H}$ stay a restricted lined factor on a Hilbert planetary $\mathcal{H}$, then T is believed to is unionist factor if $TT^* = T^*T = I$.

Definition 1.4.

Let $S, T: \mathcal{H} \longrightarrow \mathcal{H}$ stay a restricted lined factor on a Hilbert planetary $\mathcal{H}$, at that point S and T are said to be ϡ- shuttling operators if $ST = ϡTS \neq \hat{0}, ϡ \in \mathbb{C} \setminus \{0\}$.

2. Mean Result

Definition 2.1.

Agreement $T, h : \mathcal{H} \longrightarrow \mathcal{H}$ stay a restricted lined factors on a Hilbert planetary $\mathcal{H}$, like that $hT = Th, hT^* = T^*h$. The factor T is believed to be $(h, \mathcal{M})$-

hyponormal operator if there occurs a affirmatory actual figure_$\mathcal{M} \geq 1$, such that $\mathcal{M}h\mathrm{T}^*\mathrm{T} \geq \mathrm{TT}^*h$.

To clarify this acquainting, study the next paradigm.

Paradigm

1. AgreementT $= \begin{bmatrix} 5 & 2 \\ -2 & 5 \end{bmatrix}, h = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$, now T is ($h, \mathcal{M}$)-hyponormal operator as soon as $\mathcal{M} = 2$, since $\mathcal{M}h\mathrm{T}^*T$-$\mathrm{TT}^*h = \begin{bmatrix} 87 & 0 \\ 0 & 87 \end{bmatrix}$
2. Let $\mathrm{T} = \begin{bmatrix} 5 & 3 \\ 5 & 3 \end{bmatrix}, h = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$, now$T$ is not (h , $\mathcal{M}$)-hyponormal operator for any positive real numeral $\mathcal{M}$ ≥1.

Proposition 2.2.

Lease $\mathrm{T}: H \longrightarrow H$ be (h , $\mathcal{M}$)-hyponormal operator on a Hilbert space H, now :

i- Obviously that if $h = 1$ and$\mathcal{M} = 1$, now T be hyponormal operator.

ii- Condition L is latched subconcept of $\mathcal{H}$ and fixed beneath $\mathrm{T}|._L$. is (h , $\mathcal{M}$)-hyponormal operator.

Proof

(ii)

$$\begin{aligned}\mathcal{M}h(\mathrm{T}|._L)^*(\mathrm{T}|._L) &= \mathcal{M}(h|_L)(T\mathrm{T}^*|._L)(\mathrm{T}|._L) \\ &= (\mathcal{M}h\mathrm{T}^*\mathrm{T})|_L \\ &\geq (\mathrm{TT}^*h)|_L \\ &= (\mathrm{T}|_L)(\mathrm{T}^*|_L)(h|_L) \\ &= (\mathrm{T}|_L)(\mathrm{T}^*|_L)^*(h|_L)\end{aligned}$$

$= (\mathrm{T}|_L)\,(\mathrm{T}|_L)^*\ h$

Thus; $T|._L$ is $(h, \mathcal{M})$-hyponormal operator.

Proposition 2.3.

If $\mathrm{T}: \mathcal{H} \longrightarrow \mathcal{H}$ stand an $(h, \mathcal{M})$-hyponormal factor on a Hilbert planetary $\mathcal{H}$, now:

i. $ኢ\mathrm{T}$ is $(h, \mathcal{M})$- hyponormal factor, for any $ኢ \in \mathbb{C}$.

ii. $(\mathrm{T} - ኢI)$ is (h, M)-hypononmal factor, aimed at several $ኢ \in \mathbb{C}$.

iii. condition T and h are reversible factor, now T^{-1} is $(h, \mathcal{M})$-hyponormal factor.

Probative

i.

$Mh(ኢ\mathrm{T})^*(ኢ\mathrm{T}) = Mh\bar{ኢ}\,\mathrm{T}^*ኢT$

$= \bar{ኢ}ኢ(\mathcal{M}h\mathrm{T}^*\mathrm{T})$

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$\geq \bar{\lambda}\lambda(TT^*h)$
$= (\lambda T)(\lambda T)^*h$
Hence, (λT) be $(h,\mathcal{M})$-hyponormal operator.
ii.
$Mh(T-\lambda I)^*(T-\lambda I) = Mh(T^*-\bar{\lambda}I)(T-\lambda I)$
$= Mh(T^*T-\lambda T^*-\bar{\lambda}T+\bar{\lambda}\lambda I)$
$= MhT^*T - M\lambda hT^* - M\bar{\lambda}hT + M\bar{\lambda}\lambda h$
$\geq TT^*h - \lambda hT^* - \bar{\lambda}hT + \bar{\lambda}\lambda h$
$= TT^*h - \lambda T^*h - \bar{\lambda}Th + \bar{\lambda}\lambda h$
$= \left(TT^* - \lambda T^* - \bar{\lambda}T + \bar{\lambda}\lambda\right)h$
$= (T\left(T^*-\bar{\lambda}I\right) - \lambda\left(T^*-\bar{\lambda}I\right)h$
$= (T-\lambda I)(T-\lambda I)^*h$
Hence, $(T-\lambda I)$ is $(h,\mathcal{M})$-hyponormal operator.
iii.
assume T is (h,M)-hyponormal operator, now
$MhT^*T \geq TT^*\mathrm{h}$
$(TT^*h)^{-1} \geq (\mathcal{M}hT^*T)^{-1}$
$(hTT^*)^{-1} \geq \mathcal{M}^{-1}(T^*Th)^{-1}$
$M(T^*)^{-1}T^{-1}h^{-1} \geq h^{-1}T^{-1}(T^*)^{-1}$
$M(T^*)^{-1}T^{-1}\mathrm{h}.h^{-1} \geq \mathrm{h}.h^{-1}T^{-1}(T^*)^{-1}$
$Mh(T^{-1})^*T^{-1} \geq T^{-1}(T^{-1})^*h$
And so; T^{-1} be $(\mathrm{h},\mathcal{M})$-hyponormal operator.
Proposition 2.4.
Lease, $T: H \longrightarrow H$ is $(h,\mathcal{M})$-hyponormal operator on a Hilbert space, $T^*T^n = T^nT^*$, now T^n be $(h,\mathcal{M})$-hyponormal operator.
Proof
As T is $(h,\mathcal{M})$-hyponormal operator, then T^n is $(h,\mathcal{M})$-hyponormal operator for $.n = 1$.
Reason, T^n is $(h,\mathcal{M})$-hyponormal operator, $n = m$;
$Mh(T^m)^*T^m \geq T^m(T^m)^*h$
At present, showing this T^n be $(h,\mathcal{M})$-hyponormal operator if $n. = m. + 1$
$(T^{m+1})(T^{m+1})^*.h = TT^mT^*(T^m)^*.h$
$= TT^*T^m(T^m)^*.h$ (as $T^*T^n = T^nT^*$)
$\leq MTT^*h(T^m)^*T^m$
$= MhT(T^{m+1})^*T^m$
$= Mh\,(T^{m+1})^*\,T\,T^m$

$= Mh\,(T^{m+1})^*\,(T^{m+1})$

Hence; T^n be $(h,\mathcal{M})$-hyponormal operator.

Reminder 2.5.

Agreement $T,S:\mathcal{H}\longrightarrow\mathcal{H}$ is $(h,\mathcal{M})$-hyponormal operator on a Hilbert space $\mathcal{H}$, now $(T+S)$be not certainly (h,M)-hyponormal operator. To clarify this, study the resulting case:

Example 2.6.

The operator T $=\begin{bmatrix}5 & 2\\ 2 & 5\end{bmatrix}$ be $(h,\mathcal{M})$-hyponormal operator, when $h=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$ and M=1. And S= $\begin{bmatrix}0 & 1\\ 7 & -2\end{bmatrix}$ *be* (h,M)-hyponormal operator , where$h=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$ and $M=52$, however $S+T=\begin{bmatrix}5 & 3\\ 5 & 3\end{bmatrix}$ is not $(h,\mathcal{M})$-hyponormal operator $\forall\,\mathbb{R}^+$, M≥1.

Now, the resulting hypothesis to the state of affairs that are true

Theorem 2.7.

Rent $S:H\longrightarrow H$ i $(h,\mathcal{M}_1)$-hyponormal operator, let $T:H\longrightarrow H$ be $(h,\mathcal{M}_2)$-hyponormal operator, then $(S+T)$ is $(h,\mathcal{M})$-hyponormal operator if $S^*T=TS^*$, where $\mathcal{M}\geq\mathcal{M}_1\mathcal{M}_2$.

Proof

$$\begin{aligned}(S+T)(S+T)^*h &= (SS^*h+S\mathrm{T}^*h+\mathrm{T}S^*\,h+TT^*h)\\ &\leq M_1hS^*S+ST^*h+TS^*h+M_2hT^*T\\ &\leq Mh\,S^*S+MST^*h+MTS^*h+MhT^*T\\ &= Mh\,(S^*S+T^*S+S^*T+TT^*)\\ &= Mh((S+T)^*(S+T))\end{aligned}$$

Therefore, $(S+T)$ is (h,M)-hyponormal operator.

Reminder 2.8.

Agreement $T,S:\mathcal{H}\longrightarrow\mathcal{H}$ is $(h,\mathcal{M})$-hyponormal operator on a Hilbert space $\mathcal{H}$, now $(T\,S)$be not certainly (h,M)-hyponormal operator. To clarify this, study the resulting case:

Example 2.9.

The operator S=$\begin{bmatrix}5 & 2\\ -2 & 5\end{bmatrix}$, also $T=\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix}$, are (h,M)-hyponormal, where $h=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}$, and M=1, but $TS=\begin{bmatrix}10 & 0\\ -4 & 0\end{bmatrix}$, is not (h,M)-hpoynormal operator $\forall\,\mathbb{R}^+$, $M\geq 1$

Theorem 2.10.

Rent $S: H \longrightarrow H$, $(h, \mathcal{M}_1)$-hyponormal operator, let $T: H \longrightarrow H$ be $(h, \mathcal{M}_2)$-hyponormal operator, then (ST) is $(h, \mathcal{M})$-hyponormal operator if $ST = TS, ST^* = T^*S$ where $\mathcal{M} = \mathcal{M}_1\mathcal{M}_2$.

Proof

$$
\begin{aligned}
(S\,T)(S\,T)^*h &= STT^*S^*h \\
&= TT^*SS^*h \qquad (\text{since } ST = TS, \text{ and } ST^* = T^*S) \\
&\leq M_1TT^*hS^*S \\
&\leq M_1M_2hT^*TS^*S \\
&= MhT^*S^*ST \\
&= Mh(ST)^*(ST)
\end{aligned}
$$

So, (ST) is (h, M)-hyponormal operator.

Theorem 2.11.

Lease $S, T: H \longrightarrow H$ be bounded linear operators on a Hilbert space $\mathcal{H}$, too $h: H \longrightarrow H$ be unitary bounded linear operator on $\mathcal{H}$, s. t. $ST = ג TS$, $ג \in \mathbb{C} \setminus \{0\}$, now

1- Condition S^* is (h, M_1)-hyponormal operator and T is (h, M_2)-hyponormal operator, formerly $|ג| \leq (M_1\, M_2)^{\frac{1}{2}}$

2- If S is (h, M_1)-hyponormal operator and T^* *is* (h, M_2)-hyponormal operator, formerly $|\lambda| \geq (M_1\, M_2)^{-\frac{1}{2}}$.

Proof

1- As $ST = \lambda TS$,

$$
\begin{aligned}
|ג|\|TS\| &= \|\lambda TS\| \\
&= \|ST\| \\
&= \|(ST)^*(ST)\|^{\frac{1}{2}} \\
&= \|T^*S^*ST\|^{\frac{1}{2}} \\
&= \|T^*S^*SIT\|^{\frac{1}{2}} \\
&= \|T^*S^*Shh^*T\|^{\frac{1}{2}} \qquad (\text{since } hh^* = h^*h = I) \\
&= \|T^*(S^*Sh)h^*T\|^{\frac{1}{2}} \\
&\leq \|M_1T^*hSS^*h^*T\|^{\frac{1}{2}} \\
&= M_1{}^{\frac{1}{2}}\|(T^*hS)(T^*hS)^*\|^{\frac{1}{2}} \\
&= M_1{}^{\frac{1}{2}}\|T^*hS\| \\
&= M_1{}^{\frac{1}{2}}\|(T^*hS)^*(T^*hS)\|^{\frac{1}{2}}
\end{aligned}
$$

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$$= M_1^{\frac{1}{2}}\|S^*h^*(TT^*h)S\|^{\frac{1}{2}}$$
$$\leq M_1^{\frac{1}{2}}\|M_2S^*h^*hT^*TS\|^{\frac{1}{2}}$$
$$= (M_1M_2)^{\frac{1}{2}}\|S^*IT^*TS\|^{\frac{1}{2}}$$
$$= (M_1M_2)^{\frac{1}{2}}\|(TS)^*(TS)\|^{\frac{1}{2}}$$
$$= \sqrt{M_1M_2}\|TS\|$$

And so, $|\lambda|\|TS\| \leq \sqrt{M_1M_2}\|TS\|$, also $|\lambda| \leq \sqrt{M_1M_2}$

2- As $S\mathrm{T} = \lambda\mathrm{T}S$, now

$$\|ST\| = \|\lambda TS\|$$
$$= |\lambda|\|TS\|$$
$$= |\lambda|\|(TS)^*(TS)\|^{\frac{1}{2}}$$
$$= |\lambda|\|S^*T^*TIS\|^{\frac{1}{2}}$$
$$= |\lambda|\|S^*(T^*Th)h^*.S\|^{\frac{1}{2}}$$
$$\leq |\lambda|\|M_2S^*h.T.T^*.h^*.S\|^{\frac{1}{2}}$$
$$= |\lambda|M_2^{\frac{1}{2}}\|(S^*h.T)(S^*.h.T)^*\|^{\frac{1}{2}}$$
$$= |\lambda|M_2^{\frac{1}{2}}\|S^*hT\|$$
$$= |\lambda|M_2^{\frac{1}{2}}\|(S^*hT)^*(S^*hT)\|^{\frac{1}{2}}$$
$$= |\lambda|M_2^{\frac{1}{2}}\|T^*h^*(SS^*h)T\|^{\frac{1}{2}}$$
$$\leq |\lambda|M_2^{\frac{1}{2}}\|M_1T^*h^*hS^*ST\|^{\frac{1}{2}}$$
$$= |\lambda|(M_1M_2)^{\frac{1}{2}}\|T^*IS^*ST\|^{\frac{1}{2}}$$
$$= |\lambda|(M_1M_2)^{\frac{1}{2}}\|(ST)^*ST)\|^{\frac{1}{2}}$$
$$= |\lambda|(M_1M_2)^{\frac{1}{2}}\|ST\|$$

As a result, $\|ST\| \leq |\lambda|\sqrt{M_1M_2}\|ST\|$, so $1 \leq |\lambda|\sqrt{M_1M_2}$

And so; $|\lambda| \geq (M_1M_2)^{\frac{-1}{2}}$

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Reference

[1] Berberian S.K., (1961),"Introduction to Hilbert Space", 1st ed., New York Oxford University Press.

[2] Clancey K., (1979), "Seminormal Operators", Springer Veralag, Berlin Heidelderg New York.

[3] Coburn L. A, (1966), " Weyl's Theorem for Non-normal Operators", Michigan Math. J., Vol. 13,

[4] Devi S., (1972), " A New Class of Operators", Abstract of Operator, Indian Mathematical Society Conference, .

[5] Halmos P.R. ,(1950), "Normal Dilations and Extensions of Operators ",Summa Brasil Math. ,

[6] Istratescu V., (1967), "On Some Hyponormal Operators", pacific Journal of Mathematics, Vol. 22,

[7] Khalagai J.M. and Kavila M., (2013),"On Spectral Properties of λ-Commuting Operators in Hilbert Spaces", International Electronic Journal of Pure and . Applied Mathematics, Academic Publications, Ltd., Vol. 6,

[8] Patel S.M., (1974), " A Class of non-hyponormal operators on Hilbert Spaces", .

[9] Shah N.C. and Sheth I. H., (1975), "On Operators of Class (M)", The Journal of M.S. University of Baroda Science Issu, Vol. 24,

[10] Sharma J.N. and Vasishtha A.R., (1989), "Functional Analysis" Meerut India.

[11] Stampfli J.G., (1962),"Hyponormal Operators", Pacific Journal of Mathematics, Vol. 12,

[12] Tajmouati A., Elbakkali A. and Ahmed M.B., (2016), " On λ- hyponormal Operators”, .

[13] Wadhwa B.L., (1974), "M-hyponormal Operators", Duke Math. Journal, Vol.41,