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A New Iterative Scheme to the Common Fixed Point in Banach Space

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مخطط تكراري جديد للنقطـــة الصامــــدة المشتركة في فضاء بنـــاخ

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Abstract:
In this paper, introduced new iterative four step scheme in Banach spaces by used four mappings to convergence common fixed point, relied on two classes of mapping nonexpansive mapping and quasi nonexpansive mapping, we assumed T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings, proved some convergence theorems to the common fixed point based on so fundamentals for Banach spaces.

Keywords: Common fixed point, quasi nonexpansive mappings, iterative process, Banach spaces.

المستخلص:
في هذا البحث، اقدم مخطط تكراري جـــديد مكون من اربـــعه خطوات في فضـــاءات بناخ باستخدام اربعــه دوال لتقريـــب النقطة الصامدة المشتركـــــة، بالاعتماد على نمطيـــن من الدوال وهما الدوال الغير التـــــوسيعية والدوال الشبـــه غير توسيعية ، حيث نفرض ان الدالـــة الاولى والثالثة دالتين غير توسعيتين و الدالــــة الثانيـــة والرابعة دالتين شبه غير توسعيتين، نبرهن بعض مبرهنـــــــات التقارب الى النقطـــة الصامدة المشتركة بالاستناد الى الاساسيات في فضاءات بناخ.

الكلمات المفتاحية: النقطة الصامدة المشتركة، الدوال الشبـــه غير توسعيـــة، مخطط تكراري، فضاءات بناخ

1-Introduction

Suppose Y is nonempty subset of a Banach spaces X, the mappings T_1, T_2, $T_3, T_4: Y \longrightarrow Y$ and a point $s \in Y$ is fixed point of T where $Ts = s$ in general Banach spaces. Fixed point theory has many application in various fields therefore it has been a flourishing area of research[1]. Nowadays, a vigorous research activity is developed in the area of numerical reckoning fixed point for suitable classes of nonlinear operators, see [2][3][4][5], ones the existence of fixed point problems analytic is not easy and thus the need to consider and

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approximate solution is pertinent [6], Luaibi and Abed in (2021) studied the existence theorem for Voltera type equation fixed point theorem in G-metric spaces with some application [7], Mannan et al introduced applications of fixed point theorem Banach spaces for mapping defined on metric spaces with graph or partial order [8], Ullah and Arshad propose a new three step iteration scheme to convergence fixed points for Suzuki generalized non expansive mapping for uniformly convex Banach spaces [9], many researchers have discussed topic of the common fixed point, some of them worked on the common fixed point by using multivalued mapping in modular function spaces[10][14], and other worked in Banach spaces [11].

In our previous research[11], we studied the iterative scheme

let $T: E \longrightarrow E$, and E nonempty convex subset of Banach spaces, here, we introduced the sequence $\{x_n\}$ by the algorithm following.

$x_1 \in E$
$h_n = (1-\beta_n)x_n + \beta_n T x_n$
$y_n = T h_n$
$J_n = (1-\alpha_n)y_n + \alpha_n T y_n$
$f_{n+1} = T J_n, n\epsilon N$
where $\{\alpha_n\}$and $\{\beta_n\}$ in (0,1)

Now, Present the iterative algorithm that will deal with in this paper
let $T_1, T_2, T_3, T_4: Y \longrightarrow Y$, and Y nonempty convex subset of Banach spaces, here, we introduced the sequence $\{x_n\}$ by the algorithm following.

$x_1 \in Y$
$h_n = (1-\beta_n)x_n + \beta_n T_1 x_n$
$y_n = T_2 h_n$
$J_n = (1-\alpha_n)y_n + \alpha_n T_3 y_n$
$x_{n+1} = T_4 J_n, n\epsilon N$
where $\{\alpha_n\}$and $\{\beta_n\}$ in (0,1)

(1)

2-Prelimineаries

In this section review some important definitions and lemmas that we can use in the results

Definition 2-1 [6]:Let $T: E \longrightarrow E$ a mapping and E is nonempty subset of Banach space said to be nonexpansive mapping if

$$\|Tx - Ty\| \leq \|x - y\|$$

Definition 2-2 [12]:Let $T: E \longrightarrow E$ a mapping and E is nonempty subset of Banach space said to be qausi nonexpansive mapping if there exist s fixed point and

$$\|Tx - s\| \leq \|x - s\|$$

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Note that: Every nonexpansive mapping with fixed point is qausi nonexpansive mapping but the convers is not true for example.

Example 2-3: Let $T: E \longrightarrow E$ and E is nonempty subset of Banach space the function

$$Tx = \begin{cases} 0 & if\ x \neq 2 \\ 1 & if\ x = 2 \end{cases}$$

T is quasi nonexpansive mapping but not nonexpansive since if $x = 1.9, y = 2$ then $\|Tx - Ty\| = \|0 - 1\| = 1 \nleq \|1.9 - 2\| = 0.1$.

Definition 2-4 [6]: Let $T: E \longrightarrow E$ is mapping, a sequence $\{x_n\}$ in E is said to be Fajer monotone if $\|x_{n+1} - s\| \leq \|x_n - s\|$ for all s fixed point.

Lemma 2-5 [13]**:** Let X satisfy uniformly convex Banach spaces and let $\{t_n\}$ in (0,1) be bounded away from 0 and 1, if there exists $m > 0$ such that

$\lim sup_{n\to\infty} \rho(x_n) \leq m, \lim sup_{n\to\infty} \rho(y_n) \leq m$

And $lim_{n\to\infty} \rho(t_n x_n + (1 - t_n) y_n) = m$, then $\lim_{n\to\infty} \rho(x_n - y_n) = 0$

Lemma 2-6 [14]

Let $\{\rho_n\}_{n=1}^{\infty}$, $\{\theta_n\}_{n=1}^{\infty}$ and $\{\zeta_n\}_{n=1}^{\infty}$ nonnegative sequence such that

$\rho_{n+1} \leq (1 - \theta_n)\rho_n + \zeta_n$

Where $\{\theta_n\}$ sequence in $(0,1)$ and $\{\zeta_n\}$ sequence in real number such that

$\sum_{n=1}^{\infty} \theta_n < \infty$ and $\sum_{n=1}^{\infty} \zeta_n < \infty$, then $\lim_{n\to\infty} \rho_n$ is exists.

Definition 2-7 [15] **:** A Banach space X is said to be uniformly convex if $\psi_\eta(\epsilon) = \inf\{1 - \left\|\frac{x+y}{2}\right\| : x, y \in B_\eta, \|x - y\| \geq \epsilon\} > 0$ for all $0 < \varepsilon \leq 2$ say that uniformly convex Banach spaces has power P and $P \geq 1$ there exists constant c such that $\psi_\eta(\epsilon) \geq c\epsilon^P$ for all $0 < \varepsilon \leq 2$.

Definition 2-8 : Let X satisfy uniformly convex Banach spaces, let Y be nonempty convex subset of X, let $T_i: Y \longrightarrow Y$ said to be satisfy condition (II) if there exsist a nondecreasing function $\phi: [0, \infty) \longrightarrow [0, \infty)$ such that $\phi(0) = 0$ and $\phi(t) > 0$ for all t in $[0, \infty)$ and $\|x - T_i x\| \geq \phi(dist(x, \cap F_p(T_i))$ for all $x \in E$, where $dist(x, \cap F_p(T_i))$ denotes the distance from x to $F_p(T_i)$.

3- Main Rustles

In our previous research[11], we introduced the sequence $\{x_n\}$ by the algorithm following.

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let $T: E \longrightarrow E$, and E nonempty convex subset of Banach spaces, here, we introduced the sequence $\{x_n\}$ by the algorithm following.

$x_1 \in E$
$h_n = (1 - \beta_n)x_n + \beta_n T x_n$
$y_n = T h_n$
$J_n = (1 - \alpha_n)y_n + \alpha_n T y_n$
$f_{n+1} = T J_n, n \epsilon N$
where $\{\alpha_n\}$and $\{\beta_n\}$ in (0,1)

In this section will study some convergence results with the iterative scheme in equation (1) as follows

let $T_1, T_2, T_3, T_4: Y \longrightarrow Y$, and Y nonempty convex subset of Banach spaces, here, we introduced the sequence $\{x_n\}$ by the algorithm following.

$x_1 \in Y$
$h_n = (1 - \beta_n)x_n + \beta_n T_1 x_n$
$y_n = T_2 h_n$
$J_n = (1 - \alpha_n)y_n + \alpha_n T_3 y_n$
$x_{n+1} = T_4 J_n, n \epsilon N$
where $\{\alpha_n\}$and $\{\beta_n\}$ in (0,1), where T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings.

Theorem 3-1: Let X satisfy uniformly convex Banach spaces, let Y be nonempty convex subset of X, let T_1, T_2, $T_3, T_4: Y \longrightarrow Y$ where T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings. and x_n in Y define by (1) then $\lim_{n\to\infty}\|x_n - s\|$ exists for all s fixed point of T in Y.

Proof: T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings

By definitions (2-1),(2-2), equation (1), and convexity

$$\|x_{n+1} - s\| = \|T_4 J_n - s\|$$
$$\leq \|J_n - s\|$$

$$\|x_{n+1} - s\| \leq \|J_n - s\| \quad (2)$$

Also, $\|J_n - s\| = \|(1 - \alpha_n)y_n + \alpha_n T_3 y_n - s\|$

$$\leq (1 - \alpha_n)\|y_n - s\| + \alpha_n \|T y_n - s\|$$

Hence $\|T_3 y_n - s\| \leq \|y_n - s\|$

Substituting in equation we get

$$\|J_n - s\| \leq \|y_n - s\| \quad (3)$$

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By the same way,

$\|y_n - s\| = \|T_2 h_n - s\|$

$\leq \|h_n - s\|$

$\|y_n - s\| \leq \|h_n - s\|$
(4)

Similarity,

$\|h_n - s\| = \|(1-\beta_n)x_n + \beta_n T_1 x_n - s\|$

$\leq (1-\beta_n)\|x_n - s\| + \beta_n \|T_1 x_n - s\|$

Hence $\|T_1 x_n - s\| \leq \|x_n - s\|$

Substituting in equation we get

$\|h_n - s\| \leq \|x_n - s\|$
(5)

By (2),(3),(4) and (5) $\|x_{n+1} - s\| \leq \|x_n - s\|$

And by Lemma 2-6 then $\lim_{n\to\infty} \|x_n - s\|$exists

Theorem 3-2: Let X satisfy uniformly convex Banach spaces, let Y be nonempty convex subset of X, let T_1, T_2, $T_3, T_4 : Y \to Y$ where T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings. and x_n in Y define by (1) is Fajer monotone.

Proof: By Theorem 3-1

T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings

By definitions (2-1),(2-2), equation (1), and convexity

$\|x_{n+1} - s\| = \|T_4 J_n - s\|$

$\leq \|J_n - s\|$

$\|x_{n+1} - s\| \leq \|J_n - s\|$

Also, $\|J_n - s\| = \|(1-\alpha_n)y_n + \alpha_n T_3 y_n - s\|$

$\leq (1-\alpha_n)\|y_n - s\| + \alpha_n \|T y_n - s\|$

Hence $\|T_3 y_n - s\| \leq \|y_n - s\|$

Substituting in equation we get

$\|J_n - s\| \leq \|y_n - s\|$

By the same way,

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$\|y_n - s\| = \|T_2 h_n - s\|$

$\leq \|h_n - s\|$

$\|y_n - s\| \leq \|h_n - s\|$

Similarity,

$\|h_n - s\| = \|(1-\beta_n)x_n + \beta_n T_1 x_n - s\|$

$\leq (1-\beta_n)\|x_n - s\| + \beta_n \|T_1 x_n - s\|$

Hence $\|T_1 x_n - s\| \leq \|x_n - s\|$

Substituting in equation we get

$\|h_n - s\| \leq \|x_n - s\|$

$\|x_{n+1} - s\| \leq \|x_n - s\|$

And by Definition 2-4, x_n in E define by (1) is Fajer monotone.

Theorem 3-3: Let X satisfy uniformly convex Banach spaces, let Y be nonempty convex subset of X, let T_1, T_2, $T_3, T_4 : Y \longrightarrow Y$ where T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings. and x_n in Y define by (1) then $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$.

Proof: By Theorem 3-1 $\lim_{n\to\infty}\|x_n - s\|$ exists

Let $lim_{n\to\infty}\|x_n - s\| = k$ such that $k \geq 0$ (6)

By (5) and (6)

$\|h_n - s\| \leq \|x_n - s\| = k$ (7)

$\|x_{n+1} - s\| = k = lim_{n\to\infty}\|x_n - s\|$

By (2)

$\|x_{n+1} - s\| = \|T_4 J_n - s\| \leq \|J_n - s\|$

By (3)

$\|J_n - s\| \leq \|y_n - s\|$

By (4)

$\|y_n - s\| \leq \|h_n - s\|$

Then $\|x_{n+1} - s\| \leq \|h_n - s\| \Rightarrow k \leq \|h_n - s\|$ (8)

By (7) and (8) $\lim_{n\to\infty}\|h_n - s\| = k$ (9)

T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings

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$\|T_1 x_n - s\| \leq \|x_n - s\|$ (10)

$\lim_{n\to\infty}\|T_1 x_n - s\| \leq \lim_{n\to\infty}\|x_n - s\|$

So, $\lim_{n\to\infty}\|T_1 x_n - s\| \leq k$ (11)

Since $\lim_{n\to\infty}\|h_n - s\| = k$ then

$\lim_{n\to\infty}\|(1-\alpha_n)x_n + \alpha_n T_1 x_n - s\| = k$

$\lim_{n\to\infty}\|(1-\alpha_n)x_n + \alpha_n T_1 x_n - s\| = k$

$\lim_{n\to\infty}\|(1-\alpha_n)(x_n - s) + \alpha_n(T_1 x_n - s)\| = k$ (12)

By (6),(11) , (12) and by using Lemma 2-5 then $\lim_{n\to\infty}\|x_n - Tx_n\| = 0$..

Theorem 3-4: Let X satisfy uniformly convex Banach spaces, let Y be nonempty convex subset of X, let T_1, T_2, $T_3, T_4 : Y \longrightarrow Y$ where T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings. and x_n in Y define by (1), x_0 is unique common fixed point in T_i then x_n ρ-strongly convergence to fixed point of T_i in Y.

Proof: T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings

By definitions (2-1),(2-2) and, convexity

$\|x_{n+1} - x_0\| = \|T_4 J_n - x_0\|$

$\leq \|J_n - x_0\|$

$\|x_{n+1} - x_0\| \leq \|J_n - x_0\|$ (13)

Also, $\|J_n - x_0\| = \|(1-\alpha_n)y_n + \alpha_n T_3 y_n - x_0\|$

$\leq (1-\alpha_n)\|y_n - x_0\| + \alpha_n\|T_3 y_n - x_0\|$

Hence $\|T_3 y_n - x_0\| \leq \|y_n - x_0\|$

Substituting in equation we get

$\|J_n - x_0\| \leq \|y_n - x_0\|$ (14)

By the same way,

$\|y_n - x_0\| = \|T_2 h_n - x_0\|$

$\leq \|h_n - x_0\|$

$\|y_n - x_0\| \leq \|h_n - x_0\|$ (15)

Similarity,

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$\|h_n - x_0\| = \|(1-\beta_n)y_n + \beta_n T_1 x_n - x_0\|$

$\leq (1-\beta_n)\|x_n - x_0\| + \beta_n\|T_1 x_n - x_0\|$

Hence $\|T_1 x_n - x_0\| \leq \|x_n - x_0\|$

Substituting in equation we get

$\|h_n - x_0\| \leq \|x_n - x_0\|$ (16)

By (13),(14),(15) and (16)

$\|x_{n+1} - x_0\| \leq \|x_n - x_0\|$

Furthermore it

$\|x_n - x_0\| \leq \|x_{n-1} - x_0\|$

Since $\|x_1 - x_0\| \leq \|x_0 - x_0\|$, so ,$\|x_n - x_0\| \leq \|x_0 - x_0\|$

$\|x_n - x_0\| \leq \|0\| = 0$, then $x_n \longrightarrow x_0$

In our previous paper[11], we proved strong convergence as follows

Let X satisfy uniformly convex Banach spaces, let E be nonempty convex subset of X, let $T: E \longrightarrow E$ be generalized (α, β)-mean nonexpansive mapping and satisfy (I) condition, x_n in E define by (1), then x_n strongly convergence to fixed point of T in E.

Proof: : $lim_{n\to\infty}\|x_n - s\|$ exists for all s is fixed point, if $lim_{n\to\infty}\|x_n - s\| = 0$, nothing to prove,

if $lim_{n\to\infty}\|x_n - s\| = k, k \geq 0$

Since $\|x_{n+1} - s\| \leq \|x_n - s\|$, then $dist(x_{n+1}, F_p(T) \leq dist_\rho(x_n, F_p(T))$

So $lim_{n\to\infty} dist_\rho(x_n, F_p(T))$ exists, by applying condition (I)

$lim_{n\to\infty}\emptyset(dist\left(x_n, F_p(T)\right) \leq lim_{n\to\infty}\ dist\|x_n - Tx_n\| = 0$

Since $\emptyset(0) = 0$, hence $lim_{n\to\infty} dist(x_n, F_p(T))\ = 0$

$lim_{n\to\infty}\|x_n - s\|$ exists, then $lim_{n\to\infty}\|x_n - F_p(T)\|$ exists and $s \in F_p(T)$

Suppose that x_{n_k} subsequence of x_n, and u_k sequence in $F_p(T)$

$\|x_{n_k} - u_k\| \leq \frac{1}{2^k}$ since $\lim inf_{n\to\infty}\ dist(x_n, F_p(T)) = 0$

$\|x_{n_{+1}} - u_k\| \leq \|x_n - u_k\| \leq \frac{1}{2^k}$

$\|u_{k+1} - u_k\| \leq \|u_{k+1} - x_{n+1}\| + \|x_{n+1} - u_k\|$

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$$\leq \frac{1}{2^{k+1}} + \frac{1}{2^k}$$

$$\leq \frac{1}{2^{k-1}}$$

$\|u_{k+1} - u_k\| \longrightarrow 0$ as $k \longrightarrow \infty$

u_k is ρ-Cauchy, $F_p(T)$, So, u_k is ρ-converge to $F_p(T)$, then $\|u_k - s\| \longrightarrow 0$

Now,

$\|x_{n_k} - s\| \leq \|x_{n_k} - u_k\| + \|u_k - s\|$, hence, x_n converge to fixed point s in $F_p(T)$.

For the purpose of comparing the types of mapping we present the following theorem.

Theorem 3-5: Let X satisfy uniformly convex Banach spaces, let Y be nonempty convex subset of X, let T_1, T_2, $T_3, T_4: Y \longrightarrow Y$ where T_1, T_3 is nonexpansive mappings and T_2, T_4 is quasi nonexpansive mappings. $F_p = F_p(T_1) \cap F_p(T_2) \cap F_p(T_3) \cap F_p(T_4) \neq \emptyset$, and let T_1, T_2, T_3 and T_4 satisfied condition (II), and x_n in Y define by (1), then x_n strongly convergence to common fixed point of T_i in Y.

Proof: : By Theorem 3-1 $lim_{n\to\infty}\|x_n - s\|$ exists for all s is fixed point, if $lim_{n\to\infty}\|x_n - s\| = 0$, nothing to prove,

if $lim_{n\to\infty}\|x_n - s\| = k, k \geq 0$

Since $\|x_{n+1} - s\| \leq \|x_n - s\|$, then $dist(x_{n+1}, \cap F_p(T_i)) \leq dist(x, \cap F_p(T_i))$

So $lim_{n\to\infty} dist_\rho(x_n, F_p(T_i))$ exists, by applying condition (II) and Theorem 3-1

$lim_{n\to\infty} \emptyset(dist(x, \cap F_p(T_i)) \leq lim_{n\to\infty} dist\|x_n - Tx_n\| = 0$

Since $\emptyset(0) = 0$, hence $lim_{n\to\infty} dist(x_n, \cap F_p(T_i)) = 0$

By Theorem 3-1 $lim_{n\to\infty}\|x_n - s\|$ exists, then $lim_{n\to\infty}\|x_n - F_p(T)\|$ exists and $s \in F_p(T_i)$

Suppose that x_{n_k} subsequence of x_n, and u_k sequence in $F_p(T_i)$

$\|x_{n_k} - u_k\| \leq \frac{1}{2^k}$ since $\lim inf_{n\to\infty} dist(x_n, \cap F_p(T_i)) = 0$

$\|x_{n_{+1}} - u_k\| \leq \|x_n - u_k\| \leq \frac{1}{2^k}$

$\|u_{k+1} - u_k\| \leq \|u_{k+1} - x_{n+1}\| + \|x_{n+1} - u_k\|$

$$\leq \frac{1}{2^{k+1}} + \frac{1}{2^k}$$

$$\leq \frac{1}{2^{k-1}} \qquad (17)$$

$\|u_{k+1} - u_k\| \longrightarrow 0$ as $k \longrightarrow \infty$

u_k is Cauchy, $F_p(T_i)$, So, u_k is ρ-converge to $F_p(T_i)$, then $\|u_k - s\| \longrightarrow 0$

Now,

$\|x_{n_k} - s\| \leq \|x_{n_k} - u_k\| + \|u_k - s\|$, hence, x_n converge to fixed point s in $F_p(T_i)$.

4-Conclusion

As shown in the above theorems, the iterative scheme in equation (1) convergence to the common fixed point in Banach spaces, it also approaches the common fixed point if it is unique as shown in the theorem 3-4, researcher may prove some numerical example with the iterative scheme given in equation (1), while it can be used with other classes of mappings or even with usual fixed point in Banach spaces.

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