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Chaotic Function-Based Improvements in Evolutionary and Swarm-Based Algorithms: systematic review

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Abstract

The increasing intricacy of practical optimization challenges in engineering, logistics, and data science has led to the extensive use of evolutionary and swarm-intelligence metaheuristics. Yet these methods often suffer from premature convergence and poor diversity when exploring high-dimensional or multimodal landscapes. Deterministic chaotic functions—exemplified by logistic, sine, tent, and Chebyshev maps—offer ergodic, sensitive, and topologically mixing sequences that can be seamlessly integrated into metaheuristic operators to enhance both exploration and exploitation. In this survey, we first introduce the mathematical foundations of chaos theory and categorize the principal chaotic maps used in optimization contexts. We then systematically review their incorporation into two major algorithmic families: Swarm-intelligence techniques (e.g., Particle Swarm Optimisation, Ant Colony Optimisation), evolutionary algorithms (e.g., Genetic Algorithms, Differential Evolution), where chaos has been used to population initialisation, mutation and crossover strategies, and adaptive parameter management, where chaotic sequences modulate inertia weights, pheromone updates, and positional updates.

For each category, we synthesize benchmark results demonstrating improvements in convergence speed, solution accuracy, and robustness against local optima. We also discuss practical implementation issues—including map selection, parameter calibration, and computational overhead—and highlight promising future directions such as hybrid chaos-driven frameworks and domain-specific applications in robotics, wireless sensor networks, and machine learning.

Keywords
Chaotic maps · Metaheuristic optimization · Evolutionary algorithms · Swarm intelligence · Chaos theory.

1. Introduction

The pursuit of efficient optimization techniques is a fundamental challenge across scientific and engineering domains, including operations research, artificial intelligence, manufacturing, and communications. Many real-world problems—such as resource allocation, scheduling, network design, and machine learning parameter tuning—are inherently nonlinear, high-dimensional, and multimodal, often lacking analytical gradients or exhibiting discrete variables [1][2]. In such contexts, metaheuristic algorithms have emerged as indispensable tools due to their flexibility, robustness, and minimal reliance on problem-specific information.

Metaheuristics are high-level frameworks designed to guide subordinate heuristics in exploring complex search spaces and approximating global optima. Among these, evolutionary algorithms (EAs) and swarm intelligence algorithms are two of the most prominent categories, both drawing inspiration from natural phenomena [2-4]. Evolutionary algorithms, Evolution Strategies (ES), Genetic Algorithms (GAs) and Differential Evolution (DE) are examples of methods in which genetic variation and natural selection 18 i.e. by repeated application of selection, crossover and mutation to populations of candidate solutions have inspired these methodologies. Some of the swarm intelligence algorithms are Particle Swarm Optimisation (PSO), Ant Colony Optimisation (ACO) and Artificial Bee Colony (ABC). emulate the collective behaviors observed in social organisms, leveraging simple interaction rules among individuals to discover optimal solutions collectively [1].

Despite their widespread adoption and proven efficacy, both evolutionary and swarm-based metaheuristics are prone to several intrinsic limitations. Chief among these are premature convergence, where the search stagnates at suboptimal solutions due to loss of population diversity, and the exploration-exploitation trade-off, which governs the aptitude algorithm in effectively balancing the international search (survey) with fine-tuning near promising regions (exploitation) [2][5]. Other challenges include sensitivity to parameter settings, inefficiency in highly rugged or dynamic landscapes, and difficulty in escaping from local optima. These shortcomings can significantly compromise algorithmic performance, particularly for large-scale or highly complex optimization tasks.

To address these limitations, researchers have increasingly turned to chaos theory as a powerful enhancement strategy. Chaotic systems, though deterministic and governed by nonlinear dynamics, produce behavior that appears random and complex—yet is fully determined by precise mathematical rules [6]. Core characteristics of such systems—ergodicity, sensitivity to initial conditions, and topological mixing—ensure thorough exploration of the search space and generate pseudo-random sequences that maintain structured diversity [6][7]. Studies have demonstrated that substituting traditional random number generators with chaotic maps, such as the logistic, tent, or Lozi maps, significantly enhances the effectiveness of evolutionary operators and population initialization, thereby improving convergence behavior and avoiding premature stagnation (e.g., in chaotic-enhanced Genetic Algorithms for nonlinear equations and chaos-driven Differential Evolution variants) [6].

The application of chaotic functions such as the logistic map, tent map, sine map or Chebyshev map in evolutionary and swarm intelligence algorithms has given birth to a new class of "chaos-enhanced" metaheuristics. These chaos related strategies are incorporated into different steps of some meta-heuristic algorithms, including adaptive parameter control, crossover, mutation, population initialization, and dynamic control of exploration and exploitation. The final outcome is a more robust, flexible and effective search process that can successfully traverse complex fitness landscapes and avoid premature convergence 7. Numerical comparisons through benchmark and real-world test problems have demonstrated that embedded chaotic metaheuristics can provide a rapid convergence to high-quality solutions and superior overall performance in many cases 9.

Due to the increasing body of work and the rapid evolution of this multidisciplinary field, a review is needed to enable the identification of effective strategies, to recognise research areas needing further development, and to collect lessons learned. This introduction describes the general motivations, theoretical / technical foundation, practical implementation, benchmark outcomes as effects of the application of chaotic function-based techniques in evolutionary and swarm-based optimization algorithms. With an in-depth overview, this volume is intended as a reference for researchers and practitioners in the field of metaheuristic and swarm intelligence optimization.

The organizational assembly of the research is as follows:: Section 2 reviews theoretical foundations of deterministic chaos and common chaotic maps. Section 3 Theoretical Foundations of Metaheuristics: Evolutionary and Swarm Algorithms Section 4 surveys chaos-enhanced evolutionary and swarm algorithms. Section 5 discusses comparative performance analyses, implementation considerations, and open challenges. Section 6 concludes the survey and outlines future research directions.

2. Theoretical Foundations of Deterministic Chaos and Common Chaotic Maps

Deterministic chaos describes a class of dynamic systems governed by nonlinear deterministic equations whose solutions exhibit behavior that appears random, yet arises from deterministic processes. Despite the absence of true randomness, chaotic systems demonstrate high sensitivity to initial conditions, a property famously encapsulated by the "butterfly effect" [10]. This sensitivity, together with topological mixing and dense periodic orbits, gives rise to rich, complex dynamics that are neither wholly predictable nor strictly stochastic. Figure 1. Shows sorting the chaotic systems.

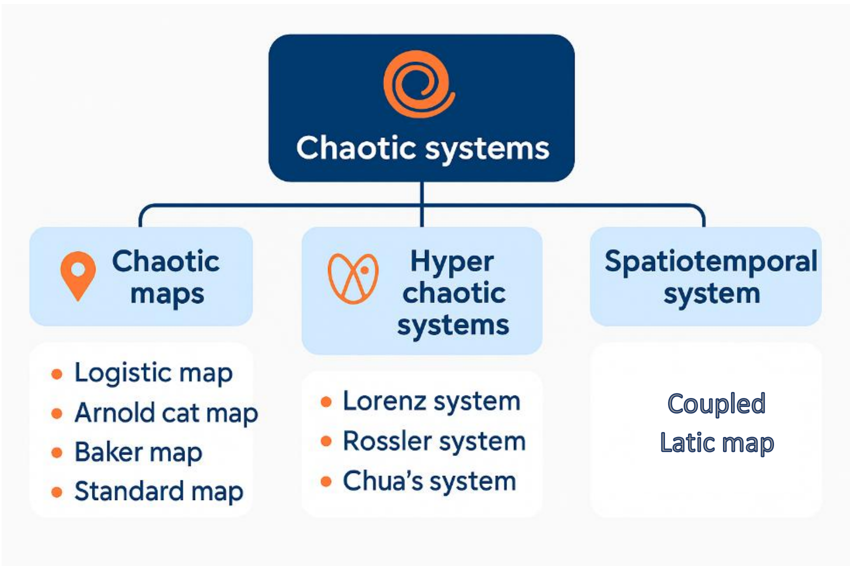

Figure 1: classification of chaotic systems

2.1. Key Properties of Chaotic Systems

Three principal properties distinguish chaotic systems from other dynamical processes: ergodicity, sensitivity to initial conditions, and topological mixing. Ergodicity ensures that chaotic orbits cover the accessible phase space densely, supporting diversity in candidate solutions. Sensitivity to initial conditions leads to exponentially diverging outcomes, enabling diverse and uncorrelated sequences. Topological mixing ensures that any region of phase space will eventually overlap with any other, promoting global exploration and discouraging premature convergence [10].

2.2. Mathematical Representation

Chaos is commonly studied using discrete-time dynamical systems, or maps, which iteratively update a variable according to a nonlinear function:

$$x_{n+1} = f(x_n)$$

Where (f) is a nonlinear transformation and x_0 is the initial condition. For suitable choices of f and x_0 , the sequence exhibits chaotic behavior [10].

2.3. Common Chaotic Maps

Metaheuristic optimisation has made substantial use of a number of simple but effective chaotic maps. Among the notable examples are the tent map and the logistic map, sine map, and Chebyshev map. These maps provide a range of ergodic and mixing behaviors and have become standard tools for introducing chaos into optimization algorithms [10][11].

- Logistic Map:

$$x_{n+1} = rx_n(1 - x_n)$$

where r∈[3.56995,4] produces chaos. The widely used of logistic map is due to ease and well-characterized behavior.

- Tent Map:

$$x_{n+1} = \begin{cases} \mu x_n, & x_n < 0.5 \\ \mu(1 - x_n), & x_n \geq 0.5 \end{cases}$$

with μ=2 for fully chaotic dynamics.

- Sine Map:

$$x_{n+1} = \lambda\, sin(\pi x_n)$$

where λ≈1 ensures chaoticity.

- Chebyshev Map:

$$x_{n+1} = cos(n\ \cos^{-1}(\ x_n))$$

These maps share the ability to produce sequences that, while entirely deterministic, exhibit properties reminiscent of random noise—an asset in stochastic search and optimization. By leveraging these properties, researchers have successfully integrated chaotic maps into evolutionary and swarm-based metaheuristics to promote diversity, avoid stagnation, and enhance global search capabilities.

The deterministic nature of chaotic maps ensures repeatability, which is desirable for scientific experimentation and algorithm benchmarking. At the same time, their pseudo-random characteristics enrich the stochastic processes underlying metaheuristics, helping algorithms escape local optima and achieve superior performance across a wide array of optimization problems [10][11].

3. Theoretical Foundations of Metaheuristics: Evolutionary and Swarm Algorithms

Metaheuristics Challenging optimisation problems can be approached by high-level algorithmic blueprints called metaheuristics, which apply adaptive and stochastic mechanics to control subsidiary heuristics. Metaheuristics are grounded in their ability to efficiently explore high-dimensional and potentially non-convex solutions spaces, and therefore are particularly suited for practical problems where standard optimization techniques are inapplicable or computationally intractable 1. The classification of metaheuristic algorithms is illustrated in Figure 2.

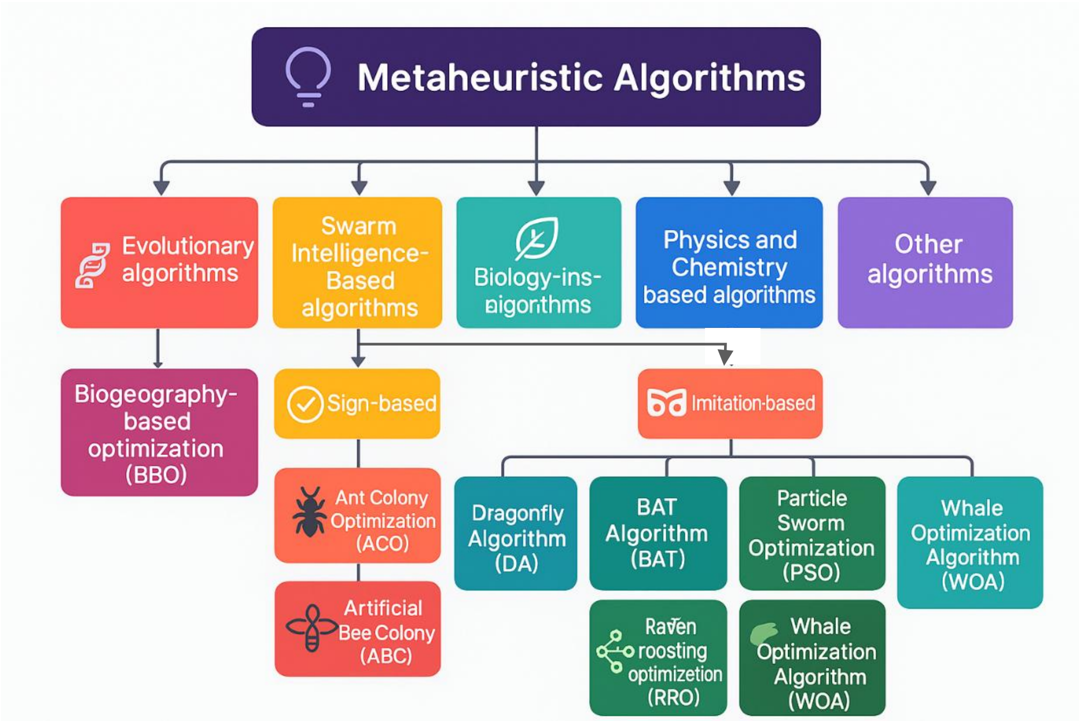

Figure 2: classification of metaheuristic algorithms

3.1. Evolutionary Algorithms

The principles of natural evolution found in Darwinian process: variation, selection and inheritance are the foundation of evolutionary algorithms (EAs). Each candidate solution that an EA keeps in memory is a point in the search space. Through pseudo-natural selection (more able individuals getting higher chances to reproduce), recombination or crossover (exchange of information between solutions), and mutation (introduction of randomness), the population evolves from one generation to the next. EAs can trade off the two goals of the exploration (search of new regions of the space) and exploitation (refinement of possible solutions), due to the combination of these operators 13 and 14.

Mathematically, EAs are modeled as Markov processes, where the population at each generation depends probabilistically on the previous generation's state and applied operators. This stochastic framework allows EAs to escape local optima and, with appropriate diversity-preserving mechanisms, to converge to global optima with high probability, especially as the number of generations increases [14][15].

3.2. Swarm Intelligence Algorithms

The aggregated and distributed behaviour of natural systems such as colonies of insects, shoals of fish, and flocks of birds are theoretical basis for swarm intelligence (SI) algorithms. Opposed to EAs, the general idea behind SI algorithms is that primitive local rules, social interactions and adaptive behaviors lead to emergent global intelligence 3 and no specific genetic operators are applied. Particle Swarm Optimisation (PSO), for example, combines the best-known positions of its neighbours with those of its own to modify the position of every agent (or particle) in search space. This mechanism can be seen as a dynamic system whose behavior is governed by the interplay of cognitive and social components, leading to self-organization and adaptive search [3][16]. Similarly, in Ant Colony Optimization (ACO), artificial ants construct solutions incrementally, guided by pheromone trails that encode collective learning. Theoretical studies show that ACO can be formalized as a stochastic process converging to high-quality solutions under [17].

3.3. Exploration–Exploitation Balance

A central theoretical principle in both EAs and SI algorithms is the balance between exploration (diversifying the search to discover new regions) and exploitation (intensifying the search near high-quality solutions). This trade-off is managed via algorithm parameters (e.g., mutation rate in EAs, inertia weight in PSO, pheromone evaporation in ACO) and is essential for avoiding premature convergence and confirming vigorous presentation crossways varied optimizing landscapes [1][12].

3.4. Convergence and Performance

While the stochastic nature of metaheuristics precludes deterministic guarantees of global optimality, theoretical analyses using Markov chains, dynamical systems, and probability theory have provided valuable insights into their convergence behavior and robustness. Properly designed metaheuristics, particularly those with diversity maintenance and adaptive mechanisms, can approach global optima with high probability over time [3][18].

4. Surveys chaos-enhanced evolutionary and Swarm algorithms

In recent years, advanced meta-heuristic and swarm intelligence algorithms have achieved significant progress in solving complex optimisation problems such as the Travelling Salesman Problem (TSP). Some of the important works in this aspect are summarized in Table1. Tubishat et al. combined singer chaotic map to enhance spatial diversity and Simulated Annealing (SA) for local search exploitation. [19] introduced an improved Sine Cosine Algorithm (ISCA). Compared with traditional SCA and other optimisation methods, the hybrid SCA technique proved to be applied with higher accuracy as well as significant feature selections for Hadith text classification and benchmark datasets. To enhance the search ability of the solution space, Aydilek et al. 20] used a hybrid firefly and particle swarm optimisation method, enhanced by chaotic maps. The method extracts random parameters from 10 different chaotic mappings with the purpose of enhancing local optimal avoidance and explorative capability. Compared to canonical and hybrid approaches, and puzzle benchmarks, we obtained successful and reliable results for CHFPSO. The Agglomerative Greedy Brain Storm Optimization (AG-BSO), proposed by Wu and Fu [21] integrates a heuristic crossover operator, exchange rules to enhance the efficiency of the proposed algorithm, hierarchical agglomeration to improve the convergence property and a greedy approach to maintain population diversity. When tested with TSP cases AG-BSO performed better than conventional BSO and other heuristic algorithms in terms of accuracy, speed of optimization and robustness. By using a chaotic neurone, sine function, linear matrix, etc., Cui et al. [22] introduced a new model for chaotic neural networks. Energy function of Travelling salesman problem (TSP) is formulated by a simple-rich chaotic dynamics, and the practical effectiveness of the proposed model is verified by the implementation with FPGA. To solve TSP, Palominos and Vera [6-1] recently proposed using a hybrid GA that incorporates the local search in the process of their working. [23] studied chaotic searches in MBO. These approaches significantly improved on the quality and runtime efficiency of solutions to difficult combinatorial optimisation problems by injecting chaos into the worker and early solution generation phases. Lawah et al. [24] have devised and optimised cryptographic S-boxes using discrete chaotic maps and the GWO. Strong cryptographic and secure S-boxes could also be generated as a result of the larger solution space and robustness of the hybrid method. QCBOA was introduced by Prasanthi et al. [25] and provides a refined version of the original butterfly optimisation procedure, whose features employ ingredients from quantum computing and chaos theory. For benchmark functions and real-world limited optimisation problems, contrastive action between chaos mapping and quantum wave-based searching method accelerated the convergence and improved the accuracy. Long, Min, and Longlong Wang [26] suggested a new S-box design way by employing the improved artificial bee colony (ABC) algorithm with a discrete chaotic map. First, an unifromly distributed population of S-box is initiated by OBO, in which chaotic sequences are generated by a mutually-coupled logistic map. To achieve the second phase of optimization process for the S-boxes and to improve the cryptographic strength, in enhanced ABC method, Gaussian mutations and dual transposition operations are employed. The good resistance to a cryptanalytic attack is guaranteed by the significance of nonlinearity and differential uniformity of the fitness function. Based on the experimental results, the proposed method can generate S-boxes of higher security (nonlinearity, avalanche effect, bit independence; resistance against both linear and differential attacks) as compared to various state of the art methods. A physical education course scheduling scheme with an improved chaotic genetic algorithm (CGA) was proposed in literature [27]. It adopts a 2D matrix as crossover and mutation, real number encoding for solutions representation, and chaos function to initialize population and mathematical transformation of scheduling restrictions and factors such as instructor, course, room and time slot. The combination allows to avoid local minima and strengthen the capability of searching globally. Experimental evidence shows that improved CGA exhibits superior efficiency, robustness, and resource requirement over standard genetic algorithm and other optimization methods, and therefore is a useful and flexible mechanisms to solve complex scheduling problems in education. A novel population-based meta-heuristic algorithm, that inspired by the chaotic dynamics, called chaotic evolution optimization (CEO) algorithm is introduced in this paper [28]. The way the two-dimensional discrete memristive map evolves in a chaotic manner provided the principal inspiration for CEO. The CEO approach is based on a theoretical model that generates random search directions for evolutionary algorithms through the compounding of a hyperchaotic attribute of the memristive map. The CEO is subsequently formed by integrating the crossover and mutation mechanisms of the DE framework. The application of combining various chaotic maps in different stages of MOEAs, in navigating NSGA-II framework, is again considered in the present study [29]. They systematically incorporate 10 different chaotic maps (e.g., logistic, tent, and cat maps) by the authors in three main algorithmic process steps: population initialization, crossover, and mutation. This article evaluates the effect these changes have on both convergence and diversity for popular benchmark multiobjective problems (ZDT series) and over a wide range of experiments. The findings indicate that chaotic maps significantly improve the performance of MOEA, especially with regard to problems that involve complex Pareto fronts or local optima. The cat map is very good in the difficult cases.

Table 1: Summary of Recent Metaheuristic Algorithms Enhanced with Chaotic Systems and Their Application Domain

Author(s)	Metaheuristic Method Used	Chaotic Algorithm Used	Application / Case Study
Tubishat et al. [19]	Enhanced Sine Cosine Algorithm (ISCA) + Simulated Annealing	Singer chaotic map	Feature selection for Hadith text classification, benchmarks
Aydilek et al. [20]	Hybrid Firefly and Particle Swarm Optimization (CHFPSO)	Ten different chaotic maps	Benchmark optimization problems
Wu and Fu [21]	Agglomerative Greedy Brain Storm Optimization (AG-BSO)	Not specified (focus on greedy & clustering)	Traveling Salesman Problem (TSP)
Cui et al. [22]	Chaotic Neural Network Model	Linear matrices, sine functions, multiple chaotic neurons	TSP optimization (FPGA implementation)
Palominos et al. [23]	Marriage in Honeybees Optimization (MBO)	Chaotic methods (in initialization & worker phases)	Traveling Salesman Problem (TSP), combinatorial optimization
Lawah et al. [24]	Grey Wolf Optimizer (GWO)	Discrete chaotic maps	Cryptographic S-box design and optimization
Prasanthi et al. [25]	Quantum-Chaotic Butterfly Optimization Algorithm (QCBOA)	mapping the Chaos, quantum wave-based exploration	Benchmark & real-world constrained optimization
Long, Min, & Longlong Wang [26]	Improved Artificial Bee Colony (ABC) Algorithm	Intertwining logistic map	Cryptographic S-box optimization
Study [27]	Improved Chaotic Genetic Algorithm (CGA)	Chaotic mapping	Physical education course scheduling
Paper [28]	Chaotic Evolution Optimization (CEO), Differential Evolution framework	Memristive hyperchaotic map	Benchmark functions

Study [29]	Multiobjective Evolutionary Algorithms (MOEAs, NSGA-II)	Ten chaotic maps (logistic, tent, cat, etc.)	ZDT benchmark problems (multiobjective optimization)

5. Discussion the performance analyses, implementation considerations, and open challenges.

The integration of chaotic systems into evolutionary and swarm intelligence algorithms has attracted considerable interest due to their potential to overcome longstanding limitations such as premature convergence and lack of diversity. Comparative studies consistently show that chaotic maps—when properly embedded in algorithmic operators—can significantly boost the performance of metaheuristics across a variety of benchmark and real-world problems [30][31].

Often, empirical benchmarks show that swarm-based algorithms (like Particle Swarm Optimisation and Ant Colony Optimisation) and chaos-enhanced evolutionary algorithms (like Genetic Algorithms or Differential Evolution) outperform their canonical counterparts, especially on multimodal, high-dimensional, or deceptive search spaces. By employing chaotic sequences for population initialization, parameter control, and operator selection, these algorithms tend to achieve:

- Improved convergence speed: Chaotic dynamics help the population rapidly escape local optima and traverse the search space more effectively.
- Enhanced solution quality: Higher diversity and pseudo-randomness in search steps result in more robust exploration and, often, better final solutions.
- Greater robustness: Chaos-based methods are less likely to get trapped in suboptimal regions and often display more consistent performance over multiple runs [32].

However, comparative performance is highly sensitive to the choice of chaotic map, the method of integration, and problem characteristics. For example, logistic and tent maps are frequently effective, but map selection may need to be tailored to the specific optimization scenario [33].

While the potential benefits of chaos-based enhancements are substantial, their successful implementation requires careful attention to several practical aspects:

- Selection and tuning of chaotic maps: Not all chaotic maps yield the same improvement; parameter settings (e.g., initial conditions, control strictures) must be sensibly selected to ensure wanted ergodic and mixing properties.
- Algorithmic embedding: Chaos can be introduced at different stages (e.g., initialization, mutation, crossover, velocity update), and its impact depends on where and how it is embedded [33].
- Computational overhead: Although chaotic maps are computationally inexpensive, improper integration can increase algorithmic complexity or introduce numerical instability.
- Repeatability and randomness: While chaos offers deterministic pseudo-randomness, the reproducibility of results requires that initial conditions and parameters be documented and controlled [10].

Despite their promise, chaos-enhanced metaheuristics present several open challenges:

- Theoretical understanding: The mechanisms by which chaos improves exploration–exploitation balance remain incompletely understood; there is a need for deeper theoretical analysis and modeling [32]
- Automated map and parameter selection: Developing adaptive or self-tuning schemes for map and parameter selection would further improve the generality and usability of chaos-based methods.
- Hybrid and dynamic approaches: Combining multiple chaotic maps, or dynamically switching between maps during runtime, could lead to further performance gains but introduces new design complexities.
- Real-world applications: Most studies focus on benchmarks; more work is needed to validate chaos-enhanced algorithms on large-scale, noisy, or dynamic real-world problems.
- Integration with other techniques: Synergizing chaos with other enhancement strategies—such as adaptive parameter control, hybridization with other metaheuristics, or the incorporation of machine learning techniques—remains an open research area [32].

6. Conclusion and Future Work

The incorporation of chaotic maps into swarm intelligence and evolutionary algorithms was examined in this investigation, highlighting how chaos can significantly enhance convergence speed, solution quality, and robustness. While the use of chaotic functions helps overcome issues like premature convergence and lack of diversity, the choice of chaotic map, integration method, and parameter tuning remain critical factors affecting performance. Future research in this area should focus on several promising directions: Developing self-adaptive and dynamic chaos integration strategies that can automatically adjust chaotic maps and parameters according to the problem landscape and search progress. Conducting more comprehensive theoretical analyses, possibly using tools from dynamical systems and probability theory, to better explain the underlying mechanisms of chaos-enhanced metaheuristics. Expanding empirical validation to include large-scale, real-world, noisy, or dynamic optimization problems to demonstrate practical benefits. Exploring hybrid frameworks that combine chaotic systems with other intelligent optimization techniques, such as machine learning-based control, to further boost performance. Overall, chaos-based improvements represent a valuable and still-evolving direction for the metaheuristic optimization community, with significant potential to address increasingly complex problems across engineering, science, and industry.

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