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# The Randomness in the Application of Pure Mathematics to Science

Israa Essa Abed

Wafaa Mohammed Ridha Shakir
Al-Furat Al-Awsat Technical University, Technical Institute of Babylon,
Department of Computer Systems, Babil, Iraq
inb. israa@abed.atu.edu.iq
wfa@atu.edu.iq.

#### **Abstract**

It is commonly stated that truth is only attributed to statements supported by factual evidence and logical reasoning. In mathematics, some valid mathematical facts exist without any discernible cause, and certain of these facts may appear arbitrary and coincidental. Nevertheless, they demonstrate the remarkable capacity of the human mind for reasoning and analysis. Experiments and practical implementations are carried out in the realm of science and physical phenomena to validate the equations used in deriving their outcomes. One prominent example of such scientific and physical applications is the field of thermodynamics. Differential and integration equations are central illustrations that elucidate the concept of randomness in pure mathematics. When formulating a random equation with several variables, mainly if the equation is of a degree greater than one and has multiple coefficients, it is conceivable to have an unlimited number of solutions that may not be feasible. This research will demonstrate how to develop a comprehensive understanding of the human mind. Specifically, we will explore the formation of relative considerations and their use as reference points for solving equations. By doing so, we want to address the challenge of dealing with the inherent unpredictability of pure mathematics in scientific applications. Relative considerations are utilized to create experimental devices that serve as references for obtaining desired equation results. Thermodynamics serves as a prominent example of this scientific approach, as it demonstrates how relative considerations can be employed as references to solve equations and overcome the inherent uncertainties of pure mathematics in scientific applications. This study aims to introduce, discuss, and analyze many theoretical and stochastic concepts and features in the field of science, with a specific focus on physical applications and thermodynamics.

**Keywords:** Mathematics, Randomness, Thermodynamics, Relative Considerations

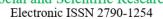
العشوائية في تطبيق الرياضيات البحتة على العلوم

اسراء عيسى عبد حبيب وفاء محمد رضا شاكر الملخص:

أن الحقيقة تُخصص حصرًا للتصريحات المدعمة بأدلة لا جدال فيها وتفكير منطقي لا يُعارض. في عالم

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الرياضيات، تظل بعض الحقائق الرياضية قائمة بصلابة، بلا سبب مميز، حيث تظهر بعض هذه الحقائق كما لو كانت تجريبية وصدفية. ومع ذلك، تعد هذه الحقائق شاهدًا على قوة العقل البشري المدهشة في مجالات التفكير والتحليل. يتم إجراء تجارب دقيقة وتنفيذات عملية في ميدان العلوم والظواهر الفيزيائية لتوثيق المعادلات المستخدمة في استقرار نتائجها.

مثال بارز على تلك التطبيقات العلمية والفيزيائية يتجلى في مجال الديناميكا الحرارية. تعتبر معادلات التفاضل والتكامل رموزًا بارزة، تسلط الضوء على مفهوم العشوائية في الرياضيات النقية. عند صياغة معادلة عشوائية تحمل عدة متغيرات، خاصة إذا كانت درجتها تفوق واحدًا وتحتوي على معاملات متعددة، يتسنى الانتقال إلى عدد لا حصر له من الحلول قد لا تكون عملية. يهدف هذا البحث إلى عرض تطور شامل لفهم العقل البشري بدقة. وبشكل خاص، سنستكشف تكوين الاعتبارات النسبية واستخدامها كمراجع لحل المعادلات. من خلال ذلك، نسعى لمواجهة تحدي التعامل مع عدم التنبؤ الذي يترتب على الرياضيات البحتة عند تطبيقها في العلوم. تُستخدم الاعتبارات النسبية لإنشاء أجهزة تجريبية تكون مرجعًا للحصول على نتائج المعادلات المرغوبة . تُعد الديناميكا الحرارية مثالًا بارزًا على هذا النهج العلمي، للحصول على نتائج المعادلات المرغوبة . تُعد الديناميكا الحرارية مثالًا بارزًا على هذا النهج العلمي، المترتبة على الرياضيات البحتة عند تطبيقها في السياقات العلمية. يهدف هذا البحث إلى تقديم ومناقشة وتحليل العديد من المفاهيم والسمات النظرية والعشوائية في ميدان العلوم، مع التركيز بشكل خاص على التطبيقات الفيزيائية و مجال الديناميكية الحرارية الشاقة.

الكلمات المفتاحية: الرياضيات، العشوائية، الديناميكا الحرارية، الاعتبارات النسبية

# 1. Introduction

This investigation aims to explore the notion of randomness in delighted subjects. Consequently, it is necessary to clearly distinguish between randomness and non-randomness in science and pure mathematics. This will enable us to comprehend the underlying theories and concepts and facilitate the identification of relevant factors and experimental methods for addressing the issues related to randomness in pure mathematics. Several prominent scientists, such as Hilbert, Euclid, Leibniz, Turing, and Whitehead, attempted to address this problem using the intuitive method. Hilbert aimed to develop a comprehensive formal axiomatic system encompassing all mathematics. Despite criticism, Hilbert's approach can be considered a significant achievement, often called the "dream of the century" by many. The user's text is "[2]". Hilbert identified many essential features a system must possess to be considered intuitive. The system must possess inherent clarity, encompassing the requirement for an intuitive framework that includes all mathematical concepts. Additionally, the system must demonstrate coherence and comprehensiveness, ensuring that all axioms and modes of mathematical reasoning we acknowledge are derived from it. The user's text is "[3]". Figure (1) illustrates the primary factors that render any system axiomatic.



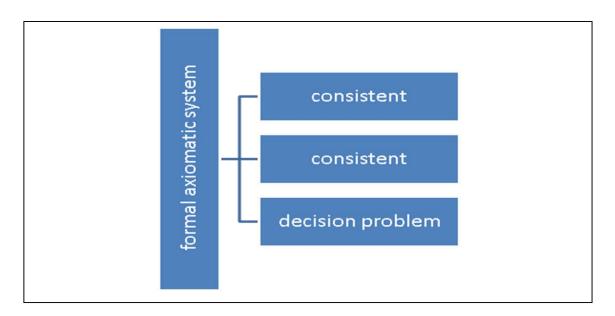


Fig. (1): Formal Axiomatic System

Consistency refers to the absence of the ability to prove a statement and its negation. Completeness here refers to the requirement that if someone makes a statement, they must be able to substantiate it with theories, logical deductions, or fundamental principles. The resolution to the issue is to verify or ascertain. An algorithm enables you to determine the validity of a given significance. Hilbert aimed to include the fundamental branches of mathematics, namely Triple Arithmetic, Calculus, and Algebra, into his axiomatic framework [4]. To assure consistency when acting, it is necessary to have a mechanical method that validates and substantiates the claim of the activity. This procedure must have a set of rules. Your formal language consists of a restricted set of symbols and regulations. While many mathematical facts may be called "random facts," it is widely acknowledged that they are not truly random in the literal sense. Instead, their seeming unpredictability stems from our current lack of understanding of their underlying reference and actual cause. Thus, mathematical theories exist that are proven false, and we subsequently uncover ideas that invalidate and reject their validity. However, our ultimate goal in this research is to identify the primary approaches employed in addressing problems related to randomness in pure mathematical applications [5]. We will accomplish this by conducting a comparative analysis of all methods and previous studies to determine the most significant among them.

#### 2. Studies and theories

#### 2.1 Gödel's theorem

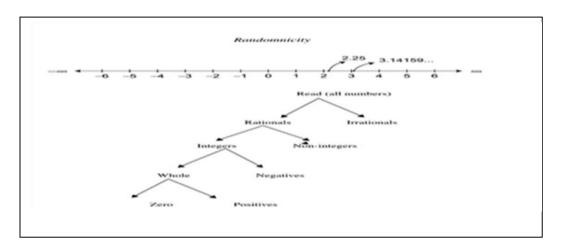
"The information that is established, organized, and based on facts is never

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sufficient to fully encompass the entirety of the truth." Mathematics relies mainly on numerical values, and researchers initially embraced natural numbers, followed by positive integers, negative integers, rational and irrational numbers, and, eventually, the concept of imaginary numbers. Refer to Figure 2, depicting a number line and the categorizations of numbers[6]. Integers are categorized into positive, negative, and zero. Negative integers are represented to the left and continue indefinitely towards negative infinity.

In contrast, positive integers are represented to the right and extend indefinitely towards positive infinity on the number line. Rational numbers encompass several representations, including decimal fractions, numerators denominators, and integers (1, -2, 0.15, 1/2, etc.). Non-rational numbers, such as  $\pi$  and others, are called e numbers. Imaginary numbers encompass negative square roots as well.



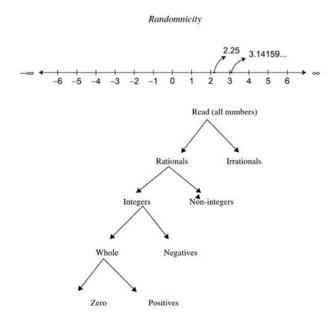


Fig (2) the number line and the classifications of numbers [6]



Self-reference is a concept that occurs in mathematics and possesses significant influence. Despite the complexity of the mathematical proof[7], we can create an approximate representation of the original form as long as we maintain simplicity and accuracy. Gödel asserted that we have theories, proposals, and conjectures in mathematics. He emphasized the importance of identifying Gödel or referential statements, which state that a particular proposition cannot be proven within the existing mathematical system. It is crucial to distinguish between a bid that is supportive but unprovable and an assumption that is not supportive. The request must be supportive for a proposition or self-reference to be considered complete. To accomplish this objective, two measures must be implemented. The initial phase involves the creation of a numerical encoding system, which is referred to as CGödelIZATION. Here, we shall illustrate. The English language consists of twenty-six letters: a, b, c, ..., z. Each letter is assigned a numerical value, with a being equal to 1, b being equal to 2, and so on. By adhering to precise guidelines, each distinct number can represent a particular word. Words can be represented by prime numbers (2, 3, 5, 7, 11, etc.) based on the number of letters they include. Certain words can be spoken in the following manner[9]: For instance, the term "if" is comprised of two characters that may be represented as follows:

# IF=2<sup>2\*</sup>3<sup>6</sup>=19.552.734.375.000

The second step is to form special equations even if the exponents are large. An example of these equations:

$$2x + 3y = 1$$

$$2x + 3y = 5$$

$$x^{3} + y^{3} = z^{3} - 3$$
$$5x^{221} + 3y^{49} - 47 = 0$$

Given these equations, some equations can be proven and solved, and other equations cannot be established or solved[10]; the first equation cannot be proven, while the second equation can be confirmed when:

# X=1 and y=1

As for the first equation, if we add an integer value to the right side and subtract an integer value to the left side, we can reach almost proof, Where the scientist Stephen Wolfram was able to computerize large numbers of integers up to a Print ISSN 2710-0952

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billion, Therefore, no matter how immense the power of the X and Y transactions is, it becomes easy to solve equations.

## 2.2 Slippery road theorem

As René Magritte said [11], "Everything hides something we want to see." The most prominent example that can clarify this is the law of the area of a circle and the law of the circumference of a circle, as

#### Circumference= $2\pi r$

# Circle area(A)= $\pi r^2$

An approximation ratio is a transcendental integer with unlimited decimal places. Here, the pi ratio was used as a benchmark, which is unquestionably favorable. The reference was determined by the measurement[12]. When establishing a law to define the circumference of a circle, which can be readily measured using any measuring method, and relating that length to the radius of the process, an estimated value of  $\pi$  may be derived. Many ratios in equations, including those in engineering and equations of integration and differentiation, have followed a similar method[13]. In essence, this approach relies on the principles of testing and measuring, making it a highly effective strategy for addressing issues related to random numbers. More specifically, it pertains to the concept of randomness within pure mathematics.

#### 2.3 Leibniz's theorem of axioms

Leibniz introduced his axiom. In his work, he presented an axiomatic set theory[14] known as the Leibniz-Mycielski axiom, LM. This theory involves ordered pairs of variables x and y, with an order  $\alpha$  that surpasses the orders of x and y. The approach also includes a formula  $\phi(v)$ , represented as  $(V\alpha, \in)$ , which fulfills the condition  $\phi(x) \land \neg \phi(y)$ . The closure is equal among the variables while considering a defined class function devoid of variables. All sets x must have at least two items, denoted as F(x). Leibniz's thesis posits that no separate materials can be completely identical [15]. They must necessarily differ from each other in at least one feature. The principle of Leibniz asserts that a logical link exists between entities, which may be described as a logical connection between them. Furthermore, suppose we possess two materials, X and Y, that exhibit similarities but diverge in a single feature or variable. In that case, it is possible to organize them in a particular manner based on their qualities.

## 2.4 Hilbert's theorem

Hilbert is regarded as the preeminent mathematician, particularly in mathematical logic and meta-mathematics. He is recognized as one of the

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pioneers in developing proof theory and mathematical reasoning. Before delving into Hilbert's view, it is necessary to address the concept of axioms. The axiomatic technique, often known as axioms, is a formal approach employed by science to establish assertions or assumptions known as axioms[18]. The axioms are interconnected, forming the foundation for the hypotheses. Criteria for a particular system: Hilbert constructed a comprehensive system encompassing all fields of mathematics and achieved the aspirations of many with his theory. His theory emphasized essential features that must be present for the system to be considered intuitive. The system must adhere to axioms, ensuring it is intuitive and consistent. It should also be comprehensive, encompassing all accepted mathematical axioms and modes of reasoning, which must be inferred. The system's intuitiveness is a crucial factor to be taken into account [19]:

a- consistency b- complet c- conclusion(decision problem)

- -Consistency means that we are not necessarily able to prove a hypothesis or a specific pattern.
- -Complete means that you have provided an affirmation to harm or prove it, whether by theories, reasoning, or hypotheses [20].
- decision problem: The solution to the problem is that you must assert or state. The algorithm allows you to determine if there is a specific meaning. There are what is known as the twenty-three Hilbert problems[21], which were posed by Hilbert and said were difficult to solve but would greatly determine the shape of mathematics in the future. These issues included continuity hypotheses, consistency of axioms, metric space, whether continuity groups are differential or not, whether physics supports the subject of axiomatic theories, the issue of central surfaces and Bernard's hypothesis, as well as the foundations of rigor and relativity of crystals[22], As well as calculating variables by analytical methods and linear differential equations, developing a general method for solving problems figuring calculus changes, and this is what we will focus on in this research.

## 2.3.1 Development of random calculus by Hilbert's theorem

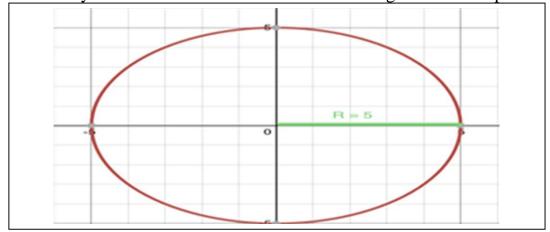
Calculus is inherently distinct from other branches of mathematics due to its dynamic Nature, in contrast to the static Nature of other mathematical disciplines. In essence, calculus may be defined as the branch of mathematics that deals with analyzing motion and examining the principles governing the change process. Alternatively, to provide a more formal explanation [23]:

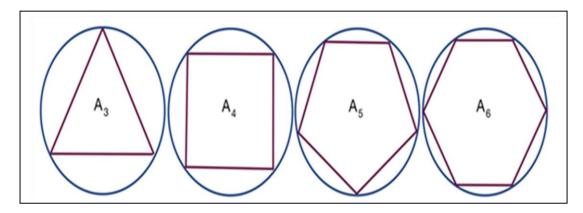
Calculus encompasses the fundamental operations of differentiation and integration, serving as a vital instrument for engineers to ascertain rates of change and areas. It serves as the mathematical foundation for addressing problems involving variables that vary with time or another reference variable. A solid grasp of calculus is indispensable for advancing one's studies and cultivating proficiency in solving real-world engineering problems. This will be



apparent in the subsequent chapter when we will model physical systems and employ differential equations, which describe rates of change. These equations will enable us to represent the physical system, analyze it, and find a solution under certain conditions.

This chapter serves as an introduction to the principles of calculus and their practical applications in engineering. The subsequent chapter delves into using differential equations to represent physical systems and their solutions for various inputs. Before the advent of calculus, mathematics was primarily static and limited in its ability to describe moving objects. However, most things are in constant motion, ranging from electrons within atoms to planets in the universe. Calculus, on the other hand, excels in addressing this dynamic Nature. It finds relevance in various fields where the significance of mathematics may not be immediately apparent. To illustrate the potential of calculus, let us consider a seemingly straightforward problem: determining the area of a circle. We are already familiar with the formula for calculating the size of a process:



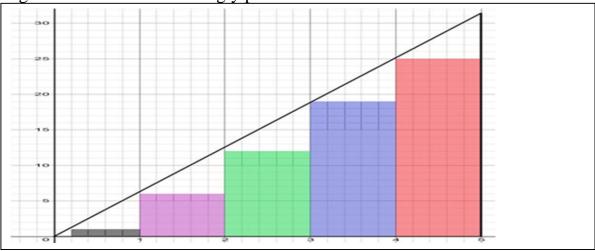


Fig(3) shows the circle left over.

Subsequently, in an attempt to acquire more forms and minimize the remaining area of the circle, let us explore an alternative approach: dividing the process into concentric rings. The width of this rectangle is equivalent to the ring's

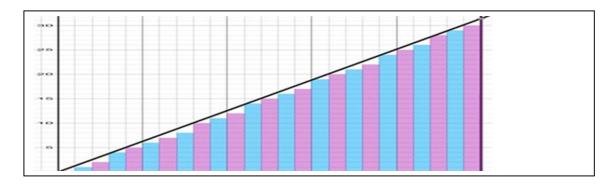


circumference, which is  $\pi 2\pi R$ . The height is determined by the smaller radius, r that you previously selected. Let's designate the variable r as dr, indicating a slight variation in radius between adjacent circle rings. What is our current situation? We possess a collection of circular rings approximated as rectangles and know how to determine their respective areas. As the options get smaller (or as the circle is divided into progressively smaller rings), our estimation of the ring's area becomes increasingly precise.



Fig(4) the rings of concentric circles

Concentric circles arranged on a graph with the equation:  $y = 2\pi R$  As the value of dr decreases, the estimation of the entire area of the process gets increasingly precise [24]:



The total areas of concentric circles are represented as the area under the graph.

We know the formula for the area of a triangle, Which in this case would be:

$$A = \frac{1}{2} bh$$
 which in this case would be:

$$A = \frac{1}{2}(5)(2\pi \times 5) = \frac{5(2\pi \times 5)}{2} = \pi(5)^2 \rightarrow A = \pi R^2$$

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However, how did we arrive at this point? Let us pause and contemplate the matter at hand. We encountered a predicament that may be resolved by approximating it by the aggregation of several smaller integers, each like  $\pi 2\pi \times R$   $\times dr$ , for R values ranging from 0 to 5. The value dr represents the selected thickness for each ring of the circle. There are two crucial aspects to consider in this context: Dr serves a dual purpose: it determines the extent of the rectangles we are summing up and signifies the intervals between the various values of R—the more limited the options for Dr, the more accurate the estimate.

However, reducing the doctor's size will result in a more precise response. By decreasing the value of dr, the approximation of the original problem improves. The sum of the areas of the rectangles becomes closer to the area under the graph. Therefore, it can be inferred that the exact solution to the original problem is equal to the area under this graph. Are these concepts intriguing? Now, you may be questioning the rationale of investing effort in a task as straightforward as determining the size of a circle. Let us pause to contemplate... Given that we successfully determined the area of a circle by rephrasing the problem as determining the space beneath a graph, is it feasible to employ the same approach for other, more intricate charts? Affirmative, we are indeed capable of doing so. Consider, for instance, the graph of the equation  $y=x^2$ , which represents a parabola.

Programming a computer to create random numbers is inherently challenging due to the deterministic Nature of computers, which often produce predictable outputs depending on their programmed instructions. Random Number Generators (RNGs) allow computers to have distinct, non-uniform, and unpredictable numbers. They are utilized in many practical applications, including The study and practice of techniques to secure communication and information from unauthorized access or modification. Random numbers play a vital role in the cryptographic protection of passwords, web browsers, and other online and digital information forms—wallets for digital currencies. Seed phrases generated using random numbers are employed in BIP39 standard procedures, which are utilized to compute encryption keys for Bitcoin wallets simulations. Pseudorandom number computer-generated sequences employed for testing and repeating simulations, such as Monte Carlo simulations, to sample things to estimate unknown ratios and regions. Machine learning (ML) is a field that focuses on developing algorithms and models that enable computers to learn and make predictions or decisions without being explicitly programmed. Random numbers and machine learning frameworks that do not require a pre-defined model, such as domain randomization (DR), are utilized in several practical applications, including robotic Hoover cleaners and the Open AI hand dexterity learning project. Random numbers and pseudorandom number generators (PRNGs) play a crucial role in game creation, encompassing gameplay mechanics and visual elements—computer science.



Random numbers play a vital role in several aspects of computer networking, such as generating TCP/IP sequence numbers, creating nonces for Transport Layer Security, generating password salts, and assigning DNS source port numbers—empirical research. Random numbers are utilized to develop probability samples from a population and derive statistical inferences from a survey. They also determine the appropriate treatment for different physical units in an experiment—the study of numbers, quantities, and shapes. Random number generators (RNGs) protect against selective bias, rendering them valuable for addressing mathematical issues that may seem entirely non-probabilistic and statistical problems that are challenging to answer by direct analysis [26].

### 5. Conclusions and Recommendations

#### **5.1 Conclusions**

- 1. Understand the concept of a limit and its significance in rate-of-change relationships.
- 2. Use calculus notation to describe a rate of change (differentiation) and understand the significance of the operation.
- 3. Solve engineering problems involving rates of change.
- 4. Understand what is involved in the calculus operation of integration.
- 5. Solve engineering problems involving integration.
- 6. All previous studies and research, even those that contradicted Godel's point of view, showed their agreement on the effectiveness of theories of axioms in solving the significant problems that we face in the randomness of mathematics.
- 7. There are some practical methods for solving the problem of random pure mathematics, such as the Silbury Road method, the test and measurement method, and the computing software method.

#### 5.2 Recommendations

Fractional calculus is an encompassing and expansive form of classical calculus. The genesis of calculus is commonly ascribed to Gottfried Leibnitz and Isaac Newton, despite the enduring acrimonious argument between the two on the true progenitor of calculus. Leibnitz introduced his initial publication on calculus in 1684, whereas Newton incorporated calculus into his Principia, published in 1687. While the essential calculus concepts had been familiar to Greek, Arab, and Persian mathematicians for many years, Leibnitz and Newton formulated a comprehensive calculus structure. They developed the fundamental theorem of calculus, which establishes the connection between derivatives and integrals. Following the advent of calculus, mathematicians promptly saw it as a critical study area. In 1695, Guillaume François Antoine, a prominent mathematician and the author of the initial French exposition on calculus, addressed a letter to Leibnitz inquiring about the consequences of considering an actual number as the order of differentiation rather than an integer. Leibnitz

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responded by stating that this would result in a paradox, which would eventually have valuable results. The dialogue between Marquis and Leibnitz is often regarded as the inception of fractional calculus.

Nevertheless, the practical advancement of fractional calculus was delayed until 1832. Joseph Liouville established the Riemann-Liouville definition of fractional derivative, which is currently known as such. This definition is based on the Riemann-Liouville fractional integral.

### **References:**

- [1] G. J. Chaitin, Information-Theoretic Incompleteness, World Sci-Scientific, 1992.
- [2] G. J. Chaitin, Information, Randomness & Incompleteness, second edition, World Scientific, 1990.
- [3] G. J. Chaitin, Algorithmic Information Theory, revised third printing, Cambridge University Press, 1990.
- [4] Chaitin, G. J. (1994). Randomness and complexity in pure mathematics. International Journal of Bifurcation and Chaos, 4(01), 3-15.
- [5] Chaitin, G. J. (1993). Randomness in arithmetic and the decline and fall of reductionism in pure mathematics. arXiv preprint chao-dyn/9304002.
- [6] Calude, C. S., & Chaitin, G. J. (1999). Randomness everywhere. Nature, 400(6742), 319-320.
- [7] Chaitin, G. J. (1990). Undecidability and randomness in pure mathematics. Information, Randomness & Incompleteness, 307-13.
- [8] Chaitin, G. J. (1985). Randomness and Godel's theorem.
- [9] Kolmogorov, A. N. (1963). On tables of random numbers. Sankhyā: The Indian Journal of Statistics, Series A, 369-376.
- [10] Tsonis, A. A. (2008). Randomnicity: Rules and Randomness in the Realm of the Infinite. World Scientific.
- [11] Magritte, R.M. (1946).
- [12] CS Calude, GJ Chaitin, nature.com, Nature 1999.
- [13] M Chow cas.mcmaster.ca, New Scientist 2000.
- [14] Enayat, A. (2004). On the Leibniz-Mycielski axiom in set theory. Fundam. Math, 181(3), 215-231.
- [15] Swoyer, C. (1995). Leibniz on intension and extension. Noûs, 29(1), 96-114.
- [16] Enayat, A., Kanovei, V., & Lyubetsky, V. (2021). On effectively indiscernible projective sets and the Leibniz-Mycielski Axiom. Mathematics, 9(14), 1670.
- [17] zsef Beck, J. (2009). Inevitable randomness in discrete mathematics (Vol. 49). American Mathematical Soc

Print ISSN 2710-0952



- [18] Morrison, F. (2012). The art of modeling dynamic systems: forecasting for chaos, randomness, and determinism. Courier Corporation.
- [19] Chaitin, G. J. (1985). Randomness and Godel's theorem.
- "A New Illustrated Science Dictionary (En/Ar)". www.ldlpdictionary.com.
- They are archived from the original on May 6, 2019. View it on 2019-05-06.
- [21] Hoffman, Lawrence D.; Bradley, Gerald L. (2004). Calculus for business, economic, social and life sciences (8th edition). Boston: McGraw-Hill. ISBN 0-07-242432-X.
- [22] Bardi, Jason Socrates (2006). The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time. New York: Thunder's Mouth Press. IS.
- [23] Katz, Victor J. (2004), A History of Mathematics, Brief Version, Addison-Wesley, p. 125–126, ISBN 978-0-321-16193-2.
- Mathematics Magazine 68 (3) [24] Victor J. Katz (1995), "Ideas of Calculus in Islam and India"
- [25] Malet, Anthony (1993). James Gregorie on tangents and the Taylor rule for series expansions. Archive for History of Exact Sciences.
- [26] Shea, Marilyn (May 2007), Biography of Zu Chongzhi, University of Maine, archived from the original on March 23, 2018, Retrieved January 2009.