

Optimizing CEED Problems using Walrus and Red-Tailed Hawk Optimization Algorithm

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ABSTRACT: Combining economics emissions power dispatch (CEED) challenging optimization issue that involves is a lowering the entire cost of electricity production yet satisfying overall environment emissions constraints. The issue at hand is complex because of the nonlinear and not convex character of the function's goals and restrictions. Fuels are the primary form of electrical power production; coal, is the globe's principal fuel, accounting for 42% of the entire electrical power produced internationally. Energy from electricity is excessively pricey because of the substantial amounts invested by generation corporations as a consequence of the high reliability of fuel for generating electricity. A new two ways to solve CEED problem is to use a Algorithms for Red-Tailed Hawk Optimize (RTHOA) and Walrus Optimize (WOA). By the advantages of nature-inspired metaheuristic techniques, we can reduce generation costs and reduce emissions, enhancing power system efficiency and sustainability. As inspired by walruses' social behavior and movement patterns, the WOA demonstrates significant potential for exploring and exploiting the solution space. RTHOA, which mimics hawks' hunting strategy and sharp vision, is just as good. Three examples have been validated by a simulated research investigated. The IEEE 30-bus with six generators in Case 1 has a power consumption of 2.834 p.u., the 10-unit in Case 2 has a power requirement of 2000 MW, and the 40-unit in Case 3 has a demand of 10,500 MW. Comparison with alternative approaches documented in the published works, the simulation outcomes of the created techniques showed interest in the area of lowering emission and the expenses of electric generation. With a tiny standard deviations and a significant correlation between the optimum and poorest fitness figures, the WOA demonstrated strong performance and great consistency, especially in Case 1. As demonstrated in instance 1, the RTHOA also demonstrated strong features, particularly in preserving stability and attaining targeted fitness goals. But in Case 3, the RTHOA showed more fluctuations, suggesting a wider capacity for investigation. These results imply that the two algorithms provide useful methods for solving the CEED phenomenon.

Keywords: Cost minimization, combined economics emissions dispatch, Walrus optimization algorithm, Red

tailed hawk optimization algorithm. Economic load dispatch



1. INTRODUCTION

Electricity is currently the energy source that is most commonly utilized because of its easy mobility and effective distribution. Its production, generally in enormous scale infrastructure, transportation, and customer distribution have traditionally been centralized in industrialized nations, although contemporary legislative changes are beginning to slant towards decentralization. Although it is less expensive to use fossil fuels to produce power, performing so emits contaminants like CO₂ and SO₂, since 40% of global CO₂ emissions come from the generation of electricity utilizing these types of fuels. In additional to introducing pricing turbulence because of variations in gasoline prices, such reliance raises ongoing economic difficulties. compared to standard sources of energy, unconventional energy sources do not run out of power. Hydro, geothermal, wind, and solar energy have been regarded as environmentally friendly since they are distributed, satisfy both requirements and supplies, do not produce greenhouse gases, and never pollute.

A wide range of optimization methods additionally have simplified the process of improving CEED in power systems which may be categorized into conventional, non-conventional and hybrid systems. Energy systems are frequently optimized for efficacy and sustainability. The CEED issue affecting electrical systems has been tackled in recent generations using various kinds of strategies for optimization. These strategies can be broadly categorized into three classes: hybrid methods, non-conventional methods, and conventional, or classic, ways.

Managing energy efficiently and sustainably is important because of growing electricity demand and environmental concerns. CEED, a critical problem for power network working, intends to lower fuel expenditures while reducing emissions. CEED problems used to be solved with traditional optimization methods, but sometimes they didn't handle modern power system complexity and dynamics. [1] proposed a method for solving Economic Load Dispatch (ELD) in micro-grids through metaheuristic algorithms, it established that the suggested procedure worked well for optimizing EED objectives. [2] came up with the Spiral Optimization Algorithm to solve CEED. A novel optimization procedure based on fluid mechanics was developed to tackle CEED [3], so it might overlook some important aspects. The algorithm works great for both economic and environmental goals, based on the study. It's not as applicable to power system optimization outside of (EED). [4] proposed a method for resolving the ELD challenges in power scheme called enhancement of the Category Leader Optimizing Process. Study shows the algorithm optimizes economic goals in power system process. It shows a particular focus on ELD. Crow Search method can optimize efficient and environmental objectives in power schemes used it to solve the EED problem [5]. These results may not be generalizable to other kinds of optimization problems in power systems because they focus on EED. In a study published by [6], the Osprey Algorithm was found to show promising results in optimizing ELD. Osprey Optimization method wasn't compared to other optimization algorithms for ELD, so it wasn't a comprehensive evaluation. By using the Grasshopper Optimization Algorithm (GOA) to solve the CEED problem [7]. the GOA optimized the CEED problem pretty well. This doesn't talk about how the GOA scales to larger power systems, which might impact its practical applicability. To resolve the CEED difficulties, [8] aimed a Probability Distribution Arithmetic Optimization procedure with varying order price utilities. The main finding was that the recommended procedure improved optimization execution in handling CEED objectives. It didn't provide a detailed comparison with further cutting-edge methods towards resolving CEED issue, which would have highlighted the algorithm's competitiveness. In [9] authors provides a comprehensive review of the techniques used to address the CEED problem. Using the Gravitational Search Algorithm, proposed a solution for EED issue [10].

This paper's remaining parts are presented as follows: The CEED definition in Section 2 takes equality and equality limitations into account. In Section 3, the suggested methods (WOA) and (RTHOA) for the CEED problem are discussed. In Section 4, the simulation results and discussions are described. Finally, we use a comparative analysis to arrive at our results. In Section 5, we draw our conclusions.

2. CEED FORMULATION

2.1 ECONOMIC DISPATCH PROBLEM DESIGN

To minimize the schemes overall production costs while meeting the schemes total power demand and several critical power system criteria, you have to select the ideal mix of power generation. The fuel cost of each unit in a generating plant is calculated using Eq. (1) [2].

$$FCi(P_i) = \sum_{i=1}^{Ng} \{a_i P_i^2 + b_i P_i + c_i\} \quad (1)$$

$FCi(P_i)$ signifies the entire production cost (\$/hr), calculated as the addition of assembly components (Ng). P_i denotes the power outcome of production entity i, and a_i , b_i , and c_i are the element's fuel price factors. The "valve point influence" denotes to the ripple influence on the generating unit curve caused by the opening of individually steam entrance outlet in a turbine. To accurately model this, an additional term imitating the valve point influence must be added to the price function, complicating the optimization due to increased local minima and non-linearity. The most comprehensive explanation of the valve point effect highlights how it makes the generator's incremental fuel cost function (IFCF) more realistic by accounting for valve-point effects.

Non-convex curves might appear if the input-output curve accounts for the valve point effect (VPE) and optimal precision is required. When non-convex input-output curves are employed, the equal incremental cost approach cannot be applied since there are several outputs for each increment cost value. Consequently, the effects of valve loading are included in the basic quadratic CF as a recurring rectified sinusoid contribution. As previously mentioned, the entire computation of fuel costs across all units is recognized as the total cost of production function of EDP are mention in Eq.(2) [2].

$$F_T = \sum_{i=1}^{Ng} \{a_i P_i^2 + b_i P_i + c_i + |d_i \sin e_i (P_i^{min} - P_i)|\} \quad (2)$$

The unit I expenditure cost variables with valve point impact are d_i and e_i . The revised function for the ED problems, which is to lower F_T , is given by equation 2. The total output cost of the manufacturing plant must be calculated using this equation, which takes into consideration the following equality and inequality restrictions.

2.2 EQUALITY CONSTRAINTS

Both the loss in transmission and the system's total demand (PD) must be met by the entire power generated. Consequently, the real power balance constraint may be found using the method below in Eq. (3):

$$\sum_{i=1}^{NG} P_i = P_D + P_L \quad (3)$$

The total load demand in MW is represented by the symbol PD, and the system's transmission losses are represented by the symbol PL. The generating power must lie between the lowest output P_{imin} and the maximum output P_{imax}, according to the inequality restrictions showed in Eq. (4).

$$P_{imin} \leq P_i \leq P_{imax} \quad (4)$$

The real power of unit I's lowest and maximum producing constraints are represented by the symbols P_{imin} and P_{imax}. It is possible to determine the exact value of the system losses using a power flow solution. Methods based on the constant loss formula coefficients or B-coefficients can be used to calculate system losses. The system loss mathematic formula, sometimes referred to as (Kron's loss formula ([10]) and established on the B-coefficients as follows in Eq. (5):

$$P_L = \sum_{i=1}^{NG} B_{0i} P_i + \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_i B_{ij} P_j + B_{00} \quad (5)$$

The *i*th component of the loss factor vector is B_{0i}, and B_{ij} is the *j*th section of the loss factor matrix. B₀₀ represents the loss constant. Matrix B influences system losses, depending on network admittances and topology.

2.3 EMISSION DISPATCH PROBLEM FORMULATION

The EDP identifies the lowest-cost distribution of power among generating units to meet a given demand. However, traditional EDP does not consider total pollution emissions. Each fossil-fuel power unit's emissions depend on its electricity output. Emissions can be modeled as a product of a term derived from output power and a quadratic function as showed in Eq. (6) [3].

$$E_T = \sum_{i=1}^{NG} \{\alpha_i P_i^2 + \beta P_i + \gamma_i\} \quad (6)$$

The pollutant EDP focuses on optimizing the total pollutant emissions with effects of valve loading are included in Eq. (7).

$$E_T = \sum_{i=1}^{NG} \{\alpha_i P_i^2 + \beta P_i + \gamma_i + \xi_i e^{\tau P_i}\} \quad (7)$$

where *ET* (emissions totaling pollutants) is measured in pounds per hour. All of the generating units are represented by *NG*. *P_i* is the power outcome of generating element *i*. The emission coefficients for unit *i* are, *β* and *ξ_i*.

2.4 CEED

As was previously stated, the ED and the emission dispatch are two separate troubles. By merging the ELDP with an emission constraint, emission dispatch is introduced to conventional economic load dispatch problems. In this research project, the two points might be joined into a specific objective function by including a cost penalty component (*h*) which can be calculated as mention in Eq. (8) [4].

$$h = \frac{\alpha_i P_{i,max}^2 + b_i P_{i,max} + c_i}{\alpha_i P_{i,max}^2 + \beta P_{i,max} + \gamma_i} \quad (8)$$

When the cost fine element (*h*) is considered, the utmost costly fuel unit is *i*, and the maximum pollution-emitting entity is *j*. Emissions and fuel expenditures are joined to form the cost penalty element *h*, with fuel cost representing the total functional expenditure in US dollars per hour. To find out the cost fine feature for a given demand from load, the following steps are engaged:

- Divide extreme fuel price of any power generator by extreme pollutants emissions to obtain initial value.
- Arrange the resulting cost factor values in increasing order.
- Starting with the smallest *h* unit, incrementally increase the maximum capacity of each unit until the total generated value exceeds the demand.
- The penalty element '*h*' for indicated load is then recognized as the penalty element '*h*' associated with last entity that meets demand. Eq. (9) defines collective objective function of ED and emissions supplies.

$$F_T = W_{eco} F_T + h W_{emi} E_T \quad (9)$$

We define the combined objective function *F_T*, where *W_{emi}* and *W_{eco}* stand for weighting factors. There exist several techniques for ascertaining the two components of weighing. For instance, when *W_{eco}* = 1.0 and *W_{emi}* = 0.0, pure emission dispatch is generated, yet the usual EDP is generated. The optimal combined objective is found using the cardinal priority ranking method. To create non-inferior solutions, this technique normalizes the weights.

2.5 MATHEMATICAL MODELLING

The following is the mathematics structure for CEED efficiency issue [4]:

Objective cost formula is specified in Eq. (10):

$$\text{Minimize } F_T = \sum_{i=1}^n (F_i \cdot P_i) + \lambda \cdot \sum_{i=1}^n (E_i \cdot P_i) \quad (10)$$

where: F_T represents overall expenditure of production (economic goal),

F_i : is fuel price factor for entity i ,

P_i : refer to produced electrical energy,

E_i : is emissions cost function for entity i ,

λ : is emissions price coefficient (balance element concerning economics and emissions goals),

n : is the total number of generating units.

3. OPTIMIZATION ALGORITHM

3.1 WALRUS OPTIMIZATION ALGORITHM (WOA)

Every walrus in inhabitants of searchers comprise the WOA procedure is a potential fix for the optimization problematic. Since individually walrus's position intimate search universe influences parameters of the delinquent, they are vectors. The walrus population is represented numerically by the population matrix. When WOA is first put into practice, the walrus populations are started randomly. The algorithm starts its search for the best solutions after this random setup. This WOA population matrix is determined by using Eq. (11) [11].

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N \times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & & \vdots & & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & & \vdots & & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N \times m} \quad (11)$$

Important components of the WOA are the amount of variables of decision-making (N) and entire number of walruses (N), both represented by m . Every walrus, denoted by X_i , offers a nominee (possible) elucidation to the issue. The qualities that every walrus proposes for the choice variables, denoted as $x_{i,j}$, impact the evaluation of the objective function. This procedure demonstrates how every walrus contributes to the algorithm's search for the best solutions. The assessed principles used for fitness function achieved as of walruses are indicated by Eq. (12)

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N \times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N \times 1} \quad (12)$$

The objective function, denoted as F_i , is evaluated for each walrus (i) in the optimization process, where F denotes object form vector. The superiority of prospective resolutions is assessed over object function quantities. The greatest member is the solution with the maximum objective function value, while the worst member has the lowest. During each iteration, participants are regrouped based on changes in their unbiased function principles.

• WOA modelling

Three procedures framework WOA's process for reviewing the walrus's location, which is founded on animal's distinctive performance.

Stage one: exploration

Walruses have a diverse diet consisting of marine invertebrates like shrimp, aquatic cucumbers, spineless coral reefs, tube worms, and several mollusks, including above sixty kinds of food. They primarily favor benthic bivalve mollusks, particularly oysters, using their searching vibrissae and vigorous flipper travels to graze sea bottom. In their exploration for food, leadership is determined by walrus with lengthiest tusks, specifying sturdiest distinct in grouping. The object function principles in optimization are equivalent to the tusk length of walruses, where the furthestmost influential walrus represents the finest impending solution with the uppermost charge for the objective function. Walruses' search patterns create individual perusing neighborhoods inside pursuit planetary, improving the universal search competence of the WOA. Equations (13) and (14) are used to quantitatively represent the behavior dynamic [11].

$$x_{i,j}^{P_1} = x_{i,j} + rand_{i,j} \cdot (SW_j - I_{i,j} \cdot x_{i,j}) \quad (13)$$

$$X_i = \begin{cases} X_i^{P_1}, & F_i^{P_1} < F_i, \\ X_i, & \text{else,} \end{cases} \quad (14)$$

In the first phase, the new location $x_{i,j}^{P_1}$ of the i th walrus, $x_{i,j}^{P_1}$ is determined by its j th dimension, , with $F_i^{P_1}$ as the estimated objective function value. $X_i^{P_1}$, is the generated location for the i th walrus. I_i , takes random integer values between 1 and 2, $rand_i$, random values in the range [0,1], and SW is measured strongest walrus and ideal candidate solution. Typically, I_i is set to 1, but in this case, it is adjusted to 2 to enhance the algorithm's exploration abilities. This adjustment leads to more focused and wider shifts in the walruses' positions, enhancing the algorithm's global search capacity. These conditions enable the algorithm to travel away from resident bests and search the preliminary issue addressing planetary more efficiently.

Stage tow: Walrus Motion

The travelling machinery of walruses, wherever they travel to stony seashores or ridges as the climate warms in utilized in the late summer by the WOA to escort the walruses in discovering appropriate places inside the exploration planetary. According to this modeling, each walrus is motivated to a diverse (at arbitrary picked) location in a distinctive portion of pursuit universe by using Eq. (15) and Eq. (16) which represent mathematical model [12].

$$x_{i,j}^{P_2} = \begin{cases} x_{i,j} + rand_{i,j} \cdot (x_{k,j} - I_{i,j} \cdot x_{i,j}), & F_k < F_i; \\ x_{i,j} + rand_{i,j} \cdot (x_{i,j} - x_{k,j}), & \text{else,} \end{cases} \quad (15)$$

$$X_i = \begin{cases} X_i^{P_2}, & F_i^{P_2} < F_i; \\ X_i, & \text{else,} \end{cases} \quad (16)$$

The location of chosen walrus may track i^{th} course of the algorithm as specified in X_k for $k \in \{1, 2, \dots, N\}$ and $k \neq i$. The symbols x_k , represent this walrus's j th dimension, and F_k gives us the value of its objective function. Centered on the second stage, $X_i^{P_2}$, is the recently discovered i th walrus site.

Stage three: Exploitation

Walruses face threats from killer whales and polar bears, prompting them to adjust their positions for defense and escape. Mimicking this behavior enhances the WOA's exploitative power within a short search region around viable solutions. The WOA adopts walrus location variations take place inside a district centered around individually walrus, by a span that decreases as the algorithm progresses. This strategy helps identify the best search space area, starting with a global search and focusing on local optimization. An adjustable radius with lower/upper boundaries is used during iterations. To replicate this in WOA, a municipal is recognized about apiece walrus, and an innovative position is arbitrarily produced inside this district. In order to simulate the phenomena, a vicinity is taken as surrounding every walrus, and according to Equations (17) and (18), an alternate location is first produced at random among the specified region. Subsequently, in accordance with Eq. (19), the current location takes the place of the prior one if the goal function's score is enhanced [11].

$$x_{i,j}^{P_3} = x_{i,j} + (lb_{local,j}^t + (ub_{local,j}^t - rand \cdot lb_{local,j}^t)), \quad (17)$$

$$Local\ bounds : \begin{cases} lb_{local,j}^t = \frac{lb_j}{t}, \\ ub_{local,j}^t = \frac{ub_j}{t}, \end{cases} \quad (18)$$

$$X_i = \begin{cases} X_i^{P_3}, & F_i^{P_3} < F_i; \\ X_i, & \text{else,} \end{cases} \quad (19)$$

In the repeated form, the low and higher bounds of the j th variable are indicated by t , lb_j , and ub_j , respectively. The variables $lb_{local,j}^t$ and $ub_{local,j}^t$, respectively, denote the allowed resident low and high bounds for the j th variable. These values mimic local exploration close to the candidate solutions. The third phase is the foundation for the recently developed i th walrus website, X_i^{P3} . The WOA implementation flowchart is presented in Figure 1, and its pseudo-code is specified in Algorithm 1.

Algorithm 1. Pseudo-code of WOA

Start WOA.

- 1: Enter entire optimizing problematic data.
- 2: Established numeral of walruses (N) then whole quantity of repetitions (T).
- 3: Setting up the walruses' primary placements.
- 4: For $t = 1: T$
- 5: Bring up to date resilient walrus established on fitness function assessment measure.
- 6: For $i = 1: N$
- 7: Phase1: Serving tactic (investigation)
- 8: Compute fresh position of the j th walrus via (13).
- 9: Inform the i th walrus position via (14).
- 10: Phase 2: Relocation
- 11: Select migration target intended for i th walrus.
- 12: Evaluate fresh position of j th walrus with (15). (15).
- 13: Update the i th walrus location using (16).
- 14: Phase 3: Escaping and fighting against predators (exploitation)
- 15: Calculate a new position in the neighborhood of the i th walrus using (17) and (18)
- 16: Update the i th walrus location using (19).
- 17: end
- 18: Save finest applicant resolution to date.
- 19: end
- 20: Produce finest quasi-optimal resolution attained by WOA designed for specified case.
- 21: End WOA

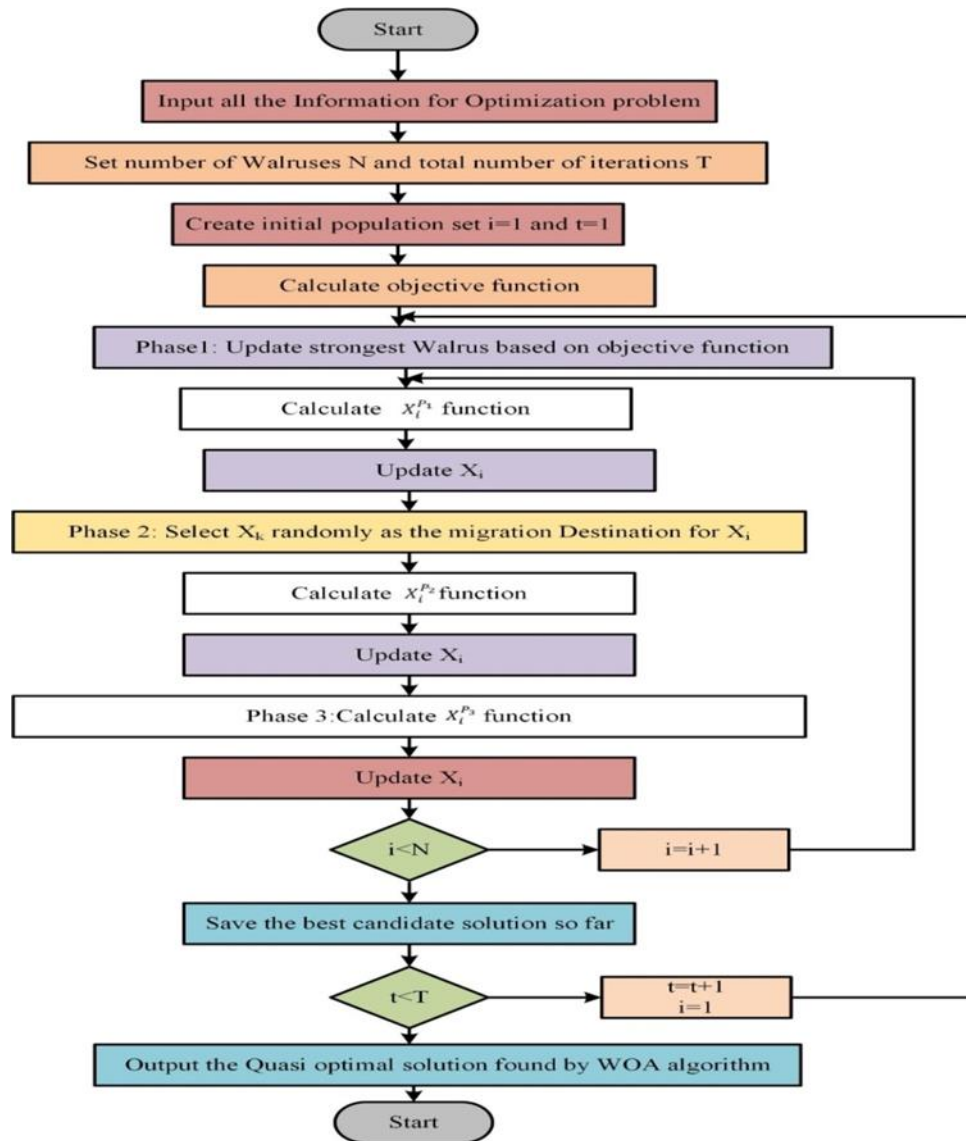


FIGURE 1 Flowchart diagram of Walrus optimization [11]

3.2 RED TAILED HAWK OPTIMIZATION ALGORITHM (RTHOA)

Based on RTH predatory behaviour, this metaheuristic algorithm optimizes CEEDs. These hawks are known for their keen hunting skills, precision, and adaptability to environmental conditions. RTHO solves complex optimization problems by mimicking hawks' hunting strategies and adaptive behaviours. Hawk populations could be a solution to CEED optimization. They explore the search space and modify their locations centered on their encounters and collective knowledge. By incorporating exploration and exploitation mechanisms, the hawks can search for promising areas in the search space and refine the best ones. By dynamically adjusting its search strategies, the algorithm captures the adaptability of RTHs, ensuring efficient convergence towards the best solution. By balancing economic costs and emission reductions while ensuring reliable power supply, RTHO is particularly good at handling CEED's diverse and complex constraints.

•Inspiration and behavior during hunting

The RTH is a carnivorous predator, primarily hunting mammals like rodents, which make up 85% of their diet, but also eating birds, fish, invertebrates and amphibians. Their diet varies with geography and season. During flight, they use a gentle dihedral wing posture to minimize flapping and conserve energy, soaring efficiently with speeds of 32-64 km/h and pitching equal to 190 km/h. They exhibit powerful flight patterns, especially when defending nests.

•High soaring: Flying at high altitudes through least dithering to preserve energy through scouting.

•Low soaring: Descending to a lower altitude and circling prey to assess finest stage and habitation to strike.

•Stooping and swooping: Diving at prey from a high speed in a curved trajectory after selecting the optimal attack position.

•Mathematical model

The hunting strategy and the inspiration source are covered in the first part. Next, a model of the RTH movements is created, and every phase of the process is scrutinized.

•High soaring: In order to find the optimum spot for accessibility to food, the red-tailed hawk will fly high in the clouds. Eq. (20) is computational representation that describes the altitude soaring phase, and Figure 2 shows how the red-tailed hawks act throughout this period of flight [13]:

$$X(t) = X_{best} + (X_{mean} - X(t-1)) \cdot Levy(dim) \cdot TF(t) \quad (20)$$

where X_{mean} is the position's mean, X_{best} is the best-obtained position, and $X(t)$ is the RTH location at repetition t . The impose flying circulation function may be computed using Eq. (21) and (22), and the transition factor function, denoted by $TF(t)$ can be calculated according to Eq. (23)

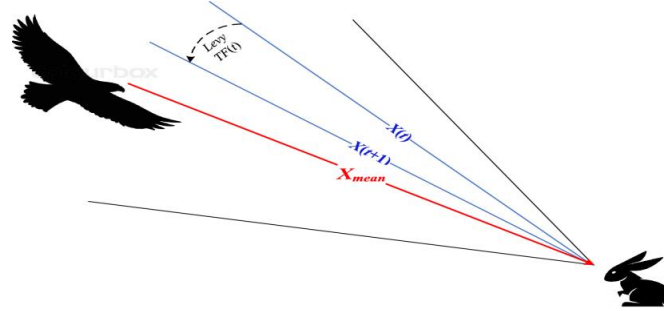


FIGURE 2 Performance of Red-Tailed Hawk throughout highest elevated phase

$$Levy(dim) = s \frac{\mu \cdot \sigma}{|v|^{\beta-1}} \quad (21)$$

$$\sigma = \left(\frac{\Gamma(1+\beta) \sin(\frac{\pi\beta}{2})}{\Gamma(1+\beta/2) \cdot \beta \cdot 2^{(\frac{\beta}{2}-1)}} \right) \quad (22)$$

As u and v are arbitrary numbers [0 to 1], β is fixed (1.5), and s is fixed (0.01). Dim is the issue width.

$$TF(t) = 1 + \sin(2.5 + \left(\frac{t}{T_{max}}\right)) \quad (23)$$

• Near to the ground soaring: The hawk soars in a series of spirals that bring it much nearer to ground, surrounding the target. This phase is demonstrated in Fig. 4, and its formula can be stated in Eq. (24), Eq. (25) and Eq. (26) [13].

$$X(t) = X_{best} + (x(t) + y(t)) \cdot Stepsize(t) \quad (24)$$

$$Stepsize(t) = X(t) - X_{mean} \quad (25)$$

$$\begin{cases} x(t) = R(t) \cdot \sin(\theta(t)) \\ y(t) = R(t) \cdot \cos(\theta(t)) \end{cases} \begin{cases} R(t) = R_0 \cdot \left(r - \frac{t}{T_{max}}\right) \cdot rand \\ \theta(t) = A \cdot \left(r - \frac{t}{T_{max}}\right) \cdot rand \end{cases} \begin{cases} x(t) = x(t) / \max|x(t)| \\ y(t) = y(t) / \max|y(t)| \end{cases} \quad (26)$$

where A is the angle gain, $rand$ is a arbitrary gain, r is a limit gain, and R_0 is the radius's initial value. Stooping and Swooping: The hawk sharply bends to attack the victim by using its best position from the low flying stage. These features let the hawk make spiraling movements around its prey. Figure 3 describes the manners of the red-tailed hawks throughout this stage.

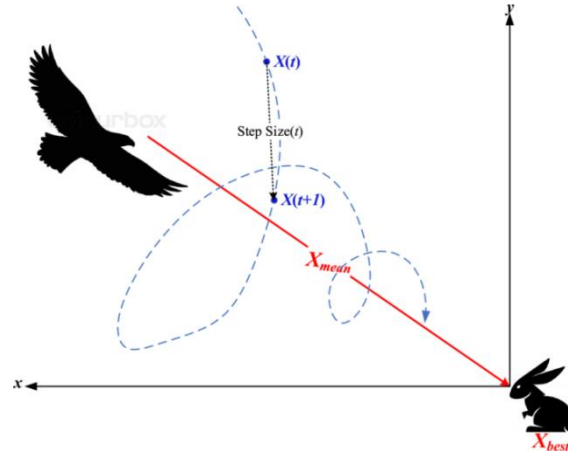


FIGURE 3 Performance of Red-Tailed Hawk throughout low elevated phase

•Stooping and Swooping: Hawk habits its best-attained location from low-slung flying point to abruptly stoop and knockout the victim. These features help hawk in flying about the target through corkscrew motions. Fig. 5 explains the performance of the RTHs throughout this phase. This stage can be modeled as follows in Eq. (27):

$$X(t) = \alpha(t) \cdot X_{best} + x(t) \cdot Stepsize1(t) + y(t) \cdot Stepsize2(t) \quad (27)$$

It tolerates for the succeeding computation of individually stage size: where individually stage extent can be computed as showed in Eq. (28) and Eq. (29).

$$Stepsize1(t) = X(t) - TF(t) \cdot X_{mean} \quad (28)$$

$$Stepsize2(t) = G(t) \cdot X(t) - TF(t) \cdot X_{best} \quad (29)$$

As α and G stance for speeding up and gravity modules, are correspondingly specified in equations (30) and (31). Conduct of RTH throughout stooping and swooping steps is defined in figure 4.

$$\alpha(t) = \sin^2\left(2.5 - \frac{t}{T_{max}}\right) \quad (30)$$

$$G(t) = 2 \cdot \left(1 - \frac{t}{T_{max}}\right) \quad (31)$$

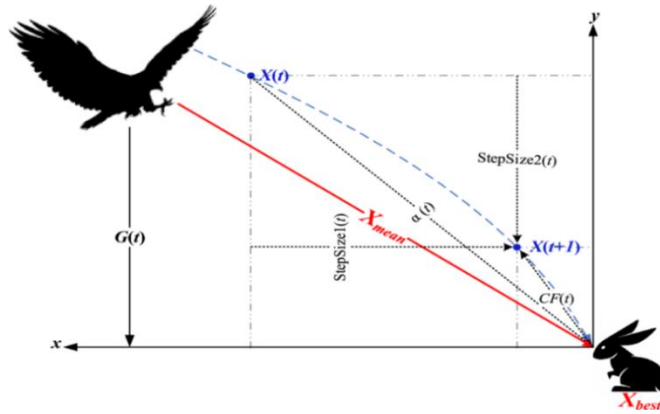


FIGURE 4 Manners of RTH throughout stooping and swooping phases [13]

where G represent earth gravity impact, that lowers for lessen form of assault when the hawk is much nearer to its victim, and α is the hawk's speed, which increases with an upsurge in t to enhance the meeting speed. Step-by-step algorithm for the RTHO technique modified for CEED optimizing.

Algorithm 2. Pseudo-code of RTHO

- 1: Begin: arbitrary peers inside the pursuit planetary.
- 2: Whereas $t < T_{max}$ do
- 3: Extraordinary soaring phase:
- 4: for $i = 1: Npop$ do
- 5: Compute Impose flight scattering
- 6: Calculate the transition factor TF
- 7: Update positions

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8: end
9: Low soaring stage:
10: for  $i = 1: Npop$  do
11: Compute direction synchronizes
12: Inform locations
13: end
14: Stooping & Swooping phase:
15: for  $i = 1: Npop$  do
16: Compute speeding up then gravity features
17: Compute stepsize
18: Inform locations
19: end
20: End Even though
    
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4. SIMULATION RESULTS and DISCUSSIONS

In this study: Three cases are considered; the suggested algorithms are implemented on standard IEEE 30-bus system bearing in mind VPE in existence of two WOA and RTHOA, in mandate to solve CEED problem. The single-line diagram of which is explained by the Figure 5: Case 1: is six unit (power demand-Pd = 2.834 MW), Case 2: is a 10-unit (power demand-Pd = 2000 MW) and Case 3: is a 40-unit (power demand-Pd = 10,500 MW). The cost function and emission data's for the above cases are taken from [14].

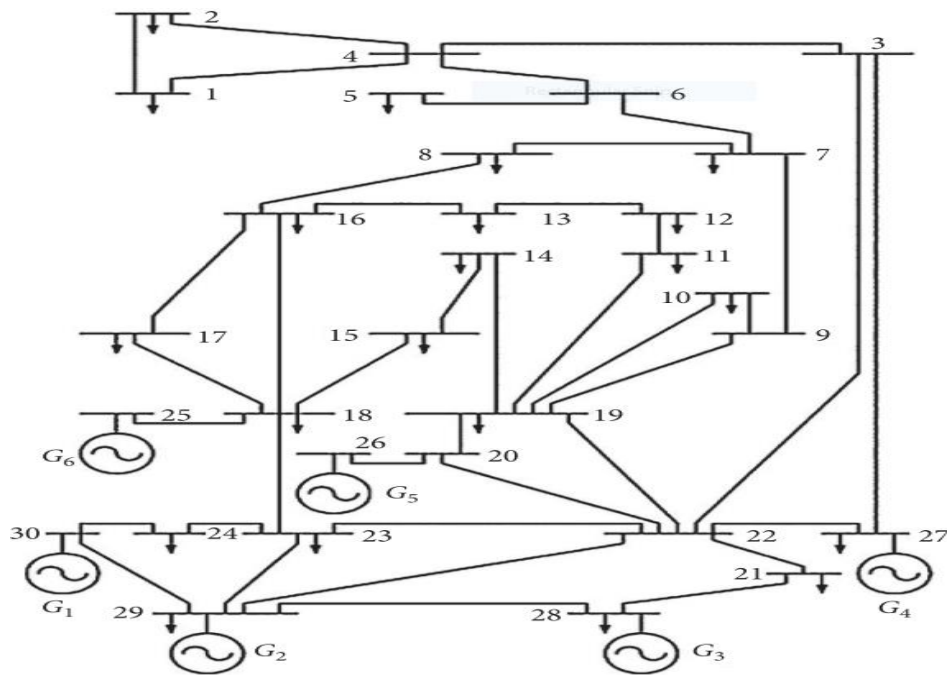


FIGURE 5 Single line diagram of IEEE 30-bus system

4.1 WALRUS OPTIMIZATION RESULTS

The convergence curves of the proposed algorithm for the best solution obtained for the 6-unit, 10-unit and 40-unit test systems is shown in Figures 6, 7 and 8 considered in this simulation using WOA.

Study case One: Six generation units

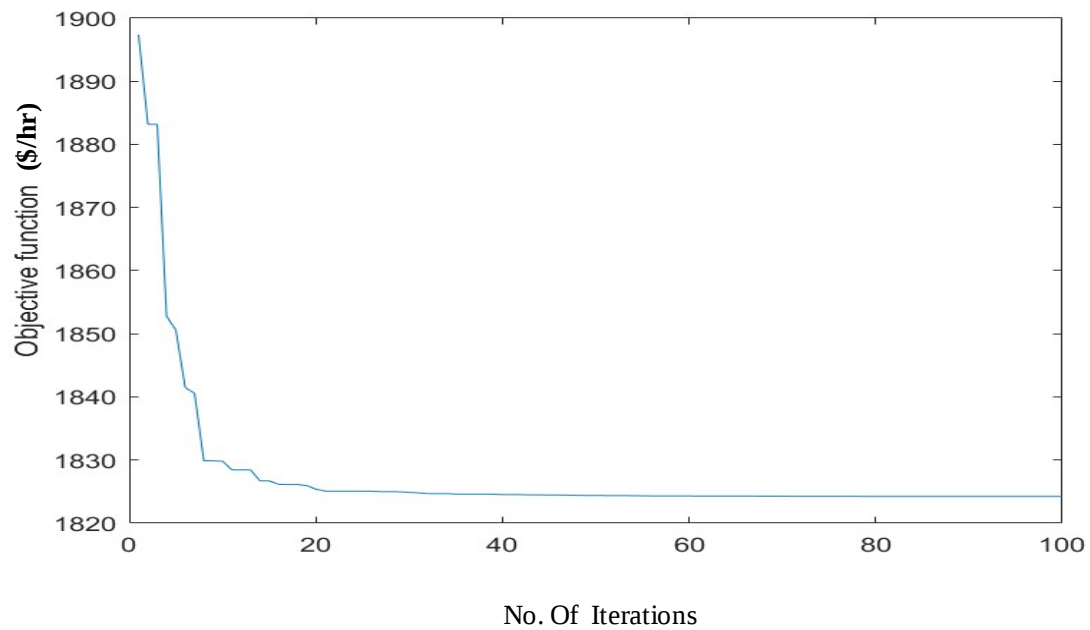


FIGURE 6 Reaching target of cost function aimed at study case one with WOA

Study case Two: Ten generation units

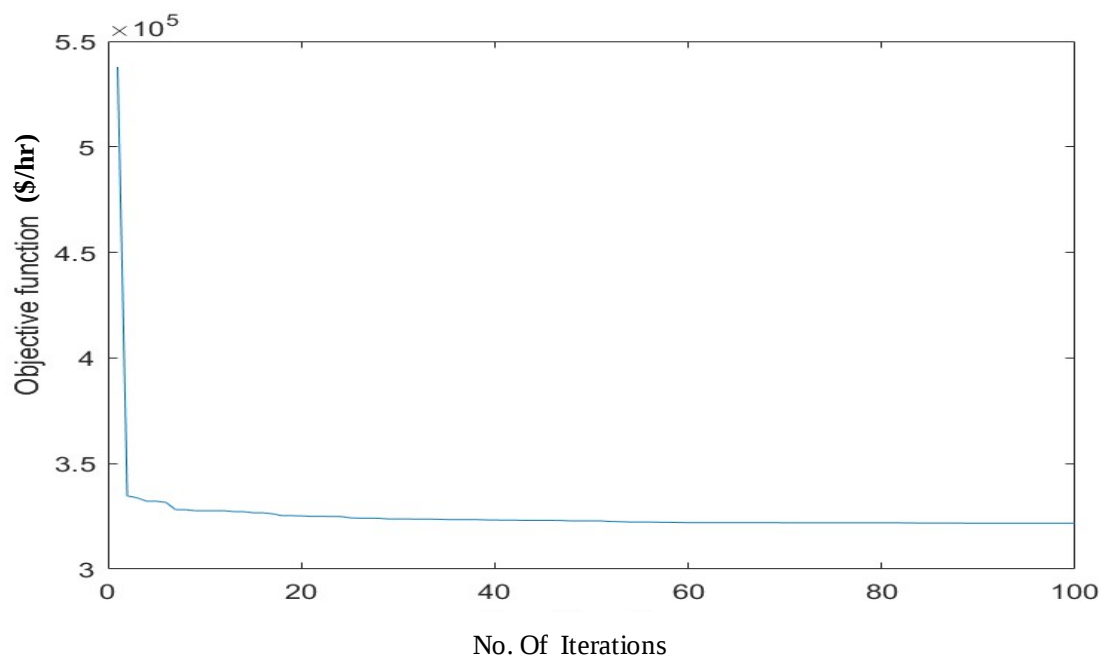


FIGURE 7 Reaching target of cost function aimed at study case tow with WOA

Study case Three: Forty generation units

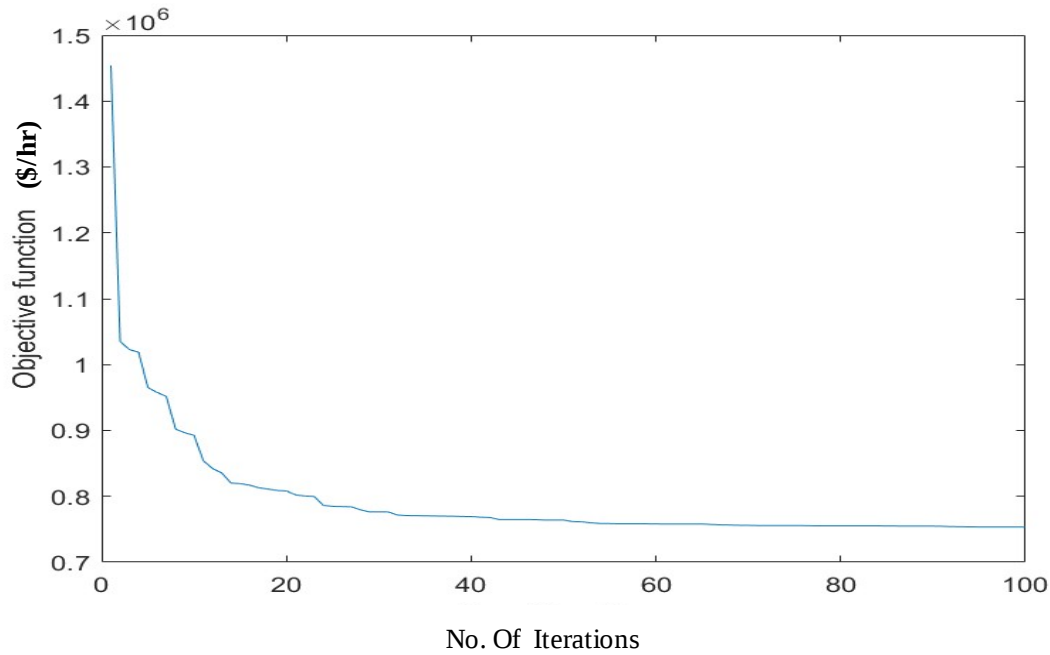


FIGURE 8 Reaching target of costfunction aimed at study case three with WOA

Table 1 WOA Results

Study Cases	average trail	S.D	finest fitness	poorest fitness
1	1.8244 e3	0.7541	1.8239 e3	1.822 e3
2	1.1997 e5	270.93	1.1872 e5	1.2029 e5
3	35.401	35.708	176.0157	340.37

Table 1 provides details on the performance of the WOA across three different cases. For each case, it reports average trail, normal deviance (SD), finest fitness, and poorest fitness values. In Case 1, the average trail is 1.8244e3 (\$/hr) with a standard deviation of 0.7541, the finest fitness achieved is 1.8239e3 (\$/hr), and the poorest fitness is 1.822e3. Case 2 shows a upper average trail of 1.1997e5 and a standard deviation of 270.93, with the best and worst fitness values being 1.1872e5 (\$/hr) and 1.2029e5 (\$/hr), respectively. Case 3 has a average trail of 35.4018 and a much higher normal deviance of 35.708, indicating significant variability, with best and worst fitness values of 176.0157 (\$/hr) and 340.37 (\$/hr), respectively. This data suggests that the algorithm's performance varies considerably depending on the case, with Case 2 showing relatively high stability and fitness values, while Case 3 displays the most variability.

4.2 OPTIMUM OUTCOMES RED-TAILED HAWK

Curves of fitness functions values of 6, 10 and 40 generating units were carried out in this simulation study with RTHO. Figures 9, 10 and 11 represent object function values vs. number of iteration for 6,10 and 40 generators correspondingly.

Study case One: Six generation units

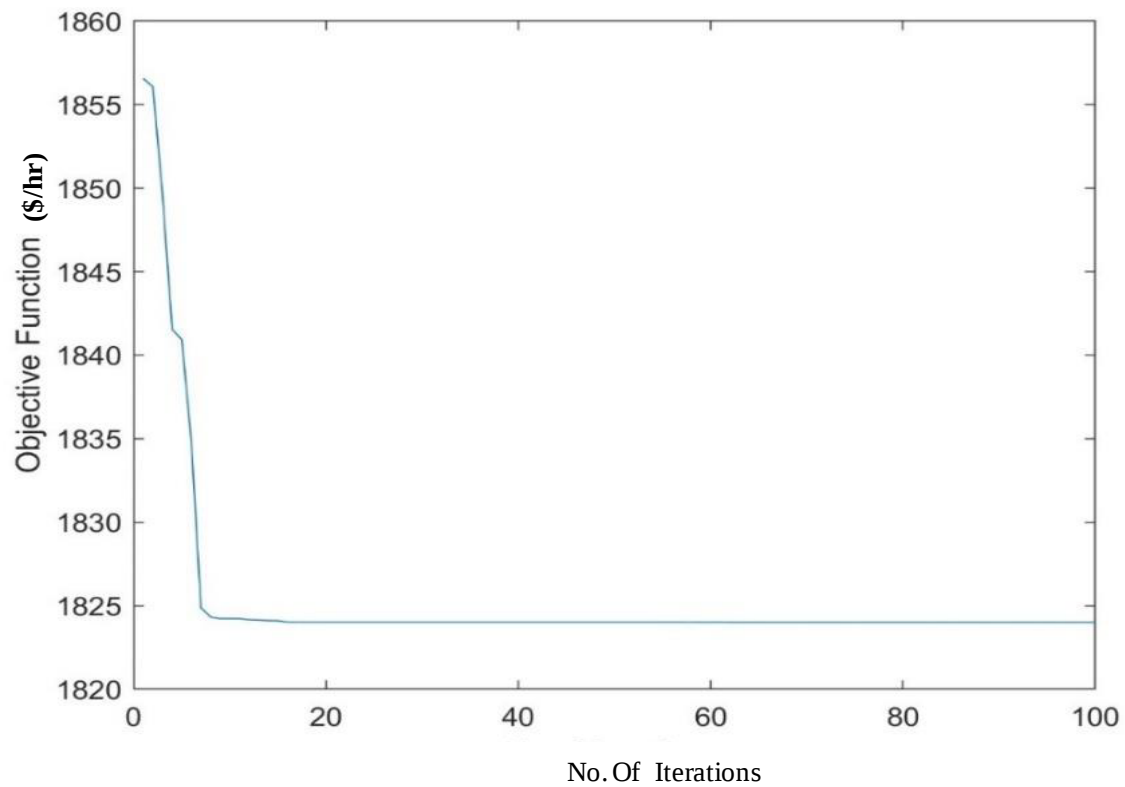


FIGURE 9 Reaching target of cost function aimed at study case one with RTHO A

Study case Two: Ten generation units

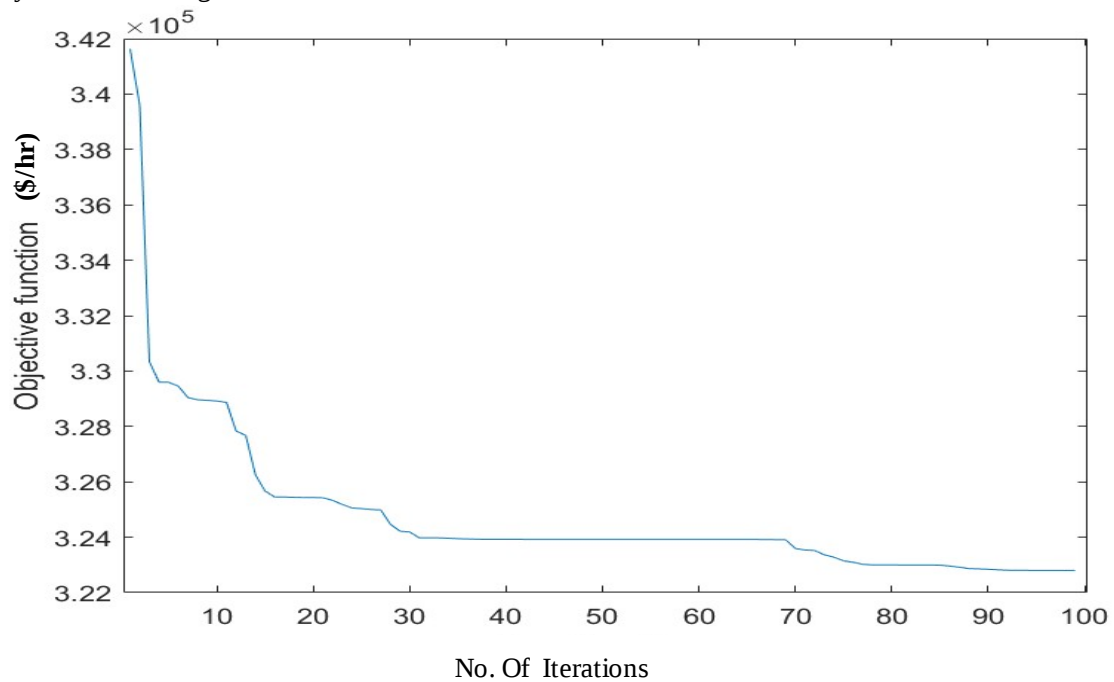
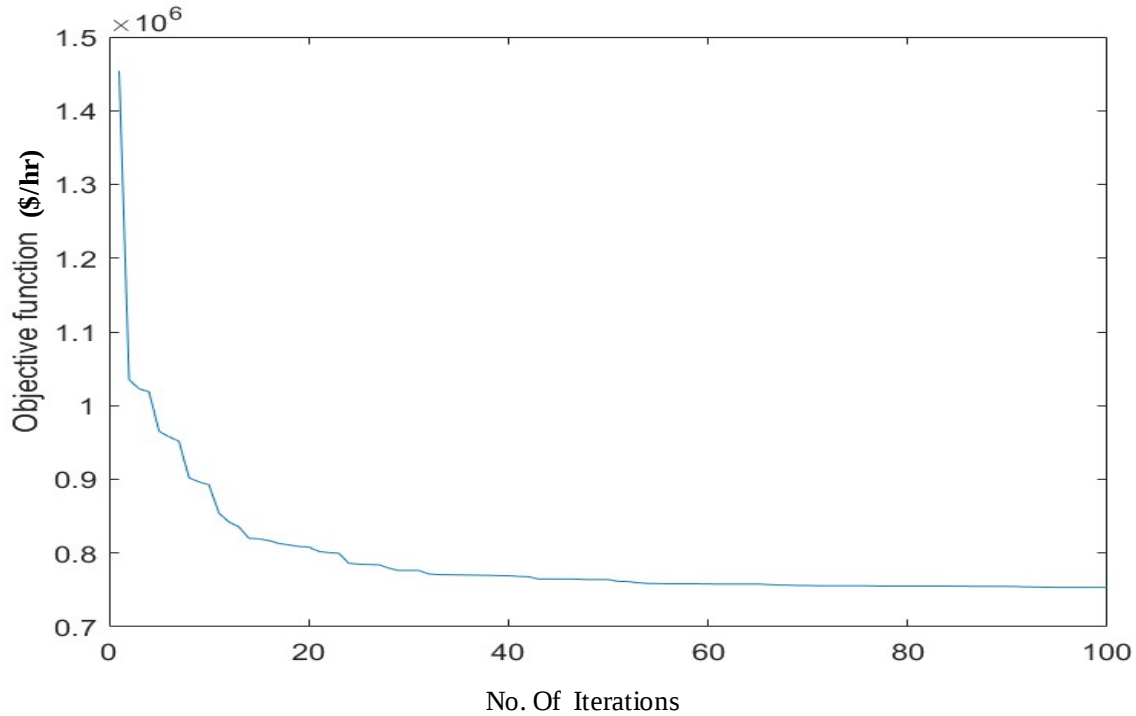


FIGURE 10 Reaching target of cost function aimed at study case two with RTHO A

Study case Three: Forty generation units

**FIGURE 11** Reaching target of cost function aimed at study case three with RTHO A**Table 2** RTHO Results

Study Cases	Average Trail	S.D	Finest Fitness	Poorest Fitness
1	1.8242 e3	0.3100	1.8239 e3	1.8250 e3
2	1.2006 e5	342.63	1.184 e5	1.2046 e5
3	295.29	14.7080	251.46	329.94

Table 2 provides the performance metrics for the RTHOA across three different cases. For each case, it lists average trail, normal deviance, finest fitness, and poorest fitness values. In Case 1, average experiment was 1.8242e3(dollar/hour) of standard deviation with 0.3100, the finest objective was 1.8239e3 (dollar/hour), and poorest objective was 1.8250e3 (dollar/hour), indicating high consistency and minimal variation. In Case 2, average trail increases significantly to 1.2006e5 with a larger standard deviation of 342.63, the finest fitness achieved is 1.184e5(\$/hr), and the poorest fitness is 1.2046e5(\$/hr), reflecting moderate variability. Case 3 shows average trail of 295.29 and a normal deviance of 14.7080, with the finest fitness documented at 251.46(\$/hr) and the poorest fitness at 329.94(\$/hr), indicating a broader array of aptness principles but less inconsistency compared to study case two.

4.3 COMPARATIVE ANALYSIS

The proposed (WOA) and (RTHOA) were calculated in contradiction of a collection of supplementary optimization methods to evaluate their usefulness. Including "Grey wolf optimization (GWO), Moth flame optimization (MFO), Multiverse optimization (MVO), (PSO), Salp swarm optimization (SSO), Whale optimization algorithm (WHOA)". Outcomes obtained by suggested systems are indicated in table 3.

Table 3 Comparison of best solutions of Generation Cost of the six unit system ($P_{\text{demand}} = 2.834 \text{ MW}$)

Generator NO.	Grey wolf optimization	Moth flame optimization	Multiverse optimization	Particle swarm optimization	Salp swarm optimization	Whale optimization	Walrus optimization	Red tailed hawk optimization
G1 (MW)	0.370957	0.336269	0.367224	0.32366	0.44411	0.232142	0.34896	0.326529
G2 (MW)	0.42794	0.458515	0.415092	0.451174	0.554778	0.354082	0.41964	0.441249
G3 (MW)	0.459082	0.408108	0.574005	0.542828	0.573484	0.923872	0.517481	0.518553
G4 (MW)	0.839106	0.623595	0.649645	0.53769	0.480638	0.228807	0.582539	0.583046
G5 (MW)	0.38333	0.455589	0.510782	0.535346	0.463586	0.682232	0.528159	0.524592
G6 (MW)	0.387833	0.587654	0.347304	0.473374	0.354821	0.435731	0.468302	0.470524
Best Score(\$/hr)	1827.906	1836.306	1833.373	1824.923	1846.322	1933.56	1824.461	1823.908

The above Table 3 displays statistics for six unit system, Here the recommended WOA and RTHOA results compared with some other algorithms, they executes superior and has a least generation cost 1824.461 (\$/hr), 1823.908 (\$/hr) respectively.

Table 4 Comparison of best solutions of Generation Cost of the ten generators system ($P_{\text{demand}} = 2000 \text{ MW}$)

Generator NO.	Grey wolf optimization	Moth flame optimization	Multiverse optimization	Particle swarm optimization	Salp swarm optimization	Whale optimization	Walrus optimization	Red tailed hawk optimization
G1 (MW)	52.33413	55	46.68295	54.99413	54.70752	54.95172	55	54.99709
G2 (MW)	80	69.03504	62.57551	79.97392	66.59121	80	79.55938	80
G3 (MW)	72.32635	66.54099	81.54476	79.86829	102.3886	73.81535	81.59243	67.47329
G4 (MW)	90.90867	106.4639	52.94127	82.26748	76.33461	28.93526	83.53782	84.44176
G5 (MW)	159.4585	130.6111	160	159.9702	141.4114	124.1096	159.9919	159.9221
G5 (MW)	227.1962	224.7971	240	239.9784	238.2162	173.5443	239.9742	239.9573
G7 (MW)	269.6645	276.2597	295.5875	292.1161	290.0548	271.7285	293.6665	299.9998
G8 (MW)	309.0811	230.4492	279.0359	303.0692	191.9015	339.9791	288.5746	338.0275
G9 (MW)	399.9091	455.5399	421.0903	384.489	462.0781	470	387.4923	394.7581
G10 (MW)	421.149	470	444.4386	405.2476	460.9165	470	412.6934	361.9683
Best Score(\$/hr)	323084.2	332162.1	326122.1	323825.2	331991	341050	320882.1	321257.9

Table 4 displays statistics intended for ten generator case, The projected WOA and RTHOA gaining finest resolution of entire budget 320882.1(\$/hr) and 321257.9 (\$/hr) correspondingly. The WOA and RTHOA supply nearly finest outcomes in decreasing together fuel expenditures and pollutants emissions for the ten generators study case. Their economical marks and effective unit allocations high spot their prospective as vigorous approaches for addressing multi objective issues.

5. CONCLUSIONS

WO and RTHO Algorithms proves that CEED problems can be solved effectively and efficiently. Based on the comparative analysis, shown in Tables 3 and 4, the performance of the proposed algorithms were the best in comparison with the other techniques, both algorithms have their strengths and their performance varies. Considering study case one, (WOA) finest fitness attained is 1.8239e3(\$/hr) least normal deviance and a adjacent space amid superior and poorest fitness values, while (RTHOA) showed impressive performance, particularly in retaining steadiness and reaching necessary fitness results 1.8250e3 (\$/hr). In Case 3, the RTHOA showed bigger changeability, which indicates a wider survey ability. These outcomes recommend that both procedures proposal appreciated methods for augmenting CEED difficulties, with the WO System presence extra reliable and RTHO Procedure providing a balance between exploration and exploitation. Table 3 and 4 provide a comprehensive analysis of different methods. Among them the proposed method delivering best solution in all aspects. It adds to the growing body of research on nature-inspired optimization, giving us insight into how they can help power systems solve their problems more economically and with fewer emissions. The promising results obtained from the WO and RTHO algorithms for addressing (CEED) problem surface the way for numerous forthcoming investigation opportunities. First of all, supplementary examination into crossbreed optimization tactics incorporating assets of both procedures can be discovered. Joining reliability of WO Procedure through survey competences of RTHO Optimization Procedure might tip to improved enactment and strength through a broader collection of CEED situations. Furthermore, piloting real-world applications and challenging on larger-scale power systems can deliver real-world visions into scalability and applicability of these procedures in compound energy administration situations.

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CONFLICTS OF INTEREST

The authors declare no conflict of interest

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