

Composite Materials Under Fatigue Loading: General Review

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Abstract

Advanced applications, such as aircraft manufacturing, require sophisticated materials. Composite materials are among these advanced materials and offer several advantages, including high strength and low weight. Given that these applications experience repeated loading, studying fatigue in composite materials is essential. This paper provides a comprehensive review of fatigue failure in composite materials, focusing on the types of fatigue loads, the characteristics of composite materials, and the damage mechanisms. Additionally, we discuss modelling and simulation techniques to understand fatigue behavior and the standards necessary for conducting fatigue failure testing in composite materials. The study of fatigue in composite materials is diverse, reflecting the materials' complexity, which varies across scales. Due to composite materials' heterogeneity, numerical modelling can be challenging. It often requires numerous constants that change with various factors, which can only be determined through experimental test. As a result, studying fatigue in composite materials can be costly.

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1. Introduction

Since the 1950s, numerous studies have examined the fatigue performance of composite materials across various engineering fields, considering different loading and environmental conditions. This includes modelling fatigue to analyze fatigue behavior and predict fatigue life [1, 2].

The composite material consists of two or more substances to give physical properties different from its constituents [3]. Some properties that are improved by making composite material are: strength [4, 5], high fatigue strength because of static strength and its slight decrease with cycle number to fracture [6], stiffness [7], slight decrease with several loading oscillations [6], weight [8], temperature dependent behavior [9], thermal insulation [10], corrosion resistance [11], fatigue life [12], low notch sensitivity, low sensitivity to the frequency of loading [4]. Hence, the increasing popularity of modern high-performance products is not surprising for composite material, even though composite are not recent innovations where the Bible referenced straw-reinforced bricks in the Old Testament [4].

The aerospace industry is the leading field for highly engineered composite applications [13]. The first airplane took its maiden flight no more than a century ago, and composite materials have been used to build structural components for more than half that time. The first usage dates back to 1940, when a main spar on Blenheim aircraft was constructed from flax thread skin infused with Phenolic resin, followed by the first fibrous composite in 1947. After that, they show a steady increase in the usage of composite in aviation, as shown in Fig. 1 for the composite material used in fixed-wing and rotary-wing aircraft industries. Today, most rotary blades are made of composite [14] after being made of metal blades in the sixties of the last century. The use of composite rotor blades lasts at least 20,000 hours, while the use of metallic rotor blades lasts

about 1000 hours due to the difference in fatigue performance [4].

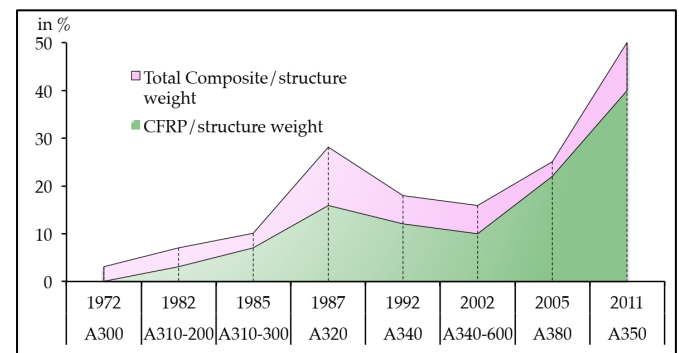


Fig. 1 Composite usage in airbus aircraft [4].

Many aircraft parts are made from carbon epoxy and aramid epoxy [15], reducing the aircraft's weight and increasing payload and economy. Therefore, composite material is used rather than metal. Furthermore, because the composite material has high fatigue resistance, it enhances the stiffness-to-density and strength-to-density ratios [16]. Table 1 illustrates the specific modulus and strength of materials used in aircraft [17].

The known behavior of composite materials under fatigue loading has evolved significantly with advancements in product designs. Carbon fiber/epoxy, in particular, has demonstrated excellent fatigue resistance [18] from the early stages of composite development, which has been the focus of extensive research. Given that composite materials are anisotropic [19], a stress system that produces only a minor strain in the main direction of the fibers may not significantly influence the formation of strains normal to the fiber/resin interface or the fibers themselves. However, such damage

must be noticed. Therefore, understanding fatigue in composites is crucial, as it is essential to identify the mechanisms of fatigue damage and the methods for assessing the accumulation and progression of this damage. This understanding will enable more reliable predictions of component lifespan [20].

Table 1. Elastic modulus and tensile strength of materials used in aircraft [17].

Material	Modulus of elasticity/density (GPa/g.cm ⁻³)	Tensile strength/density (MPa/g.cm ⁻³)
Steel (AISI 4340)	25	230
Aluminum (7075-T6)	25	180
Titanium (Ti-6Al-4V)	25	250
E Glass/Epoxy composite	21	490
S Glass/Epoxy composite	47	790
Axamid/Epoxy composite	55	890
HS (High Tensile Strength) Carbon/Epoxy composite	92	780
HM (high modulus) Carbon/Epoxy composite	134	460

However, composite materials still have disadvantages that must be taken into account during design, like weak compressive load resistance, squeezing [21], corrosion sensitivity when in applications that have contact with aluminum alloys and steel, moisture absorption [22], and consequent deterioration of mechanical properties over time. However, a suitable design can overcome this disadvantage [6].

Composite materials can be categorized based on their matrix and reinforcement types [23]. According to the matrix, there are three main classifications: polymer matrix composites (PMCs), metal matrix composites (MMCs), and ceramic matrix composites (CMCs). On the other hand, composites can be classified by their reinforcement types into particle-reinforced composites, short fiber or whisker-reinforced composites, continuous fiber or sheet-reinforced MMCs, and laminate composites [17]. Since laminated composite materials are the most extensively studied under the influence of fatigue loads, this research will focus specifically on composite laminates.

Replacing metal with composite materials in engineering structures makes studying the fatigue phenomena associated with composites essential. Unlike metals, which are homogeneous [24] and isotropic materials with a single failure mode [4], composites exhibit more complex fatigue behaviors. While fatigue in metals has been extensively studied for over a century, resulting in a comprehensive set of design rules for various engineering metals and alloys, the same understanding is still developing for composites [20]. Figure 2 and Table 2 show the reason to use composite rather than metal.

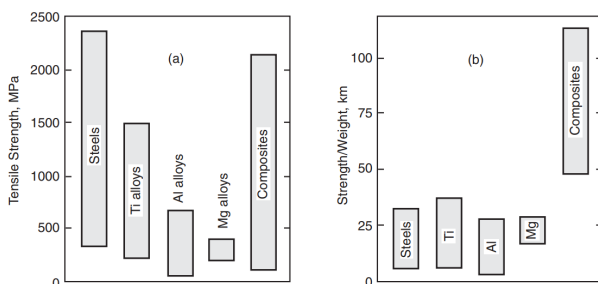


Fig. 2 Strength comparison between various structural metals and polymer matrix composite classes showing ranges for (a) tensile strength and (b) tensile strength per unit weight [25].

Table 2. Comparison between metals and composite laminate.

Metals	Composite laminate
The fatigue dominant timespan is the crack's initiation, after initiation of crack the crack will propagate from one or a few cracks and dominates [26].	The fatigue-dominant timespan is the crack's propagation, where the crack initiation has to pass through many stages. The crack initiation starts when many microscopic cracks develop from Matrix weak, fiber-matrix bonds and voids at low cycles. Neighboring fibers stop it, which causes load path redistribution due to local stiffness degradation and stiffness degradation in other regions. The final failure occurs when macroscopic cracks accumulate when a certain number of microscopic cracks saturate the matrix. The delamination (damage to the inter-laminar matrix) may occur due to inter-laminar matrix cracks [26].
The direction of propagation of these macroscopic cracks usually propagated normal to the principle stress direction at the crack tip (mode I crack propagation) [26].	Delamination spreads very quickly between the layers due to the absence of inhibitors such as fibers in the inter-laminar region; furthermore, the fiber-supporting effect in compression loading is decreased due to intra-laminar fiber-matrix debonding and delamination, which leads to micro-buckling [27], [28], therefore, the fatigue due to inter-laminar damage is significant to study [26].
Isotropic material, it has just one failure mode [4], [29].	Composite anisotropy makes the fatigue analysis sophisticated because there are four Independent failure mechanisms in composite material caused by fatigue, matrix cracking [30], delamination, fiber breakage, and interfacial debonding [31]. The complex understanding of fatigue in composite material comes from stress complex state nonlinear behavior, anisotropies and different ways of failure. Figure 3 and 4 illustrate the differences between fatigue damage and stiffness reduction between composites and metals [29].

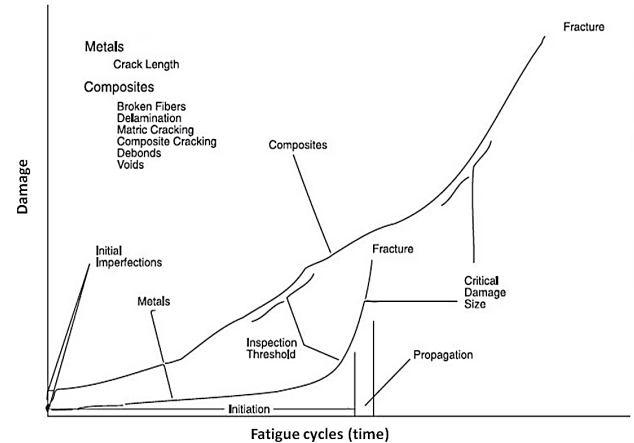


Fig. 3 Fatigue damage between composite and metals [29], [32].

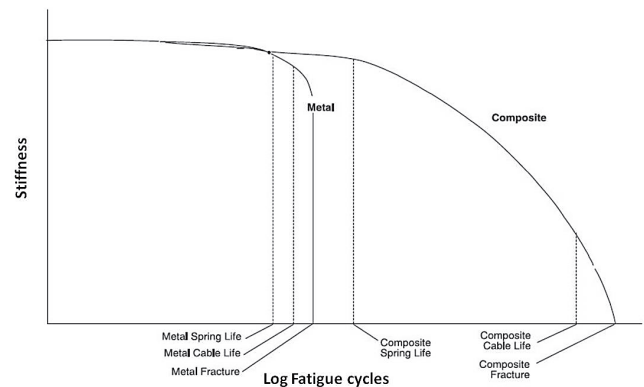


Fig. 4 Difference in stiffness reduction between composite materials and metal [29].

Most of the research on reviews focuses on a specific topic in reviewing the fatigue of composite materials. Like Talreja [33] studied the composite material using the strain approach, while Burhan and Kim [34] presented a review about modeling by the S-N curve for composite materials. Alam et al. [35] presented a review on the fatigue of carbon fiber-reinforced plastics where the significance of environmental factors on fatigue performance and service life was discussed, fatigue and the mechanics of cyclically-loaded composites were defined, the fatigue response and fatigue properties of CFRP in various forms were clarified, and the various methods used to model fatigue in CFRP were summarized, while in [36] Mortazavian and Fatemi studied the short fiber-reinforced polymer composite from the aspect of fatigue behavior and modeling. Pascoe [37] presented a critical review of methods for the prediction of fatigue delamination growth in composites and adhesive bonds, including stress/strain-based models, fracture mechanics-based models, cohesive-zone models, and models using the extended finite element method. Based on the observed macro-scale behavior of test specimens, Bak et al. [38] reviewed the fatigue delamination of composite laminate using observed phenomenology and computational methods. Also, Tabiei and Zhang [39] studied the experimental and simulation aspects of fatigue delamination in composite laminate. Deng [40] reviewed and assessed laminated composite structures fatigue delamination damage. Khan et al. [41] showed the effect of mean stress or stress ratio on the growth of fatigue delamination in composite materials. Gao et al. [42] presented a review on mode I fatigue of fiber-reinforced polymeric composites, where the variables influencing failures were taken into account in relation to fiber and matrix breakdowns, A review is conducted on numerical modeling techniques for predicting the life of composites under fatigue stress, Additionally, included are the testing methods utilized to confirm the composite's fatigue performance under mode I load. Strategies for extending the life of composites under mode I fatigue loading have also been compiled. Ansari et al. [43] studied the analysis of fiber-reinforced polymer composite fatigue damage. Vikram and Kumar [44] presented a review on fatigue crack growth and the finite element method published since the 19th century and identified new research lines, while Vassilopoulos [1] presented a review about the history of fatigue in fiber-reinforced polymer composite laminate in the period between 1950-2020. Post et al. [45] review the composite material under variable loading and assess the current state of the art in spectrum loading. It is not surprising that there are many reviews about fatigue delamination, as it is the most common type of failure in composite laminate [39]. However, in this paper, we will focus on providing a general idea about fatigue failure and fracture in the macro-mechanical scale of composite laminate as a starting point for anyone who wants to start research about fatigue in composite.

2. Fatigue loading

The repeated load to which the material is exposed in structures, vehicles, and machines components [46], can cause microscopic physical damage, even to resulting cyclic stresses. Under the stress below the ultimate strength of the material, microscopic damage can occur and then accumulate under continuous cyclic loading [47], which leads to crack initiation or damage that leads to component failure. This failure is

called fatigue [25]. The figure below show the periods of fatigue life.

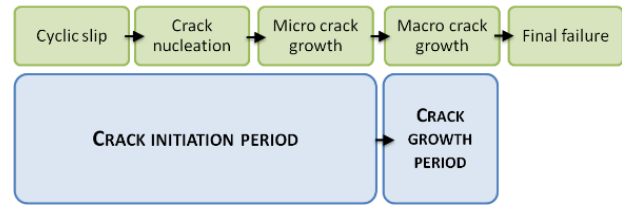


Fig. 5 Fatigue life periods [48].

2.1. Fatigue parameters

Important fatigue parameters must define before the study of fatigue. Illustrate in Fig. 6, are as follows:

- The cyclic stress range:

$$\Delta\sigma = \sigma_{max} - \sigma_{min} \quad (1)$$

- The cyclic stress amplitude or alternative stress:

$$\sigma_a = \frac{(\sigma_{max} - \sigma_{min})}{2} \quad (2)$$

- Mean stress:

$$\sigma_m = \frac{(\sigma_{max} + \sigma_{min})}{2} \quad (3)$$

- Stress ratio:

$$R = \frac{\sigma_{min}}{\sigma_{max}} \quad (4)$$

Where σ_{max} and σ_{min} are the maximum and minimum stress levels, respectively [17].

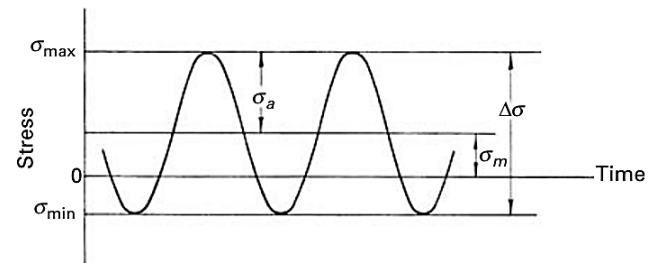


Fig. 6 Fatigue parameter [17].

There are many ways to apply cyclic loading, as shown in Fig. 7 with the value of the stress ratio, where C and T refer to compression and tension, respectively.

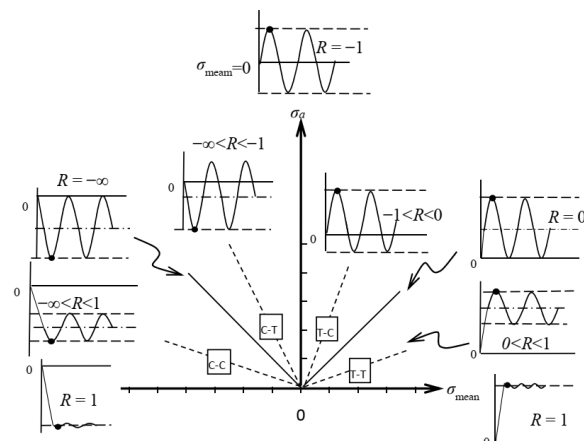


Fig. 7 Loading types and stress ratio ranges [34], [49].

3. Composite laminate

In this paper, the focus is on the use of composite laminate, where the most research on fatigue loading uses it because it has high fatigue resistance. Composite laminates consist of two or more fabrics [50], [51] unidirectional, biaxial, or multiaxial, the fabric can be oriented in different directions, in the fiber direction, unidirectional fabrics have a high modulus and strength [52], while in the matrix there are the closest fibers packing density and lower fiber undulation, while the multiaxial fabrics used reduce the time of production and enhance toughness and inter-laminar strength [53]. Some types of composite laminates are shown in Fig. 8 and Table 3.

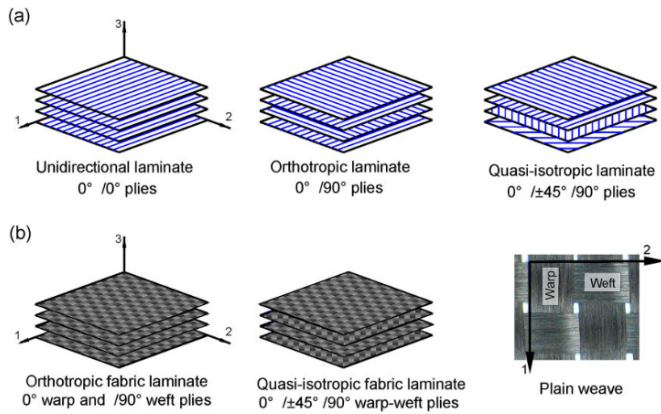


Fig. 8 Laminates: (a) stack-up of unidirectional and (b) biaxial woven composite laminates [53].

Table 3. Special cases of laminates [54].

Composite laminate type	example
Symmetric laminates (plies angle and thickness are the same above and below the midplane) Ex: [0/30/60]	<div style="display: flex; align-items: center;"> <div style="width: 20px; height: 20px; background-color: yellow; margin-right: 5px;"></div> <div> 0 30 60 30 0 Mid-plane </div> </div>
Cross-Ply laminates (only 0 and 90° plies were used) Ex: [0/90 ₂ /0/90]	0 90 90 0 90
Angle ply laminates (plies of the same material and thickness, only oriented at +θ and -θ directions) Ex: [-40/40/-40/40]	-40 40 -40 40
Antisymmetric Laminates (the material and thickness of the plies are the same above and below the midplane, but the ply orientations at the same distance above and below the midplane are negative of each other) Ex: [45/60/-60/-45]	45 60 -60 -45
Balanced Laminate (layers at angles other than 0 and 90° occur only as plus and minus pairs of +θ and -θ. The plus and minus pairs do not need to be adjacent to each other, but the thickness and material of the plus and minus pairs need to be the same) Ex: [30/40/-30/30/-30/-40]	30 40 -30 30 -30 -40

It is important to know the following terms to study composite laminate

- Isotropic: the material has the same mechanical properties in all directions. Composite laminates aren't isotropic [55].
- Transversely isotropic or quasi-isotropic [56]: the laminate have the same stress-strain behavior at all direction of material plane due to one plane has same mechanical properties at any direction in that plane.

- Orthotropic: three perpendicular planes have different mechanical properties. Therefore, the properties of materials different with each direction [57], [58]. All unidirectional laminae are separately orthotropic [59]. Most laminated composites are orthotropic [60].
- Homogeneous: the properties of a material do not change at any point in the material [61]. Composite laminates are heterogeneous because they consist at least of fiber and matrix, while in the study of linear elastic response on the macroscopic scale of a composite laminate, the material can be classified as homogeneous. This assumption is called Smearing of matrix and fiber.
- Directions of Principal Material: directions perpendicular and parallel to the fibers in a lamina, Fig. 9 show the difference between lamina and laminate. The directions described should be noted, as they may not align with the principal stress directions as defined within the framework of continuum mechanics [62].

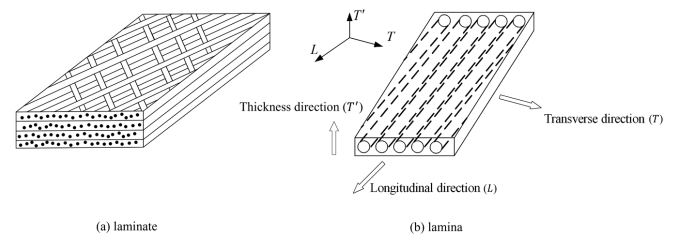


Fig. 9 Difference between lamina and laminate [63].

The study of composite generally change with change scale of study, Fig. 5 show the different scale of composite material. Figure below show the difference between scales.

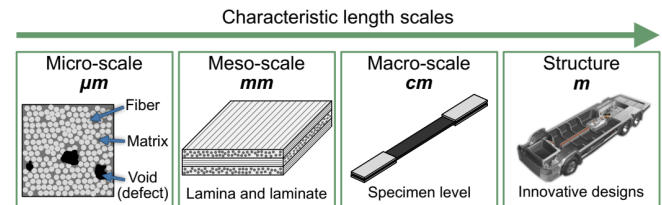


Fig. 10 The difference between scales in composite material [32], [64].

Composite laminate have advantage and disadvantage illustrated in Table 4

Table 4. Advantages and disadvantages of composite laminates used in the industry [65].

Advantage	Disadvantage
High resistance to impact damage [66].	Composite laminates are more brittle than wrought metals, which makes composites much easier to damage.
Have resistance to corrosion degradation and fatigue.	Environmental degradation can be exhibited by the matrix.
High strength-to-weight ratio.	Can be weak in transverse properties.
The tailored fiber in different patterns can increase efficiency and sustain itself under applied loads. This directional tailoring capability can meet design requirements.	Fabrication and raw materials cost and are expensive.

3.1. Damage mechanisms in composite laminate

The damage to composite laminates occurs due to heterogeneity. Damage can be classified into two categories:

1. Intralaminar (inside one ply).
2. Interlaminar, damage occurs between the plies that lead to separation between them [67], [68].

Figure 11 show the possible modes of damage and the shapes of its.

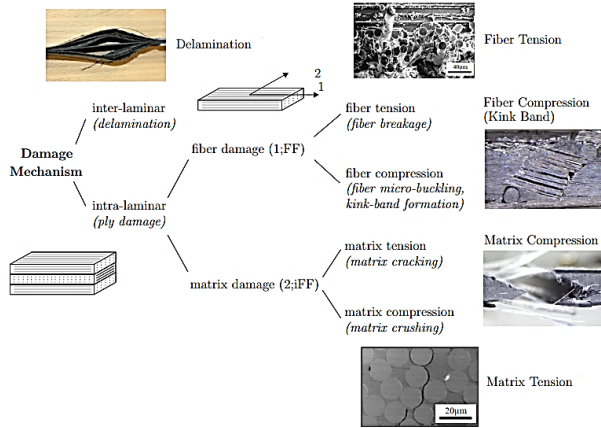


Fig. 11 Damage mechanisms classification in composite materials [69].

The explain of damage of composite material will be according to coordinate system of ply below

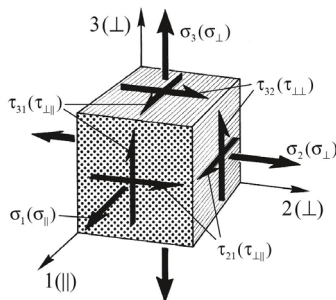


Fig. 12 Coordinate system of ply.

3.1.1. Intralaminar damage

Occur in the matrix, fiber, or interface between them), there are different mechanisms of damage in tension and compression compared to metals. This is due to the inability of the fibers to bear the compression load on their own [67].

➤ Fiber failure:

- Static load tension:
 1. It occurs when the load is in 1-direction of the ply.
 2. The fiber failure is quasi-ductile.
 3. Start gradually as a result of stiffness degradation due to fracture of the weakest individual fiber, then lead to whole fiber bundle failure under a higher load.
- Static load compression:
 1. It occurs when the load is in 1-direction of the ply.
 2. Failure occurs like rod buckling, but at the micromechanical level. Due to stability losses. Because of the low shear stiffness of the material, buckling can occur in shear mode, this is called micro-buckling, as shown in the Fig. 13.

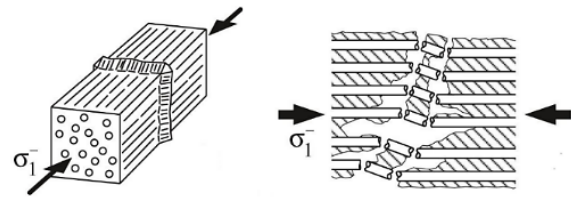


Fig. 13 Micro-shear-buckling [67].

3. The hole's edges are critical areas of micro-buckling, where buckling occurs in the fiber due to a lack of support on the edges, and shear stress leads to increased stress near the hole.

➤ Matrix failure or IFF (in plane inter-fiber-failure) It is more sophisticated, where it separated between the planes of max. Load (action plane) and plane of fracture.

- Static load tension:
 1. It occurs when the load is in the 2-direction of the ply.
 2. The plane of fracture is parallel to fiber (the action plane and fracture plane fall together) and occurs at low strains (about 5%), where there are high differences between fiber and matrix modulus.
- Static load compression:
 1. It occurs when the load is in the 2-direction of the ply.
 2. The angle of fracture plane is slightly above 45°.
- In plane shear loading τ_{12}
 1. The present of shear stress caused two action planes: the 13 plane and the 23 plane, because the matrix has the minimum fracture resistance, which leads to fracture parallel to the fiber, as shown in the Fig. 14.

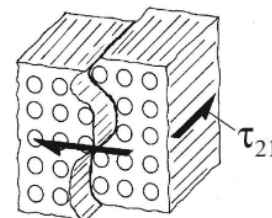


Fig. 14 Fracture-plane [67].

2. The out-of-plane shear stress (13-plane) can show the same behavior, while shear in the 23-plane leads to fracture due to failure of the matrix in tension in the direction under 45° normal to the maximum principal stress.

3.1.2. Intralaminar damage

Occur due to interlaminar stresses, including normal stress σ_3 and shear stress τ_{13} and τ_{23} , it is like to matrix fracture in intra-Laminar but with planar propagation. The crack parry's absence of fiber can cause delamination due to matrix damage, manufacturing defects, and drilling as a result of the push-out and peel-up mechanisms shown in the figure below, the areas of interlaminar stress such as free edges, curved sections, and ply-dropoffs [67].

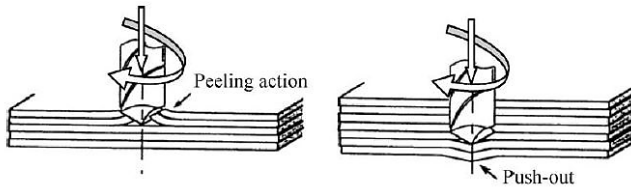


Fig. 15 Peel-up delamination and push-out delamination [67].

Table 5 shows the damage of composite materials under fatigue loading.

Table 5. Composite damage under fatigue load.

Damage of cross-ply laminate under tension-tension fatigue [42].	
Matrix and fiber failure under mode I fatigue [42] [70].	
1. Initiation of micro-crack in matrix and fiber separation. 2. The striation and hackle per unit length. The fiber imprint and hackles on the fracture surface, where the fiber imprints evidence the fabric-matrix separation, and hackles show the appear of shear stress state as a result of fiber pull-out. The striations formed in the fiber-matrix interface of the matrix are the result of the extension of micro-cracks or molecular chain breakages [71].	
woven fabric composites, under tension-tension fatigue loading in the weft direction [72].	

In their book, Ramesh Talreja and Janis Varna show a comprehensive study of damage in different types of composite material under different types of fatigue load [73].

4. Fatigue modelling

To study the fatigue behavior and fatigue life, the fatigue can be modeled in three different ways, as shown in Fig. 16.

As mentioned in Fig. 16, the fatigue modeling can be classified according to three categories.

1. Fatigue life model: which includes the use of the S-N curve and presents fatigue failure criteria but doesn't take into account mechanisms of actual damage.
2. Phenomenological model: for residual stiffness or strength, don't provide the development of damage and include

macro-stress. Based on empirical criteria, the cycle-by-cycle change in strength or stiffness can be predicted.

3. Progressive model: interested in measurable manifestations of damage like delamination [74].

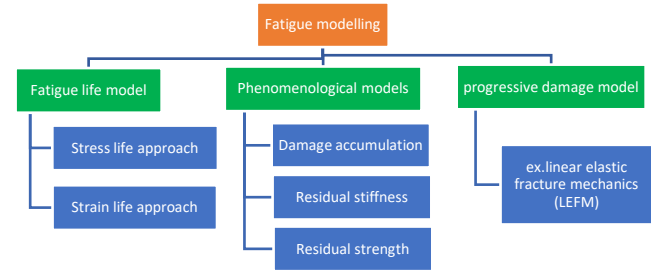


Fig. 16 Classification of fatigue modelling.

4.1. Fatigue life model

4.1.1. Stress-life approach

The oldest and most common way in researches to deal with fatigue data since the 19th century, but much research for composite materials chose this approach arbitrarily, and in this approach cannot distinguish between the initiation and propagation of damage in fatigue life [17], [34]. Can benefit from this approach when stress and strains are mostly elastic [17], and it is used in high cyclic fatigue more than 10^6 cycles [75].

This approach used the S-N curve, wöhler curve [17], which shows the contact between stress and the number of cycles to failure [76]. There is no fatigue limit in composite material as a metallic alloy, despite the high fatigue resistance of composite material [77]. By fitting the S-N curve (log-log plot), the following mathematical equation can be obtained [25].

$$\sigma_a = C + D \log N_f \quad (5)$$

C, D: Curve fitting constant, can obtain equation below by approximating the log-log curve.

$$\sigma_a = A N_f^B \quad (6)$$

Equation 6 in different form:

$$\sigma_a = \sigma'_f (2N_f)^b \quad (7)$$

$$\text{Where: } A = 2^b \sigma'_f, \quad B = b$$

The effect of stress ratio in another form of stress approach equation [1].

$$\sigma_{max} = \sigma_o N^b \quad (8)$$

Table 6 Illustrate the parameters of eq. (8) in different stress ratio in composite material.

Table 6. Parameters of eq. (8) in different stress ratio in composite material [1].

designation of Material		Stress ratio	σ_o	b
P2BT-T	Glass and carbon/epoxy [$\pm 45/90_4$]	0.10	87.2	-0.060
		0.50	91.4	-0.046
		0.70	83.7	-0.026
DD16-C	E-glass/polyester [90/0/ $\pm 45/0$] _s	10.0	480.7	-0.068
		1.43	437.2	-0.021
		1.10	490.7	-0.020
UT500/135-T	Twill-woven UT500 carbon fiber and 135 epoxy	0.05	782.2	-0.074
		0.50	676.6	-0.036
		1.00	649.0	-0.027
DLJ-T	Pultruded GFRP bonded double-lap joint	0.10	38.5	-0.083
		0.50	43.1	-0.075
		0.90	31.7	-0.031
T800S-25 C-T	Carbon/epoxy [(45, -45)/(0, 90)] _s	0.05	1285.1	-0.051
		0.50	1134.6	-0.030
		1.00	994.1	-0.018
DD16-T	E-glass/polyester [90/0/ $\pm 45/0$] _s	0.10	732.8	-0.102
		0.50	814.6	-0.094
		0.90	688.2	-0.045
DLJ-C	Pultruded GFRP bonded double-lap joint	10.0	32.5	-0.043
		2.00	30.3	-0.032
		1.10	30.3	-0.016
T800S-170 °C-T	Carbon/epoxy [(45, -45)/(0, 90)] _s	0.05	757.5	-0.093
		0.50	690.6	-0.065
		1.00	709.7	-0.058
QQ1T-T	E-glass/epoxy [$\pm 45/0_2$] _T	0.10	147.6	-0.082
		0.50	156.9	-0.073
		0.70	144.5	-0.050
T400/3601-T	Satin-woven CFRP laminates	0.10	1033.8	-0.040
		0.50	1026.8	-0.025
		0.80	1048	-0.019

Fazlali et al. [78] highlight the importance of various damage mechanisms and their interactions of UD composites under tension-tension fatigue while Movahedi-Rad [79] studies the damage of angle-ply GFRP laminate under fatigue load. Harris [20] presented in his book a comprehensive study of composite materials at different scales using a stress approach. Burhan and Kim [34] reviewed the S-N curve in composite material. Pertuz et al. [80] study the behavior of continuous fiber-reinforced thermoplastic composites under different types of loads. Zhou [81] studied the behavior of FRP composites, using different types of fibers to model fatigue behavior using genetic algorithm (GA). Djeghader and Redjel [82] presented experimental work to study the behavior of random short glass fiber/polyester under a cyclic bending load. Park et al. [83] developed a nonlinear formulation of constant life diagrams to find a more accurate S-N curve. Ropalekar et al. [84] show the development of E-glass epoxy by adding graphene oxide (GO) under a flexural fatigue test. Ma et al. [85] show the effect of stress ratio, orientation of fiber and frequency in various composite materials while Ferdous et al. [86] show the influence of stress level, stress concentration and frequency on glass fiber- reinforced polymer. Xu and Bhamidipati [87] propose a new curve fitting model by adding the formulation of equivalent static loading and the equivalent number of cycles. Kim and Huang [88] study the behavior and fatigue life of polyethylene terephthalate glycol-modified (PETG).

4.1.2. Strain-life approach

The need to create an approach to predict short fatigue life, especially in ductile material, shows the strain-life approach, which developed between the 1950-1960. The strain approach used (ϵ - N) curves, which include elastic and plastic strains

from cyclic stress-strain curves, as a data source for ϵ - N curve. This approach differs from stress approach, where the latest used average stresses rather than local strains and local stresses and use factors of elastic-stress concentration and modifications of empirical therefrom [25].

The equation of strain amplitude consists of two parts, elastic and plastic strains, as shown in eq. (9).

$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} \quad (9)$$

Where

$$\epsilon_{ea} = \frac{\sigma_a}{E} = \frac{\sigma_f'}{E} (2N_f)^b, \quad \epsilon_{pa} = \epsilon_f' (2N_f)^c \quad (10)$$

By sub. eq. (10) in eq. (9) obtain eq. (11) below.

$$\epsilon_a = \frac{\sigma_f'}{E} (2N_f)^b + \epsilon_f' (2N_f)^c \quad (11)$$

Where the σ_f' , ϵ_f' , b and c are material constant.

Figure 17 shows the regions of the strain life approach, including the following regions:

1. Region I
 - Non-progressive failure.
 - Represented by interfacial debonding and fiber breakage.
2. Region II
 - Finite life region.
 - Progressive damage.
 - Represented by fiber-bridged cracks.
3. Region III
 - Region of fatigue limit.
 - The load is low enough to stop the crack growth in matrix due to the heterogeneity of the composite materials.

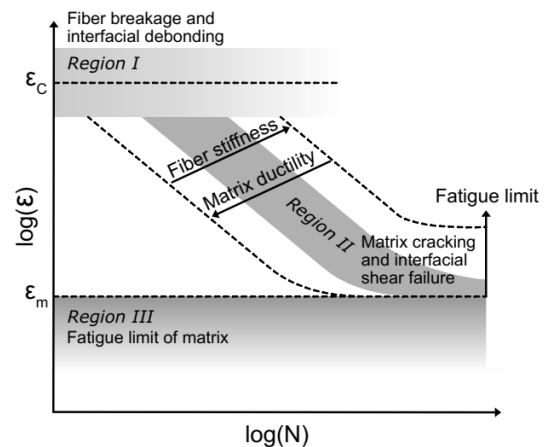


Fig. 17 Strain-life approach for composite materials [32].

There is little research about this approach and it needs more details with regard to plastic strains [25] and most composite material have brittle matrices [89].

Talreja [33] adopted the strain approach to study the fatigue in unidirectional composites loaded parallel to the fibers for each polymer matrix composite, metal matrix composite and ceramic matrix composite. Studied unidirectional composites loaded inclined to the fibers and fatigue in different types of laminates. Where adapted the strain approach rather than the stress approach because of the following factors: (1) Regardless of the fiber volume fraction,

the first cycle fails when the composite strain reaches the fiber's failure strain. (2) The matrix fatigue limit determines the composite fatigue limit. Because of the fiber limitation, even though the composite is being tested under load control, the matrix inside the composite is exposed to strain-controlled fatigue. Therefore, strain can be used to define the composite fatigue limit.

Eleftheroglou et al. [90] used stochastic modeling and structural health monitoring (SHM) data obtained from measurements of strain, to get remaining useful life (RUL) online in composite materials under fatigue loading by assent of strain data, while Kolasangiani et al. [91] used strain-life curve in their study to predict the fatigue life of composite laminate made from flax-epoxy in different staking sequence. Yadav and Thapa [92] develop strain approach by internal variable theory on woven glass/epoxy.

4.2. Phenomenological models

4.2.1. Damage accumulation

The present of applications with variable amplitude shows the need to appear cumulative or accumulates fatigue damage [93] model by Palmgren in 1924, which was called the Miner rule [94], [95] represented below.

$$\sum_i \left(\frac{n_i}{N_i}\right) = 1 \quad (12)$$

Where the n_i/N_i represents the consumption of resistance of fatigue as a result of stress amplitude S_{a1} when apply n_1 during N_1 of fatigue life endurance, the same process repeated in next cycles which have different amplitude, the final failure occur when $(n/N)_i$ become 100% [95].

The plot of fatigue damage consists of three regions: the rapidly increasing region, the plateau region and the burst-out region, which represent final failure, respectively [96], as shown in Fig. 18.

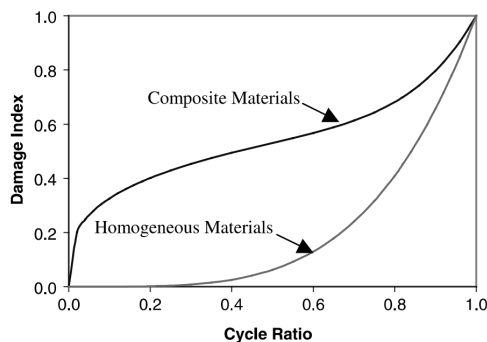


Fig. 18 Comparison between accumulation behavior between heterogeneous and homogeneous materials [97].

Epaarachchi [98] represents many relations in the development of Miner rule with suitable for the nature of composite materials in glass fiber-reinforced composites. Post [45] presented a review of the variable load modeling and last developments in this field. Liao et al. [99] study impact response and damage accumulation in composite laminate. Hassanifard and Feyzi [100] presented an experimental and numerical investigation of fatigue damage accumulation in composite bolted joints. Kharrat et al. [101] show the effect of damage accumulation on local fracture mechanisms acoustic

signatures in carbon fiber/epoxy. Batsoulas and Giannopoulos [102] presented a new theory of accumulation damage based on CDM.

4.2.2. Residual stiffness and residual strength

Residual stiffness and resided strength, or stiffness and strength degradation, is one of the way to predict and model fatigue life in composite material. Where the state of actual damage is used by the material state damage metric. The damage metric in residual strength is when the material's residual strength reaches the maximum applied stress level during cycles where material failure occurs, while the stiffness degradation used to predict its behavior isn't related to macroscopic failure. Failure can be expressed in different ways, like when reaching the critical level of predetermined stiffness degradation, when meeting the minimum stiffness of the design requirement of deformation, or by measuring the strains of cyclic [103].

The disadvantage of residual stiffness and strength is that they can't deal with complex patterns of load and fields develop multiaxial stress [103].

The stiffness degradation curve shown in the figure below consists of three stages: stage I, the redaction of stiffness (2-5%), where the transverse cracks in matrix are developed; stage II, additional redaction occurs (1-5%), where take a linear pattern, the damage occurs due to edge delamination development and the appearance of longitudinal cracks along the 0° fiber; stage III, the failure occurs when the first break of fiber [104]. The stiffness curve can be plotted by the S-N curve and Sc-N (stiffness-controlled curves)[103].

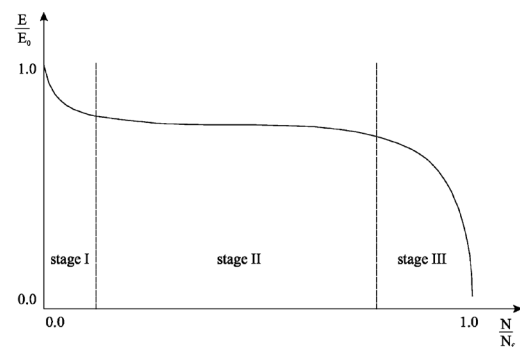


Fig. 19 Curve of stiffness degradation in fibrous composite materials [104].

The following research's used stiffness degradation model, Kim [105] to study the sleeve of composite material behavior, while Wu [106] studied the behavior of $[\pm 45^\circ]_2$ s graphite/epoxy. Yadav and Thapa [107] studied the model on woven glass fiber/epoxy. Khan et al. [108] modeled on Woven Carbon Fabric/Polyester. Koricho et al. model the twill E-Glass/Epoxy composite under bending fatigue [109]. Zhao et al. [110] studied unidirectional composite materials of different types of fibers. Herrmann et al. [111] study the model of arbitrarily oriented tunneling composites and cracks of delamination. Liu et al. [112] modify the model for fatigue in wind turbine blades. Carraro and Quaresimin [113] presented a framework to study off-axis crack initiation and propagation. Drvoderic et al. [114] study crack density in off-axis plies.

The following researches uses the residual strength model: D'Amore [115] aims to find a relation between variable amplitude and constant amplitude. Whitworth [116] models the behavior of graphite/epoxy composite laminates, while

D'Amore et al. [117] model the behavior of carbon fiber-reinforced composites. D'Amore and Grassia [118] presented a comparative study of the use of residual strength to model fatigue damage under constant amplitude.

Beyene and Belingardi presented bending fatigue behavior of twill fabric E-glass/epoxy composite [119] were used both stiffness degradation and residual strength.

Wu et al. and Shiri et al. [120], [121] combined between damage accumulation and stiffness degradation, respectively, while Khan et al. and Suwarta et al. [122], [123] combine the strength degradation with damage accumulation.

4.3. Progressive damage model

4.3.1. Fracture mechanics approach

Fracture mechanics approach or linear elastic fracture method (LEFM) it was the approach of interest in Irwin (1957) and Anderson (1995), where suppose the crack is already exists in components and by increasing crack length with the number of cycles, the damage can be analyzed. The benefit of this approach will be when the cracks are recognized as a result of fatigue [76].

Paris used the equation correlation between fracture mechanics and fatigue in the early 1960s [124], [125], where the fatigue crack growth behavior can be expressed by the relation between crack propagation rate during the cycle and stress intensity factor range. The figure below illustrates the process of obtaining this equation [25].

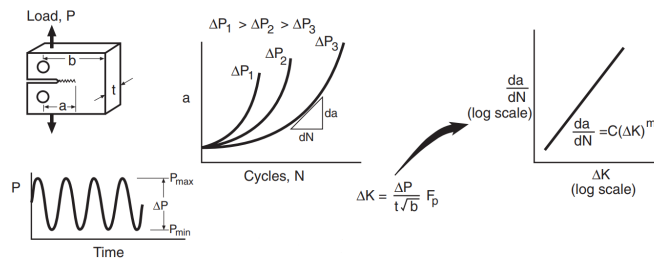


Fig. 20 Process of the empirical equation (Paris law) [25].

Where Paris equation is

$$\frac{da}{dN} = C(\Delta K)^m \quad (13)$$

$\frac{da}{dN}$: crack growth rate during cycles

ΔK : stress intensity factor range $\Delta K = F \Delta S \sqrt{\pi a}$

F : shape function, can calculated mathematically or by using finite element software's to including complex shape or show the influenced of fiber orientation as in this researches [126]-[128].

ΔS : stress range.

a : crack length

C, m material constant, where m is slope in log-log plot between stress intensity factor range and crack growth rate, it higher in brittle material than ductile material. Nittur et al. [129] provide way to find Paris parameters.

There are limits to the application of Paris law in fiber-reinforced composites. Almost all research on composite material used the same Paris equation but use change in energy release rate rather than stress intensity factor as a result of the orthotropic properties of composite laminate, which make it difficult to obtain (k) where major failure is delamination at least in initial stages [130].

It is important to know the relation between the stress intensity factor and the energy release rate.

$$G = \frac{K_I^2}{E} \quad (14)$$

$E = E'$ in plane stress, $E = E' / (1 - \nu^2)$ in plane strain, where eq. (12) become [131], [132].

$$\frac{da}{dN} = C_3(\Delta G)^{C_4} \quad (15)$$

$$C_3 = C(E')^{c^4}$$

$$C_4 = m/2$$

Tables 7 and 8 show different versions of the Paris law by different researchers and show the value of the Paris equation various with volume fraction, stress ratio, stacking sequence, fiber bridging, and all factor effects on crack propagation rate.

The curve of Paris law typically steepens and looks to approach the fatigue crack growth threshold (ΔK_{th}) a vertical asymptote at low crack growth rates. ΔK_{th} represent a bottom limit of K below which fracture formation is not typically observed. High growth rates can cause the curve to steepen once more because of the unstable cracks that start to expand quickly right before the test specimen finally fails [25]. As shown in Fig. 21.

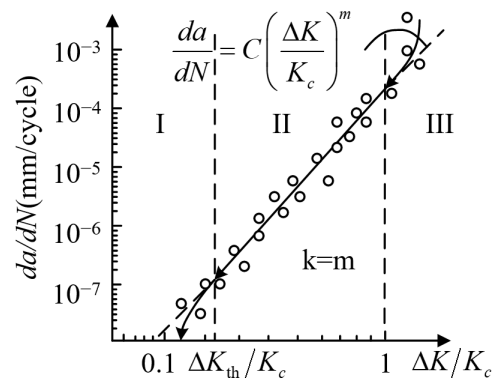


Fig. 21 the regions of fatigue crack growth [40].

There are studies related to fatigue crack growth threshold (ΔK_{th}) as in [133]-[135]

Table 7. Researches of Paris law relation with stress intensity factor.

Specimen and Material	Relation	Relation parameter value		RF.		
		Paris Law coefficient m/cycle	Paris Law exponent			
*CT (ASTM E-647) specimen of Randomly oriented short carbon fiber/ PEEK(150CA30)	$\frac{da}{dN}$ $= C \left(\sqrt{\Delta k \cdot k_{average}} \right)^m$	5.43×10^{-20}	17.4	[130]		
**Plate of bi-layered FGMs aluminum alloy and FGM of aluminum alloy and alumina	$\left(\frac{da}{dN}\right) = c(x)(\Delta K_{Ieq})^{m(x)}$ $c(x) = C_{alloy}e^{vx}$ $m(x) = m_{alloy}e^{sx}$ Where $v = \frac{1}{L} \ln \left(\frac{C_{ceramic}}{C_{alloy}} \right)$ $s = \frac{1}{L} \ln \left(\frac{m_{ceramic}}{m_{alloy}} \right)$	Aluminum alloy		[136]		
		10^{-12}	3			
		Alumina		[137]		
		2.8×10^{-10}	10			
Notch and un-notch plate and CT specimens of carbon fiber laminate	$\left(\frac{da}{dN}\right) = c(\Delta k)^m$	$(0^\circ/90^\circ/90^\circ/90^\circ/90^\circ)_s$		[138]		
		2.29×10^{-19}	4.867			
		$(45^\circ/-45^\circ)_{2s}$				
		3.55×10^{-19}	5.84			
***DCB specimen of carbon/PA6	$\frac{da}{dN} = C \left[\frac{\Delta K - \Delta k_{thr}}{1 - \sqrt{\frac{k_{max}}{A}}} \right]^m$	2.58×10^{-8}	0.6	[139]		
CT specimen of PP+SGF samples with wt. %(10-40)-vol. %(3.9-19.4)	$\frac{da}{dN} = C(\Delta k)^m$	Notch L		[140]		
		5.5×10^{-8} 3.7×10^{-10}	7.8-9			
Notch T						
3×10^{-8} 2.88×10^{-13}		8.7-11.5				
Notch L						
3.8×10^{-7} 9×10^{-13}		5.3-10.1				
Notch T						
10^{-9} 5.9×10^{-14}		7.8-13				
Plates with center cracks made from GLARE-2A2/1		$\frac{da}{dN} = C(\Delta k)^m$	1.78×10^{-15}		2.97	[141]
CT specimen of Sheet molding compound (SMC) of chopped E-glass/polyester		$\frac{dA}{dN} = C(\Delta k^\alpha \cdot k_{ave}^\beta)^m$	2.06×10^{-20}		11.2	[142]

* CT: Compact tension specimen.

** FGM: Functionally graded material composite its property various with its direction [143].

*** DCB: Double cantilever beam.

The fatigue delamination studied with the effect of fiber bridging [152]-[156], and influenced of Z-pins [157]-[161] and use digital image correlation technique to study it [162]-[164].

5. Numerical modelling

The fatigue behavior can be simulated by using software's based on finite element methods like ABAQUS and ANSYS to simulate crack initiation and propagation by means of various models, including the virtual crack closure technique (VCCT), cohesive zone modeling (CZM), Extended Finite Element Method (XFEM) and phase field explain below.

Table 8. Researches of Paris law relation with strain energy release rate.

Specimen and Material	Relation	Relation parameter value		RF.
		Paris Law coefficient m/cycle	Paris Law exponent	
I-beams of carbon fiber/epoxy (T300/914)	$\left(\frac{dA}{dN}\right) = CG_{max}^m \left[\frac{1 - \left(\frac{G_{th}}{G_{max}}\right)^{n_1}}{1 - \left(\frac{G_{max}}{G_c}\right)^{n_2}}\right]$	Unidirectional CFRP		[144]
		1.17×10^{-28}	9.97	
		Multidirectional CFRP		
		5.62×10^{-18}	3.75	
DCB of T700/QY811 carbon/bismaleim ide prepreg	$\left(\frac{da}{dN}\right)_a = C \left(\frac{GI_{max}(a)}{GI_{cf}(a)}\right)^m$	DCB mode I		[145]
		Specimen layup with interface 016//(+5/-5/0/6)s, 0°/5°		
		1.8×10^{-5}	7.3	
		(+45/-45/0/6)s//(-45/+45/0/6)s, +45°/-45°		
		0.55×10^{-3}	9.1	
		(90/0/90/0/5)s, 90°/90°		
		7.8×10^{-7}	5.4	
		015/(45/45/015), 0°/45°		
		2.1×10^{-6}	6.0	
DCB and MMB of T700/QY811 carbon/bismaleim ide prepreg	$\left(\frac{da}{dN}\right)_a = C(g_{max}(a))^m$	Mode mixture ratio (0.00)		[146]
		5.5×10^{-4}	9.1	
		Mode mixture ratio (0.25)		
		1.6×10^{-5}	7.9	
		Mode mixture ratio (0.50)		
		4.0×10^{-5}	8.0	
		Mode mixture ratio (0.75)		
		8.3×10^{-5}	7.4	
DCB of carbon fiber 5HS/RTM6 epoxy (mode I)	$\frac{da}{dN} = CG_{eff}^m$	R ² (0.75-0.86)		[147]
		4.99×10^{-18} - 3.46×10^{-21}	4.73- 5.66	
DCB and *3ENF of unidirectional carbon-fiber prepreg Cytec MTM 46 with HTS5631 fibers	$\frac{da}{dN} = c \cdot G_{max}^m$	At mean values of GI _{max} Mode I testing		[148]
		1.66×10^{-31}	12.40	
		At mean values of GI _{max} Mode II testing		
		1.75×10^{-30}	10.26	
DCB of carbon/PA6	$\frac{da}{dN} = C\Delta G^m$	1.03×10^{-10}	1.02	[139]
DCB and 3ENF samples of glass/epoxy laminated composite	$\frac{da}{a_N} = c \left(\frac{G_{I\max}(a)}{G_{IR}(a)}\right)^m$	mode I		[149]
		4.47×10^{-5}	5.27	
		mode II		
		13.49	4.0	
DCB of IM7/MTM45 carbon-epoxy	$\frac{da}{dN} = C(\Delta G)^m$	Mixed mode		[150]
		6.798×10^{-6}	4.161	
		Mode I		
		8.319×10^{-13}	2.11	
		Mode II		
		5.38×10^{-11}	1.86	
DCB of Thermosetting unidirectional carbon/epoxy M30SC/DT120	$\Delta g \frac{da}{dN} = c^1(\Delta g)^{m^1}$ $= c^1 \left(\frac{\sqrt{\Delta G} - \sqrt{\Delta G_{th}}}{\sqrt{[1 - G_{max} / \sqrt{\Delta}]}}\right)^{m^1}$	R ² = 0.766		[151]
		1.58×10^{-9}	1.96	
		R ² = 0.693		
		1.84×10^{-9}	1.71	
	R ² = 0.852			
	8.66×10^{-19}	5.90		
	R ² = 0.792			
	$\frac{da}{dN} = c^2(\Delta G_{eff})^{m^2}$ $\Delta G_{eff} = \frac{G_0}{G_{Ic}(a - a_0)} \Delta G$	1.75×10^{-18}	5.64	

*3ENF: Three points End-Notched Flexure specimen.

5.1. Virtual crack closure technique (VCCT)

This model was used in 1977 by Rybicki [165], [166] as part of the development of Irwin's crack closure integral, which involves modeling the damage to delamination based on LEFM to obtain strain energy release rate and evaluate the beginning of crack growth. Figure 22 shows the calculation of the strain energy release rate for a 2D model of a crack tip. The technique assumes the following hypotheses:

1. The energy ΔE required to close the crack is equal to the energy required to open the crack in the line between nodes 1 and 2.
2. Self-similar crack propagation, the crack propagation is in the same condition from node 2 to 3, where the opening displacements at node 1 ($\Delta u_1, \Delta w_1$) and displacement after crack propagation at node 2 ($\Delta u_2, \Delta w_2$) are the same.

The strain energy can be calculated by:

$$\Delta G = \frac{\Delta E}{\Delta A} \quad (16)$$

Where ΔA is the area of new crack surface, ΔE the total energy can be calculated by opening and shear displacement.

$$\Delta E = \frac{1}{2} [F_{x,2} \Delta u_1 + F_{y,2} \Delta w_1] \quad (17)$$

Where $F_{x,2}$ and $F_{y,2}$ shear and opening force.

Sub. eq. (17) in eq. (16)

$$G_I = \frac{1}{2\Delta a_1} F_{y,2} \Delta w_1 \quad (18)$$

$$G_{II} = \frac{1}{2\Delta a_1} F_{x,2} \Delta u_1 \quad (19)$$

Because the thickness equal to one in 2D model lead to $\Delta A = \Delta a_1$ [67], [69].

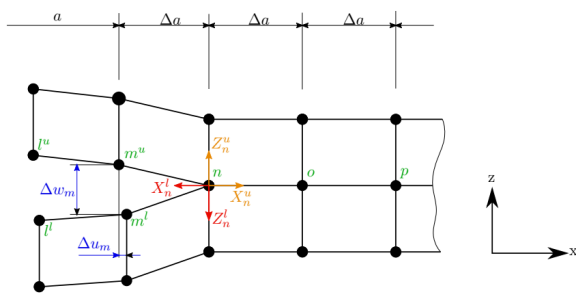


Fig. 22 VCCT in 2D model.

5.2. Cohesive zone modeling (CZM)

The first develop of CZM was shown by Dugdal [167] (1960) and Barenblatt [168] (1962), where the model was used for delamination simulation based on collegial damage mechanics. This model combines the principles of fracture mechanics and degradation of stiffness, not only the principles of fracture mechanics. The method is represented as an adhesive layer between laminae to calculate crack initiation in this model, the strength failure criteria are used.

While crack growth is presented in the crack tip front by the cohesive zone illustrated in Fig. 23, which shows the bonding force reduction between the plies, Fig. 23(b) shows the cohesive on traction separation behavior between displacement of crack and bonding residual strength, the damage change between $d = 0$, no damage, and $d = 1$, full damage, and the stiffness k reduce based on $(1 - d)k$ [69].

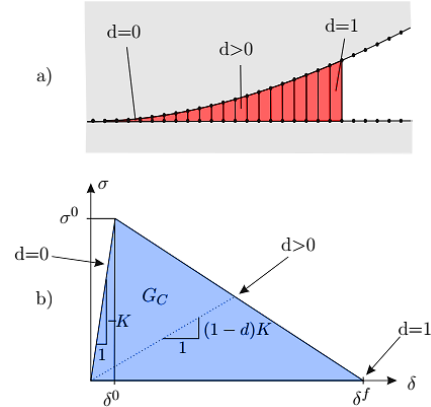


Fig. 23 CZM, behavior of traction separation [69].

The table below shows the difference between VCCT and CZM [169].

Table 9. Comparison between VCCT and CZM.

NO.	VCCT	CZM
1	Simulate crack growth on a known surface of crack	Simulate crack growth on a known surface of crack and can use a cohesive element to simulate the separation between the faces of the element.
2	Simulate the brittle fracture (LEFM)	Simulate a ductile or brittle fracture (LEFM or EPFM)
3	This technique doesn't need additional elements and uses a surface-based framework.	This technique needs to define cohesive places that are connected and interconnected with the rest of the components.
4	Need pre-crack to simulate crack growth	Can simulate crack initiation and propagation without the need to pre-crack, the crack will initiate when the stress of cohesive traction is greater than the critical value.
5	Propagation of cracks will occur when the strain energy release rate is greater than the fracture toughness.	Propagation of cracks will occur based on the model of cohesive damage, in which the usually a full crack opens, leading to energy release equal to the critical energy release rate.
6	Many crack surfaces/ fronts enable to included	Many crack surfaces/ fronts enable to included

5.3. Extended finite element method (XFEM)

Belytschko and Black (1999) [170], [171] show this technique is used to simulate crack propagation in the mesh without any need to re-mesh due to the ability to incision elements into separated parts. In this model, the degree of freedom is improved by the use of enrichment functions in the vector of displacement in the FE model, which presents the nodes surrounding the notch or material interface. The technique is shown in Fig. 24 and the equations below.

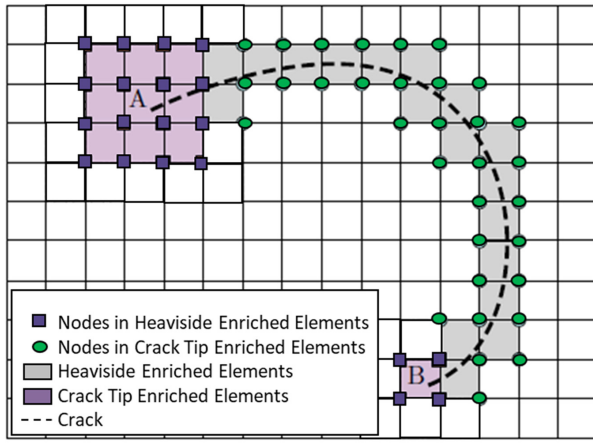


Fig. 24 XFEM technique in presence of crack and inclusion [172].

$$u^h(X) = u^{FE}(X) + u^{enr}(X) \quad (20)$$

$$u^{enr}(X) = u^H(X) + u^{tip}(X) + u^{mat}(X) \quad (21)$$

$$u^H(X) = \sum_{i=1}^l N_i(X) H(X) a_i \quad (22)$$

$$u^{tip}(X) = \sum_{g=1}^m N_g(X) \left(\sum_{\alpha=1}^4 F_{\alpha}(X) b_g^{\alpha} \right) \quad (23)$$

$$u^{mat}(X) = \sum_{h=1}^n N_h(X) X_m(X) c_h \quad (24)$$

Where,

u^{FE} : Vector of elasticity displacement.

u^{enr} : All enrichment function displacement vector.

$N_i(X)$, $N_g(X)$, $N_h(X)$: nodal shape function of classical FE.

$H(X)$: Heaviside function (strong discontinuity regard to crack partition).

$F_{\alpha}(X)$: functions of singularity (field of singularity around crack tip).

$X_m(X)$: weak discontinuity of interface of material.

a_i , b_g^{α} , c_h : additional degrees of freedom.

Sigh function consider Heaviside-Function

$$H(\xi) = \begin{cases} 1 & \forall \xi > 0 \\ -1 & \forall \xi < 0 \end{cases}$$

ξ : Function of signed distance, show whether the actual node is in side of positive or negative of crack partition [69].

Enrichment functions change from isotropic to orthotropic material; more details are found in [173].

5.4. Phase field

Phase-field fracture models have proven to be quite effective at modeling the initiation, propagation, branching, and joining of cracks in brittle and ductile materials that are stimulated externally. Phase-field fracture models are quite

flexible and can encompass many features of the material, such as anisotropy, elastoplasticity, viscoelasticity, hyperelasticity, piezoelectricity, etc. One of the most prevalent material failure processes in structural engineering, fatigue, has recently been included in the models. Figure 25 show the ability of phase field model.

The phase field fracture models' governing equations are derived from the variational principle of total energy. These equations are similar to those of other mathematical models, such as the Ginzburg-Landau and Allen-Cahn equations. These formulas aid in explaining how cracks develop in materials under different circumstances.

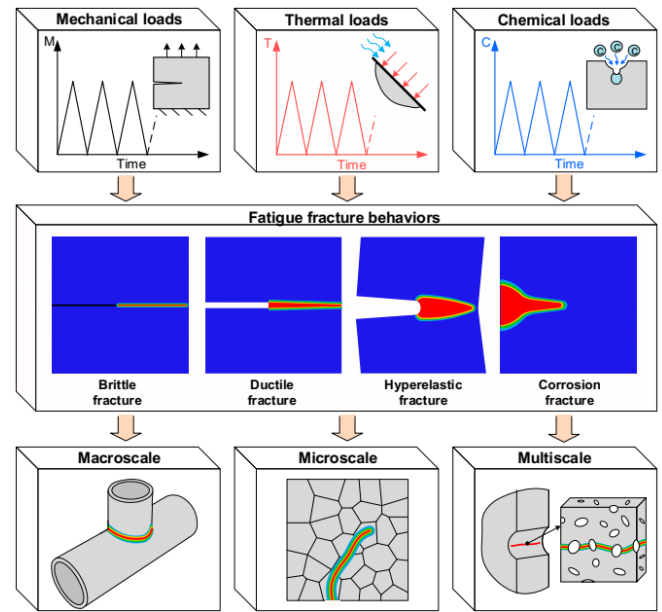


Fig. 25 Application of phase field [174].

Phase field model numerical implementation has undergone substantial development. Researchers such as Amor and Miehe have made significant contributions to the discretization of the governing equations through the use of the finite element method (FEM).

After determining the increments of displacement and fracture phase field variables, the Newton-Raphson approach is frequently employed to update the variables in the phase field model [175].

Table below show different research used numerical modelling.

Table 10. Research of numerical modelling.

Numerical modeling method	References
VCCT	[176-180]
CZM	[4, 181-194], [195]
XFEM	[136, 196-200]
XFEM + CZM	[201]
XFEM + VCCT	[202]
VCCT + CZM	[203]
Phase field	[204-208]

There more details of simulation by using FE model in [37], [40], [175] and [175].

6. ASTM used to find composite property and fatigue behavior

The heterogeneous composite material required many tests to find mechanical properties, listed below, with ASTM required to model fatigue life and behavior.

Table 11. ASTM for composite materials properties and fatigue testing.

ASTM	Description	References
D6115-97	Fatigue delamination growth (mode I) onset of unidirectional fiber-reinforced polymer matrix composites	[209]
E647	Measurement of fatigue crack growth rates	[210]
D3479/D3479M-12	For tension-tension fatigue of polymer matrix composite materials,	[211]
D 6873	Bearing fatigue	[212]
D5528	Measure of interlaminar fracture toughness (mode I) of unidirectional fiber-reinforced polymer matrix composites.	[213]
D7905	Measure of the interlaminar fracture toughness (mode II) of unidirectional fiber-reinforced polymer matrix composites	[214]
D6671M-06	Measure of interlaminar fracture toughness (mixed mode I-mode II) of unidirectional fiber reinforced polymer matrix composites	[215]
D 3039	In plane tension (E_x , E_y , V_{xy} , ST_x , ST_y)	[216]
D7291	Out of plane tension (E_z , ST_z)	[217]
D 6641	In plane compression (EC_x , EC_y , VC_{xy} , SC_x , SC_y)	[218]
D 3410	Compression test with unsupported gage section by shear loading	[219]

Table 12. shear test for composite material [220].

	Uniform shear	All stress state	Shear strength	Shear modulus	Ref.
Short beam shear (D2344)			×		[221]
Isipescu D 5379	×	×	×	×	[222]
± 45 tension shear D 3518			×	×	[223]
2 rail shear D 4255			×	×	[224]
3 rail shear D 4225			×	×	[224]
Double notch shear D 3846			×	×	[225]
Tube torsion D 5448	×		×	×	[226]
V-notched rail shear D 7078	×	×	×	×	[227]

7. Conclusions

Fatigue in composite material is considered a tremendous challenge because of the numerous influences on it, including matrix and fiber material, volume fraction, orientation of fiber, moisture content, porosity, rate of applied stress and strain, stress ratio and frequency, which make the numerical simulation is difficult due to the need for an empirical base before simulation, which leads to cost-ineffectiveness.

The study of composites is different from scale to scale, which increases the difficulty of modeling and the number of

tests required for it, due to its heterogeneity. Eliasson [32] presented a good framework to study composite fatigue at different scale lengths.

Almost all studies of fatigue in composites used the stress approach, while the rest of the approaches had limit studies, where the strain approach is more suitable for low-cycle fatigue. Almost all studies by fatigue crack growth method study the delamination failure as the most common failure in composite laminate, while there are limit application for failure by fracture.

The numerical modelling by phase field show good result comparison to XFEM, where the latest has no ability to simulate branching of crack, and consider XFEM better than VCCT and CZM which have specific path.

Still the study of fatigue and modeling it spearheaded with of composite development material, like using Nano material as in [228-231] or including more details in composite like influenced of stitch density [232-234], effect on holes on it [235, 236], add viscoelasticity effect [237], study delamination migration [238], etc.

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